QCD in Heavy Ion Collisions: II

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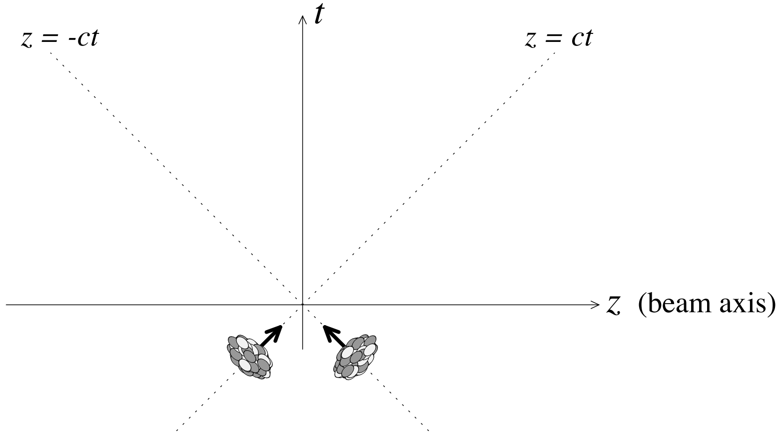
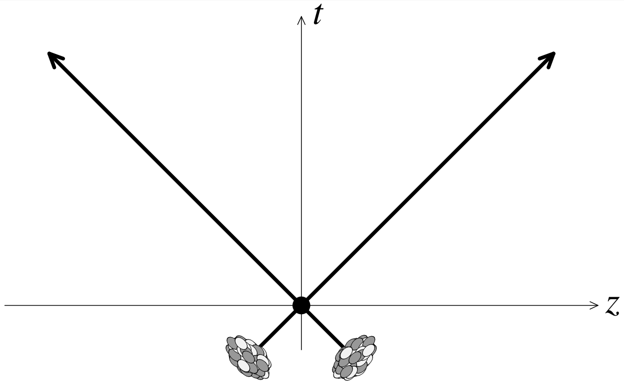
Lecture I: Initial conditions



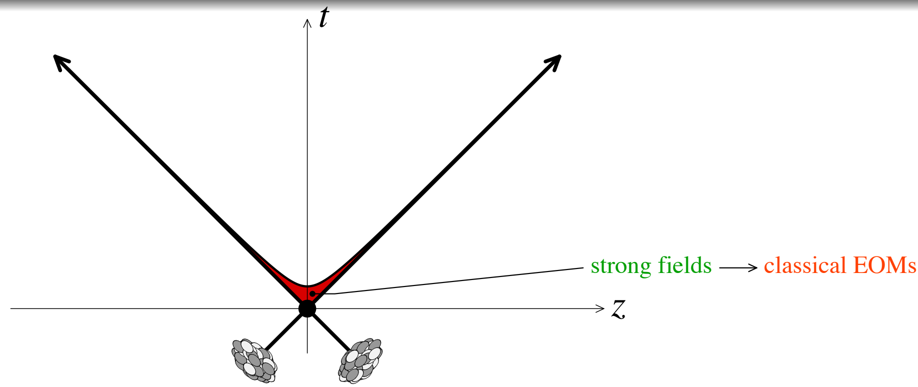
- $\tau < 0$: hadronic wavefunctions prior to the collision
  - high–energy evolution & the Color Glass Condensate
  - it applies to any highly energetic hadron (proton or nucleus)

Lecture I: Initial conditions



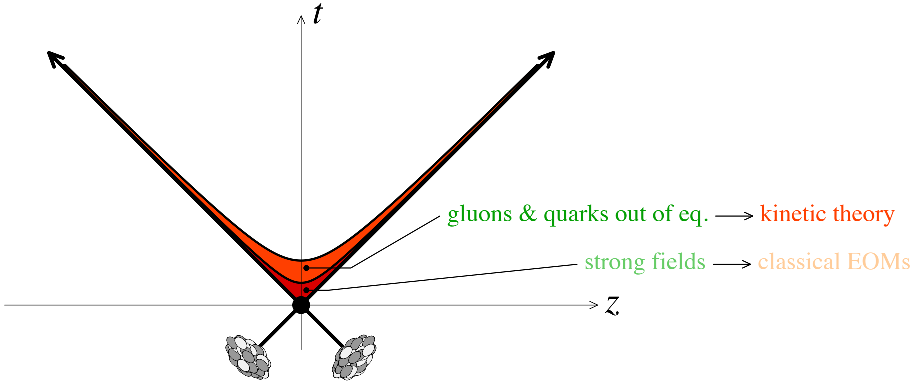
- $\tau < 0$: hadronic wavefunctions prior to the collision
- $\tau \sim 0$ fm/c : the hard scattering
  - production of hard particles: jets, direct photons, heavy quarks
  - calculable within (standard) perturbative QCD ('leading twist')
  - 'hard probes' of the surrounding medium

Lecture I: Initial conditions



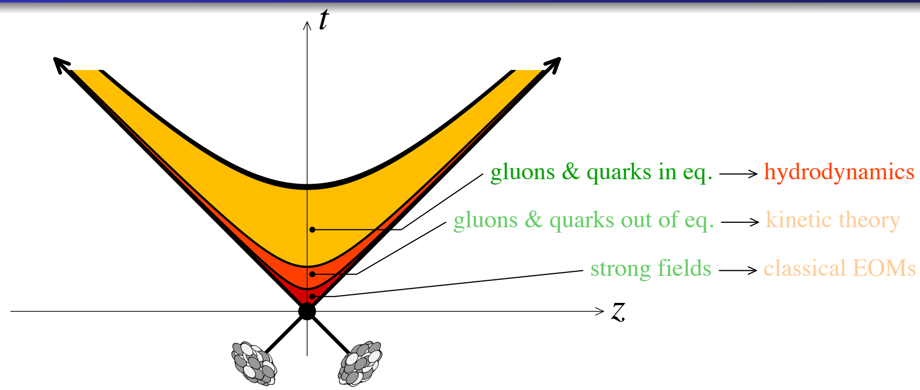
- $\tau < 0$: hadronic wavefunctions prior to the collision
- $\tau \sim 0$ fm/c : the hard scattering
- $\tau \sim 0.2$ fm/c : strong color fields (or 'glasma')
  - semi–hard quanta ($p_\perp \lesssim 2$ GeV): gluons, light quarks
  - make up for most of the multiplicity
  - sensitive to the physics of saturation ('higher twist')

Lecture II: Quark–Gluon Plasma



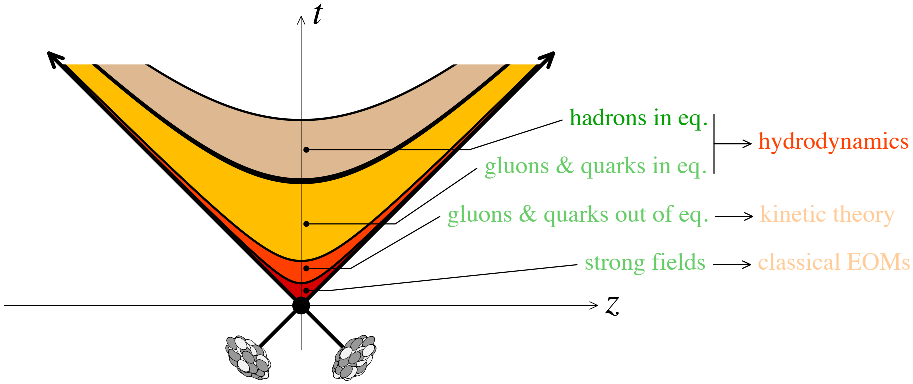
- $\tau \sim 1$ fm/c : thermalization
  - experiments suggest a fast thermalization
  - ...but this is not yet firmly understood within QCD
  - weak or strong coupling ?
  - kinetic theory, plasma instabilities, AdS/CFT

Lecture II: Quark–Gluon Plasma



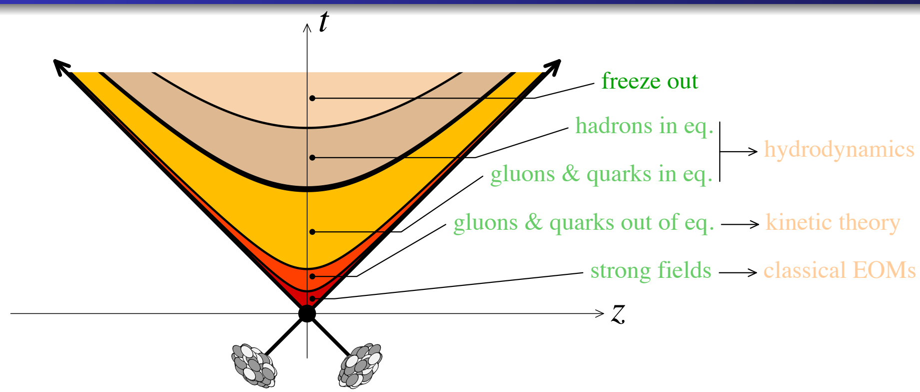
- $\tau \sim 1$ fm/c : thermalization
- $1 \lesssim \tau \lesssim 10$ fm/c : quark–gluon plasma
  - thermodynamics: lattice QCD vs. perturbative QCD
  - transport phenomena: kinetic theory, hard thermal loops
  - flow : hydrodynamics
  - jet quenching: medium–induced gluon radiation, AdS/CFT

Lecture II: Quark–Gluon Plasma



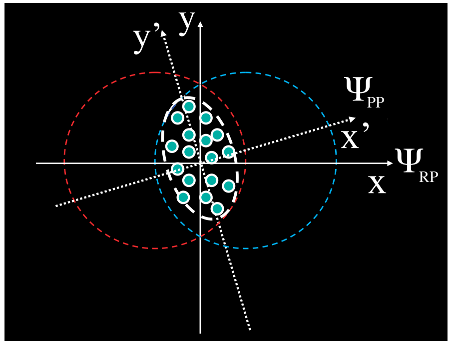
- $\tau \sim 1$ fm/c : thermalization
- $1 \lesssim \tau \lesssim 10$ fm/c : quark–gluon plasma
- $10 \lesssim \tau \lesssim 20$ fm/c : hot hadron gas
  - hadronisation: confinement
  - the hadron gas keeps expanding and cooling down

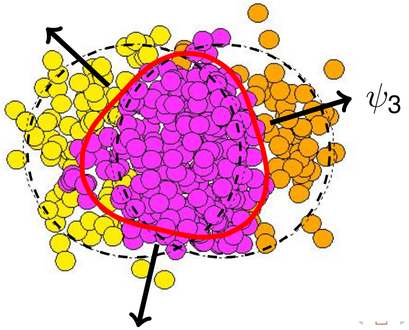
Lecture II: Quark–Gluon Plasma



- $\tau \sim 1$ fm/c : thermalization
- $1 \lesssim \tau \lesssim 10$ fm/c : quark–gluon plasma
- $10 \lesssim \tau \lesssim 20$ fm/c : hot hadron gas
- $\tau > 20$ fm/c : freeze out
  - the density becomes too small to have interactions
  - the produced hadrons exhibit a thermal spectrum

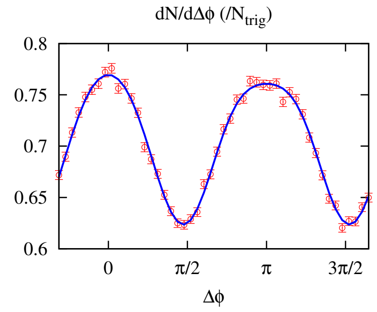
Flow and Thermalization

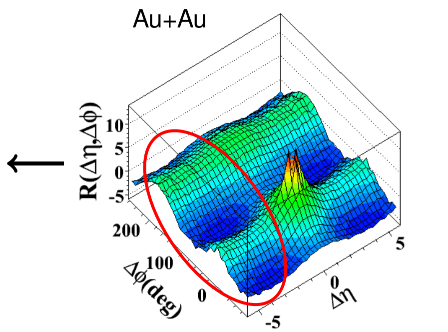




From ridge to flow

- What is the origin of the double peak structure ($\Delta\phi = 0$ and $\Delta\phi = \pi$) seen in di–hadron correlations in Au+Au ?





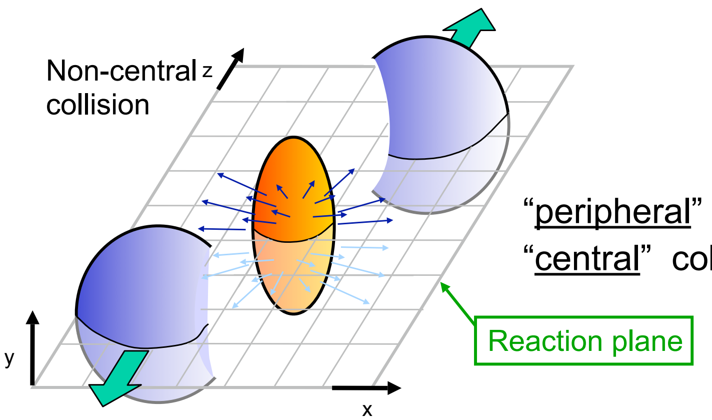
(STAR, arXiv:1010.0690)

(PHOBOS, Phys.Rev. C81 (2010) 024904)

$$\mathcal{R} \equiv \frac{\langle N_1\, N_2\rangle - \langle N_1\rangle\, \langle N_2\rangle}{\langle N_1\rangle\, \langle N_2\rangle} \propto\, v_2^2\, \cos\left(2\Delta\phi\right)$$

- This is elliptic flow !

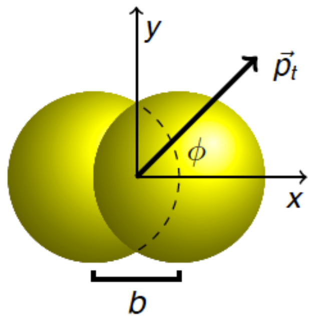
The geometry of a HIC



"peripheral" collision (b ~ b_{max})
"central" collision (b ~ 0)

Number of participants (N_{part}): number of incoming nucleons (participants) in the overlap region

Elliptic flow v_2



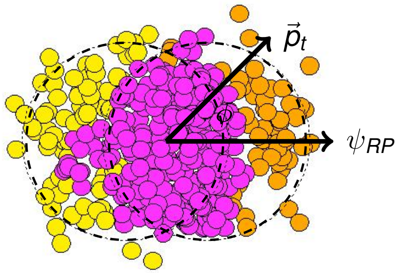
$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto\, 1\, +\, 2v_2 \cos 2\phi$$

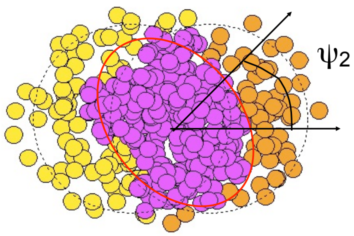
v_2 : the 'coefficient of the elliptic flow'

- Non–central AA collision: impact parameter $b_\perp > 0$
- The interaction region is (roughly) elliptic
- Pressure gradient is larger along the smaller axis (x)
- Fluid velocity is proportional to the pressure gradient
- Particle emerge predominantly parallel to the fluid velocity

$\Longrightarrow$ the particle distribution is not axially symmetric !

The role of fluctuations (1)



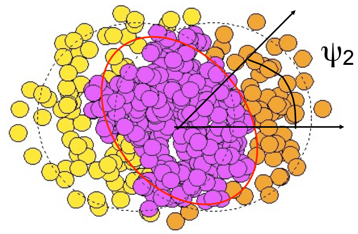


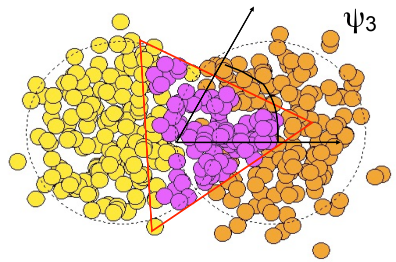
- Nucleons are randomly distributed inside a nucleus.
- The participants (nucleons which undergo at least one collision) do not make exactly an ellipse ...
- ... and the minor axis of that (approximate) ellipse needs not be exactly along the x axis !

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2v_2 \cos 2(\phi - \Psi_2)$$

- The event plane is not the same as the reaction plane !

The role of fluctuations (2)



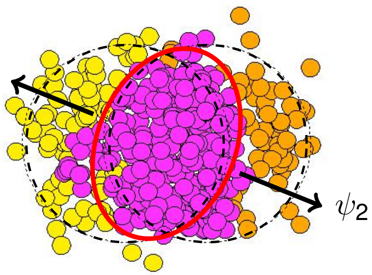


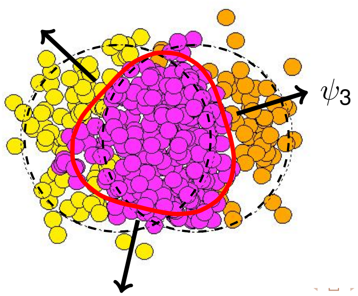
- In some events, the shape of the interaction can be quite different from an ellipse !
- Then one speaks about triangular flow ...

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2v_2 \cos 2(\phi - \Psi_2) + 2v_3 \cos 3(\phi - \Psi_3) + \ldots$$

- ... or even higher harmonics

The role of fluctuations (3)





- And of course all these harmonics can coexistent (in different proportions) within a same event !

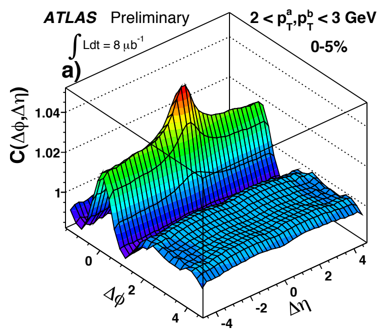
$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \Psi_n)$$

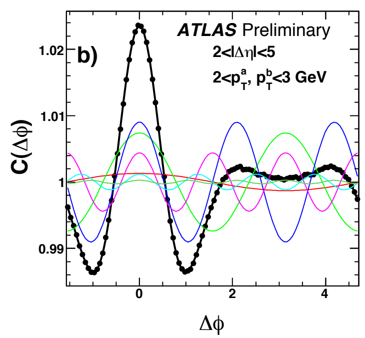
- This amounts to a Fourier decomposition of the azimuthal distribution of the participants !
- The most amazing: all these v_n's can actually be measured

v_n from 2–particle correlations

$$\left\langle \frac{\mathrm{d}N_{pairs}}{\mathrm{d}\Delta\phi} \right\rangle \propto 1 + 2\sum_{n=1}^{\infty} \langle v_n^2 \rangle \cos n(\phi - \Psi_n)$$

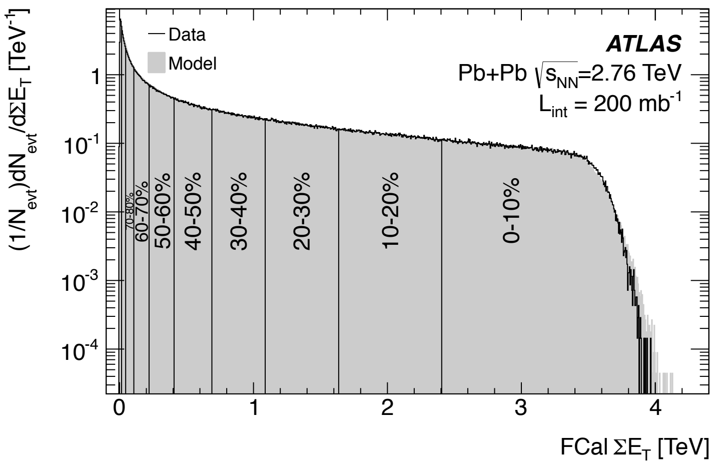
- The reference phases Ψ_n drop out in the convolution !





- Integrate the data within slices of $\Delta\eta$, perform a Fourier transform per slice, then present v_n as functions of $\Delta\eta$, $p_\perp$ and in bins of centrality

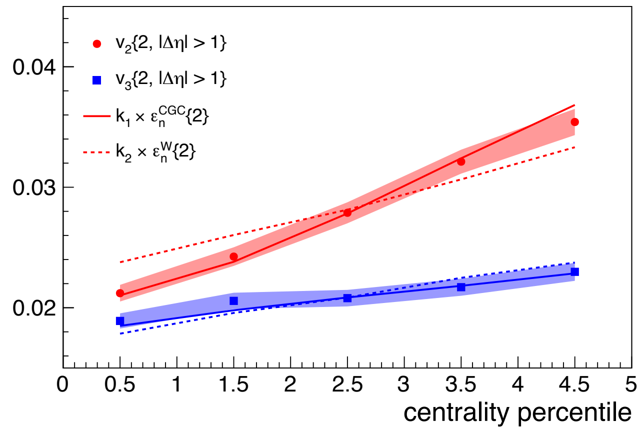
Centrality bins in a HIC



- The more central an event is, the higher the (transverse) energy deposited in the forward calorimeter
- The 10% events with the highest energy deposit ≡ 'the 10% most central events

Centrality dependence for v_2

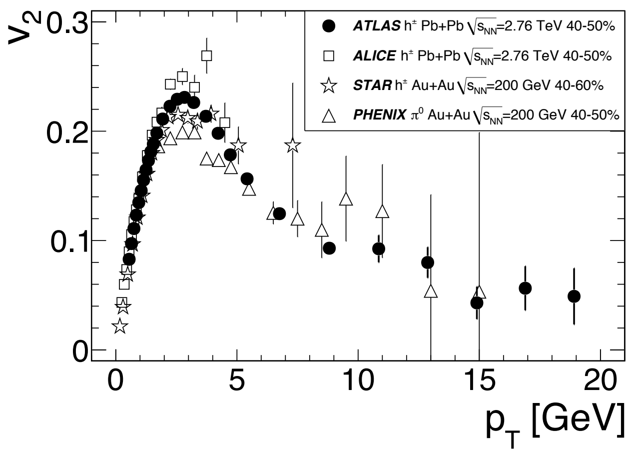
ALICE, arXiv:1105.3865



- The larger the centrality, the smaller v_2 !

for central collisions, the interaction region has spherical symmetry
$\Longrightarrow$ no flow !

Momentum dependence for v_2

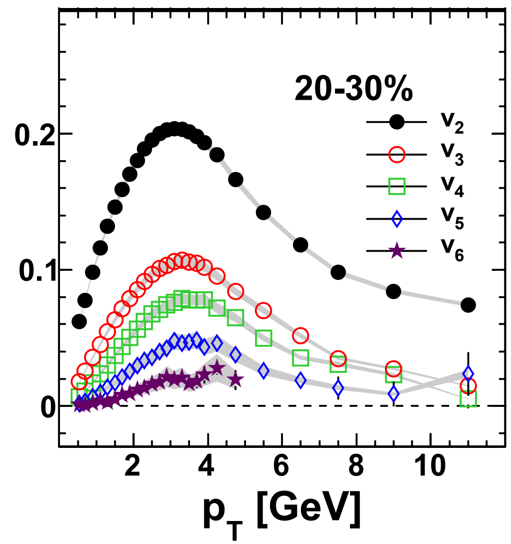


- v_2 first rises up to $3 \div 4$ GeV, then decreases again.

  ▷ relatively hard/fast particles cannot be driven by the flow *(imagine a bullet flowing with the wind)*
- No significant increase in v_2 from RHIC to LHC

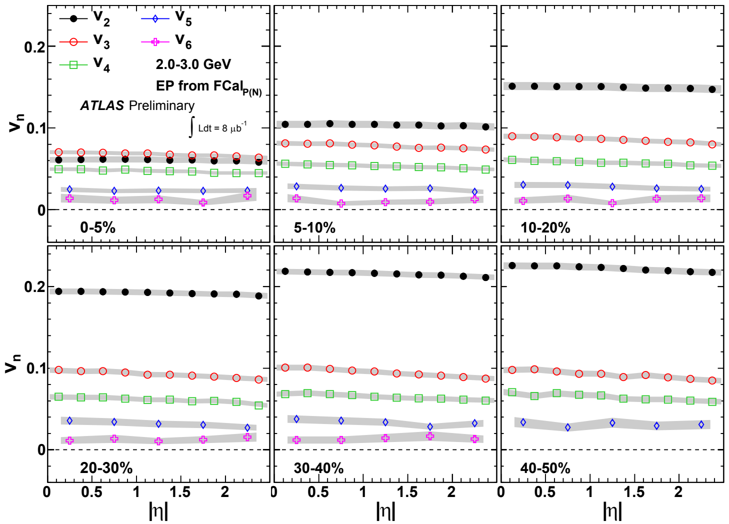
$p_\perp$ dependence for v_n, $n = 2 - 6$

(Talk by J. Jia for the ATLAS Collaboration at Quark Matter 2011)



- Similar $p_\perp$ dependence for all n: rise up to 3-4 GeV, then fall

Pseudorapidity dependence for v_n



- Weak η dependence for all v_n's !
- Distributions which are boost–invariant (independent of η) at early times flow in the same way and give rise to 'ridge' and 'hump'

From flow to Hydro

- What can we learn out of the flow data (concerning QCD) ?
- We first learn that this matter is a fluid (it flows !)
  - 'this matter': hadrons until freeze–out
  - partonic matter in the intermediate stages
- Non–trivial ! It implies relatively strong interactions !
  - dust (no interactions) does not flow !
  - a liquid flows better than a gas (weak interactions)
- If it flows, one can use hydrodynamics
  - the effective theory for flow (see below)
- Hydro involves initial conditions and transport coefficients, which teach us about the state of the system
- Success of hydro strongly suggests (but not necessarily implies) local thermal equilibrium

Hydrodynamics in a nut shell

- Standard thermodynamics: a system in global thermal equilibrium
  - pressure (P), temperature (T), chemical potential (μ) are independent of time ...
  - and uniform throughout the volume V of the system
- Hydrodynamics is about (quasi) local thermal equilibrium
  - P, T and μ can vary with space and time ...
  - ... but they vary so slowly that one can still assume thermal equilibrium to hold locally, in the neighborhood of any point
  - the velocity $\mathbf{v}$ can be different for different fluid elements
- Hydrodynamics : effective theory of small gradients
- It holds when the mean free path of the particles in the system is much smaller than any system size.
  - 'mean free path' : distance between two successive collisions

Hydro equations = the conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J^\mu_B = 0$$

- $T^{\mu\nu}$ (energy–momentum tensor) and J^μ_B (baryonic current) :
  - fluid velocity: $u^\mu = \gamma(1, \mathbf{v}),\ \gamma = 1/\sqrt{1 - v^2}$
  - energy density $\varepsilon = E/V$ & pressure P
  - additional parameters ('viscosities') for a non–ideal fluid
- 'Ideal fluid' $\equiv$ local thermal equilibrium

$$T_{(0)} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix} \qquad \text{in the local rest frame :} \quad u^\mu = (1, 0)$$

- After a boost to the laboratory frame, this becomes:

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

Viscous hydrodynamics

- Ideal hydro assumes that there is no dissipation (no friction)
- You may think this means the coupling is weak...but you'd be wrong !
  - ▷ it actually means that the coupling is infinite ! (see below)
- Real fluids have no infinite coupling, so they have dissipation.
- This is described by transport coefficients known as viscosities

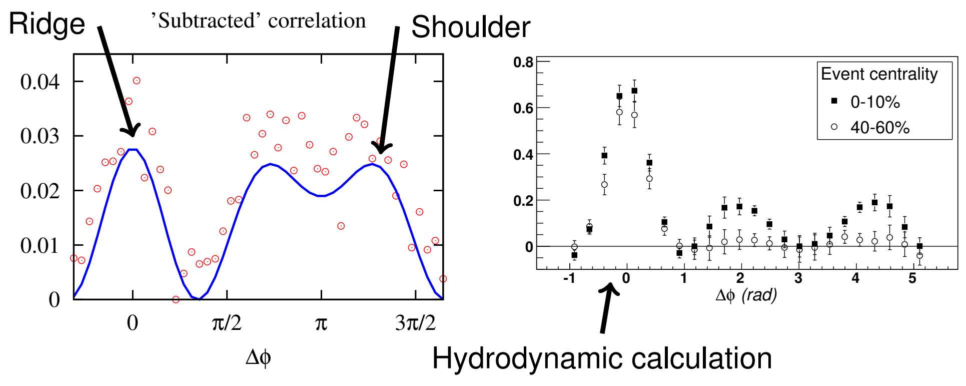
$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu} \oplus (\eta,\, \zeta) \otimes \partial u \oplus$$

N.B. Viscous effects enter $T^{\mu\nu}$ as gradient corrections

- For the hydro problem to be well defined, one needs to specify:
  - the equation of state which relates ε to P
  - the initial conditions (at $\tau = \tau_0$) for ε and $\mathbf{v}$
  - the viscosities η, ζ

Hydro calculations ...

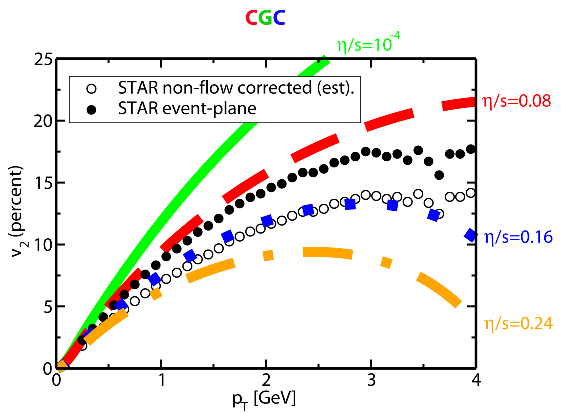
- ... do a good job in qualitatively explaining the 'ridge'...



(STAR, arXiv:1010.0690) (Takahashi, Tavares, Andrade, Grassi, Hama, Kodama, Xu, Phys.Rev.Lett.103, 242301 (2009))

Hydro simulations for v_2

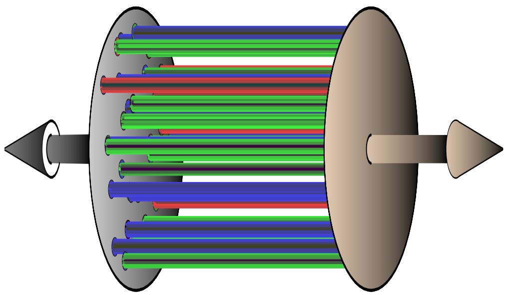
- ... and quantitatively reproducing the elliptic flow !
  (Luzum and Romatschke, 08)

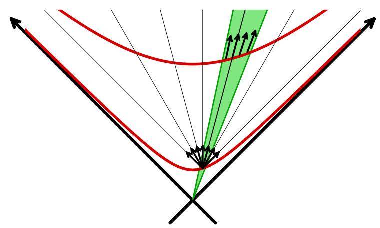


- However, a good hydro description of the data requires :
  - a very short equilibration (isotropisation ?) time $\tau_0 \lesssim 1$ fm/c
  - a very small viscosity/entropy ratio $\eta/s < 0.2$
- Both properties are puzzling ... at least at weak coupling !

The thermalization puzzle

- The energy–momentum distribution right after the collision is maximally anisotropic : longitudinal expansion, glasma flux tubes



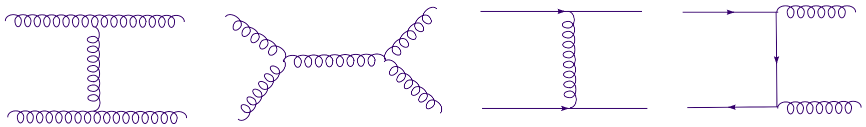


$$T_{\rm eq} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix} \qquad T_{\rm initial} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & -\varepsilon \end{pmatrix}$$

- How can the system become isotropic over such a short time $\tau_0 \lesssim 1$ fm/c ??

Thermalization at weak coupling

- To evolve towards isotropy and thermal equilibrium, particles must exchange energy and momentum with each other.
- They can do that through collisions.
- Weak coupling: the dominant mechanism is $2 \to 2$ elastic scattering



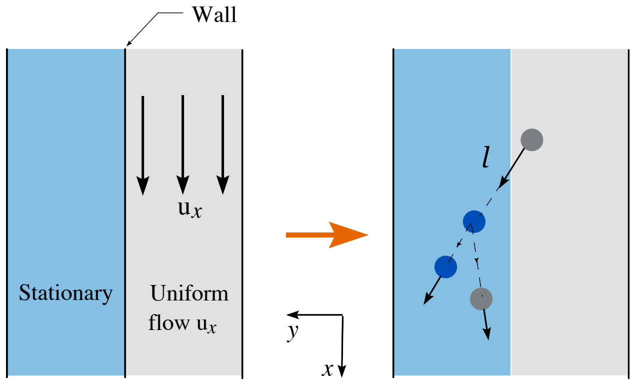
- Cross–section (σ) scales like $|\text{amplitude}|^2$, hence like $g^4 \sim \alpha_s^2$
- Mean free path (ℓ) = average distance between successive collisions

$$\ell \sim \frac{1}{\text{density} \times \sigma} \sim \frac{1}{\alpha_s^2}$$

- Typical equilibration time: $\tau_{\text{eq}} \sim \ell/v \sim 1/\alpha_s^2$
- Weakly coupled systems have large equilibration times !

Shear viscosity

- Weakly coupled systems also have large viscosity/entropy ratio !
- η : a measure of a fluid ability to transfer p_x in the y direction



$$\frac{1}{A}\frac{\mathrm{d}p_x}{\mathrm{d}t} = -\eta\,\frac{\mathrm{d}u_x}{\mathrm{d}y}$$

- Proportional to the mean free path $\ell \propto 1/\sigma \sim 1/g^4$
  $\Longrightarrow$ larger at weak coupling ! (Maxwell, 1860)

Viscosity over entropy density ratio

- $\eta \sim \ell \times \varepsilon$ (ℓ : mean free path; ε : energy density). Thus,

$$\frac{\eta}{s} \sim \ell\,\frac{\varepsilon}{s} \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

(since $\varepsilon/s \sim$ energy per particle $\sim 1/\lambda_B$)

- Heisenberg's uncertainty principle forbids ℓ/λ_B to be smaller than one
  (actually smaller than $\hbar$, but we work in 'natural units' : $\hbar = 1$)
- Hence, $\frac{\eta}{s} \gtrsim \mathcal{O}(1)$
- Weakly interacting systems have $\eta/s \gg 1$
- The matter produced in HIC has $\eta/s \sim \mathcal{O}(1)$
  $\Longrightarrow$ 'strongly–coupled quark–gluon plasma' (sQGP), or 'perfect liquid'

RHIC serves us the perfect liquid !

RHIC Scientists Serve Up "Perfect" Liquid

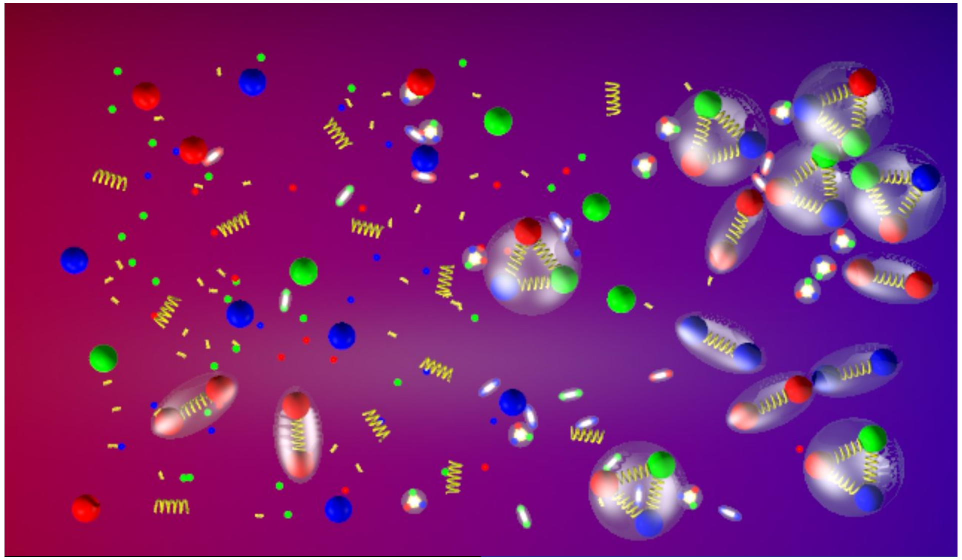
New state of matter more remarkable than predicted -- raising many new questions

Monday, April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

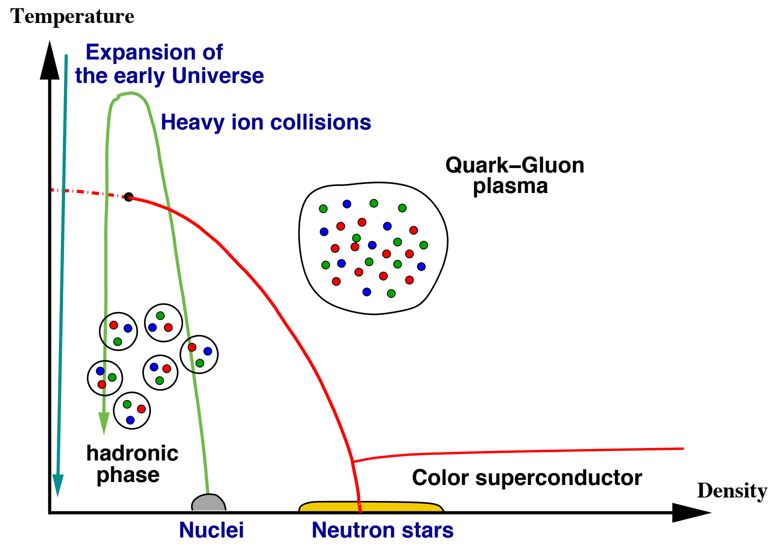
- Still under debate ... more to come !

Quark–Gluon Plasma



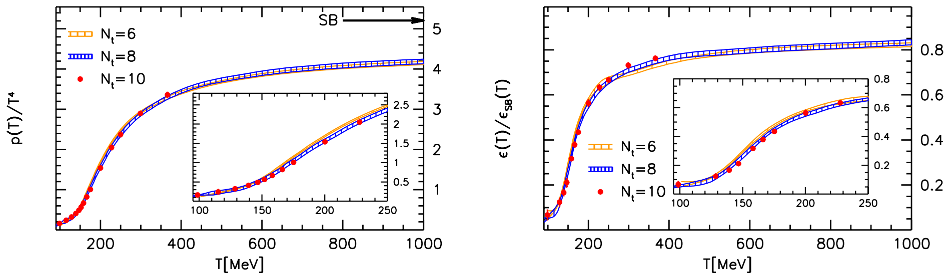
Phase–diagram for QCD

- ... as explored by the expansion of the Early Universe ...



- ... and in the ultrarelativistic heavy ion collisions.

QCD thermodynamics: lattice

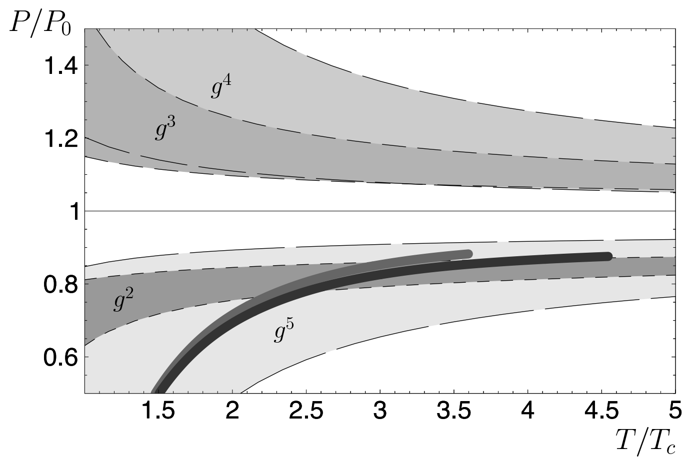


- With increasing temperature, the coupling $g(T)$ decreases, so the exact result approaches towards the Stefan–Boltzmann limit

$$P_{SB} = \frac{\pi^2}{90}\left\{2(N_c^2-1) + \frac{21}{6}N_c N_f\right\}$$

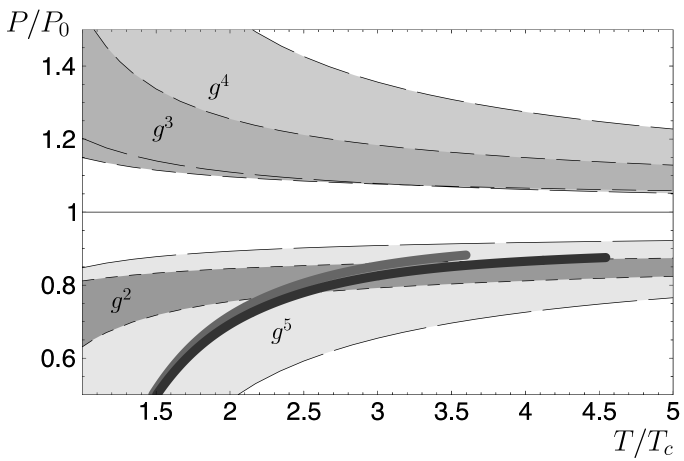
- Can one understand this approach in perturbation theory ?
- For $T \gtrsim 2.5T_c$, $\varepsilon(T) - \varepsilon_{SB}(T)$ is about 20%
- The first perturbative correction to $\varepsilon_{SB}(T)$, of $\mathcal{O}(g^2)$, is numerically about 20% as well !

QCD thermodynamics: perturbation theory



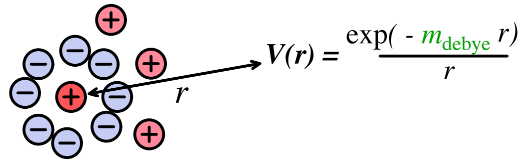
- By itself, the $\mathcal{O}(g^2)$ seems to do a pretty good job. However...
- Successive perturbative approximations — $\mathcal{O}(g^2)$, $\mathcal{O}(g^3)$, $\mathcal{O}(g^4)$, $\mathcal{O}(g^5)$ — jump up and down, without any sign of convergence.

QCD thermodynamics: perturbation theory

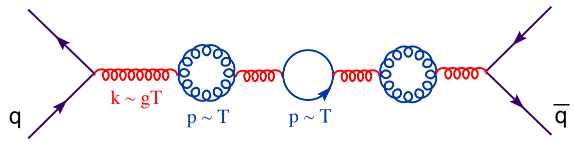


- This problem appears for any field theory, including weakly coupled QED, or scalar ϕ^4 theory !
- In QCD, $\mathcal{O}(g^6)$ and higher cannot be computed in perturbation theory anymore (infinitely many diagrams)

Recall : Debye screening



- Thermal effect associated with dressing the propagator: $m_{\text{Debye}} \sim gT$



- The electric gluon acquires a mass which is 'non–perturbative' at 'soft' momenta $k \sim gT$:

$$G_{00}(k) = \underbrace{\frac{1}{k^2 + m_D^2}}_{\text{fine !}} = \underbrace{\frac{1}{k^2}\left[1 - \frac{m_D^2}{k^2} + \left(\frac{m_D^2}{k^2}\right)^2 \cdots\right]}_{\text{not fine !}}$$

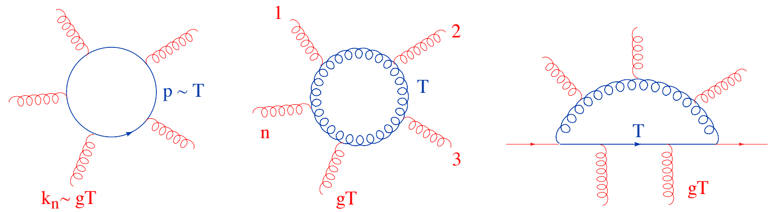
Hard Thermal Loops

- In a field theory at finite T, strict perturbation theory makes no sense
- The plasma develops collective phenomena ...

  Debye screening, Landau damping, waves ('plasmons')...

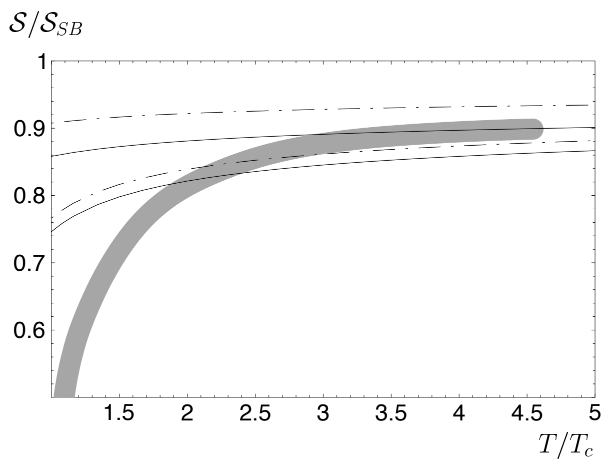
  ... which in general can be computed in perturbation theory, but whose effects are non–perturbative

  $\Longrightarrow$ they need to be resummed to all orders



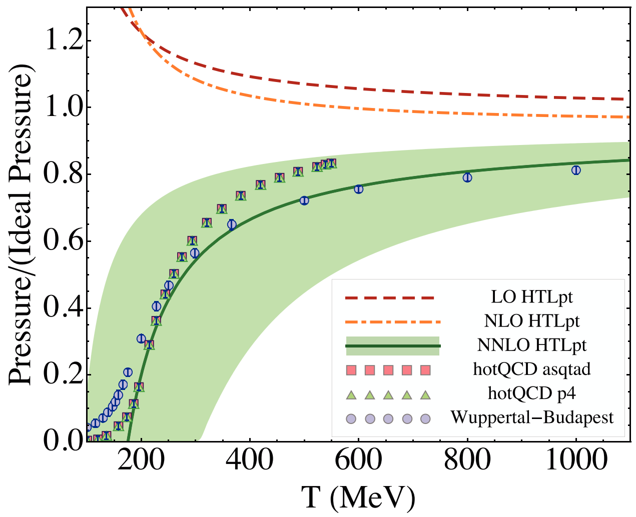
- Hard Thermal Loops : one loop diagrams with internal momenta $p \sim \mathcal{O}(T)$ ('hard') and external momenta $k_i \sim \mathcal{O}(gT)$ ('soft')
- This requires reorganizations of the perturbative expansion

HTL–resummed entropy



- 'Two–particle–irreducible' resummation of the HTL self–energies *(J.-P. Blaizot, A. Rebhan, E. I., 2000)*
- Good agreement with the lattice data (Bielefeld) for $T \gtrsim 2.5T_c$

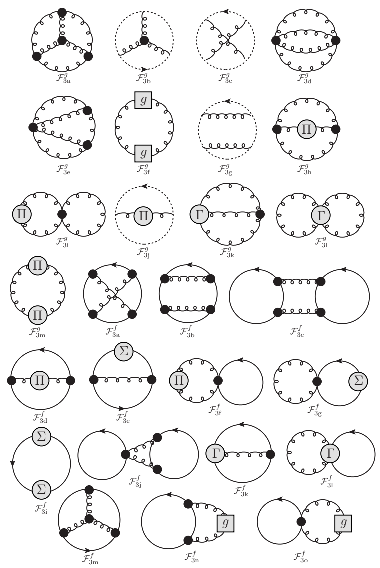
HTL–resummed pressure



- 'Screened perturbation theory' up to 3 loop order.
  (Andersen, Leganger, Strickland, Nan Su, 2011)
- Good convergence & good agreement with lattice data at 3–loop level

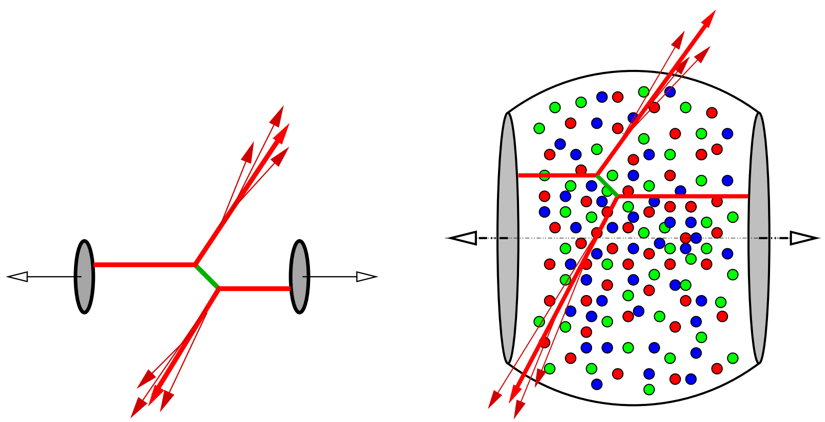
HTL–resummed pressure at 3 loop order

- Not an easy job though ! ☺



Jet quenching

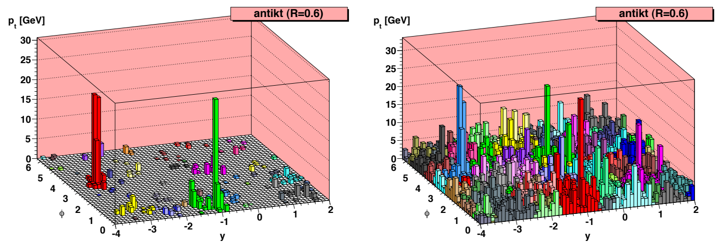
- How to probe the properties of the QGP in HIC ?



- Study the effects of the medium on the propagation of a 'hard probe', so like a jet

'Jets' vs. 'leading particles'

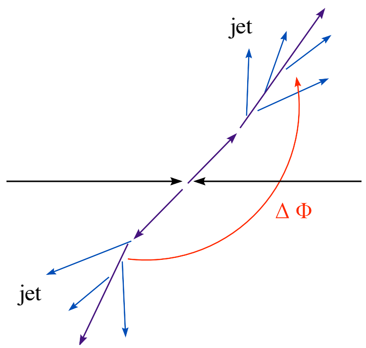
- A 'jet': the ensemble made by the 'leading particle' (a virtual parton which initiated the jet) and the products of its 'fragmentation'
- The definition of a 'jet' is also a matter of conventions ...
  - it depends upon the maximal rapidity (ΔY) and azimuthal ($\Delta\phi$) separation between the particles that we associate with a given 'jet'
  - ... and also upon the jet reconstruction algorithm
- Jet reconstruction is particularly delicate in the context of HIC...

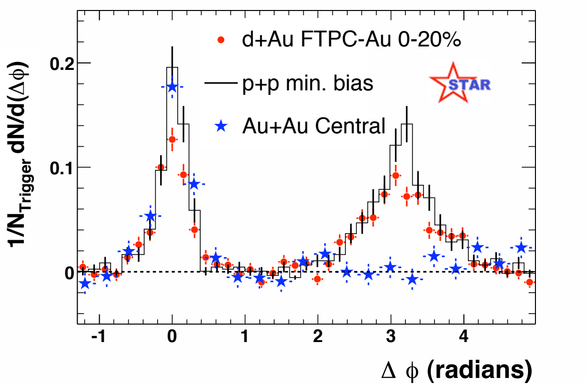


- ... and of course it requires a good, specialized, detector !

Jet quenching at RHIC

- Studies of jet quenching at RHIC have focused on 'leading particles'



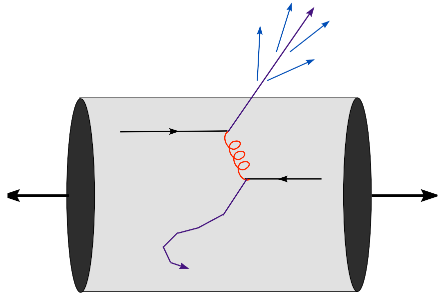


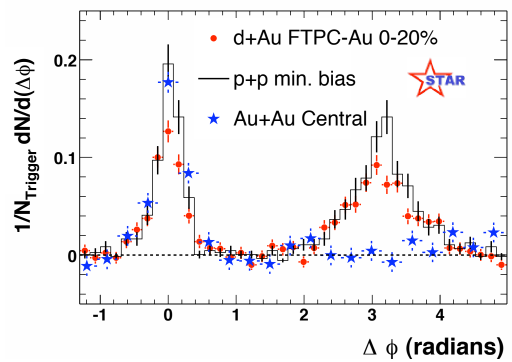
[Nucl.Phys.A783:249-260,2007]

- Azimuthal correlations between the produced jets:

  p+p or d+Au : a peak at $\Delta\Phi = 180°$

Jet 'quenching' in nucleus–nucleus collisions





- The "away–side" jet has disappeared !

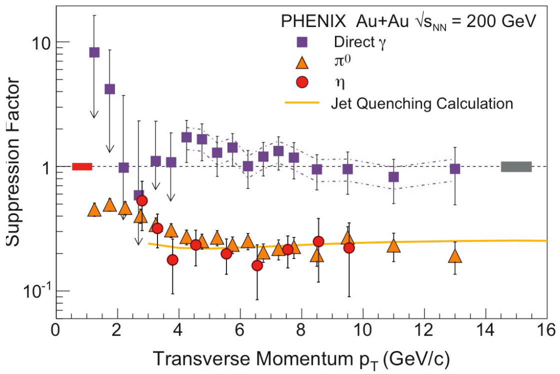
  absorbtion (or energy loss, or "jet quenching") in the medium

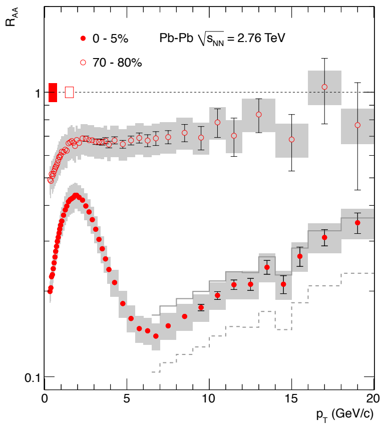
- The matter produced in a heavy ion collision is opaque

  high density, or strong interactions, ... or both

Nuclear modification factor at RHIC & the LHC

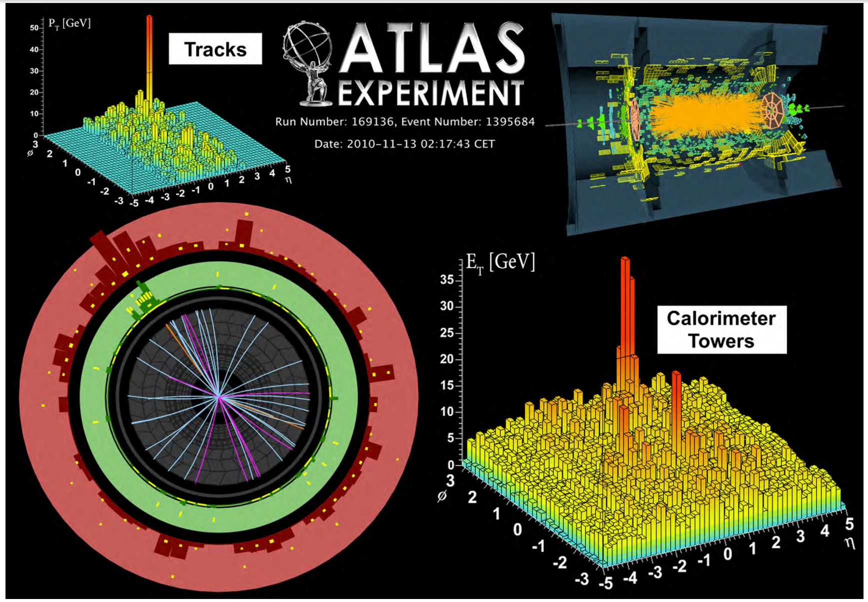
$$R_{\rm A+A} \equiv \frac{1}{A^2}\,\frac{dN_{\rm A+A}/d^2p_\perp d\eta}{dN_{\rm p+p}/d^2p_\perp d\eta}$$





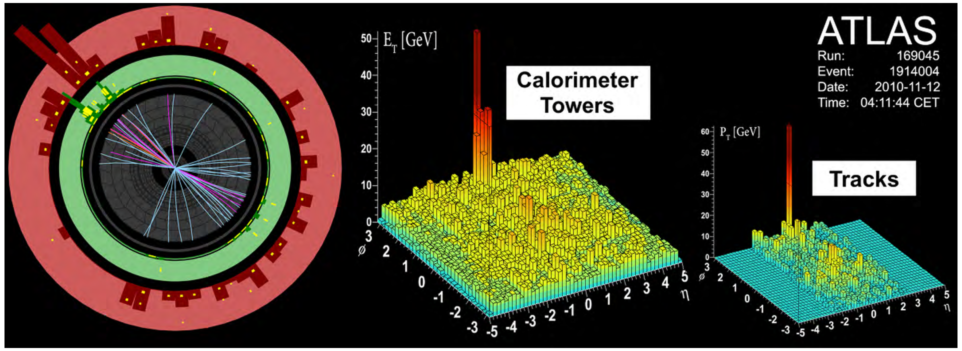
- Strong suppression $(R_{AA} \lesssim 0.2)$ in central collisions
- Large energy loss in the medium

Jets in HIC at the LHC



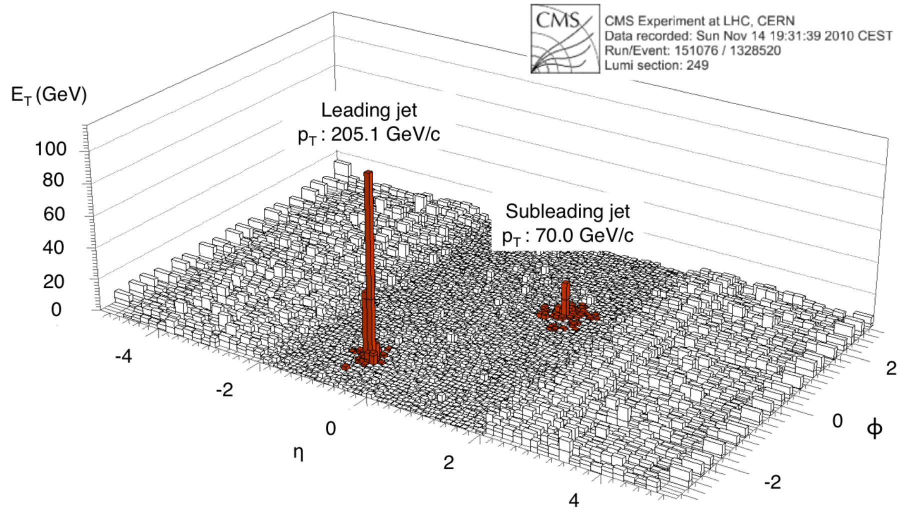
- Pb+Pb collision at $\sqrt{s_{NN}} = 2.76$ TeV

Di–jet asymmetry *(ATLAS)*



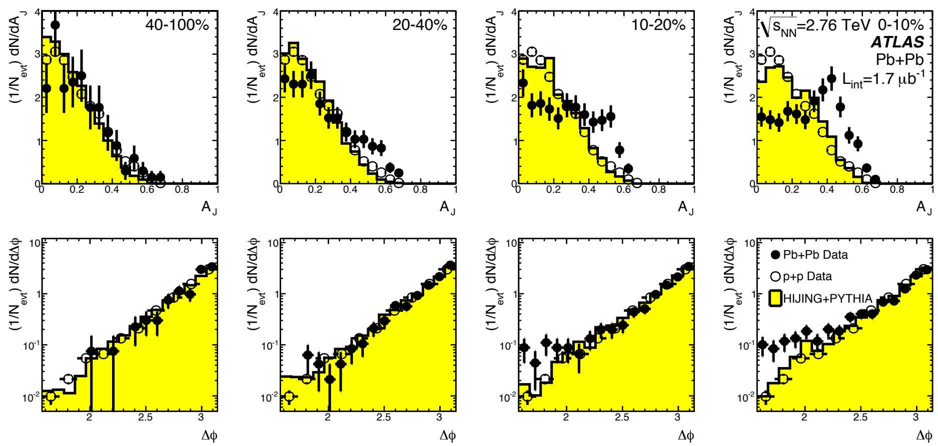
- Central Pb+Pb: mono–jet events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

Di–jet asymmetry *(CMS)*



- Central Pb+Pb: the secondary jet is barely visible
- The jet energy has been redistributed in the transverse plane

Di–jet asymmetry *(ATLAS)*

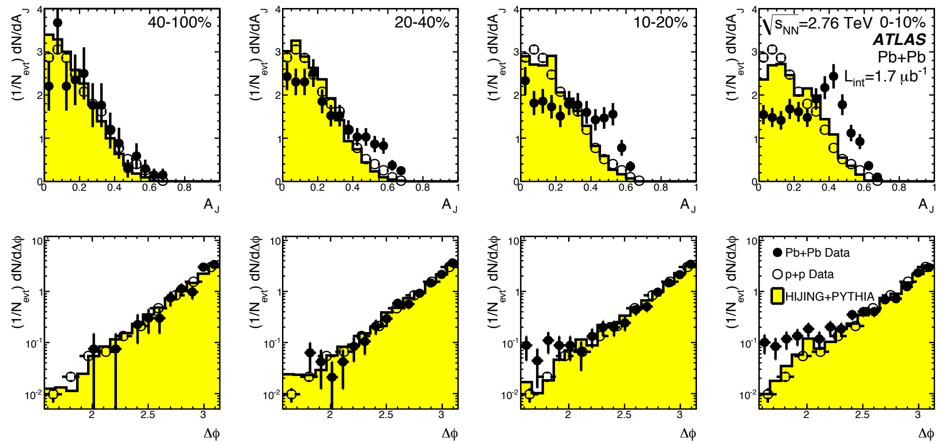


- Event fraction as a function of the di–jet energy imbalance

$$A_J = \frac{E_{T1} - E_{T2}}{E_{T1} + E_{T1}}$$

- ...and of the azimuthal angle $\Delta\phi$, for different centralities.

Di–jet asymmetry *(ATLAS)*

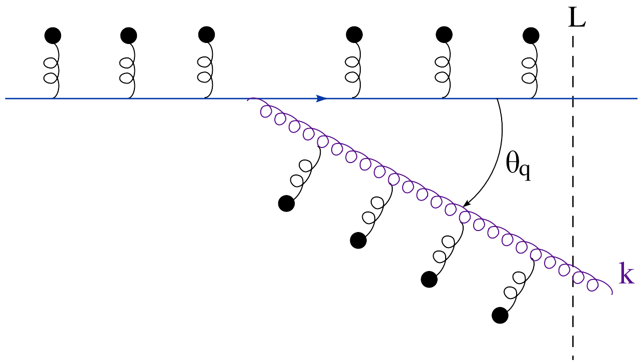


- Additional energy loss of 20 to 30 GeV due to the medium
- Typical event topology: still a pair of back–to–back jets
- The secondary jet loses energy without being deflected
- Medium–induced emissions of soft gluons at large angles

Medium–induced gluon radiation *(BDMPS-Z)*

- Additional radiation triggered by interactions in the medium

  (Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov ∼ 1995)



- A complicated problem: medium effects must be included to all orders
- Results (at least) qualitatively consistent with the LHC data !
- 2 fundamental concepts: formation time & momentum broadening

The formation time

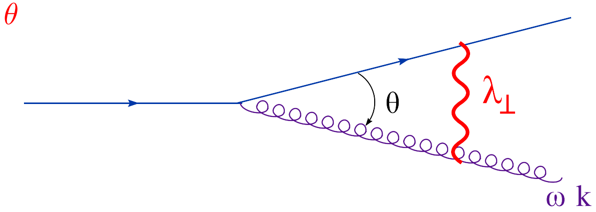
- By the uncertainty principle, it takes some time to emit a gluon !

  ▷ the gluon must lose quantum coherence with respect to its source
- Gluon with energy ω and transverse momentum $k_\perp$:

  ▷ the quark–gluon transverse separation $b_\perp$ at the emission time τ_f must be larger than the gluon transverse wavelength $\lambda_\perp$

$b_\perp \simeq \theta\,\tau_f \gtrsim \lambda_\perp \simeq 1/k_\perp$

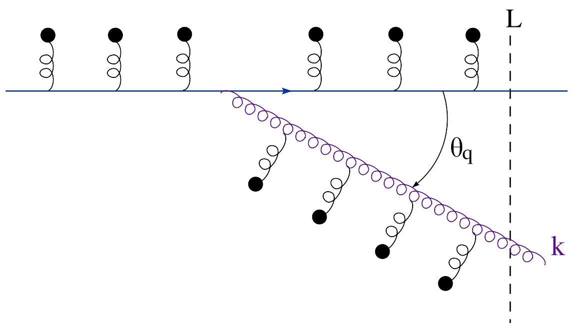
$k_\perp \simeq \omega\,\theta$



$$\tau_f \simeq \frac{\omega}{k_\perp^2} \simeq \frac{1}{\omega\theta^2}$$

Transverse momentum broadening

- The gluon receives random kicks from the plasma constituents
- Parton mean free path ℓ ($\ell \sim 1/g^2T$ for a QGP)
- Average (momentum)^2 transfer per scattering m_D^2 ($m_D \sim gT$)



$$\frac{\mathrm{d}\langle k_\perp^2\rangle}{\mathrm{d}t} \simeq \frac{m_D^2}{\ell} \equiv \hat{q} \qquad \text{‘jet quenching parameter’}$$

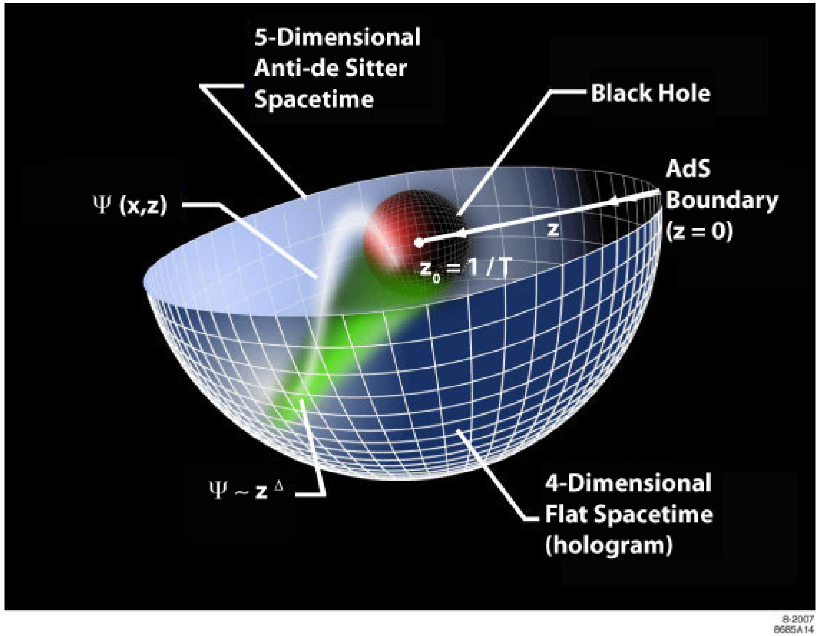
In–medium formation time

- The gluon acquires a (momentum)^2 $\sim \hat{q}$ per unit time ...
- ... and hence a momentum $k_f^2 \simeq \hat{q}\,\tau_f$ during its formation
- The formation time τ_f is determined by the condition of quantum decoherence as $\tau_f \simeq \omega/k_f^2$

$$\tau_f \simeq \sqrt{\frac{\omega}{\hat{q}}}\,, \qquad \theta_f \equiv \frac{k_f}{\omega} \simeq \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- The smaller the energy ω, the shorter the formation time τ_f and the larger the formation angle θ_f !
- This has the right characteristics to explain the LHC data ! ✓

The AdS/CFT correspondence



The evidence for strong coupling

- Three main experimental signatures:
  - small viscosity–over–entropy (η/s) ratio ('perfect fluid')
  - early thermalization $\tau_{\rm eq} \lesssim 1$ fm/c
  - strong 'jet quenching' (energy loss, momentum broadening)
- A rather shaky paradigm ...
  - a large elliptic flow v_2 can also be explained by a larger initial eccentricity together with a larger value for η/s
  - instead of early thermalization, it is enough to assume early expansion, like free streaming
  - so far, perturbative calculations were too crude to be convincing ... but progress is along the way !
- ... but a fascinating one !

The AdS/CFT correspondance

- A 'duality' (equivalence) between two very different theories
  - a conformal field theory (CFT) at strong coupling;
  - a string theory in Anti-de-Sitter (AdS) space–time at weak coupling.

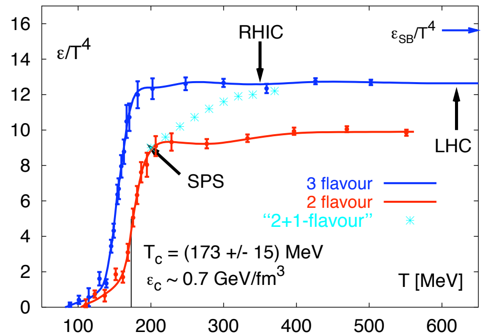
(Maldacena, 97; Gubser, Klebanov, Polyakov, 98; Witten, 98)

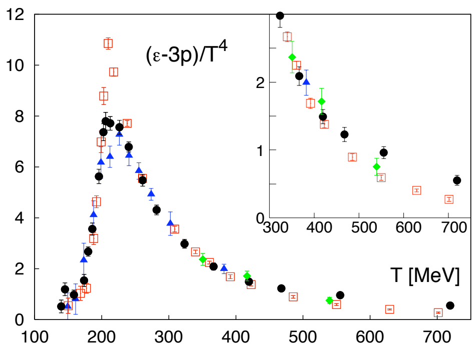
- The CFT : $\mathcal{N}=4$ Supersymmetric Yang–Mills
  - color gauge group SU(N_c)
  - conformal invariance $\Longrightarrow$ fixed coupling g
  - no confinement
  - strong 't Hooft coupling : $\lambda \equiv g^2 N_c \gg 1$ & $g^2 \ll 1$
- Is this a good model for QCD ??
- Perhaps better suited for studies of the quark–gluon plasma
  - deconfined, nearly conformal, relatively strong coupling

'Trace anomaly' in lattice QCD

- Remember: $T^{\mu\nu} = {\rm diag}\,(\varepsilon, P, P, P)$:

$\triangleright$ this would be traceless ($\varepsilon = 3P$) in a CFT





$$\text{QCD}: \qquad \langle T^\mu_\mu \rangle \equiv \varepsilon - 3P \propto \beta(g)$$

- $(\varepsilon - 3P)/\varepsilon_0 \lesssim 10\%$ for any $T \gtrsim 2T_c \simeq 400$ MeV
- $g \approx 1.5 \div 2 \Longrightarrow \lambda \equiv g^2 N_c \simeq 6 \div 10$

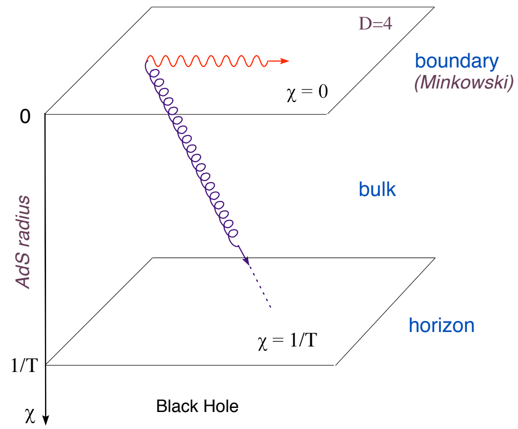
The AdS/CFT correspondance (2)

- The String Theory : type IIB in the $\mathrm{AdS}_5 \times S^5$ space–time
- Finite–T plasma in the CFT $\leftrightarrow$ adding a Black Hole in AdS_5
  ▷ a Black Hole has entropy and thermal (Hawking) radiation
- The strong 't Hooft coupling regime of the gauge theory:
  - $\lambda \equiv g^2 N_c \gg 1 \;\; \& \;\; g^2 \ll 1$ (large N_c)
- ... corresponds to the 'supergravity' regime of the string theory:
  - weak coupling & weak curvature
  - classical equations of motion in a curved space–time
- Well defined rules for computing quantum correlations in the CFT at strong coupling via semi–classical calculations in the string theory

AdS_5 Black Hole space–time

- AdS_5 : our Minkowski world $\times$ a 'radial' dimension χ

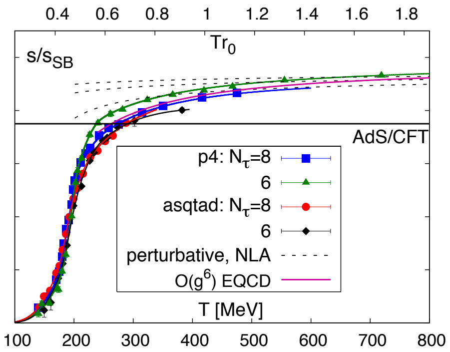
- 'radial', or '5th', coordinate : $0 \le \chi < \infty$
- the gauge theory lives at the Minkowski boundary $\chi = 0$
- finite temperature T: black hole horizon at $\chi = 1/T$

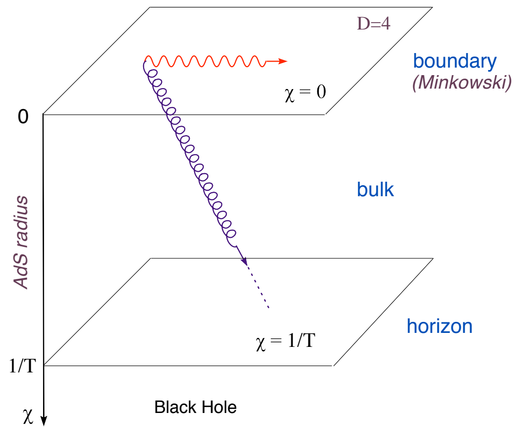


$$S_{\mathrm{BH}} = \frac{\text{Horizon area}}{4G_{10}} \Longrightarrow s \equiv \frac{S_{\mathrm{BH}}}{V_{3D}} = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0$$

AdS_5 Black Hole space–time

- AdS_5 : our Minkowski world $\times$ a 'radial' dimension χ





$$S_{\mathrm{BH}} = \frac{\text{Horizon area}}{4G_{10}} \Longrightarrow s \equiv \frac{S_{\mathrm{BH}}}{V_{3D}} = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0$$

Viscosity over entropy density ratio

(Policastro, Son, Starinets, 2001)

- Viscosity = the response of a fluid under shear forces ...
- ... hence, to a gravitational wave :

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int \mathrm{d}t \mathrm{d}^3\boldsymbol{x}\, \mathrm{e}^{-i\omega t} \langle [T_{xy}(t,\boldsymbol{x}), T_{xy}(0,\boldsymbol{0})] \rangle_T$$

= the absorbtion cross section for a low–energy graviton

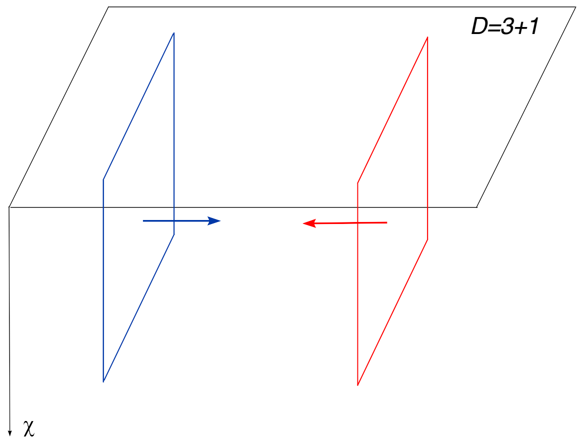
- Absorption cross section = area of horizon (known in GR)
- Entropy is also proportional to the area of the horizon

$$\frac{\eta}{s} \to \frac{\hbar}{4\pi} \quad \text{as} \quad \lambda \to \infty$$

- Universality follows from properties of black hole horizons

Heavy Ion Collisions

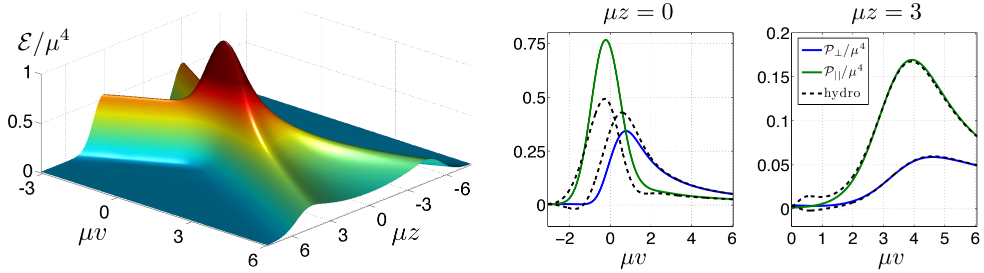
- Ultrarelativistic Heavy Ion Collision in 4D $\longleftrightarrow$
  The scattering between two gravitational shock–waves in AdS_5



- Thermalization $\longleftrightarrow$ Formation of a BH horizon

Thermalization from shock–wave scattering

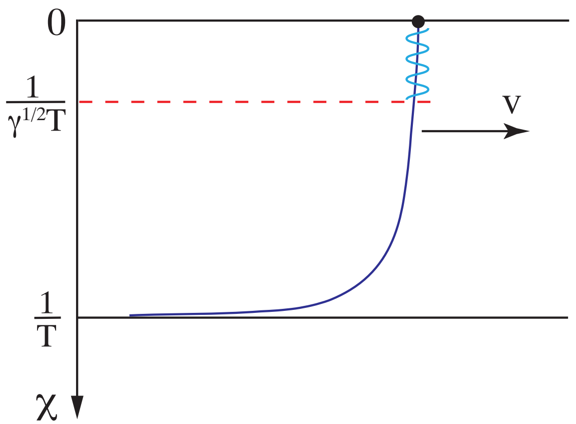
(Chesler and Yaffe, 2010)



- The remnants of the two shock waves move away from each other, but with velocities $v < 1$.
- The pressure shows isotropisation.

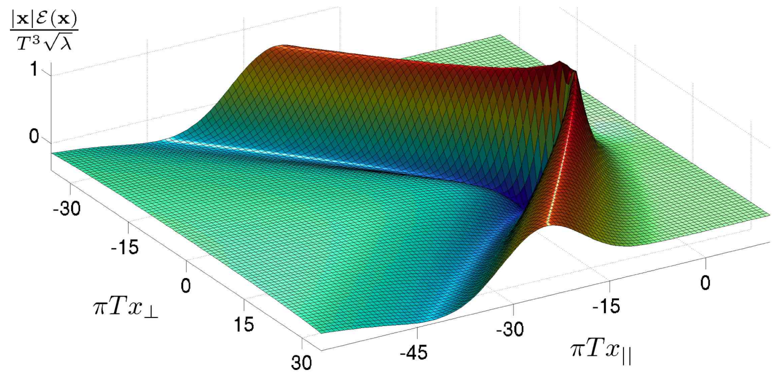
Heavy Quark in a strongly–coupled plasma

- Heavy quark in 4D $\longleftrightarrow$ 'Trailing string' in AdS_5 BH
- Energy loss $\mathrm{d}E/\mathrm{d}t$ $\longleftrightarrow$ Energy flux down the string



Herzog, Karch, Kovtun, Kozcaz, and Yaffe; Gubser (2006)
Casalderrey–Solana, Teaney (2006); Giecold, E.I., Al Mueller (2009)

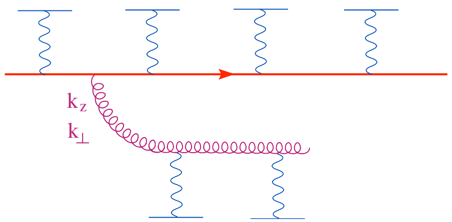
Energy loss by the heavy quark

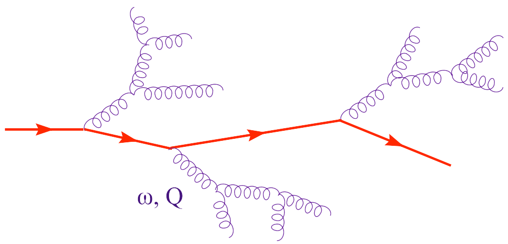


- If the quark velocity is larger than the speed of sound ($c_s = 1/3$)
  $\Longrightarrow$ Mach cone *(Chesler and Yaffe, 2007)*
- The experimental evidence at RHIC is still under debate

Medium–induced radiation at strong coupling

- Remember: Weak coupling: thermal rescattering





- Strong coupling: medium induced parton branching
- There are no plasma constituents to scatter off !

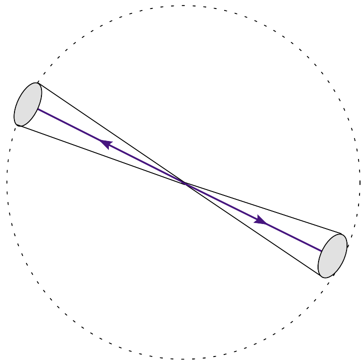
  ▷ at strong coupling, the plasma looks like a jelly, without pointlike constituents !
- All the partons branch down to very small values of x:

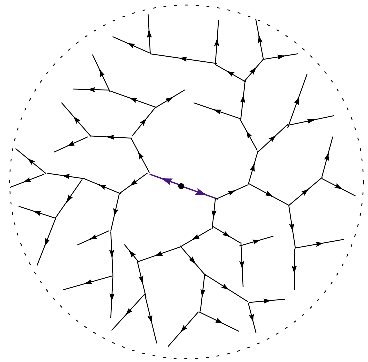
  no 'valence quarks' *(Hatta, E.I., Mueller, 2008)*

There are no jets at strong coupling !

- e^+e^- annihilation in COM frame: $q^\mu = (\omega, 0, 0, 0)$
- Typical final state at weak coupling :

  a pair of back to back jets with high momenta $k \simeq \omega/2$





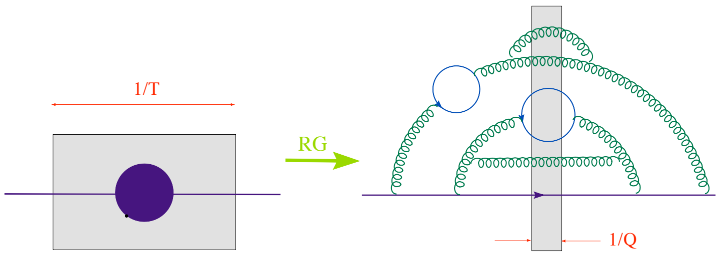
- Typical final state at strong coupling :

  an isotropic distribution of many soft particles ($k_i \sim \omega_i \sim \Lambda$)

  (Hatta, Mueller & E.I, 08; Hofman and Maldacena, 2008)

Instead of conclusions: Why gravity ?

- Why should gravity describe gauge theory at strong coupling ?
- OPE for DIS: Partons ⟷ 'twist–2' operators
- The operators depend upon the resolution scale



- $\lambda \to \infty$: rapid evolution ⇒ all operators are suppressed
- ... with one exception: the energy momentum tensor $T^{\mu\nu}$

  ⟹ the effective theory for scattering must be gravity !