Cosmology

V.A. Rubakov

Institute for Nuclear Research
of the Russian Academy of Sciences, Moscow

Outline of Lecture 1

- Expanding Universe
- Dark matter: evidence
- WIMPs
- Warm dark matter: gravitinos?

Expanding Universe

- The Universe at large is homogeneous, isotropic and expanding. 3d space is Euclidean (observational fact!)
All this is encoded in space-time metric

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

$\mathbf{x}$: comoving coordinates, label distant galaxies.

$a(t)dx$: physical distances.

$a(t)$: scale factor, grows in time; a_0: present value (matter of convention)

$$z(t) = \frac{a_0}{a(t)} - 1 : \quad \text{redshift}$$

Light of wavelength λ emitted at time t has now wavelength $\lambda_0 = \frac{a_0}{a(t)}\lambda = (1+z)\lambda$.

$$H(t) = \frac{\dot{a}}{a} : \quad \text{Hubble parameter, expansion rate}$$

- Present value

$$H_0 = (71.0 \pm 2.5)\ \frac{\text{km/s}}{\text{Mpc}} = (14 \cdot 10^9\ \text{yrs})^{-1}$$

$1\ \text{Mpc} = 3 \cdot 10^6\ \text{light yrs} = 3 \cdot 10^{24}\ \text{cm}$

- Hubble law (valid at $z \ll 1$)

$$z = H_0 r$$

Fig.

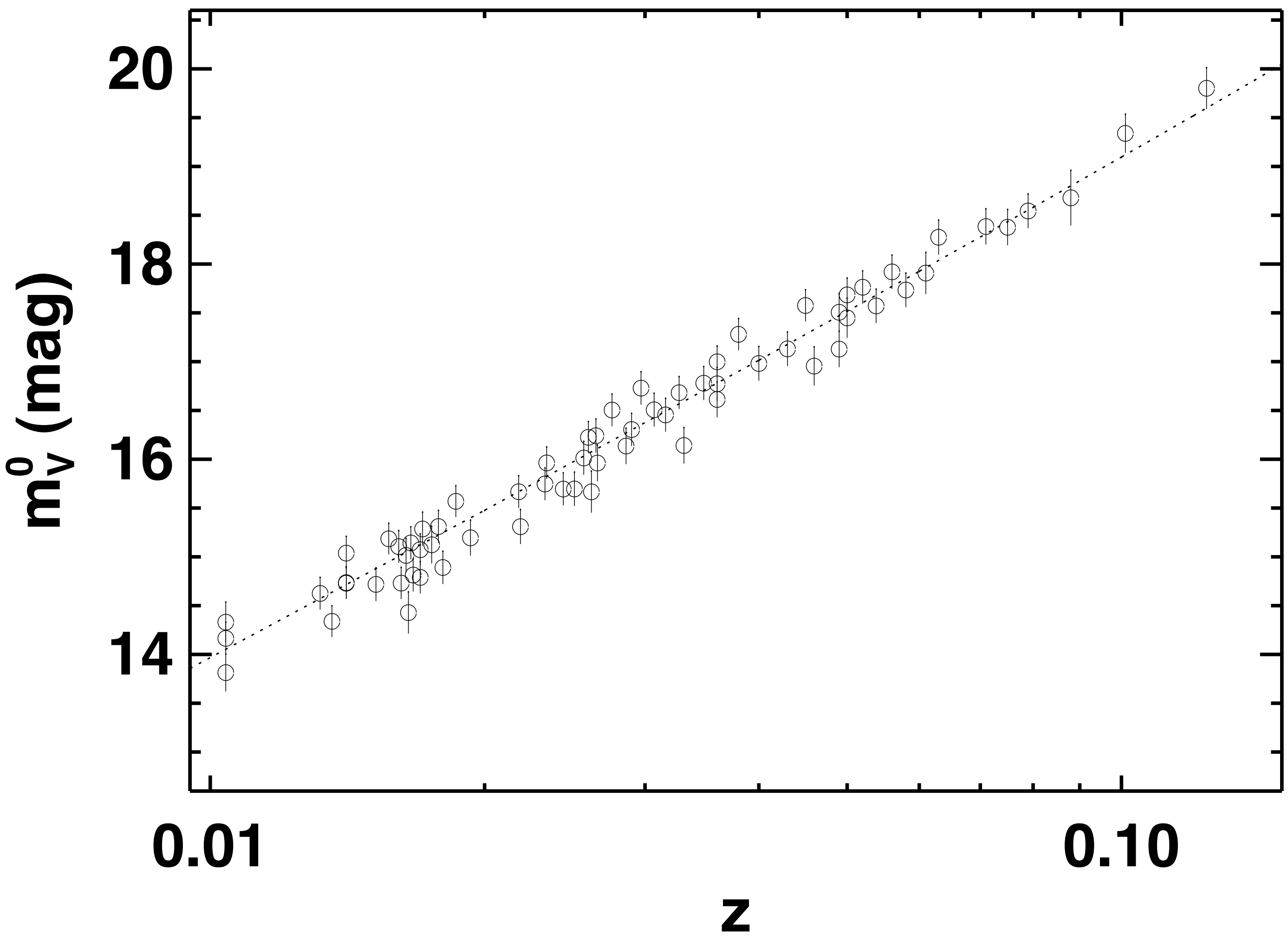
- The Universe is warm: CMB temperature today

$$T_0 = 2.726\ \text{K}$$

Fig.

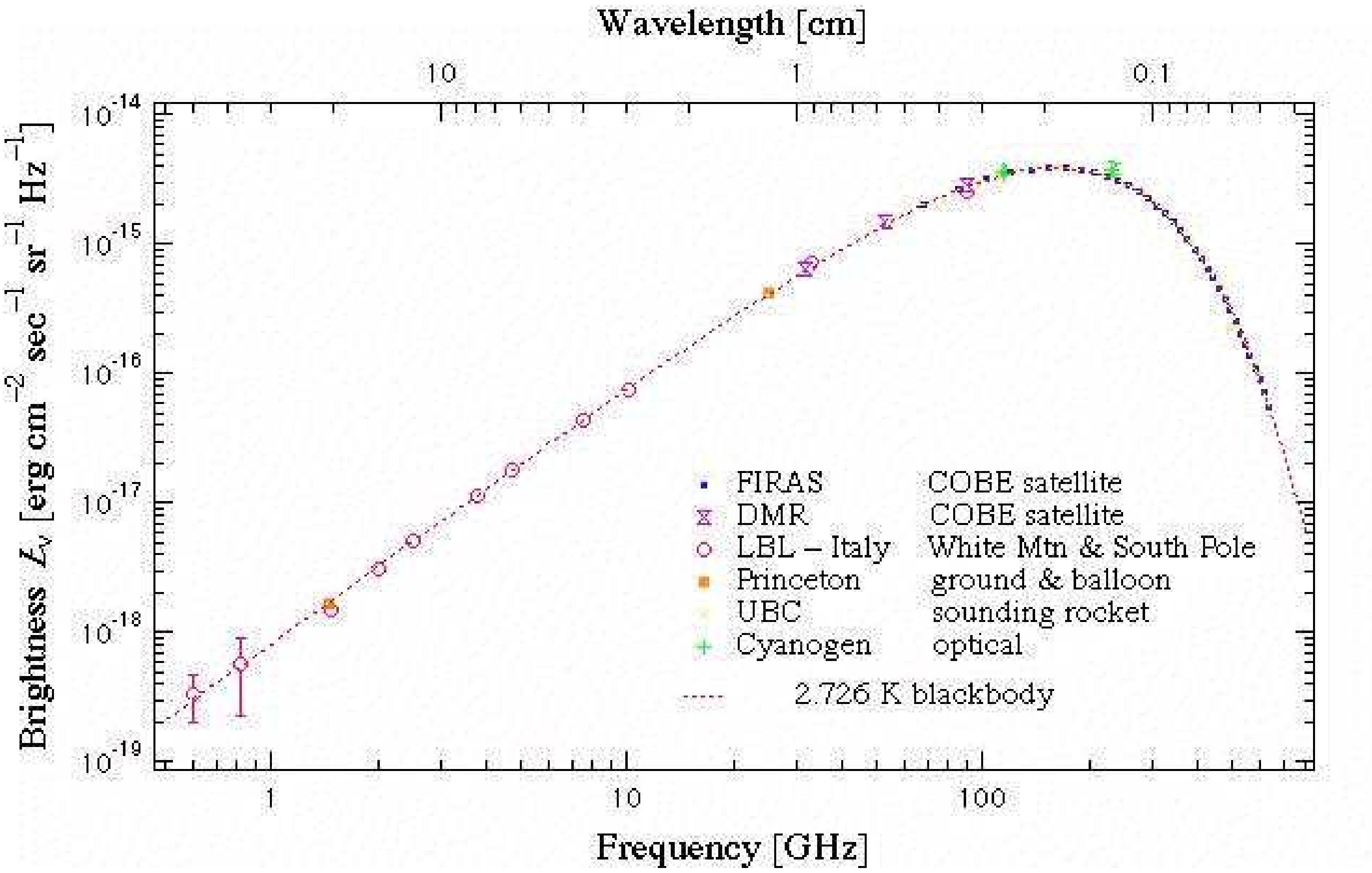
It was denser and warmer at early times.

Hubble diagram for SNe1a



$\mathrm{mag} = 5\log_{10} r + \mathrm{const}$

CMB spectrum



$T = 2.726$ K

- Present number density of photons

$$n_\gamma = \# T^3 = 410 \frac{1}{\text{cm}^3}$$

- Present entropy density

$$s = 2 \cdot \frac{2\pi^2}{45} T_0^3 + \text{neutrino contribution} = 3000 \frac{1}{\text{cm}^3}$$

In early Universe

$$s = \frac{2\pi^2}{45} g_* T^3$$

g_*: number of relativistic degrees of freedom with $m \lesssim T$; fermions contribute with factor $7/8$.

Entropy density scales exactly as a^{-3}

Temperature scales approximately as a^{-1}.

- Friedmann equation: expansion rate of the Universe vs total energy density ρ ($M_{Pl} = G^{-1/2} = 10^{19}$ GeV):

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$

Einstein equations of General Relativity specified to homogeneous isotropic space-time with zero spatial curvature.

- Present energy density

$$\rho_0 = \rho_c = \frac{3M_{Pl}^2}{8\pi} H_0^2 = 5 \cdot 10^{-6} \ \frac{\text{GeV}}{\text{cm}^3}$$

Present composition of the Universe

$$\Omega_i = \frac{\rho_{i,0}}{\rho_c}$$

present fractional energy density of i-th type of matter.

$$\sum_i \Omega_i = 1$$

- Dark energy: $\Omega_\Lambda = 0.72$
  ρ_Λ stays (almost?) constant in time
- Non-relativistic matter: $\Omega_M = 0.28$
  $\rho_M = mn(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^3$
  - Dark matter: $\Omega_{DM} = 0.23$
  - Usual matter (baryons): $\Omega_B = 0.046$
- Relativistic matter (radiation): $\Omega_{rad} = 8.4 \cdot 10^{-5}$ (for massless neutrinos)
  $\rho_{rad} = \omega(t)n(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^4$

Friedmann equation

$$H^2(t) = \frac{8\pi}{3M_{Pl}^2}\left[\rho_\Lambda + \rho_M(t) + \rho_{rad}(t)\right] = H_0^2\left[\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a(t)}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a(t)}\right)^4\right]$$

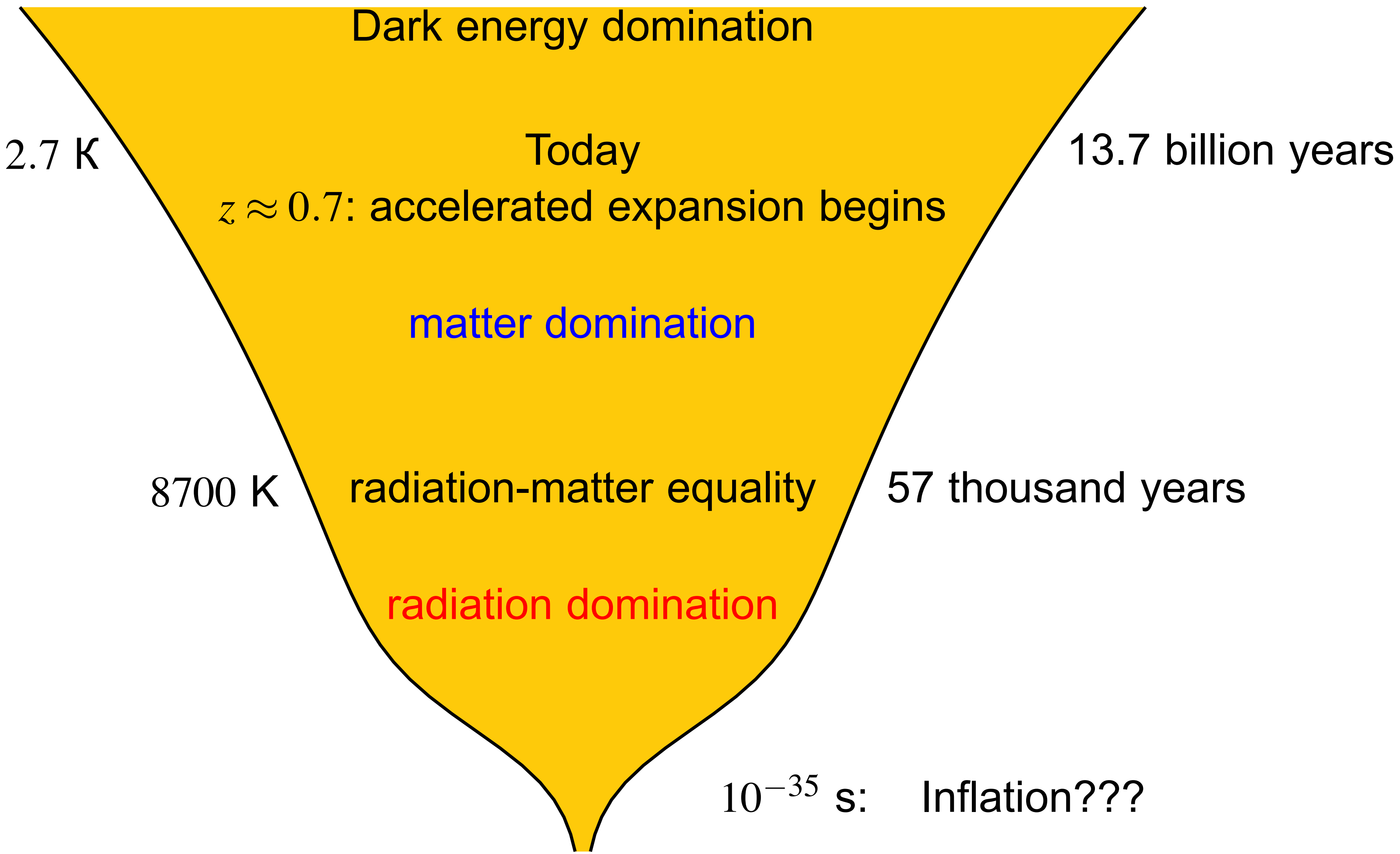
$\ldots \Longrightarrow$ Radiation domination $\Longrightarrow$ Matter domination $\Longrightarrow$ Λ–domination

now

$$z_{eq} = 3200$$

$$T_{eq} = 8700\ \text{K} = 0.75\ \text{eV}$$

$$t_{eq} = 57 \cdot 10^3\ \text{yrs}$$



Dark energy domination
2.7 K
Today
13.7 billion years
$z \approx 0.7$: accelerated expansion begins
matter domination
8700 K
radiation-matter equality
57 thousand years
radiation domination
10^{-35} s: Inflation???

Expansion at radiation domination

- Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}}\rho$$

- Radiation energy density: Stefan–Boltzmann

$$\rho = \frac{\pi^2}{30} g_* T^4$$

g_*: number of relativistic degrees of freedom (about 100 in SM at $T \sim 100$ GeV). Hence

$$H(T) = \frac{T^2}{M^*_{Pl}}$$

with $M^*_{Pl} = M_{Pl}/(1.66\sqrt{g_*}) \sim 10^{18}$ GeV at $T \sim 100$ GeV

- Expansion law:

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho \quad \Longrightarrow \quad \frac{\dot{a}^2}{a^2} = \frac{\text{const}}{a^4}$$

Solution:

$$a(t) = \text{const} \cdot \sqrt{t}$$

- $t = 0$: Big Bang singularity

$$H = \frac{\dot{a}}{a} = \frac{1}{2t}\,, \qquad \rho \propto \frac{1}{t^2}$$

- Decelerated expansion: $\ddot{a} < 0$.
  - NB: Λ-domination

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3M_{Pl}^2}\rho_\Lambda = \text{const} \Longrightarrow a(t) = \mathrm{e}^{H_\Lambda t}$$

accelerated expansion.

Cosmological (particle) horizon

Light travels along $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = 0 \Longrightarrow dx = dt/a(t)$.
If emitted at $t = 0$, travels finite coordinate distance

$$\eta = \int_0^t \frac{dt'}{a(t')} \propto \sqrt{t} \quad \text{at radiation domination}$$

$\eta \propto \sqrt{t} \Longrightarrow$ visible Universe increases in time

Fig.

Physical size of causally connected region at time t (horizon size)

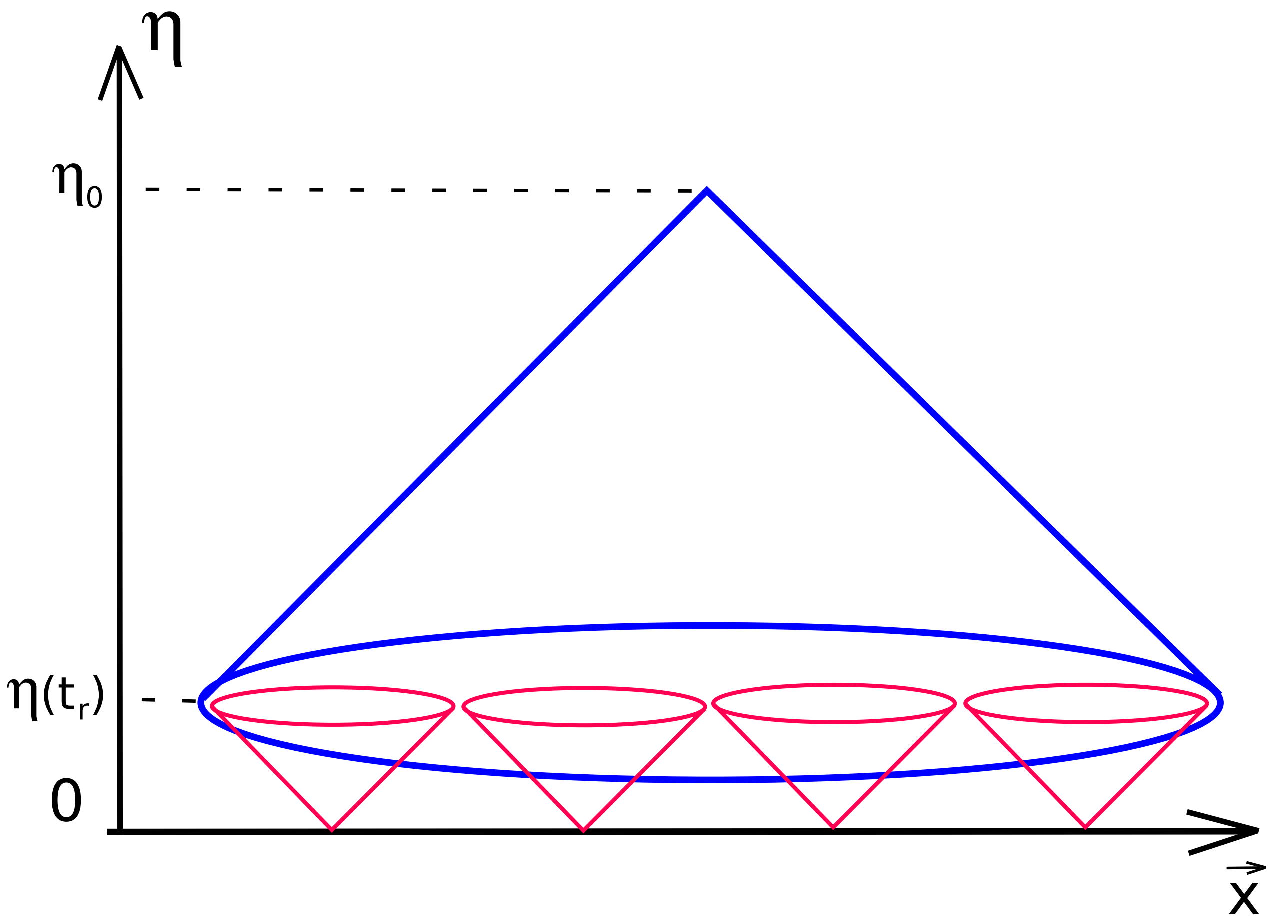
$$l_{H,t} = a(t) \int_0^t \frac{dt'}{a(t')} = 2t \quad \text{at radiation domination}$$

In hot Big Bang theory at both radiation and matter domination

$$l_{H,t} \sim t \sim H^{-1}(t)$$

Today $l_{H,t_0} \approx 15$ Gpc $= 4.5 \cdot 10^{28}$ cm

Causal structure of space-time in hot Big Bang theory



We see many regions that were causally disconnected by time t_r.
Why are they all the same?

Cornerstones of thermal history

- Big Bang Nucleosynthesis, epoch of thermonuclear reactions

$$\begin{array}{rcl} p+n & \to & {}^2H \\ {}^2H+p & \to & {}^3He \\ {}^3He+n & \to & {}^4He \\ \text{up to} & & {}^7Li \end{array}$$

  Abundances of light elements: measurements vs theory

  $T = 10^{10} \to 10^9$ K, $t = 1 \to 300$ s

  Earliest time in thermal history probed so far Fig.

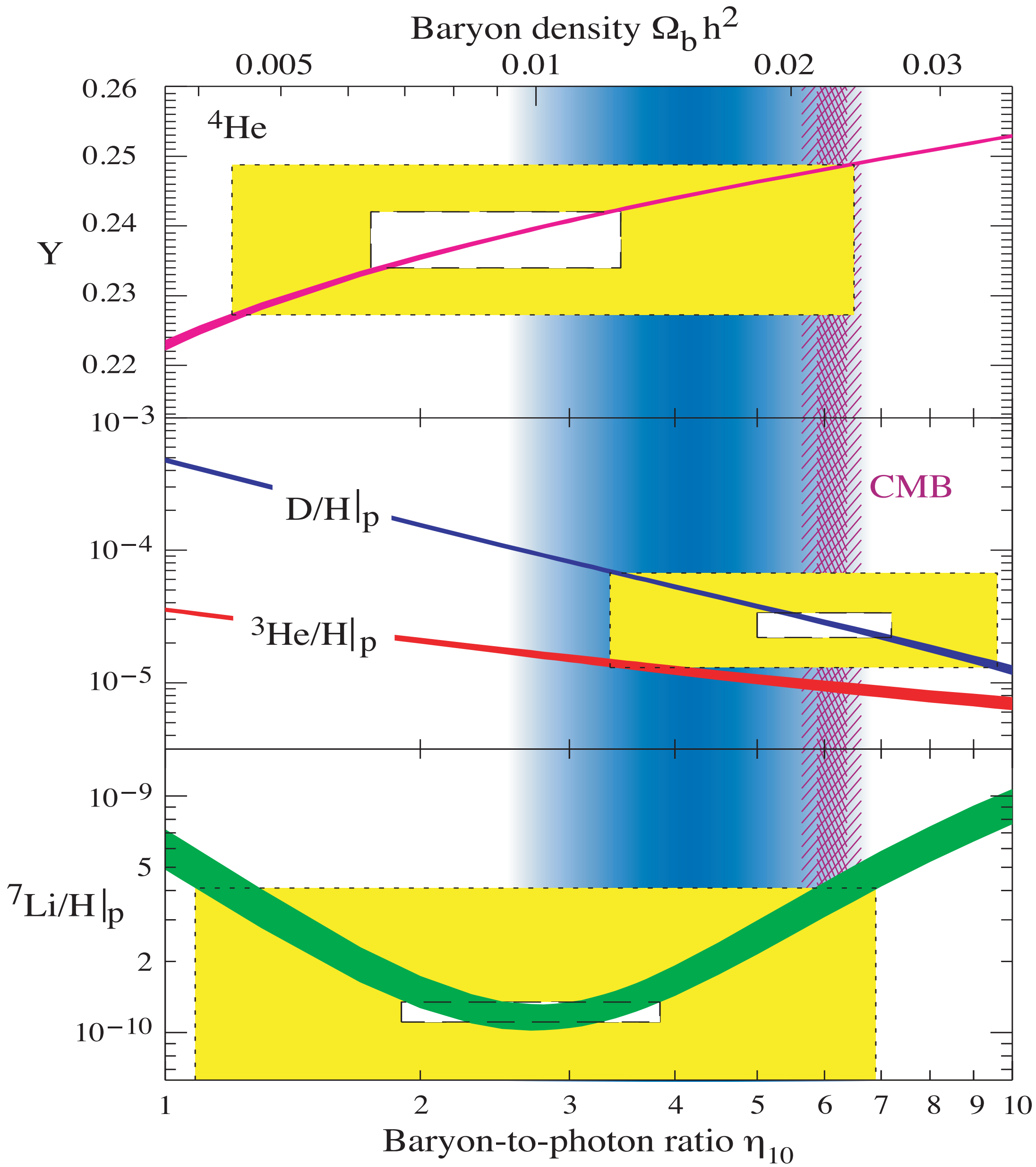
- Recombination, transition from plasma to gas.

  $z = 1090$, $T = 3000$ K, $t = 370\,000$ years

  Last scattering of CMB photons Fig.

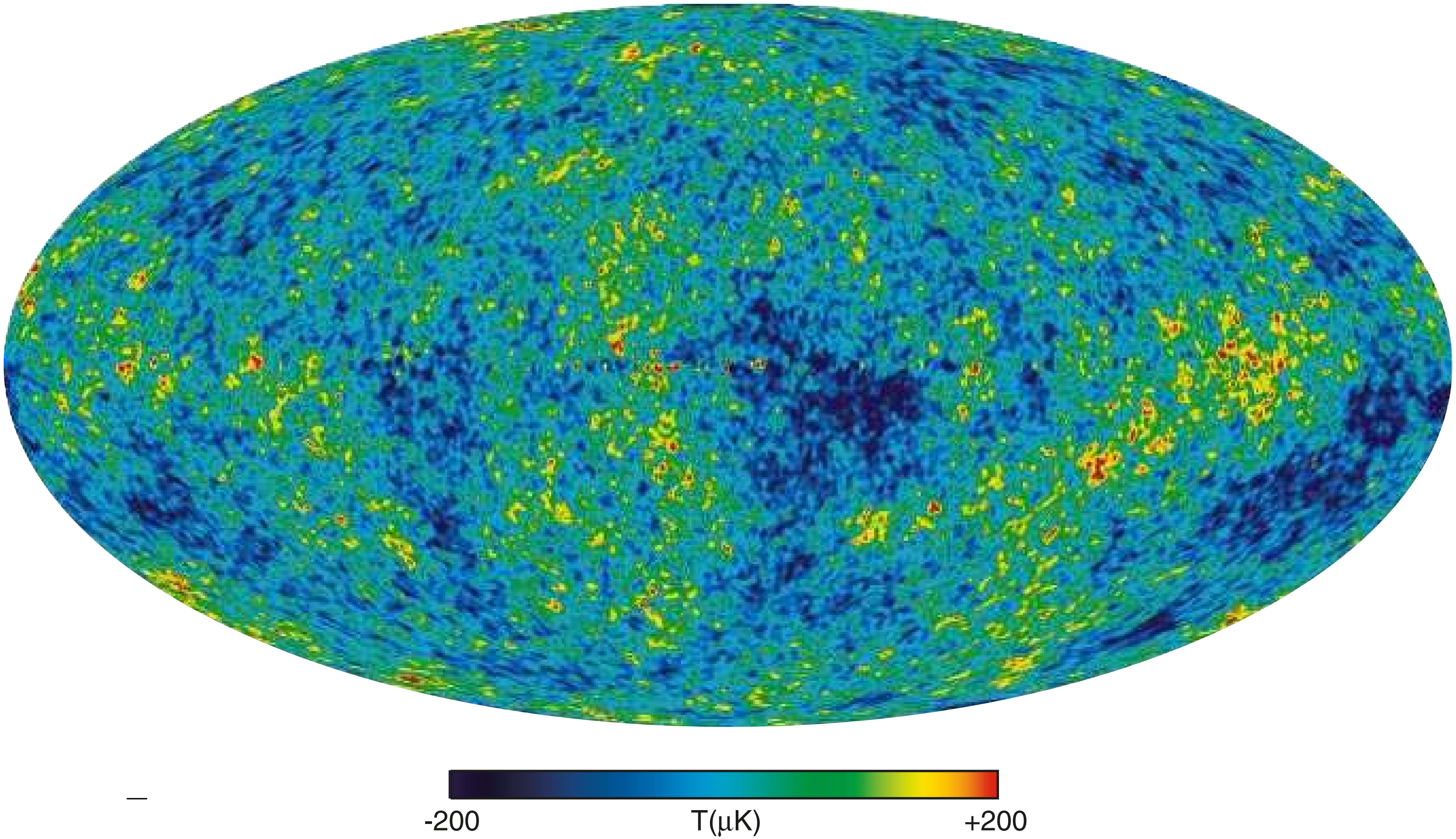
- Neutrino decoupling: $T = 2-3$ MeV $\sim 3 \cdot 10^{10}$K, $t \sim 0.1-1$s
- Generation of dark matter*
- Generation of matter-antimatter asymmetry*

*may have happend before the hot Big Bang epoch

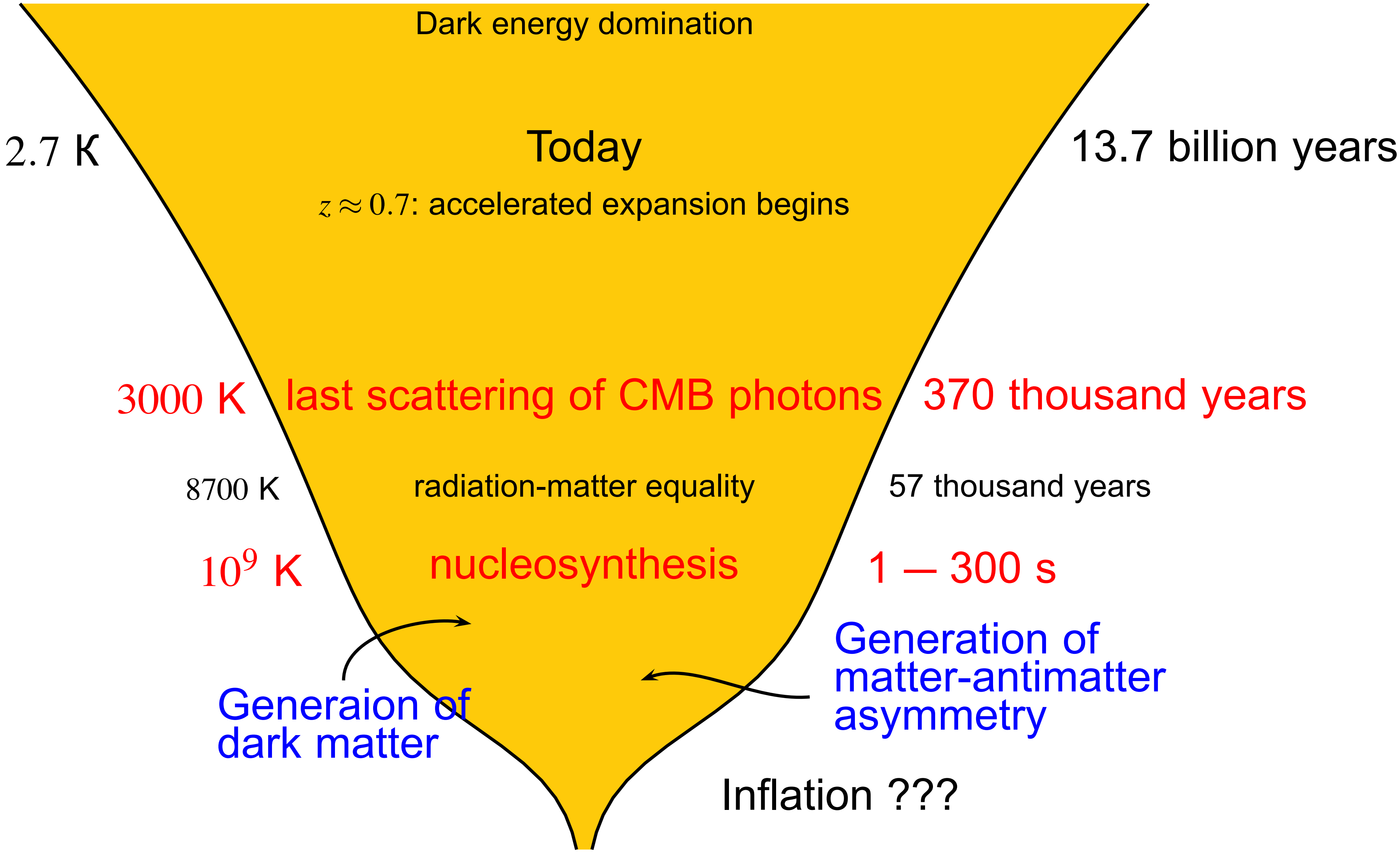


$\eta_{10} = \eta \cdot 10^{-10}$ = baryon-to-photon ratio. Consistent with CMB determination of η

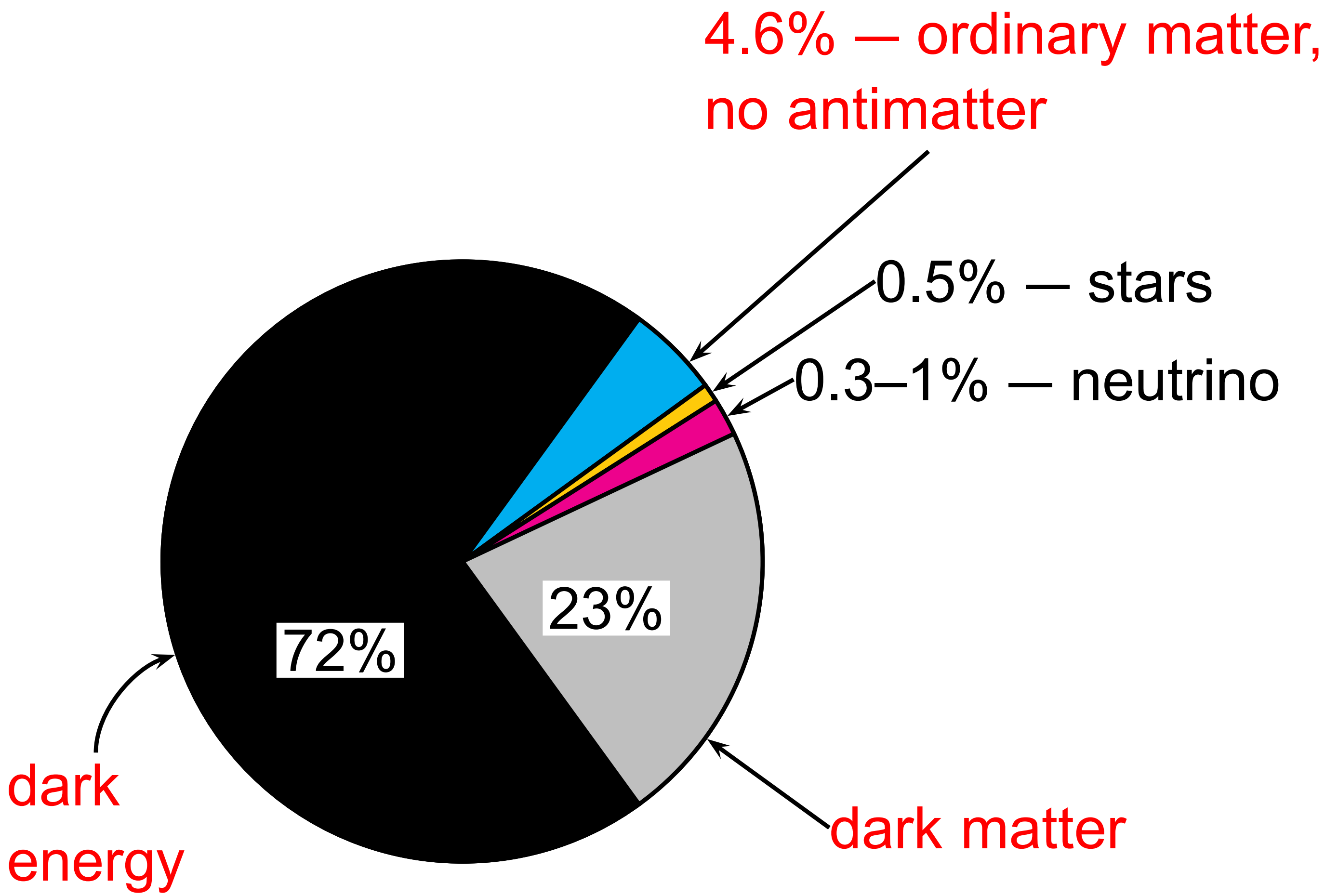
$$T = 2.725^{\circ}K, \quad \frac{\delta T}{T} \sim 10^{-5}$$



WMAP


Dark energy domination
2.7 K
Today
13.7 billion years
$z \approx 0.7$: accelerated expansion begins
3000 K
last scattering of CMB photons
370 thousand years
8700 K
radiation-matter equality
57 thousand years
10^9 K
nucleosynthesis
1 — 300 s
Generation of matter-antimatter asymmetry
Generaion of dark matter
Inflation ???

Unknowns


4.6% — ordinary matter,
no antimatter
0.5% — stars
0.3–1% — neutrino
23%
72%
dark
energy
dark matter

Dark matter

- Astrophysical evidence: measurements of gravitational potentials in galaxies and clusters of galaxies
  - Velocity curves of galaxies

    Fig.
  - Velocities of galaxies in clusters

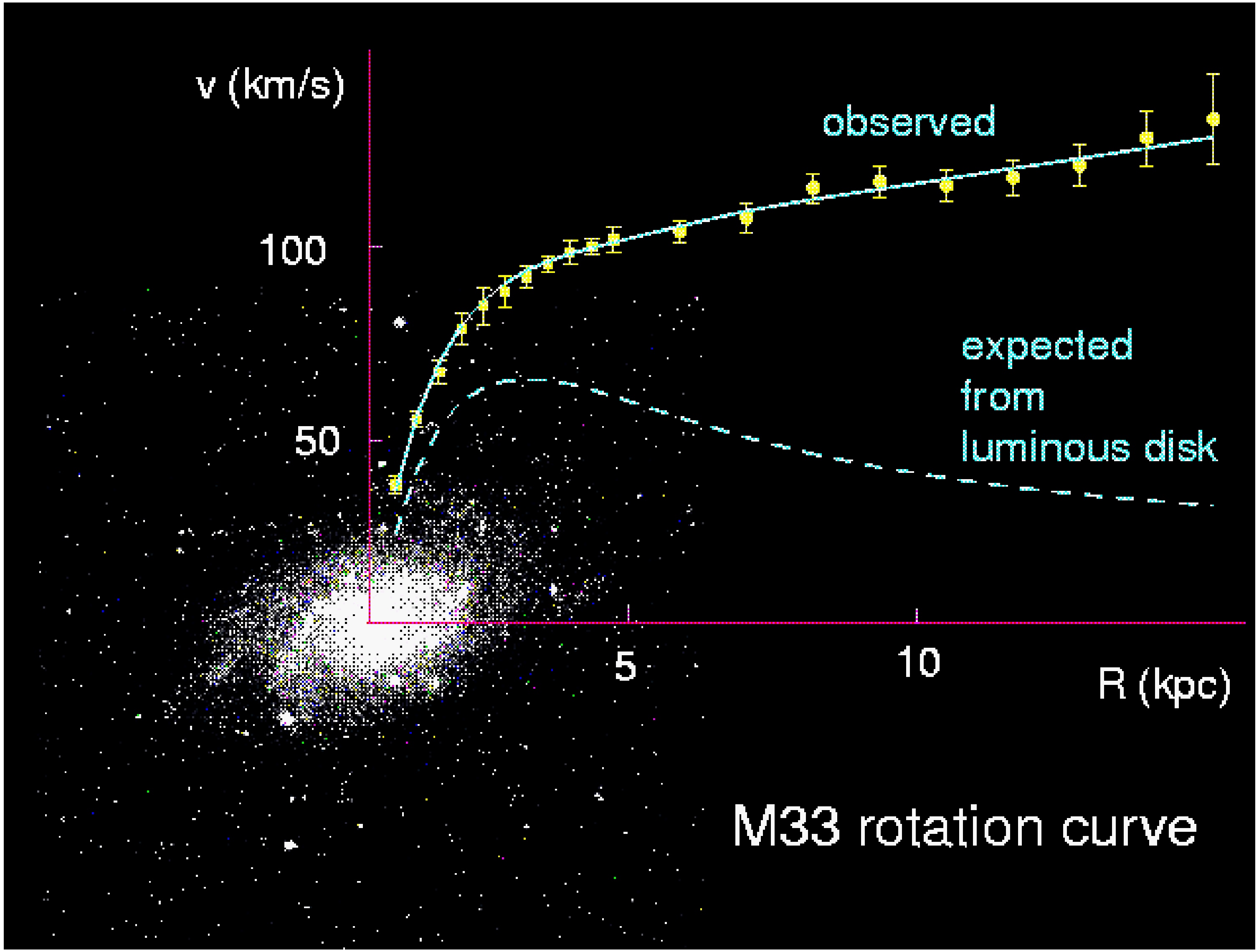
    Original Zwicky's argument, 1930's

$$v^2 = G\frac{M(r)}{r}$$

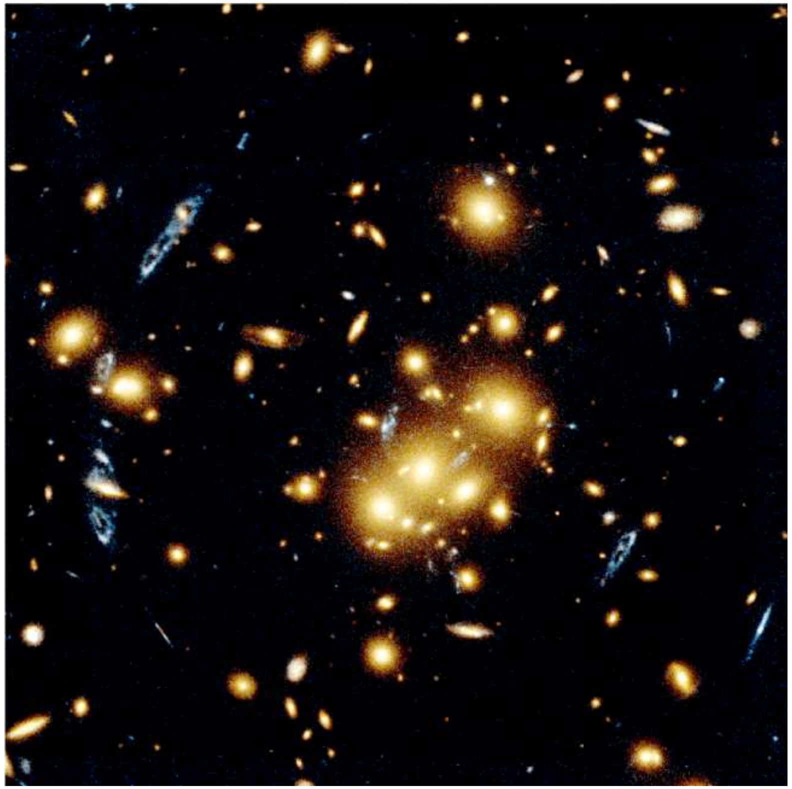
  - Temperature of gas in X-ray clusters of galaxies
  - Gravitational lensing of clusters

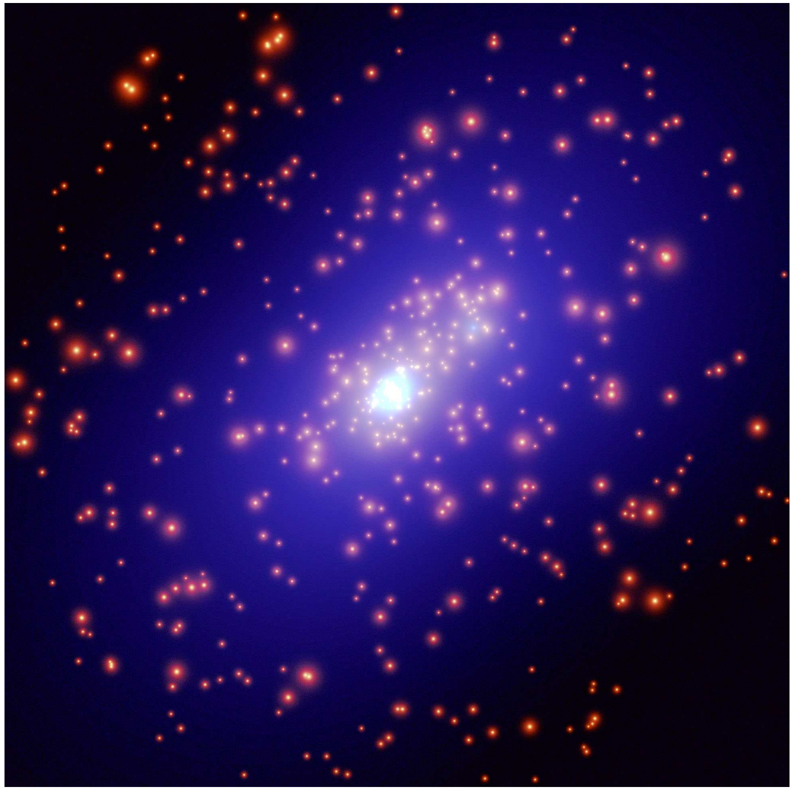
    Fig.
  - Etc.

Rotation curves



Gravitational lensing





Outcome

$$\Omega_M \equiv \frac{\rho_M}{\rho_c} = 0.2 - 0.3$$

Assuming mass-to-light ratio everywhere the same as in clusters
NB: only 10 % of galaxies sit in clusters

Nucleosynthesis, CMB:

$$\Omega_B = 0.046$$

The rest is non-baryonic, $\Omega_{DM} \approx 0.23$.

Physical parameter: mass-to-entropy ratio. Stays constant in time.
Its value

$$\left(\frac{\rho_{DM}}{s}\right)_0 = \frac{\Omega_{DM}\rho_c}{s_0} = \frac{0.2 \cdot 0.5 \cdot 10^{-6} \text{ GeV cm}^{-3}}{3000 \text{ cm}^{-3}} = 3 \cdot 10^{-10} \text{ GeV}$$

Cosmological evidence: growth of structure

CMB anisotropies: baryon density perturbations at recombination $\approx$ photon last scattering, $T = 3000$ K, $z = 1100$:

$$\delta_B \equiv \left(\frac{\delta\rho_B}{\rho_B}\right)_{z=1100} \simeq \left(\frac{\delta T}{T}\right)_{CMB} \sim 10^{-4}$$

In matter dominated Universe, matter perturbations grow as

$$\frac{\delta\rho}{\rho}(t) \propto a(t)$$

Perturbations in baryonic matter grow after recombination only

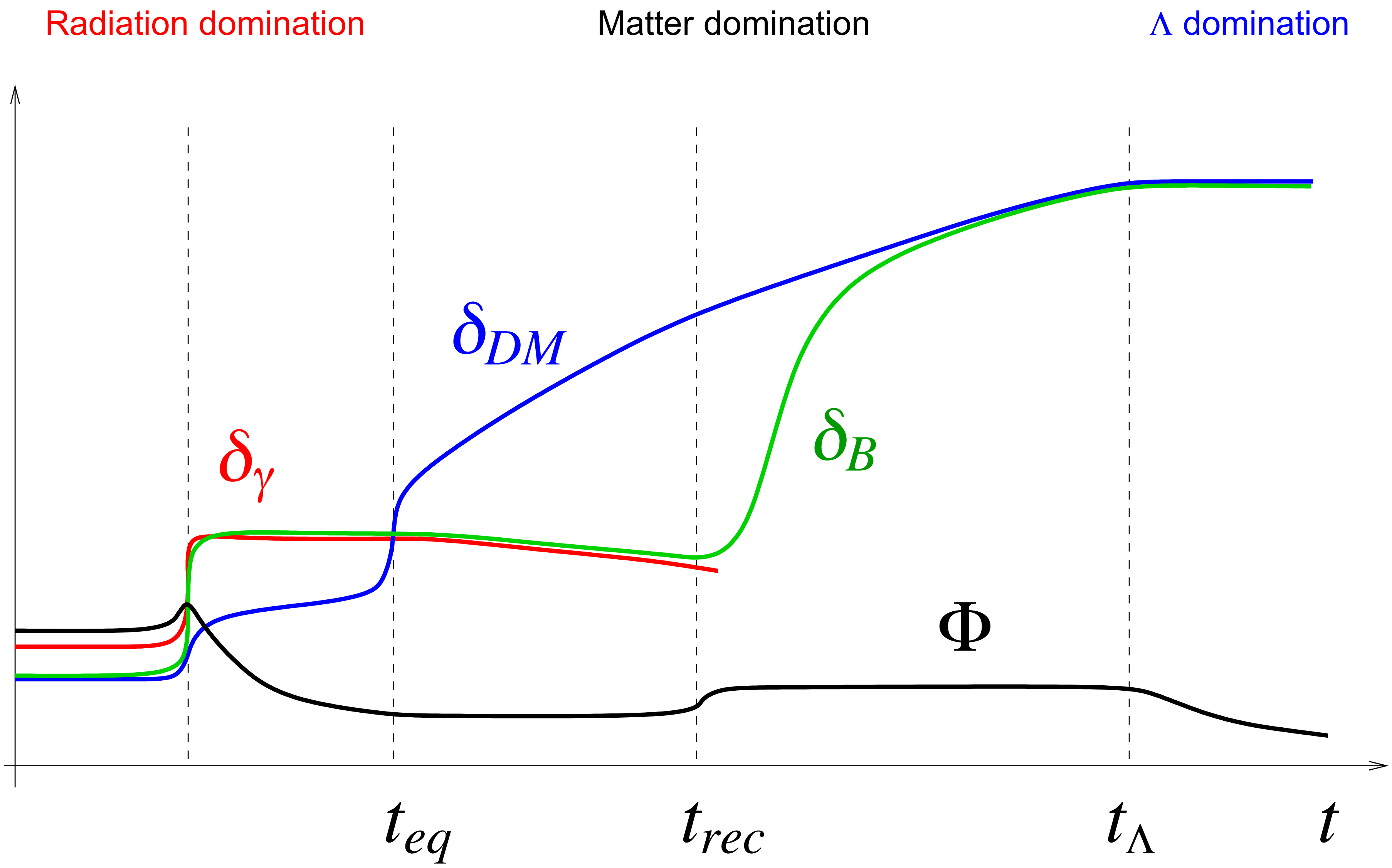
If not for dark matter,

$$\left(\frac{\delta\rho}{\rho}\right)_{today} = 1100 \times 10^{-4} \sim 0.1$$

No galaxies, no stars...

Perturbations in dark matter start to grow much earlier (already at radiation-dominated stage)

Growth of perturbations (linear regime)



NB: Need dark matter particles non-relativistic early on.

Neutrinos are not considerable part of dark matter
(way to set cosmological bound on neutrino mass,
$m_\nu < 0.2$ eV for every type of neutrino)

UNKNOWN DARK MATTER PARTICLES ARE
CRUCIAL FOR OUR EXISTENCE

If thermal relic:

Cold dark matter, CDM

$$m_{DM} \gtrsim 100 \text{ keV}$$

Warm dark matter

$$m_{DM} \simeq 1-30 \text{ keV}$$

WIMPs

Simple but very suggestive scenario

- Assume there is a new heavy stable particle X
  - Interacts with SM particles via pair annihilation (and crossing processes)

$$X + X \leftrightarrow q\bar{q} \text{ , etc}$$

  - Parameters: mass M_X; annihilation cross section at non-relativistic velocity $\sigma(v)$
- Assume that maximum temperature in the Universe was high, $T \gtrsim M_X$
- Calculate present mass density
  - Recall

$$H(T) = \frac{T^2}{M_{Pl}^*}$$

with $M_{Pl}^* = M_{Pl}/(1.66\sqrt{g_*}) \sim 10^{18}$ GeV at $T \sim 100$ GeV

- Number density of X-particles in equilibrium at $T < M_X$: Maxwell–Boltzmann

$$n_X = g_X \int \frac{d^3p}{(2\pi)^3} \mathrm{e}^{-\frac{\sqrt{M_X^2+p^2}}{T}} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{M_X}{T}}$$

- Mean free time wrt annihilation: travel distance $\tau_{ann} v$, meet one X particle to annihilate with in volume $\sigma \tau_{ann} v \Longrightarrow$

$$\sigma \tau_{ann} v n_X = 1 \quad \Longrightarrow \quad \tau_{ann} = \frac{1}{n_X \langle \sigma v \rangle}$$

- Freeze-out: $\tau_{ann}^{-1}(T_f) \sim H(T_f) \Longrightarrow n_X(T_f)\langle \sigma v \rangle \sim T_f^2 / M_{Pl}^* \Longrightarrow$

$$T_f \simeq \frac{M_X}{\ln(M_X M_{Pl}^* \langle \sigma v \rangle)}$$

NB: large log $\Longleftrightarrow T_f \sim M_X/30$

Define $\langle \sigma v \rangle \equiv \sigma_0$ (constant for s-wave annihilation)

- Number density at freeze-out

$$n_X(T_f) = \frac{T_f^2}{\sigma_0 M_{Pl}^*}$$

- Number-to-entropy ratio at freeze-out and later on

$$\frac{n_X(T_f)}{s(T_f)} = \#\frac{n_X(T_f)}{g_* T_f^3} = \#\frac{\ln(M_X M_{Pl}^* \sigma_0)}{M_X \sigma_0 g_* M_{Pl}^*}$$

where $\# = 45/(2\pi^2)$.

- Mass-to-enropy ratio

$$\frac{M_X n_X}{s} = \#\frac{\ln(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g_*(T_f)} M_{Pl}}$$

- Most relevant parameter: annihilation cross section $\sigma_0 \equiv \langle \sigma v \rangle$ at freeze-out

$$\frac{M_X n_X}{s} = \# \frac{\ln(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g_*(T_f)} M_{Pl}}$$

- Correct value, mass-to-entropy$= 3 \cdot 10^{-10}$ GeV, for

$$\sigma_0 \equiv \langle \sigma v \rangle = (1 \div 2) \cdot 10^{-36} \text{ cm}^2$$

  - Weak scale cross section.

    Gravitational physics and EW scale physics combine into

$$\text{mass-to-entropy} \simeq \frac{1}{M_{Pl}} \left(\frac{\text{TeV}}{\alpha_W} \right)^2 \simeq 10^{-10} \text{ GeV}$$

  - Mass M_X should not be much higher than 100 GeV

Weakly interacting massive particles, WIMPs.

Cold dark matter candidates

SUSY: neutralinos, $X = \chi$

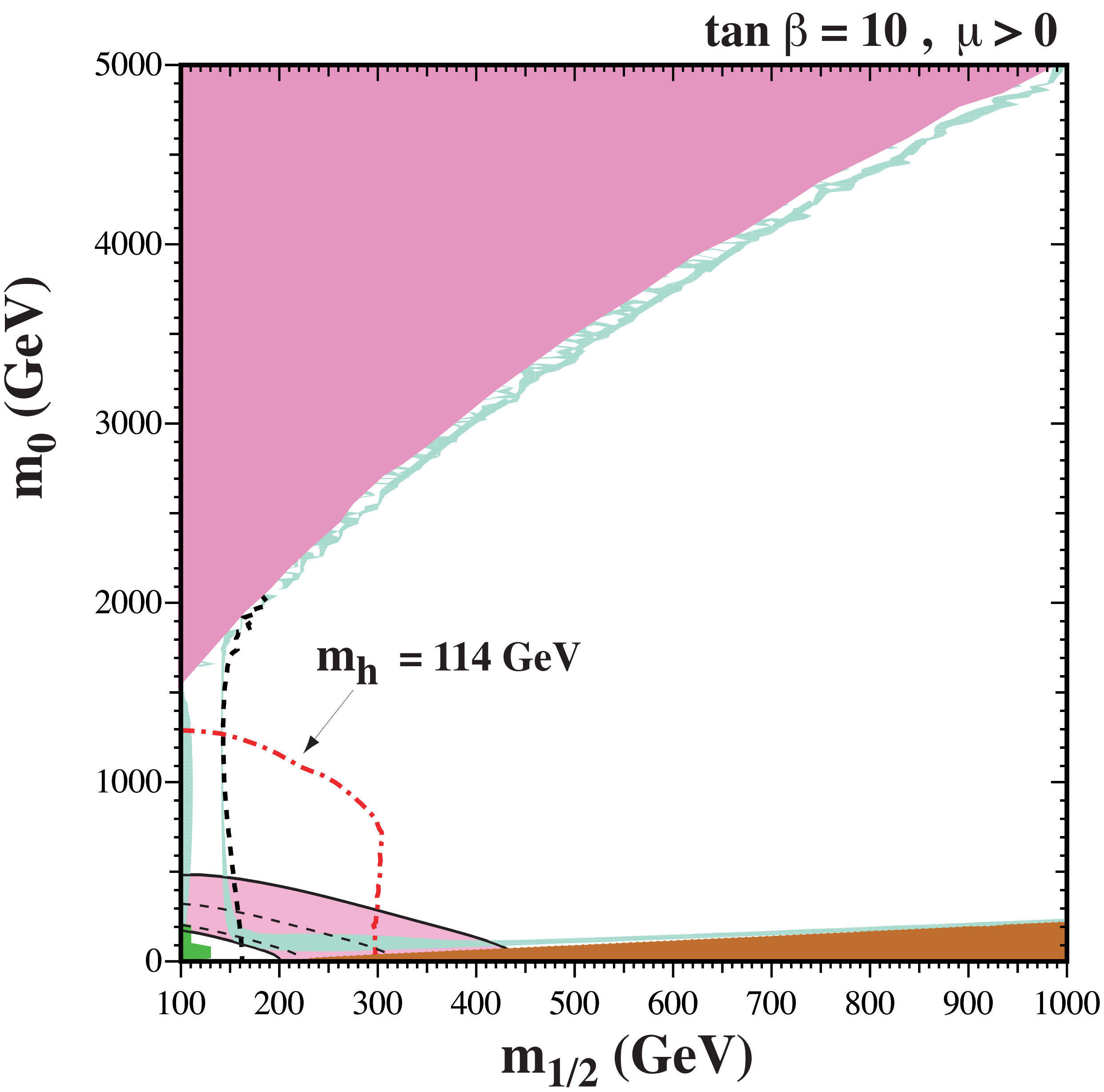
But situation is rather tense already: annihilation cross section is often too low

Important supperssion factor: $\langle \sigma v \rangle \propto v^2 \propto T/M_\chi$ because of p-wave annihilation in case $\chi\chi \to Z^* \to f\bar{f}$:

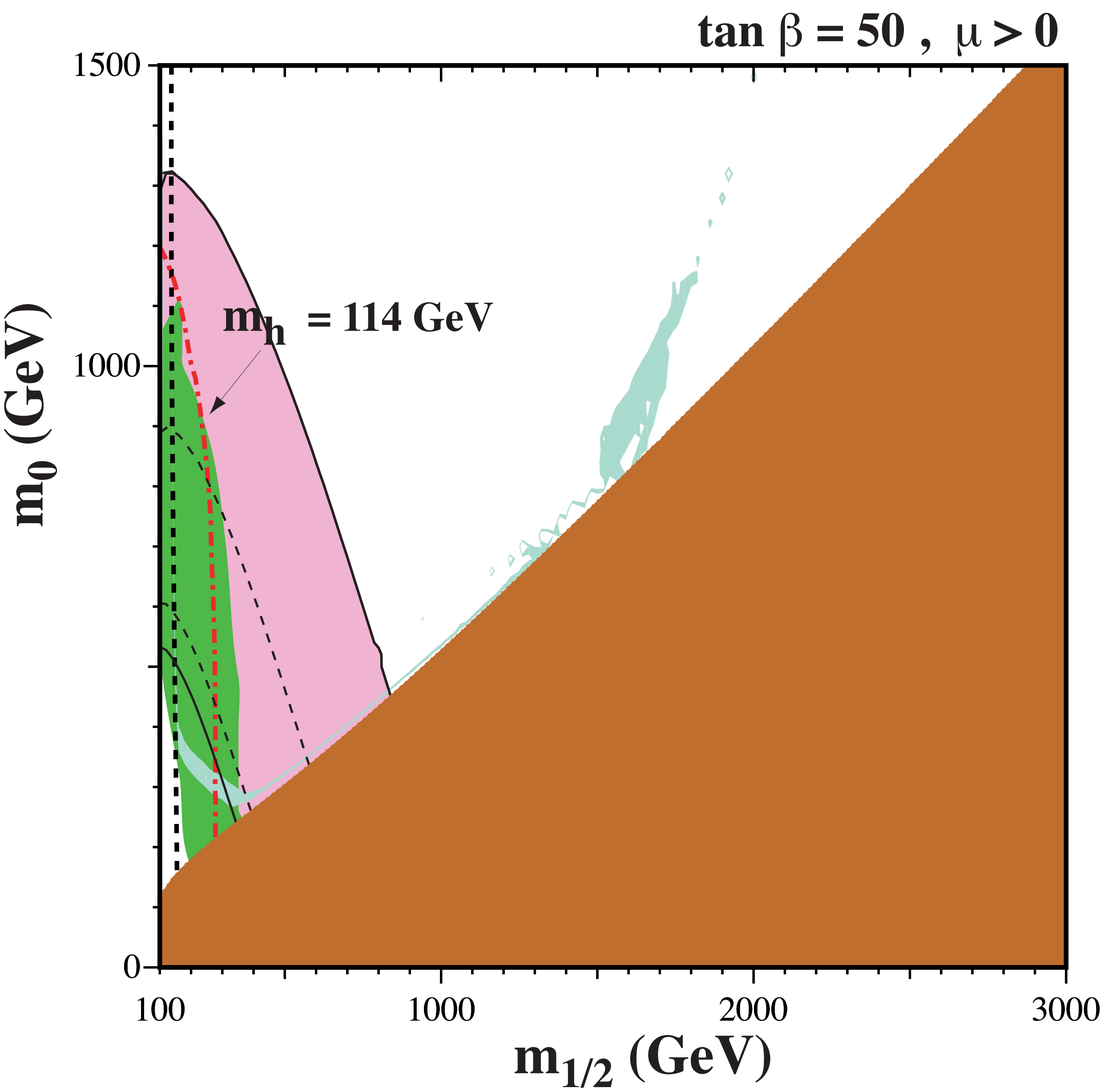
Relativistic $f\bar{f} \implies$ total angular momentum $J = 1$

$\chi\chi$: identical fermions $\implies L = 0$, parallel spins impossible $\implies$ p-wave

mSUGRA at fairly low $\tan\beta$



Larger $\tan\beta$ is better



Warm dark matter: gravitino?

- Clouds over CDM

  Numerical simulations of structure formation with CDM show

  - Too many dwarf galaxies

    A few hundred satellites of a galaxy like ours —

    But only dozens observed so far

  - Too high density in galactic centers (“cusps”)

- Not crisis yet

  But what if one really needs to suppress small structures?

High initial momenta of DM particles $\Longrightarrow$ Warm dark matter

Warm dark matter

- Decouples when relativistic, $T_f \gg m$.
- Remains relativistic until $T \sim m$ (assuming thermal distribution). Does not feel gravitational potential before that.
- Perturbations of wavelengths shorter than horizon size at that time get smeared out $\Longrightarrow$ small size objects do not form ("free streaming")
- Horizon size at $T \sim m$

$$l(T) = H(T \sim m)^{-1} = \frac{M_{Pl}^*}{T^2} = \frac{M_{Pl}^*}{m^2}$$

Present size of this region

$$l_c = \frac{T}{T_0} l(T) = \frac{M_{Pl}}{mT_0}$$

(modulo g_* factors).

Objects of initial comoving size smaller than l_c are less abundant

- Initial size of dwarf galaxy $l_{dwarf} \sim 100$ kpc $\sim 3 \cdot 10^{23}$ cm
  Require

$$l_c \simeq \frac{M_{Pl}}{mT_0} \sim l_{dwarf}$$

$\Longrightarrow$ obtain mass of DM particle

$$m \sim \frac{M_{Pl}}{T_0 l_{dwarf}} \sim 3 \text{ keV}$$

($M_{Pl} = 10^{19}$ GeV, $T_0^{-1} = 0.1$ cm).

- Particles of masses in 1 – 10 keV range
  are good warm dark matter candidates

Gravitinos

- Mass $m_{3/2} \simeq F/M_{Pl}$
  $\sqrt{F} =$ SUSY breaking scale.
  $\Longrightarrow$ Gravitinos light for low SUSY breaking scale.
  E.g. gauge mediation

- Light gravitino = LSP $\Longrightarrow$ Stable

- Decay width of superpartners into gravitino + SM particles

$$\Gamma_{\tilde{S}} \simeq \frac{M_{\tilde{S}}^5}{F^2} \simeq \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2}$$

$M_{\tilde{S}}$ = mass of superpartner $\tilde{S}$

Gravitino production in decays of superpartners

$$\frac{d(n_{3/2}/s)}{dt} = \frac{n_{\tilde{S}}}{s}\Gamma_{\tilde{S}}$$

$n_{\tilde{S}}/s = \text{const} \sim g_*^{-1}$ for $T \gtrsim M_{\tilde{S}}$, while $n_{\tilde{S}} \propto \mathrm{e}^{-M_{\tilde{S}}/T}$ for $T \ll M_{\tilde{S}}$ $\Longrightarrow$ production most efficient at $T \sim M_{\tilde{S}}$ (slow cosmological expansion with unsuppressed $n_{\tilde{S}}$)

$$\frac{n_{3/2}}{s} \simeq \frac{\Gamma_{\tilde{S}}}{g_* H(T \sim M_{\tilde{S}})} \simeq \frac{M_{Pl}^*}{g_* M_{\tilde{S}}^2} \cdot \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2}$$

Mass-to-entropy ratio

$$\frac{m_{3/2} n_{3/2}}{s} \simeq \frac{M_{\tilde{S}}^3}{m_{3/2}} \frac{1}{g_*^{3/2} M_{Pl}}$$

$$\frac{m_{3/2} n_{3/2}}{s} \simeq \sum_{\tilde{S}} \frac{M_{\tilde{S}}^3}{m_{3/2}} \frac{1}{g_*^{3/2} M_{Pl}}$$

For $m_{3/2} =$ a few keV, mass-to-entropy$= 3 \cdot 10^{-10}$ GeV

$$M_{\tilde{S}} \simeq 100 \div 300 \text{ GeV}$$

Need light superpartners

and low maximum temperature in the Universe, $T_{max} \lesssim 1$ TeV to avoid overproduction in collisions of superpartners (and in decays of squarks and gluinos if they are heavy)

Rather contrived scenario, but generating warm dark matter is always contrived

NB: $\Gamma_{NLSP} \simeq \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2} \Longrightarrow c\tau_{NLSP} =$ a few $\cdot$ mm $\div$ a few $\cdot\, 100$ m

for $m_{3/2} = 1 \div 10$ keV, $M_{\tilde{S}} = 100 \div 300$ GeV

Longer lifetime for heavier gravitino (CDM candidate)

TeV SCALE PHYSICS MAY WELL BE RESPONSIBLE FOR GENERATION OF DARK MATTER

Is this guaranteed?

By no means. Another good DM candidate: axion.

Plus a lot of exotica...

Crucial impact of LHC to cosmology,
direct and indirect dark matter searches

- WIMP, signal at LHC:
  - Strongest possible motivation for direct and indirect detection
    - Inferred interactions with baryons $\Longrightarrow$ strategy for direct detection
  - A handle on the Universe at

$$T = (\text{a few}) \cdot 10 \text{ GeV} \div (\text{a few}) \cdot 100 \text{ GeV}$$

$$t = 10^{-11} \div 10^{-8} \text{ s}$$

cf. $T = 1$ MeV, $t = 1$ s   at nucleosynthesis

- Gravitino-like
  - A lot of work to make sure that it is indeed DM particle
  - Hard time for direct and indirect searches
- No signal at LHC
  - Need luck to figure out who is dark matter particle
  - Need more hints from cosmology and astrophysics