What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

\[ p + p \rightarrow d + e^+ + \nu \]

Spin: \( \frac{1}{2} \quad \frac{1}{2} \quad 1 \quad \frac{1}{2} \quad \frac{1}{2} \)

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……

The Neutrinos

Neutrinos and photons are by far the most abundant elementary particles in the universe. There are 340 neutrinos/cc.

The neutrinos are spin – 1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that their interactions with other matter have very low strength.

Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution

(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!

Neutrino masses suggest, via the See-Saw picture, new physics far above the LHC energy scale.
The discovery of neutrino masses and leptonic mixing has come from the observation of neutrino flavor change (neutrino oscillation).

The Neutrino Flavors

We define the three known flavors of neutrinos, $\nu_e, \nu_\mu, \nu_\tau$, by W boson decays:

As far as we know, neither $\nu_e \rightarrow \nu_\mu$ nor any other change of flavor in the $\nu \rightarrow \ell$ interaction ever occurs. With $\alpha = e, \mu, \tau, \nu_\alpha$ makes only $\ell_\alpha (\ell_\nu = e, \ell_\mu = \mu, \ell_\tau = \tau)$.

Neutrino Flavor Change

If neutrinos have masses, and leptons mix, we can have —

Give $\nu$ time to change character

The last decade has brought us compelling evidence that such flavor changes actually occur.
There must be some spectrum of neutrino mass eigenstates $\nu_i$:

$$\nu_3$$

$$\nu_2$$

$$\nu_1$$

(Mass)$^2$

Mass ($\nu_i$) = $m_i$

This mixing is easily incorporated into the Standard Model (SM) description of the $\ell \nu W$ interaction.

For this interaction, we then have —

\[ L_{\text{SM}} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left( \bar{\nu}_{L\alpha} \gamma^\lambda \nu_{L\alpha} W^\lambda - \bar{\nu}_{L\alpha} \gamma^\lambda \ell_{L\alpha} W^{\lambda*} \right) \]

Taking mixing into account

If neutrino masses are described by an extension of the SM, and there are no new leptons, $U$ is unitary.

**The Meaning of $U$**

The $e$ row of $U$: The linear combination of neutrino mass eigenstates that couples to $e$.

The $\nu$ column of $U$: The linear combination of charged-lepton mass eigenstates that couples to $\nu_1$. 

\[ U = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix} \]
Neutrino Flavor Change (Oscillation) in Vacuum

Approach of B.K. & Stodolsky

Amp $[\nu_\alpha \rightarrow \nu_\beta] = \sum U_{\alpha i}^* \text{Prop}(\nu_i) U_{\beta i}$

What is Propagator $(\nu_i) \equiv \text{Prop}(\nu_i)$?

In the $\nu_i$ rest frame, where the proper time is $\tau_i$,

$$\frac{i}{\partial \tau_i} |\nu_i(\tau_i) > = m_i |\nu_i(\tau_i) > .$$

Thus,

$$|\nu_i(\tau_i) > = e^{-im_i \tau_i} |\nu_i(0) > .$$

Then, the amplitude for propagation for time $\tau_i$ is —

$$\text{Prop}(\nu_i) \equiv < \nu_i(0) |\nu_i(\tau_i) > = e^{-im_i \tau_i} .$$

In the laboratory frame —

The experimenter chooses $L$ and $t$.

They are common to all components of the beam.

For each $\nu_i$, by Lorentz invariance,

$$(E_i, p_i) \times (t, L) = m_i \tau_i = E_i t - p_i L .$$
Neutrino sources are ~ constant in time.

Averaged over time, the
\[ e^{-iE_1t} - e^{-iE_2t} \]
interference

is —

\[ < e^{-i(E_1 - E_2)t} > = 0 \]

unless \( E_2 = E_1 \).

Only neutrino mass eigenstates with a common energy \( E \) are coherent. (Stodolsky)

For each mass eigenstate,

\[ p_i = \sqrt{E^2 - m_i^2} \approx E - \frac{m_i^2}{2E}. \]

Then the phase in the \( \nu_i \) propagator \( \exp[-im_i x_i] \) is

\[ m_i x_i = E_i t - p_i L \equiv Et - (E - m_i^2 / 2E)L \]

\[ = E(t - L) + m_i^2 L / 2E. \]

Irrelevant overall phase —

Probability for Neutrino Oscillation in Vacuum

\[ P(\nu_\alpha \to \nu_\beta) = |\text{Amp}(\nu_\alpha \to \nu_\beta)|^2 = \]

\[ = \delta_{\alpha \beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^*U_{\beta i}U_{\alpha j}U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 L / 4E) \]

\[ + 2 \sum_{i>j} \text{Im}(U_{\alpha i}^*U_{\beta i}U_{\alpha j}U_{\beta j}^*) \sin(\Delta m_{ij}^2 L / 2E) \]

where \( \Delta m_{ij}^2 \equiv m_i^2 - m_j^2 \).
For Antineutrinos –

We assume the world is CPT invariant. Our formalism assumes this.

Thus,

\[ P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = P(\nu_\beta \rightarrow \nu_\alpha; U \rightarrow U^*) \]

A complex U would lead to the CP violation

\[ P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta) . \]

Must we assume all mass eigenstates have the same E?

No, we can take entanglement into account, and use energy conservation.

The oscillation probabilities are still the same.

— Comments —

1. If all \( m_i = 0 \), so that all \( \Delta m^2_{ij} = 0 \),

\[ P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = \delta_{\alpha\beta} \]

**Flavor change \( \Rightarrow \) v Mass

2. If there is no mixing,

\[ \Rightarrow U_{\alpha i} U_{\beta \neq \alpha i} = 0, \text{ so that } P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = \delta_{\alpha\beta} . \]

**Flavor change \( \Rightarrow \) Mixing
3. One can detect \( \nu_\alpha \to \nu_\beta \) in two ways:

   See \( \nu_\beta \to \nu_\alpha \) in a \( \nu_\alpha \) beam (Appearance)

   See some of known \( \nu_\alpha \) flux disappear (Disappearance)

4. Including \( h \) and \( c \)
\[
\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2 (eV^2) \frac{L(km)}{E(GeV)}
\]
\[
\sin^2[1.27 \Delta m^2 (eV)^2 \frac{L(km)}{E(GeV)}]
\]
becomes appreciable when its argument reaches \( \mathcal{O}(1) \).

An experiment with given \( L/E \) is sensitive to
\[
\Delta m^2 (eV^2) \gtrsim \frac{E(GeV)}{L(km)}.
\]

5. Flavor change in vacuum oscillates with \( L/E \). Hence the name “neutrino oscillation”. {The \( L/E \) is from the proper time \( \tau \).}

6. \( P(\nu_\alpha \to \nu_\beta) \) depends only on squared-mass splittings. Oscillation experiments cannot tell us

\[
\sum_{\text{All } \beta} P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 1
\]

But some of the flavors \( \beta \neq \alpha \) could be sterile.

Then some of the active flux disappears:
\[
\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}}
\]

7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.
\[
\sum_{\text{All } \beta} P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 1
\]

Important Special Cases

Three Flavors

For \( \beta \neq \alpha \),
\[
e^{-i m_1^2 \frac{L}{4E}} \text{Amp}^* (\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i} U_{\beta i}^* e^{i m_i^2 \frac{L}{4E}} e^{-i m_1^2 \frac{L}{4E}}
\]
\[
= U_{\alpha 3} U_{\beta 3}^* e^{2 i \Delta_{31}} + U_{\alpha 2} U_{\beta 2}^* e^{2 i \Delta_{21}} - (U_{\alpha 3} U_{\beta 3}^* + U_{\alpha 2} U_{\beta 2}^*) \text{ Unitarity}
\]
\[
= 2i [U_{\alpha 3} U_{\beta 3}^* e^{i \Delta_{31}} \sin \Delta_{31} + U_{\alpha 2} U_{\beta 2}^* e^{i \Delta_{21}} \sin \Delta_{21}]
\]
where \( \Delta_{ij} \equiv \Delta m^2_{ij} \frac{L}{4E} \equiv (m_i^2 - m_j^2) \frac{L}{4E} \).
\[ P(\nu_\alpha \rightarrow \nu_\beta) = |e^{-im_2^\beta \delta_3} \text{Amp}^*(\nu_\alpha \rightarrow \nu_\beta)|^2 \]
\[ = 4|U_{\alpha 3}U_{\beta 3}|^2 \sin^2 \Delta_{31} + |U_{\alpha 2}U_{\beta 2}|^2 \sin^2 \Delta_{21} \]
\[ + 2|U_{\alpha 3}U_{\beta 3}U_{\alpha 2}U_{\beta 2}| \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \pm \delta_{32}) \].

Here \( \delta_{32} \equiv \arg(U_{\alpha 3}U_{\beta 3}U_{\alpha 2}U_{\beta 2}) \), a CP-violating phase.

Two waves of different frequencies, and their CP interference.

When the Spectrum Is—
\[ \nu_3 \quad \Delta m^2 \quad \nu_1 \]
\[ \nu_2 \quad \Delta m^2 \quad \nu_3 \]
\[ \{ \text{Invisible if } \Delta m^2 \frac{L}{E} = O(1) \} \]

For \( \beta \neq \alpha \),
\[ P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \approx 4|U_{\alpha 3}U_{\beta 3}|^2 \sin^2(\Delta m^2 \frac{L}{4E}) \).

For no flavor change,
\[ P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha) \approx 1 - 4|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2) \sin^2(\Delta m^2 \frac{L}{4E}) \].

Experiments with \( \Delta m^2 \frac{L}{E} = O(1) \) can determine the flavor content of \( \nu_3 \).

When There Are Only Two Flavors and Two Mass Eigenstates
\[
U = \begin{bmatrix}
U_{\alpha 1} & U_{\alpha 2} \\
U_{\beta 1} & U_{\beta 2}
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
e^{i\xi} & 0 \\
0 & 1
\end{bmatrix}
\]

Mixing angle

Majorana CP phase

For \( \beta \neq \alpha \),
\[ P(\nu_\alpha \leftrightarrow \nu_\beta) = \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E}) \].

For no flavor change,
\[ P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E}) \].

Neutrino Flavor Change In Matter

Coherent forward scattering via this W-exchange interaction leads to an extra interaction potential energy —

\[
V_W = \begin{cases}
+\sqrt{2}G_F N_e, & \nu_e \\
-\sqrt{2}G_F N_e, & \bar{\nu}_e
\end{cases}
\]

This raises the effective mass of \( \nu_e \), and lowers that of \( \bar{\nu}_{\bar{e}} \).
The fractional importance of matter effects on an oscillation involving a vacuum splitting $\Delta m^2$ is —

$$\frac{\sqrt{2} G_F N_e}{\Delta m^2 / 2E} \equiv x .$$

The matter effect —

- Grows with neutrino energy $E$
- Is sensitive to $\text{Sign}(\Delta m^2)$
- Reverses when $\nu$ is replaced by $\bar{\nu}$

This last is a “fake CP violation”, but the matter effect is negligible when $x \ll 1$.

### Evidence For Flavor Change

<table>
<thead>
<tr>
<th>Neutrinos</th>
<th>Evidence of Flavor Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar Reactor (L $\sim$ 180 km)</td>
<td>Compelling Compelling</td>
</tr>
<tr>
<td>Atmospheric Accelerator (L = 250 and 735 km)</td>
<td>Compelling Compelling</td>
</tr>
<tr>
<td>Stopped $\mu^+$ Decay (LSND, L $\sim$ 30 m)</td>
<td>Does MiniBooNE see this too??</td>
</tr>
</tbody>
</table>

*Very recent evidence to be discussed soon.*

### Solar Neutrinos

**History** –

Nuclear reactions in the core of the sun produce $\nu_e$. Only $\nu_e$. 
Theorists, especially John Bahcall, calculated the produced $\nu_e$ flux vs. energy E.

Ray Davis’ Homestake experiment measured the higher-E part of the $\nu_e$ flux $\phi_{\nu_e}$ that arrives at earth.

The Homestake experiment could detect only $\nu_e$. It found:

$$\frac{\phi_{\nu_e} \text{ (Homestake)}}{\phi_{\nu_e} \text{ (Theory)}} = 0.34 \pm 0.06$$

The Possibilities:

The theory was wrong.

The experiment was wrong.

Both were wrong.

Neither was wrong. Two thirds of the $\nu_e$ flux morphs into a flavor or flavors that the Homestake experiment could not see.

The Resolution —

Sudbury Neutrino Observatory (SNO) measures, for the high-energy part of the solar neutrino flux:

$$\nu_{\text{sol}} d \to e p p \Rightarrow \phi_{\nu_e}$$

$$\nu_{\text{sol}} d \to \nu n p \Rightarrow \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$$

From the two reactions,

$$\frac{\phi_{\nu_e}}{\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}} = 0.301 \pm 0.033$$

Clearly, $\phi_{\nu_\mu} + \phi_{\nu_\tau} \neq 0$. Neutrinos change flavor.

$$P(\nu_e \to \nu_e) = 0.3$$
Change of flavor does not change the total number of neutrinos.

The total flux, $\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$, should agree with Bahcall’s prediction.

SNO: $\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} = (5.54 \pm 0.32 \pm 0.35) \times 10^6$/cm$^2$/sec

Theory*: $\phi_{\text{total}} = (5.69 \pm 0.91) \times 10^6$/cm$^2$/sec

*Bahcall, Basu, Serenelli

John Bahcall and Ray Davis both stuck to their results for several decades, and both were right all along.

**Reactor (Anti)Neutrinos**

**In KamLAND**

The KamLAND detector studies $\overline{\nu}_e$ produced by Japanese nuclear power reactors $\sim 180$ km away.

Our understanding of solar neutrino behavior implies that a considerable fraction of these reactor $\overline{\nu}_e$ should disappear before reaching KamLAND.

KamLAND does see a disappearance of about 1/3 of the $\overline{\nu}_e$.

The solar and KamLAND data are both described by the same, single set of neutrino parameters:

$$\Delta m^2_{\text{sol}} = 7.50 \times 10^{-5} \text{eV}^2, \quad \tan^2 \theta_{\text{sol}} = 0.44$$

Analysis by KamLAND
KamLAND Evidence for \( \text{Oscillatory Behavior} \)

For KamLAND, \( x_{\text{Matter}} < 10^{-2} \). Matter effects are negligible.

The \( \bar{\nu}_e \) survival probability, \( P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \), should oscillate as a function of \( L/E \) following the vacuum oscillation formula.

In the two-neutrino approximation, we expect —

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2 \left( \frac{L}{E} \frac{1.27 \Delta m^2 (eV^2)}{E(\text{GeV})} \right).
\]

For KamLAND, \( x_{\text{Matter}} < 10^{-2} \). Matter effects are negligible.

\( L_0 = 180 \) km is a flux-weighted average travel distance.

\( P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \) actually oscillates!

Atmospheric Neutrinos

Isotropy of the \( \geq 2 \) GeV cosmic rays + Gauss’ Law + No \( \nu_\mu \) disappearance

\[
\frac{\phi_{\nu_\mu} (\text{Up})}{\phi_{\nu_\mu} (\text{Down})} = 1.
\]

But Super-Kamiokande finds for \( E_\nu > 1.3 \) GeV

\[
\frac{\phi_{\nu_\mu} (\text{Up})}{\phi_{\nu_\mu} (\text{Down})} \approx 1/2.
\]
Voluminous atmospheric neutrino data are well described by —

\[ \nu_\mu \rightarrow \nu_\tau \]

with —

\[ \Delta m^2_{\text{atm}} = 2.4 \times 10^{-3} \text{ eV}^2 \]

and —

\[ \sin^2 2\theta_{\text{atm}} = 1 \]

**Accelerator Neutrinos**

Two experiments: K2K and MINOS

![Diagram showing neutrino oscillations](diagram)

**Region of the “atmospheric” oscillation parameters allowed by MINOS and Super-Kamiokande**

(From MINOS paper 1103.0340)

A single pair of parameters, with ~ maximal mixing, fits both the atmospheric and accelerator neutrino data.