The Dirac and Maxwell equations can be derived from the
Lagrangian

\[ L_{\text{QED}} = \frac{i}{2} \bar{\Psi} \gamma^\mu D_{\mu} \Psi - M \bar{\Psi} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

where \( \bar{\Psi} \gamma^\mu D_{\mu} \Psi \) is the electromagnetic current of the charged fermion. From (12) we can derive the charge conservation,

\[ \mu \gamma^\mu \Phi = \mu \gamma^\mu \Phi \]

where \( j_n \) is the electromagnetic current of the charged fermion. From (12) we can derive the charge conservation.

The coupled Euler-Lagrange field equations are then (10),

\[ \frac{\partial}{\partial x^\mu} F_{\mu\nu} - \mu (\mu - \mu \gamma^\nu \psi) \Phi = \mu \gamma^\mu \Phi \]

Lagrangian
\[ (\forall I) \ (\hbar - x) \phi [u\phi^u - (\frac{\partial}{\partial W} + \Box)^{u\mu}] = (\hbar - x)^{u\mu} \nabla \]

\[ uV[u\phi^u - (\frac{\partial}{\partial W} + \Box)^{u\mu}]wV \frac{\partial}{\partial I} = wV \frac{\partial}{\partial W} + w\mu \frac{\partial}{\partial I} \]

The propagator of a massless photon is found from

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]

\[ \text{The propagator of a massless photon is found from} \]
\[ Y(\Omega) \text{ is a particular case of unitary abelian transformations.} \]

### 2.3. Non-abelian gauge theories

Experimental limit photon mass, \( m_\gamma \lesssim 10^{-18} \text{eV} \). Experimental properties of the massless and massive photon theories are the same. Tidational polarization does not contribute to amplitudes. Due to the current conservation, \( \theta_{\mu\nu} = 0 \), the longitudinal amplitudes are

\[
\left( \frac{V_{\lambda\mu}}{w_\lambda w_\mu} - \frac{\gamma_{\lambda\mu}}{w_\lambda w_\mu} \right) = (\gamma_{\mu\nu}) \nabla
\]

Therefore, in momentum space (exerice)
SU transformations are described by matrices $U$ satisfying $U^\dagger U = UNIT$.

The simplest case is $SU(2)$ proposed by Yang and Mills.

The simplest representation is a doublet $\Psi^c = \begin{pmatrix} \Psi^p \\ \Psi^d \end{pmatrix}$, where $U_{\alpha} = e^{i\tau_{\alpha}w_{\alpha}}$ are the Pauli matrices.

The number of gauge bosons equals the number of generators of $SU(2)$.

In 1954, simplest representation is a doublet in $SU(2)$.

The simplest case is $SU(2)$ proposed by Yang and Mills.

\begin{align*}
\begin{pmatrix}
\frac{w_3 - w_2}{\sqrt{2}} & \frac{w_3 + w_2}{\sqrt{2}} \\
\frac{w_3 - w_2}{\sqrt{2}} & -\frac{w_3 + w_2}{\sqrt{2}}
\end{pmatrix} & = \begin{pmatrix}
\frac{w_3 - w_2}{\sqrt{2}} & \frac{w_3 + w_2}{\sqrt{2}} \\
\frac{w_3 - w_2}{\sqrt{2}} & \frac{w_3 + w_2}{\sqrt{2}}
\end{pmatrix} = \frac{2}{\sqrt{2}} w_3 M = w_3 M
\end{align*}

\begin{align*}
(\hat{\Omega})^T \equiv (\theta) \hat{\Omega} \quad \text{where} \quad \hat{\Omega} = \begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_3 \end{pmatrix}
\end{align*}

\begin{align*}
\hat{\Omega}(\theta) \hat{\Omega} = \hat{\Omega} \quad \Rightarrow \quad \begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_3 \end{pmatrix} = \tau_1 \hat{\Omega}
\end{align*}

\begin{align*}
\tau_1 = \hat{\Omega} \quad \Rightarrow \quad \hat{\Omega} = \tau_1 
\end{align*}

\begin{align*}
\text{Ising}
\end{align*}

\begin{align*}
\begin{array}{l}
\text{transformations are described by matrices } \hat{\Omega}, \text{ sat-}
\end{array}
\end{align*}
Exercice : show that

\[ [uD,WM] - [uW,UM] = u\omega G \]

The field strength is built from

\[ \omega G = [uD,WM] \]

\[ \theta^u M \theta^p \theta^b \theta^a + \theta^u \theta^p \theta^a \equiv \theta^u D = \frac{u}{\partial} M \theta \]

and the infinitesimal variation in component form

\[ \Gamma (\omega G) = \omega G \]

Exercice : show that
The Yang-Mills Lagrangian is

\[ L_{YM} = \frac{1}{4} F_{ab}^\mu F_\mu^{ab} - g (W - w D_w \phi) \phi = \mathcal{L} \]

Non-abelian gauge bosons have self-interactions, unlike the photon. Full Lagrangian describing interaction of Yang-Mills fields with charged fermions:

\[ w^e M^u_{w'} M^u_{q} M^{\rho q}_{\rho q} \frac{\mathcal{L}}{\mathcal{I}} - w^e M^u_{w'} M^u_{q} M^{\rho q}_{\rho q} \frac{\mathcal{L}}{\mathcal{I}} = \mathcal{L} \]

The Yang-Mills Lagrangian is

\[ \frac{1}{4} M^u_{q} M^{\rho q}_{\rho q} \mathcal{L} + \frac{1}{4} M^u_{q} M^{\rho q}_{\rho q} = \mathcal{L} \]

For Exercise (2) this implies (exercise:)

\[ \]
Exercise: show that for a SU(2) doublet
\[ \Psi \]
and the longitudinal polarization does contribute to
\[ (24) \quad \frac{V \gamma - \gamma q q}{\gamma m - m n} = (\gamma q q \gamma q) \nabla \]

\[ (23) \quad \cdots = (\gamma q q \gamma q) \nabla \]

Here \( \nabla \) is a massive field propagator is
\[ u \nabla \]
on the r.h.s. is the fermionic current
\[ \cdots \]
\[ 0 = m (W - w D w) \]

Field eqs. are
\[ \gamma = [\gamma (w) w M w \gamma]_q \]
\[ + (W - w E w) \gamma _q \]
\[ = m (W - w D w) \gamma \]

\[ \cdots \]
UV properties of the massless and massive YM theories are different. The Yang-Mills boson masses should not be added by hand.

There are however two ways the symmetries are realized in nature:

(i) Nambu-Goldstone: vacuum state not invariant under
symmetry manifest in the spectrum and interactions.

(ii) Weyl-Wigner: vacuum state is invariant under the
symmetry not manifest in the spectrum and interactions.

EX: translations (momentum), rotations (angular momen-
tum), (electric charge)...

There are however two ways the symmetries are real-
Symmetries (Noether theorem) → conserved charges.

has a $\mathbb{Z}_2$ symmetry $S \rightarrow -S$.

For zero magnetic field $B = 0$, the system with $S = \pm 1$. The simplest example of the baryon number $B$ as the symmetry of the vacuum is the Ising model dimension $d$.

Simplest example of the NG realization is the Ising model of the world. Coleman : "the symmetry of the vacuum is the symmetry of the world".

Colored : Chiral symmetry of strong interactions.

\begin{equation}
H = \frac{1}{2} \sum_i S_i^2 - \sum_i S_i T^{\alpha}_i T^{\beta}_i S_j^\alpha S_j^\beta f_{\alpha\beta}\end{equation}

\begin{equation}
0 \rightarrow \mathbb{Z}_2 \times SU(2) \times SU(2)
\end{equation}

\begin{equation}
S \times T^{\alpha}_i T^{\beta}_i
\end{equation}

where $\gamma$ is the gamma matrix in Fermi-Diracজবল সম্ভাবনা.
The magnetization $M_{pl i m}$ should therefore vanish. However, for $T > T_c$, where $kT_c < T_c$, $M = 0$. For $T < T_c$, $M = M$. 

\[
\langle \langle S \rangle \rangle \lim_{N \to \infty} \frac{1}{N} = \lim_{N \to \infty} = M
\]

The magnetization
3.1 The Goldstone theorem.

In a theory with continuous symmetry \( Y \) for every generator which does not annihilate the vacuum

\[ T_a \Phi \]

there is a massless particle \( \Phi \). An example is the linear sigma model

\[ \Phi \rightarrow R \Phi \]

acting as a rotation matrix. The potential is

\[ \frac{1}{2} \varepsilon_{

The model has a continuous symmetric action as

\[ \mathcal{L} \varepsilon (\Phi) + \frac{1}{2} \varepsilon (\Phi) \Lambda \left( \varepsilon (\Phi) \Lambda - \varepsilon (\Phi \omega \epsilon) \frac{2}{I} \right) = \mathcal{J} \]

with Lagrangian

\[ \mathcal{L} \varepsilon (\Phi) \]

\[ \Phi \rightarrow R \Phi \]

with

\[ R \]

scalar fields

\[ \Phi \]

with Lagrangian

\[ \mathcal{L} \varepsilon (\Phi) \]

\[ \Phi \rightarrow R \Phi \]

with

\[ R \]

massless, NO particle.

EX: The linear sigma model.

There is a massless, NO particle which does not annihilate the vacuum.

\[ \langle \Phi_{\nu} \rangle \neq 0 \]

In a theory with continuous symmetry, for every generator

3.1 The Goldstone theorem.
The vacuum manifold is \( O^T \) invariant under a rotation. The ground state can be chosen to be \( \Phi_0 \) preserving an \( O^T \) subgroup. Goldstone's theorem:

\[
(\eta \cdots 0, 0) = 0 \Phi
\]

The vacuum manifold is \( (N^T)O/(N)O \) \( N \) massless particles. By a rotation, the vacuum manifold is \( (N^T)O/(N)O \) invariant.
In order to check this, we define a set of shifted fields
\( \Phi_T \).

The relations are massless, they are the NG bosons.

\[ 0 = \frac{\gamma_{\mu} m}{2}, \quad \varphi = \frac{\varphi}{2} \]

The physical masses are

\[ m_{\sigma} = m_\pi \]

The manifest symmetry is indeed \( O(N-1) \), rotating

\[ \varphi = \varphi (m_{\sigma} + \varphi) \]

The Lagrangian becomes

\[ I = \sum_{\sigma} (x)_{\mu} + \lambda (x)_{\gamma} \mu ) = (x) \Phi \]

In order to check this, we define a set of shifted fields:
\[ 0 = \frac{\mathcal{L}_\Phi}{\Lambda^2} \]

that implies

\[ \mathcal{L}_\Phi = \frac{1}{2} m \mathcal{C} \theta_i = -\Phi \theta_i \]

with \( \mathcal{L}_\Phi = \frac{1}{2} m \mathcal{C} \theta_i = -\Phi \theta_i \)

and a global continuous symmetry

\[ (\mathcal{L}_\Phi) \Lambda = (\mathcal{L}_\Phi + \mathcal{L}_\Phi) \Lambda \]

sider

\[ (\mathcal{L}_\Phi) \Lambda = \mathcal{L}_\Phi \mathcal{L}_\Phi \]

General (classical) proof of the Goldstone theorem. Con-
What happens if the symmetry is local (gauge)?

The Goldstone theorem.

If the vacuum is not invariant under the symmetry generator then \( \mathcal{W} \) is an eigenvector of the mass matrix corresponding to a zero eigenvalue.

If the vacuum is not invariant under the symmetry generator then

\[
0 = \mathcal{W} \left( \frac{i}{\Lambda} \frac{\partial \Phi}{\partial \xi} \right)
\]

whereas

\[
0 = \left\langle \frac{i}{\Lambda} \frac{\partial \Phi}{\partial \xi} \Phi \Phi^\dagger \right\rangle
\]

In the vacuum, \( \Phi \) is the scalar mass matrix,

\[
0 = \left\langle \frac{i}{\Lambda} \frac{\partial \Phi}{\partial \xi} \Phi \Phi^\dagger \right\rangle + \left\langle \frac{i}{\Lambda} \frac{\partial \Phi}{\partial \xi} \Phi \Phi^\dagger \right\rangle
\]

Differentiating again and taking the vev, we get
We expand around the vacuum state

$$\Phi(x) \rightarrow \Phi$$

invariant under the local transformations

$$\Phi \rightarrow e^{i\alpha} \Phi, \quad A_m \rightarrow A_m - e^{i\alpha} \partial_m \alpha$$

potential

$$\langle \bar{\varphi} \varphi + \bar{\varphi} \varphi \rangle = \phi, \quad \langle \phi \rangle = \phi$$

with

$$\langle \bar{\varphi} \varphi \rangle = \phi, \quad \langle \varphi \rangle = \phi$$

Consider an abelian gauge theory

$$\frac{\partial \varphi}{\partial \theta}$$

3.2 The Higgs mechanism.
This is indeed true and can be seen in various ways:

\[(0 \neq \mathcal{V} \mathcal{W})^{\mu} \leftarrow \mathcal{Z} \phi + (0 = \mathcal{V} \mathcal{W})^{\mu} \]

this can only happen if

\[
\nonumber \mathcal{Z} \mathcal{W}^{\mu} = \mathcal{V} \mathcal{W} \quad \text{But} \quad \mathcal{Z} \mathcal{W}^{\mu} \leftarrow \mathcal{Z} \phi \quad \text{the gauge boson acquired a mass} \leftarrow
\]

\[(0 = \mathcal{Z} w) \quad \text{so} \quad \mathcal{Z} \phi \quad \text{is the Goldstone boson. New features appear from the kinetic term}
\]

\[
\frac{w^{\mu} \mathcal{Z} \phi}{\mathcal{Z} \mathcal{W}^{\mu}} + \mathcal{Z} \phi \mathcal{W}^{\mu} \mathcal{W} a \mathcal{W} + \mathcal{Z} \left( \mathcal{W}^{\mu} \mathcal{W} \mathcal{W} \mathcal{W} \right) = \mathcal{Z} \mathcal{W} \mathcal{W} \mathcal{W} \mathcal{W} \mathcal{W}
\]

From the quadratic mass terms we find we (I) find

\[
\frac{\mathcal{Z} \phi + a^{\mu} \phi + a}{a} = \mathcal{Z} \mathcal{W}^{\mu} = \mathcal{Z} \mathcal{W}^{\mu} \mathcal{W} a = 0 \phi
\]
where 

\[ \theta^{\mu \nu} \frac{\delta^2 S}{\delta^2 \phi} + \mu_\nu = \mu_\nu \left( \Phi \frac{\delta S}{\delta \mu} \frac{\delta \mu}{\delta \phi} - \Phi \frac{\delta S}{\delta \phi} \frac{\delta \phi}{\delta \mu} \right) = (x) \Phi \]

by the trans. \( \Phi \left( \Phi \frac{\delta S}{\delta \mu} \frac{\delta \mu}{\delta \phi} - \Phi \frac{\delta S}{\delta \phi} \frac{\delta \phi}{\delta \mu} \right) = (x) \Phi \)

The quadratic part and is absorbed into the longitudinal component of the gauge field. \( \Phi \) can be eliminated altogether in the unitary gauge.

The Goldstone can be eliminated altogether in the unitary gauge.

(44) \[
\begin{align*}
\theta^{\mu \nu} \frac{\delta^2 S}{\delta^2 \phi} + \mu_\nu &= \mu_\nu \left( \Phi \frac{\delta S}{\delta \mu} \frac{\delta \mu}{\delta \phi} - \Phi \frac{\delta S}{\delta \phi} \frac{\delta \phi}{\delta \mu} \right) \\
&= (x) \Phi
\end{align*}
\]

(45) \[
\begin{align*}
\frac{\partial}{\partial \theta} \Phi &+ \frac{\partial}{\partial \phi} \Phi = 0 \\
\therefore \Phi &= 0
\end{align*}
\]

The quadratic term can be diagonalized.
In the unitary gauge, the lagrangian is
\begin{align*}
L &= -c f T F_{mn} U_d W^m \partial_m - i e a A_t^a U_\Phi |^T T \partial_m W_i^a U_f T f Higgs\text{ mechanism, non-abelian case}
\end{align*}

Consider a gauge group $G$ of rank $r$ and scalar fields in some irreducible $n$-dim. representation $\Phi$, and with the scalar potential minimized for $\langle \Phi \rangle = \langle \phi \rangle V$ and $H \in G$ the subgroup of rank $s$ leaving $\Phi$ invariant.

\begin{align*}
(\forall i \cdots \forall i + s = a \quad \quad 0 \neq \alpha \nu L) \\
(\forall i \cdots \forall i + s = a \quad \quad 0 = \alpha \nu L)
\end{align*}

Higgs mechanism, non-abelian case
\begin{align*}
(\forall i \cdots \forall i + s = a \quad \quad 0 \neq \alpha \nu L) \\
(\forall i \cdots \forall i + s = a \quad \quad 0 = \alpha \nu L)
\end{align*}

In the unitary gauge, the lagrangian is
gauge bosons become massive.

\[(50) \quad (\mathcal{L}^1_{\Phi})_T (\mathcal{L}^p_{\Phi})_T \xi^b = q^p W\]

resulting mass matrix of the vector fields is then

eliminates the Goldstone's from the Lagrangian. The

\[(49) \quad \xi^a \mathcal{L}^b \Phi^c \mathcal{W}^d \mathcal{\partial}^m U_{\Phi^c} \mathcal{A}_m \rightarrow \mathcal{A}^p_{\Phi^c} \mathcal{W}^d \mathcal{\partial}^m U_{\Phi^c} \mathcal{A}_m \]

\[(48) \quad \frac{\xi^a}{\mathcal{L}^b \Phi^c \mathcal{W}^d \mathcal{\partial}^m U_{\Phi^c} \mathcal{A}_m} \rightarrow (x)\Phi\]

unitary gauge parametrization (Goldstone's)
\[
\cdots + a \partial_\omega g^\omega_{\mu} - u^\omega_{\mu} V = u^\rho_{\rho} V \leftarrow a \tilde{\psi} + u^\rho_{\rho} A
\]
4. The electroweak sector of the Standard Model.

4.1. Gauge group and matter content.

Standard model = "unified" description of weak and electromagnetic interactions. From the Fermi theory.

We know that we need at least a charged gauge boson $W^\pm$ and the photon $A^\mu$.

The Feynman diagram shows the interaction of these particles with quarks and leptons.