Introduction to Accelerator Physics

A Short Introduction …

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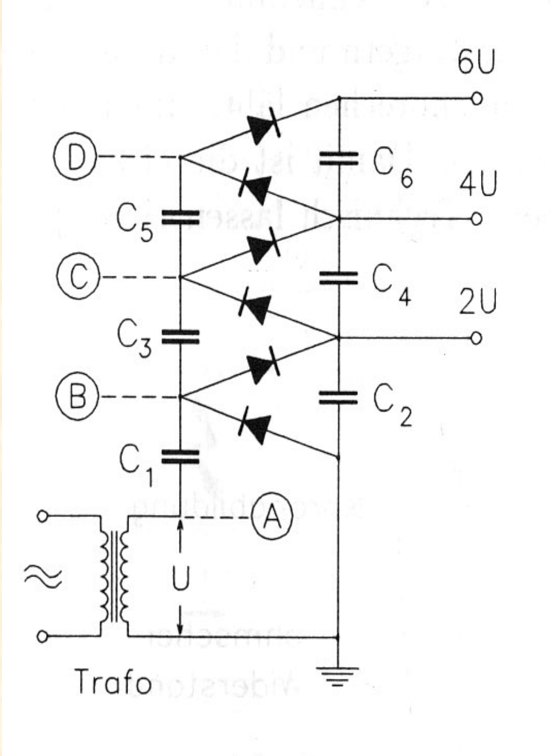
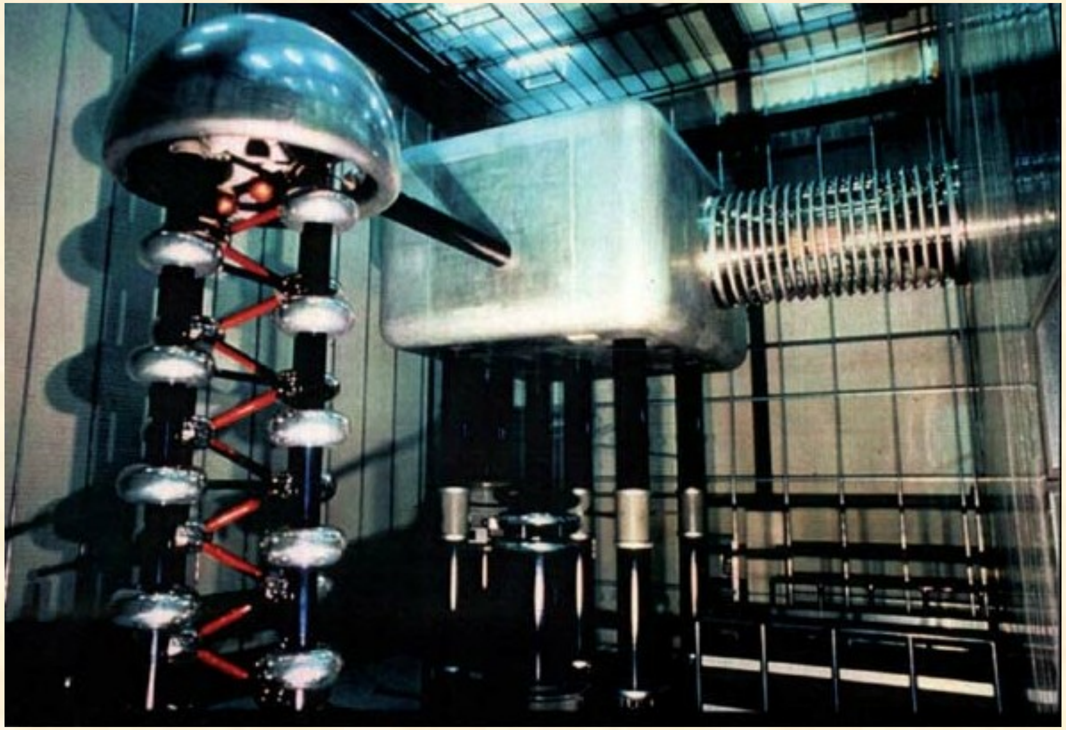
H

1.) Electrostatic Machines: The Cockcroft-Walton Generator



1928: ***Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam***

1932: ***First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV***



Particle source: ***Hydrogen discharge tube on 400 kV level***

Accelerator: ***evacuated glas tube***

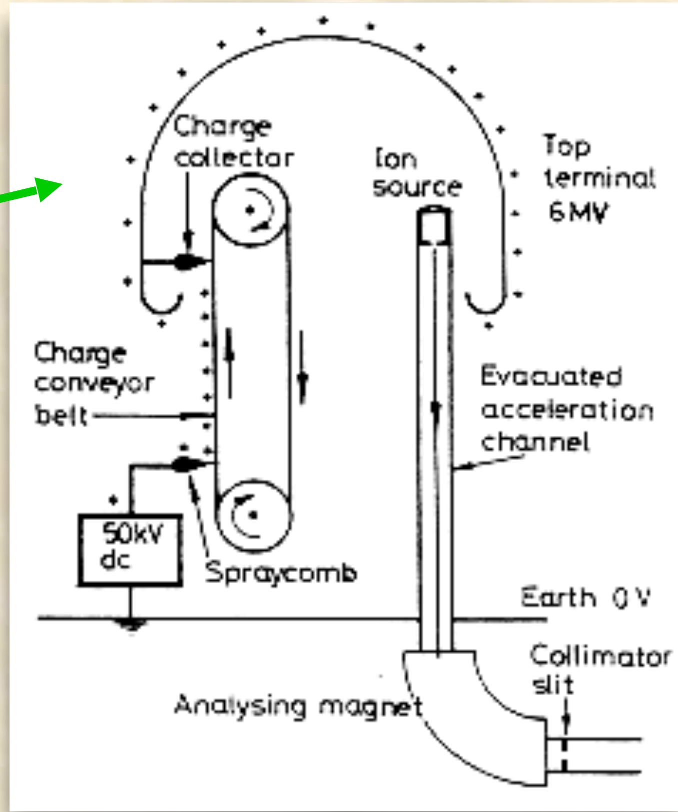
Target: ***Li-Foil on earth potential***

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:
DC Voltage can only be used once

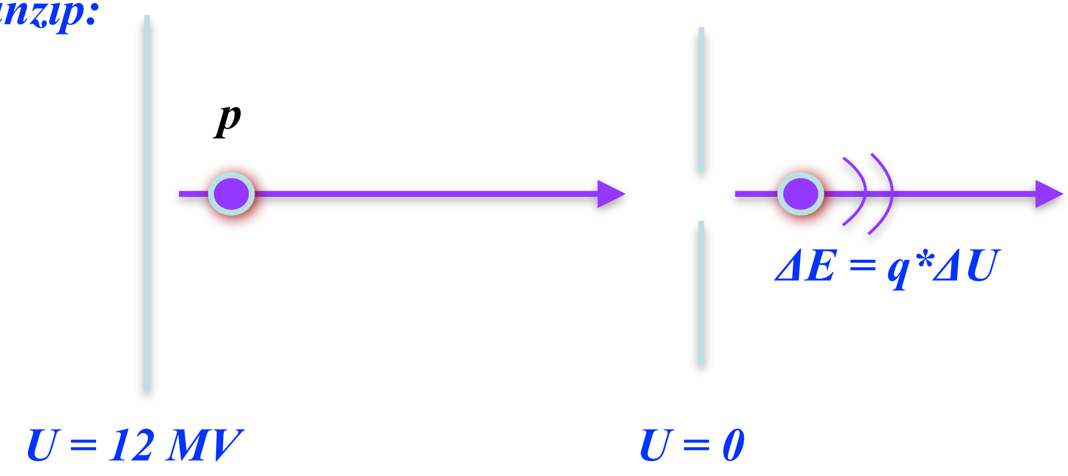
2.) Electrostatic Machines:
van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



** Terminal Potential: $U \approx 12 \ldots 28$ MV*
using high pressure gas to suppress discharge (SF_6)

Das Prinzip:



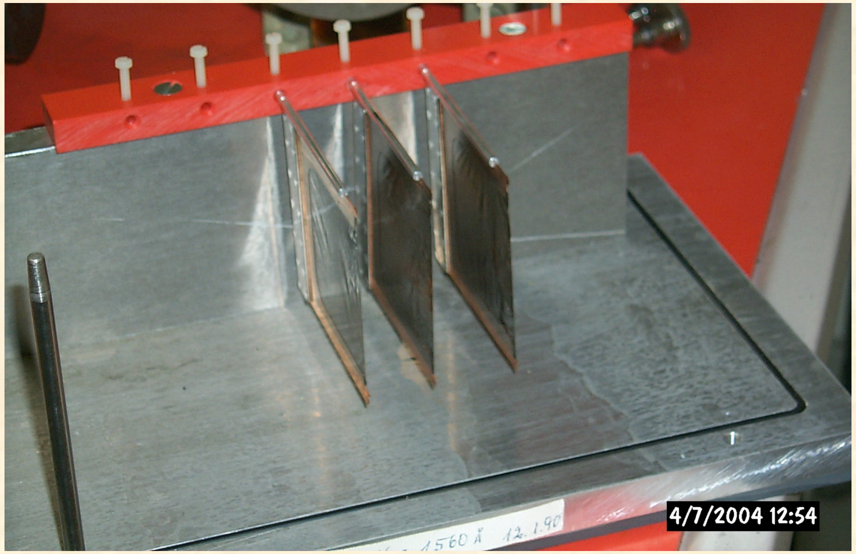
*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

The „Tandem principle":

Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine": 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

Innen-Ansicht einer solchen „Dampfmaschine“



Poggensee, DESY

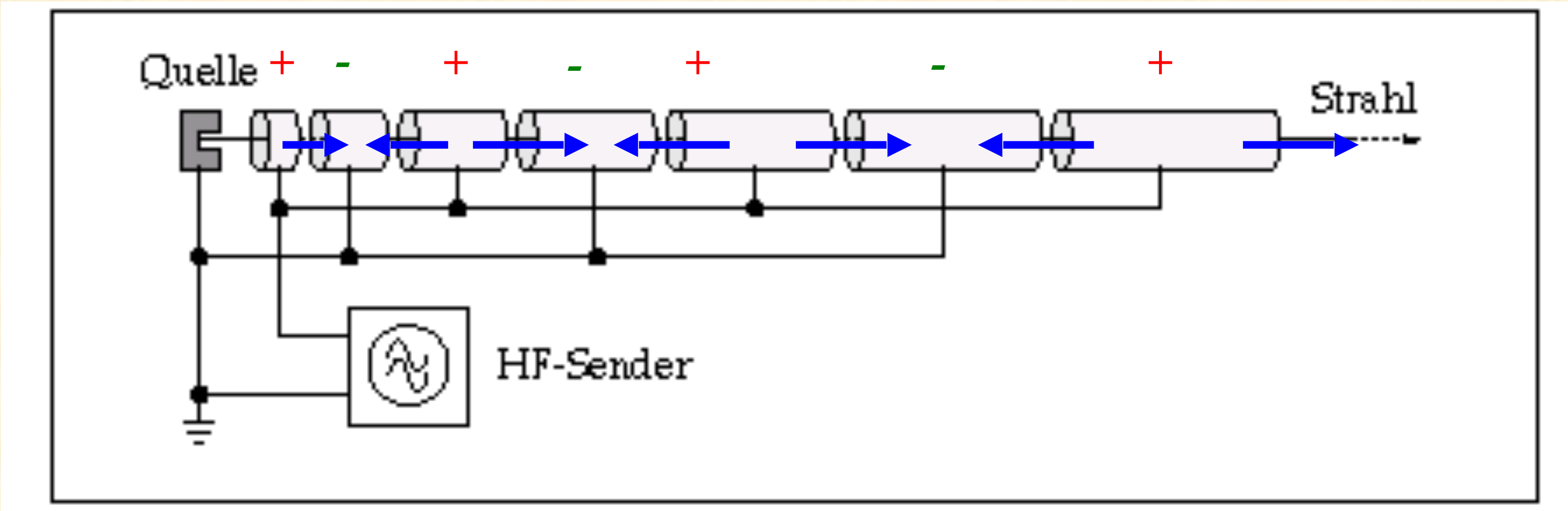
stripping foils: 1500 Å
… so nebenbei: das sind ca 1000 Atomlagen.

mechanischer Transport der Ladung mittels Glasfaserband

3.) The first RF-Accelerator: „Linac“

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



Energy gained after n acceleration gaps

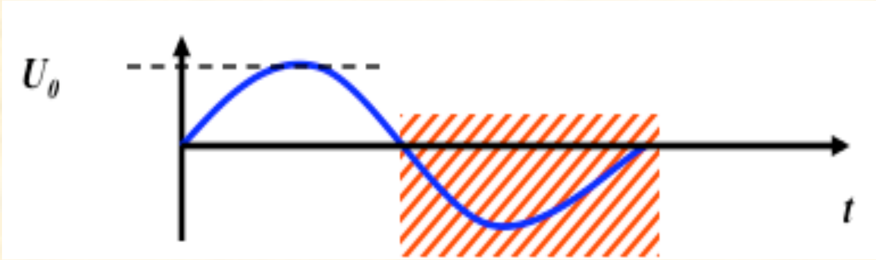
$$E_n = n \cdot q \cdot U_0 \cdot \sin \psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped“ to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



$\tau_{rf}/2$

Time span of the negative half wave: $\tau_{rf}/2$

Length of the Drift Tube: $l_n = v_n \cdot \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_n = \frac{1}{2} m v^2$

$$v_n = \sqrt{2E_n/m}$$

$$l_n = \frac{1}{f_{rf}} \cdot \sqrt{\frac{n \cdot q \cdot U_0 \cdot \sin\psi_s}{2m}}$$

Achtung !!! valid for non relativistic particles ...

Energy: ≈ 20 MeV per Nucleon β ≈ 0.04 … 0.6, Particles: Protons/Ions

Und jetzt a bissi Einstein:

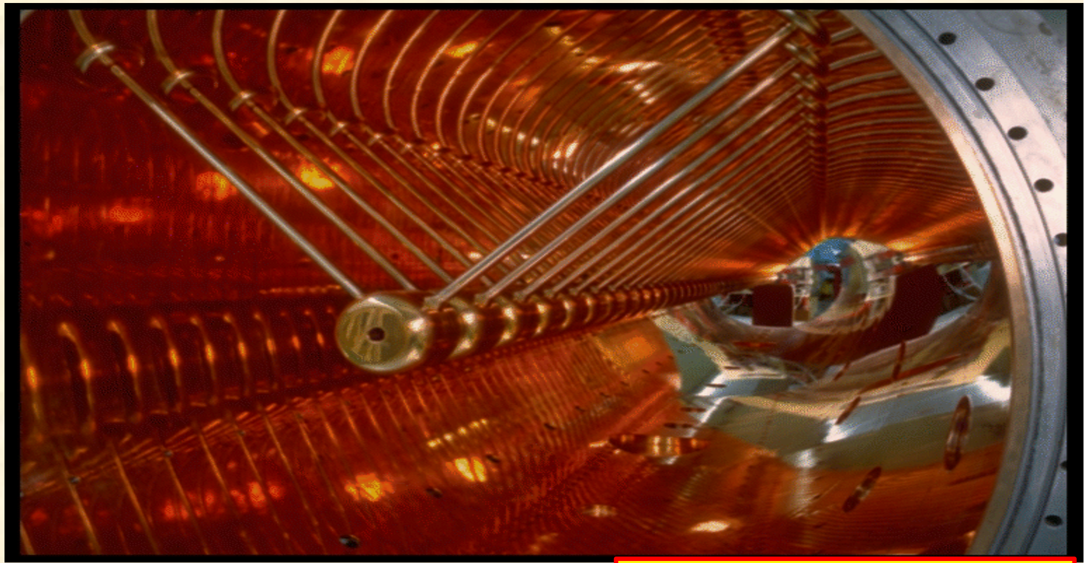
[Beam energies]

1.) reminder of some relativistic formula

rest energy $E_0 = m_0 c^2$

total energy $E = \gamma \cdot E_0 = \gamma \cdot m_0 c^2$

kinetic energy $E_{kin} = E_{total} - m_0 c^2$



Energy Gain per „Gap“: $\boxed{\boldsymbol{W = q U_0} \sin \omega_{\boldsymbol{RF}} \boldsymbol{t}}$

Linac III:

$$E_{total} = 988\ MeV$$

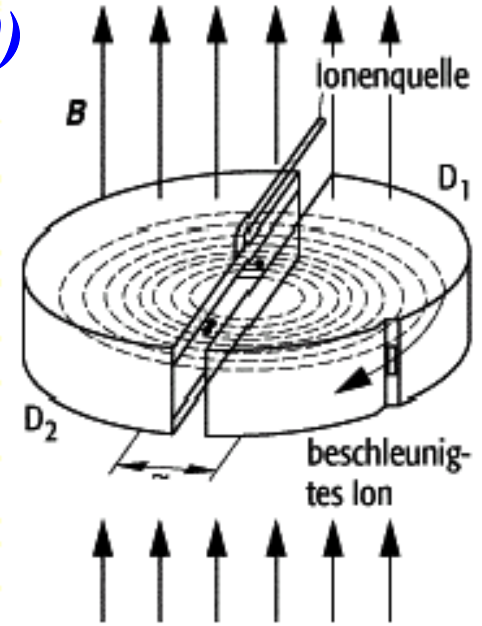
$$m_0 c^2 = 938\ MeV$$

$$E_{kin} = 50\ MeV$$

$$\gamma = \frac{E_{ges}}{E_0} = 988/938 = 1.05$$

—> im klassischen Bereich

3.) The Cyclotron: (Livingston / Lawrence ~1930)



Problem:
Linacs werden bei v=c sehr schnell sehr lanngggg.

—> Man erhaelt ne kompakte (d.h. billigere) Maschine, wenn man den Orbit der Teilchen aufwickelt.

Idea: Apply a magnetic field: B = const

Lorentzforce

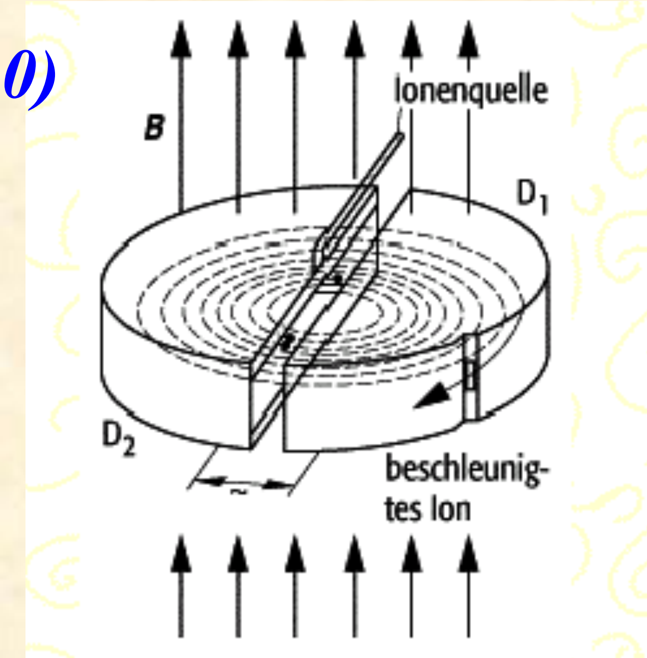
$$F = q \cdot v \cdot B$$

geladene Teilchen in Bewegung werden im Magnetfeld abgelenkt.

Kreisbahn-Bedingung:
Zentrifugalkraft wird durch die entgegengesetzte Lorentz-Kraft aufgehoben.

$$\left.\begin{array}{l} F_{Lorentz} = F_{zentrifugal} \\ q \cdot v \cdot B = \dfrac{mv^2}{r} \end{array}\right\} \quad B \cdot R = \frac{mv}{q} \quad \longrightarrow \quad B \cdot R = \frac{p}{q}$$

3.) The Cyclotron: (Livingston / Lawrence ~1930)



revolution frequency

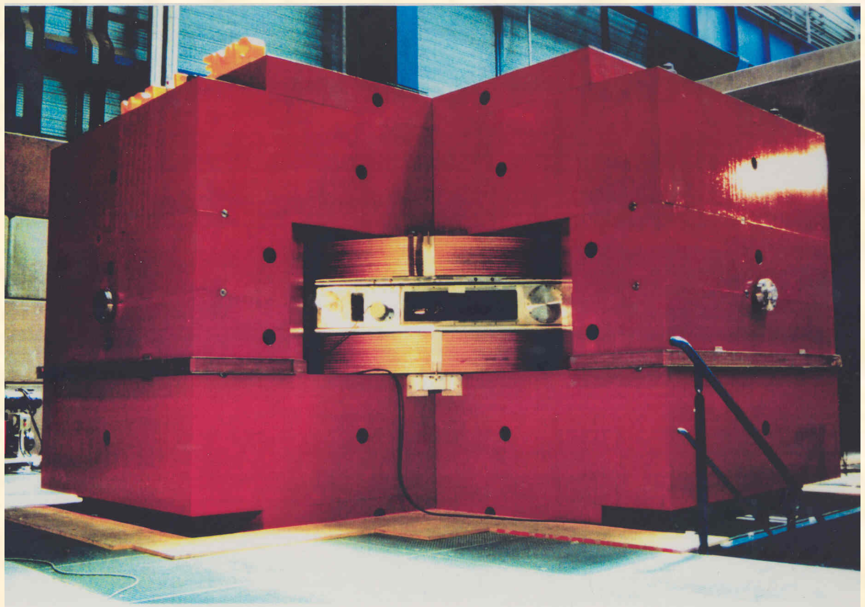
$$\omega_{revol} = \frac{v}{r} = \frac{q}{m} \cdot B = \quad const!!!$$

Die Umlaufs-frequenz im Cyclotron ist konstant.
Wir lassen eine gleich-grosse konstante RF frequenz auf die Teilchen los und die Kiste funktioniert.

increasing radius for increasing momentum ➔ ***Spiral Trajectory***

$$\omega_{rf} = \omega_{revolution} \quad \textbf{oder} \quad \omega_{rf} = h \cdot \omega_{revolution}$$

rf-frequency = h* revolution frequency,
h = "harmonic number"



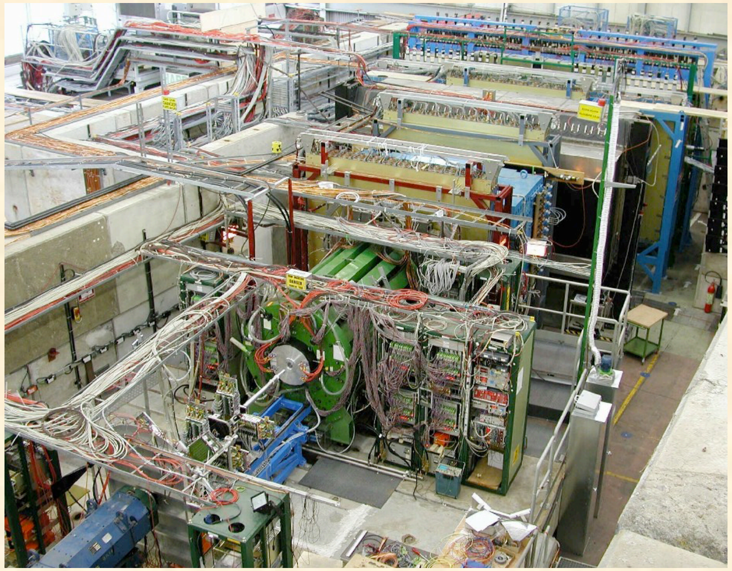
Problem: Albert !!!

$$\omega_{revol} = \frac{q}{\gamma\ m} \neq \ const$$

$$\omega_s(t) = \omega_{rf}(t) = \frac{q}{\gamma(t) \cdot m_0} B$$

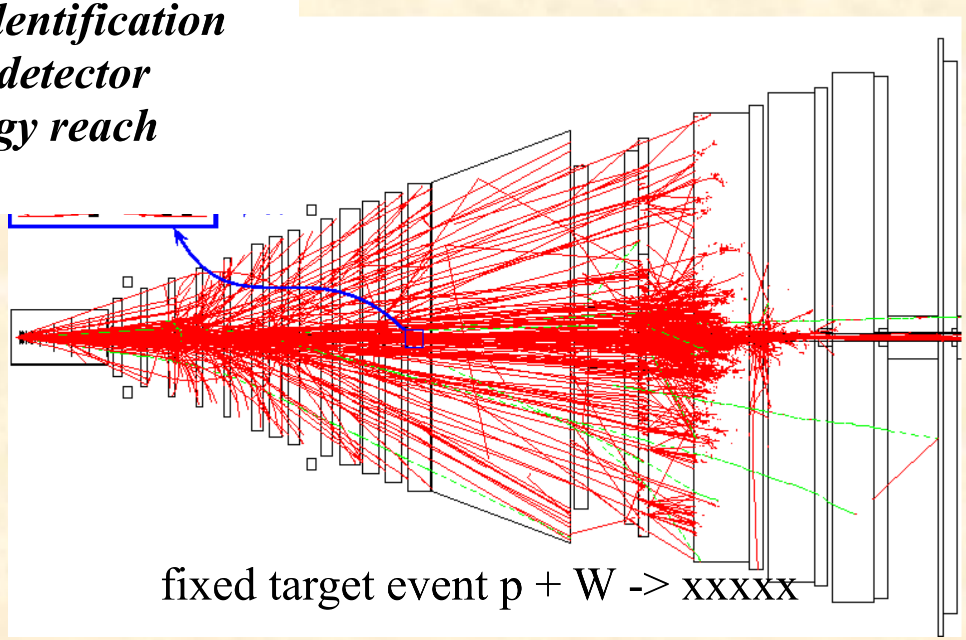
Synchro-Cyclotron
Nachfahren der RF Frequenz

Fixed target experiments:



HARP Detector, CERN

high event rate
easy track identification
asymmetric detector
limited energy reach

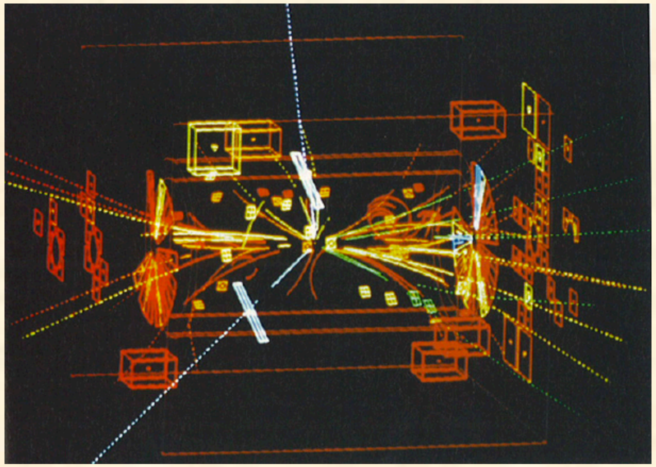


fixed target event p + W -> xxxxx

Collider experiments:

E=mc²

low event rate (luminosity)
challenging track identification
symmetric detector
$E_{lab} = E_{cm}$



Z_0 boson discovery at the UA2 experiment (CERN).
The Z_0 boson decays
into a e+e− pair, shown as white dashed lines.

II.) A Bit of Theory

The big storage rings: „Synchrotrons“

1.) Introduction and Basic Ideas

„ ... in the end and after all it should be a kind of circular machine“

→ ***need transverse deflecting force***

Lorentz force $\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$

typical velocity in high energy machines: $v \approx c \approx 3*10^8 \, m/s$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q*3*10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q * \underbrace{300 \frac{MV}{m}}_{E}$$

equivalent electrical field: E

Technical limit for electrical fields:

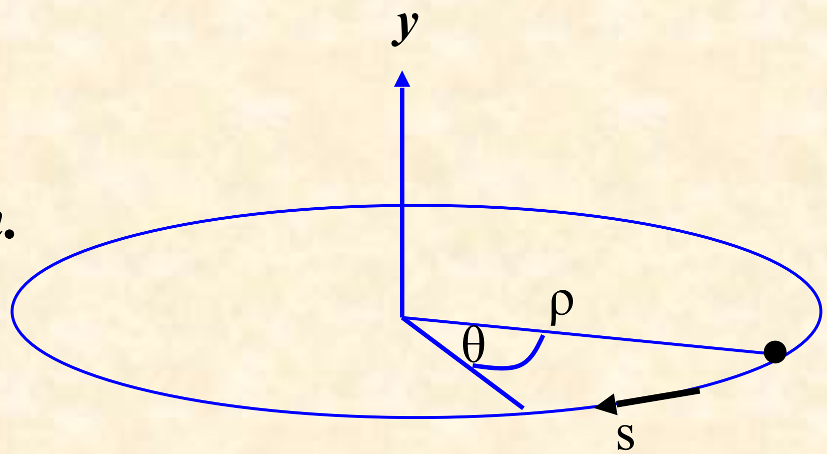
$$E \leq 1 \frac{MV}{m}$$

Ein Speicherring besteht aus
Magneten, Magneten und Magneten

und ein wenig Vakuum-Kammern, Strahldiagnose, ***und RF Systemen***

The ideal circular orbit

… das hatten wir schon.



circular coordinate system

condition for circular orbit:

Lorentz force $$F_L = e\, v\, B$$

centrifugal force $$F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$$

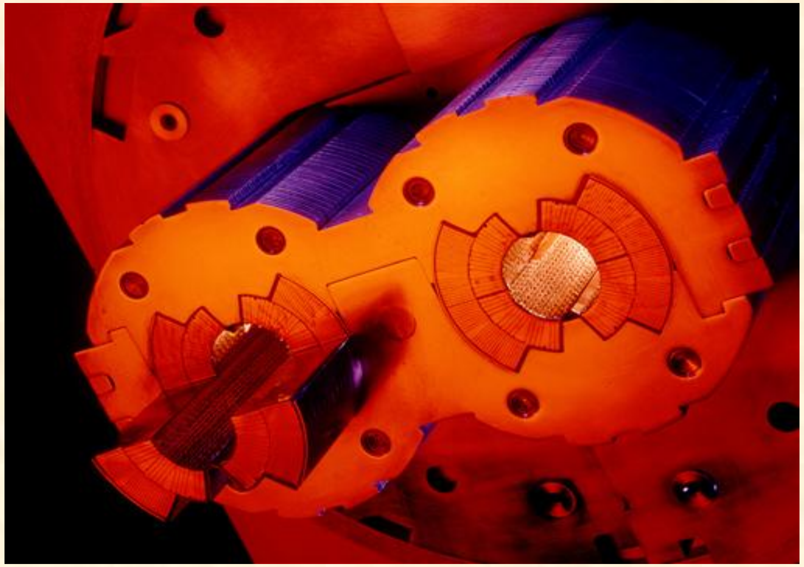
$$\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\, \cancel{v}\, B$$

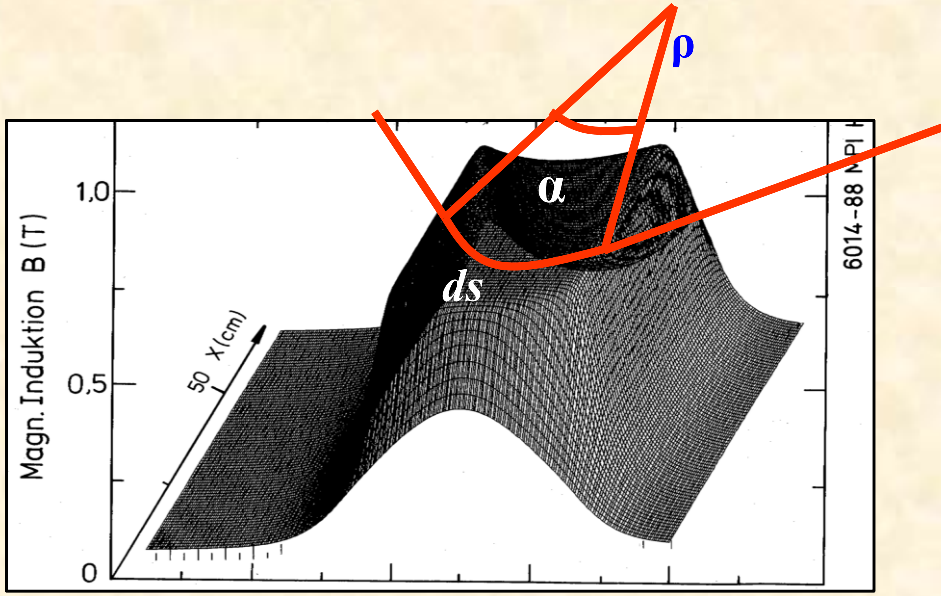
$$\boxed{\frac{p}{e} = B\,\rho}$$

B ρ = "beam rigidity"

… und jetzt isses sogar relativistisch korrekt.

The Magnetic Guide Field





field map of a storage ring dipole magnet

Dipole erzeugen mit zwei parallelen Polschuhen ein konstantes (!) Magnetfeld

$\boldsymbol{B} \approx 1 \ldots 8\ \boldsymbol{T}$

Achtung: um zum Pluto zu kommen muessen wir höchste Präzision fordern.

$$\frac{\Delta B}{B} \approx 10^{-4}$$

Ablenkradius:

$$\rho = \frac{p}{e\ B} = \frac{7000 \cdot 10^{9}\ eV}{3 \cdot 10^{8} m/s * 8 Vs/m^{2}}$$

nota bene:
die allgemeinste Ausdruck fuer die Energie ist

$$\rho = 2.8\ km$$

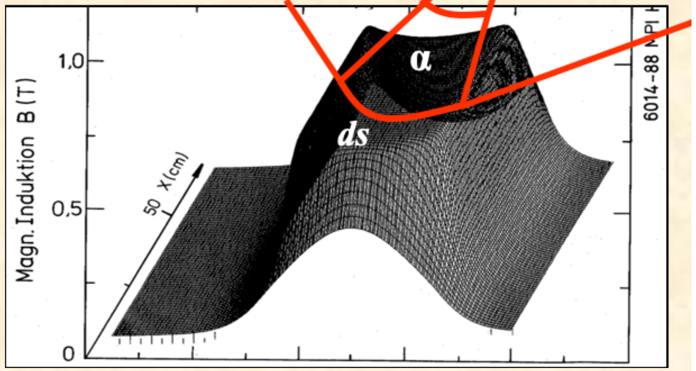
$$E^2 = p^2c^2 + m^2c^4 \quad \rightarrow \quad p \approx \frac{E}{c}$$

Achtung:
um Energie unabhängige Gleichungen zu erhalten teilen wir die Felder durch "p"

„normalised bending strength"

$$1/\rho = \frac{B}{p/e}$$

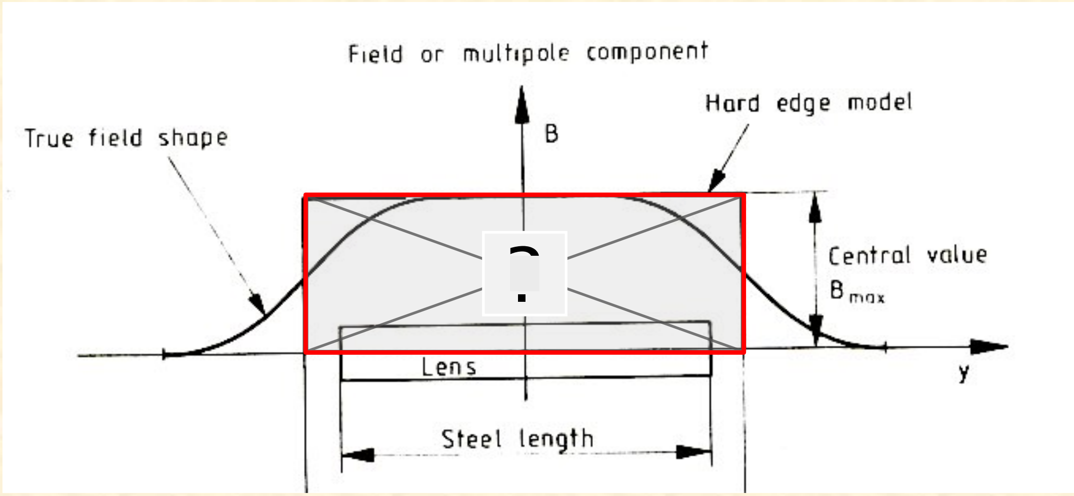
Bending Angle



„integrated field strength"

$$\alpha = \frac{B * dl}{B * \rho}$$

$$B\, l_{eff} = \int_0^{l_{mag}} B\, ds$$



The angle swept out in one revolution must be 2π, so

$$\alpha = \frac{\int B dl}{B * \rho} = 2\pi \qquad \rightarrow \qquad \int B dl = 2\pi * \frac{p}{q}$$

… for a full circle

2.) Focusing Forces: Hook's law

... keeping the flocs together:
In addition to the pure bending of the beam
we have to keep 10^{11} particles close together

focusing force

Um auf unserem Weg zu Pluto unsere 10^{11} Teilchen zusammen zu halten wenden wir den Trick der linearen rücktreibenden Kraft an:

D.h. unsere Teilchen werden wie 10^{11} Pendelchen hin und her schwingen

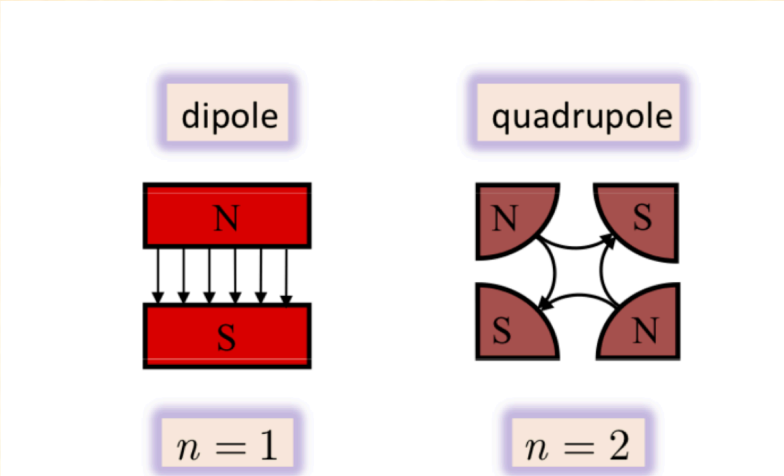
Wir müssen es nur schaffen, ein Magnetfeld zu erzeugen, das linear ansteigt.

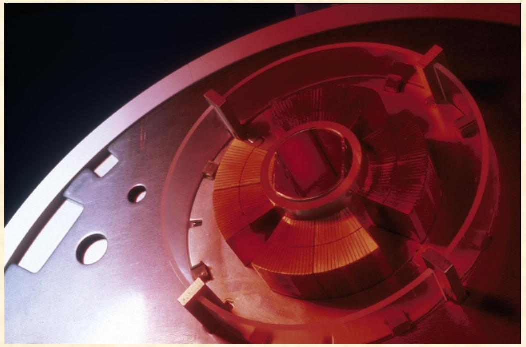
2.)Focusing Forces: Quadrupole Fields

Apply this concept to magnetic forces: we need a Lorentz force that rises as a function of the distance to ...

… the design orbit

$$F(x) = q * v * B(x)$$





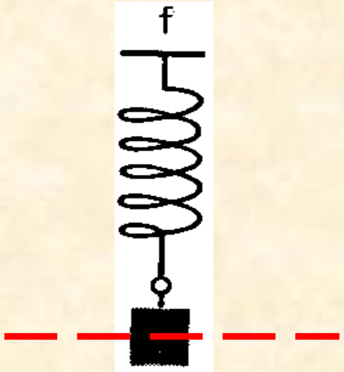
Dipoles: Create a constant field

$$B_y = const$$

Quadrupoles: Create a linear increasing magnetic field:

$$B_y(x) = g \cdot x, \quad B_x(y) = g \cdot y$$

Federpendel im Physik Buch



there is a restoring force, proportional to the elongation x:

$$F = m * a = -\,const * x$$

$$F = m * \frac{d^2x}{dt^2} = -\,const * x$$

Hook's Federgesetz: $F = -\,k * x$

Integration liefert uns eine cos- artige Lösung $x(t) = A \cdot cos(\omega t)$

oder eine sinus artige $x(t) = B \cdot sin(\omega t)$

oder eine Kombination aus beiden $x_{allg}(t) = A \cdot cos(\omega t) + B \cdot sin(\omega t)$

Vorteil:

harmonische Schwingungen sind sehr (!!) stabil,
haben eine wohldefinierte Frequenz
sind in der Natur (i.,e. Physik) weitverbreitet

Focusing forces and particle trajectories:

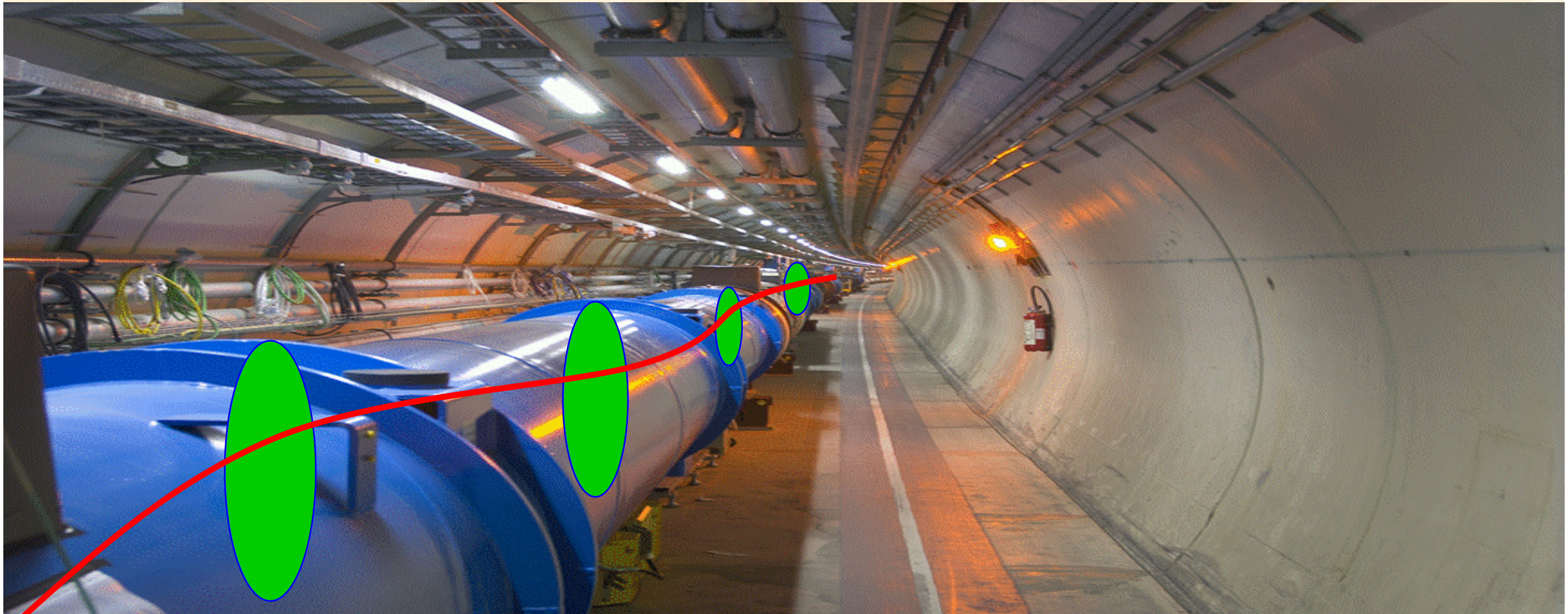
normalise magnet fields to momentum
*(remember: **B*ρ = p / q**)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

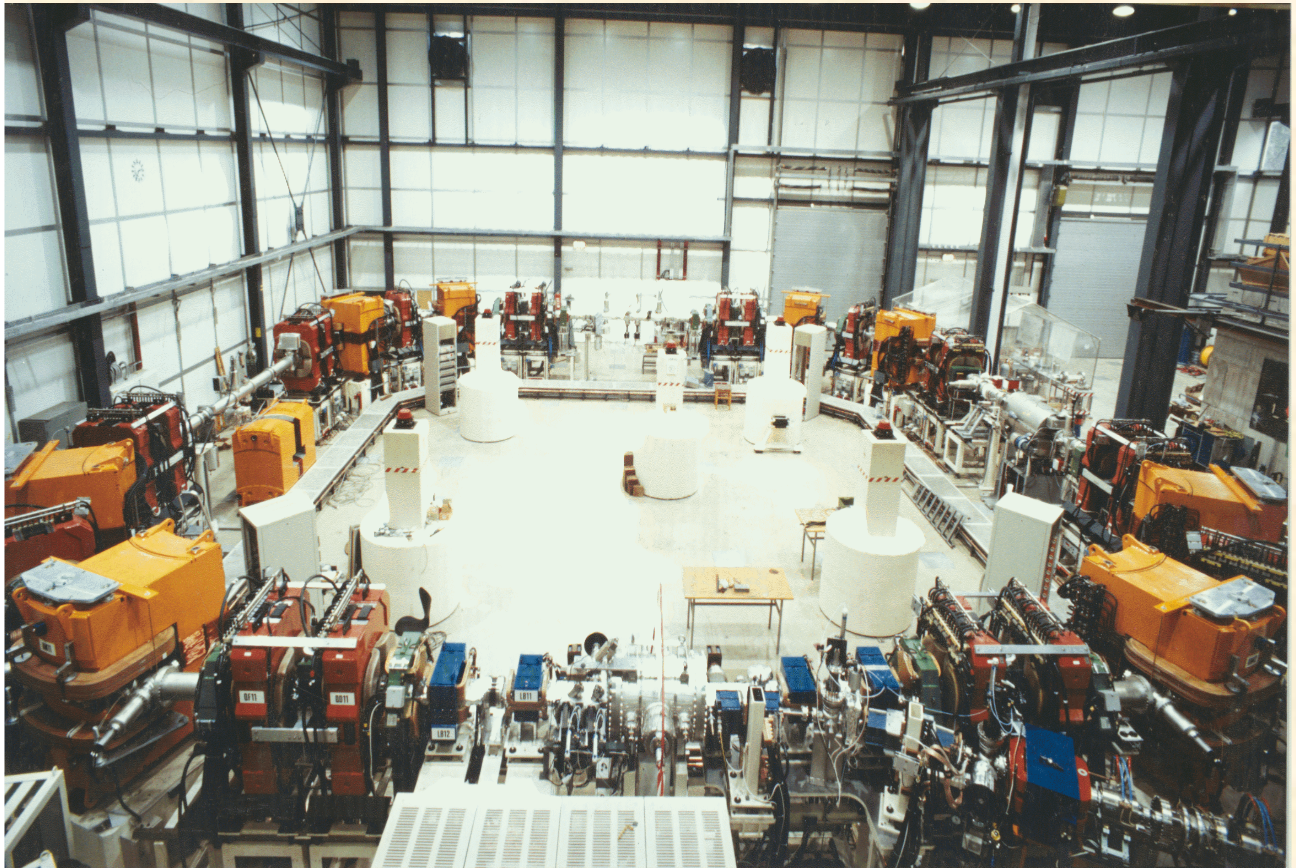
$$k := \frac{g}{p/q}$$



3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}m\,x^2} + \cancel{\frac{1}{3!}n\,x^3} + \dots$$

only terms linear in x, y taken into account *dipole fields*
quadrupole fields



Separate Function Machines:

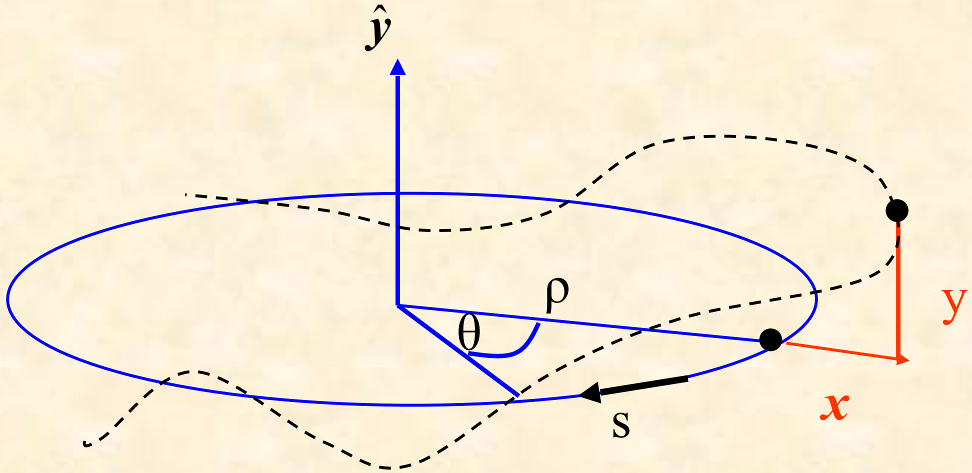
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* ***man sieht nur dipole und quads → linear***

The Equation of Motion:



* *Equation for the horizontal motion:*

$$x'' + x \cdot \left(\frac{1}{\rho^2} + k\right) = 0$$

***x* = particle amplitude**
***x'* = angle of particle trajectory (wrt ideal path line)**

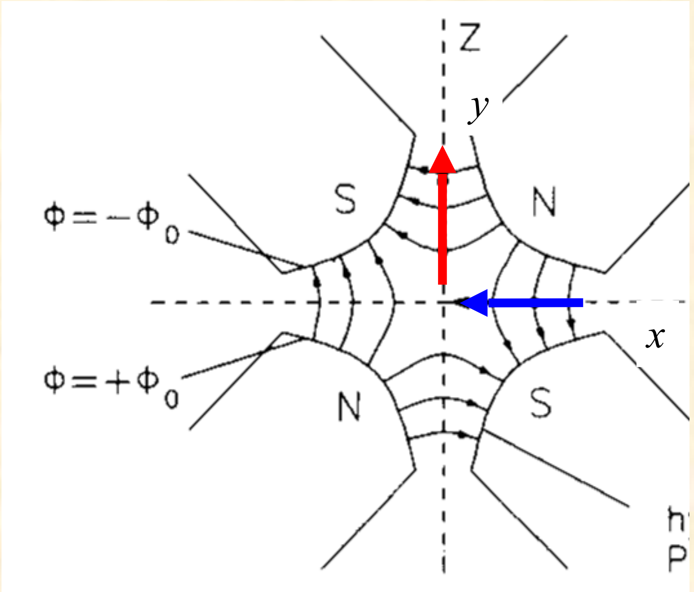
$$x'' = -\,x \cdot \underbrace{\left(\frac{1}{\rho^2} + k\right)}$$

$$x'' = \quad -K \cdot\ x$$

Hook's Gesetz fuer Speicherringe

… es gibt da nur ein kleines Problem:

In der vertikalen Ebene drehen sich die Magnetfeld-Linien um



* ***Equation for the vertical motion:***

$$\frac{1}{\rho^2} = 0$$ ***no dipoles … in general …***

$$\boldsymbol{k} \leftrightarrow -\boldsymbol{k}$$ ***quadrupole field changes sign***

$$y'' - k \cdot y = 0$$

… und Teilchen, die in der horizontalen Ebene fokussier werden,
werden im gleichen Atemzug in der vertikalen Ebene aus der Maschine befördert.

4.) Solution of Trajectory Equations

Define … hor. plane: $K = 1/\rho^2 + k$

… vert. Plane: $K = -k$

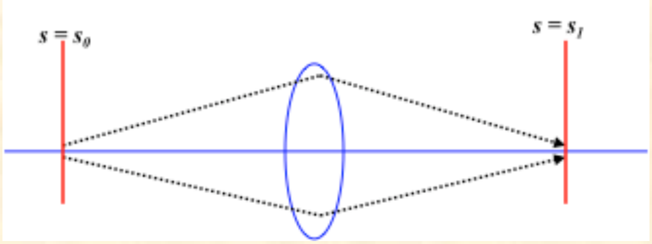
$$x'' + K\,x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole $K > 0$:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$



… da ist wieder unsere Kuckucksuhr.

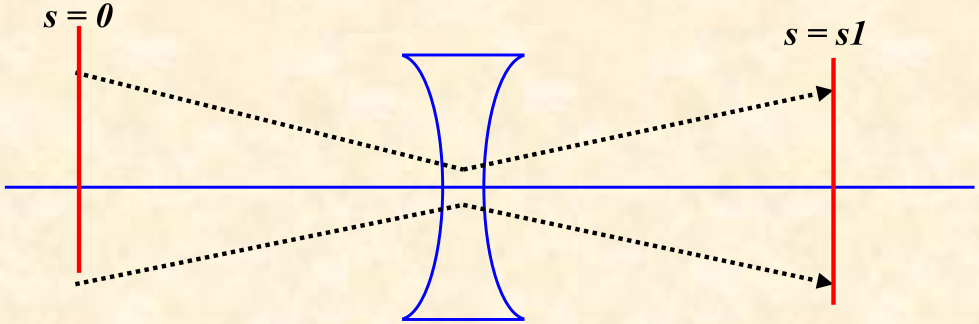
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}\,l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}\,l\right) \\ -\sqrt{|K|} \sin\left(\sqrt{|K|}\,l\right) & \cos\left(\sqrt{|K|}\,l\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$\boldsymbol{x}'' - \boldsymbol{K}\ \boldsymbol{x} = 0$$



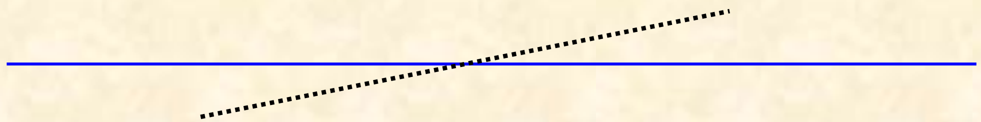
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega\, s) + a_2 \cdot \sinh(\omega\, s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\,l \\ \sqrt{|K|}\sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}$$

drift space:

$$K = 0$$



$$x(s) = x_0' \cdot s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

… zur Erinnerung:

hyperbolische Funktionen führen leicht zu Panik Attacken !

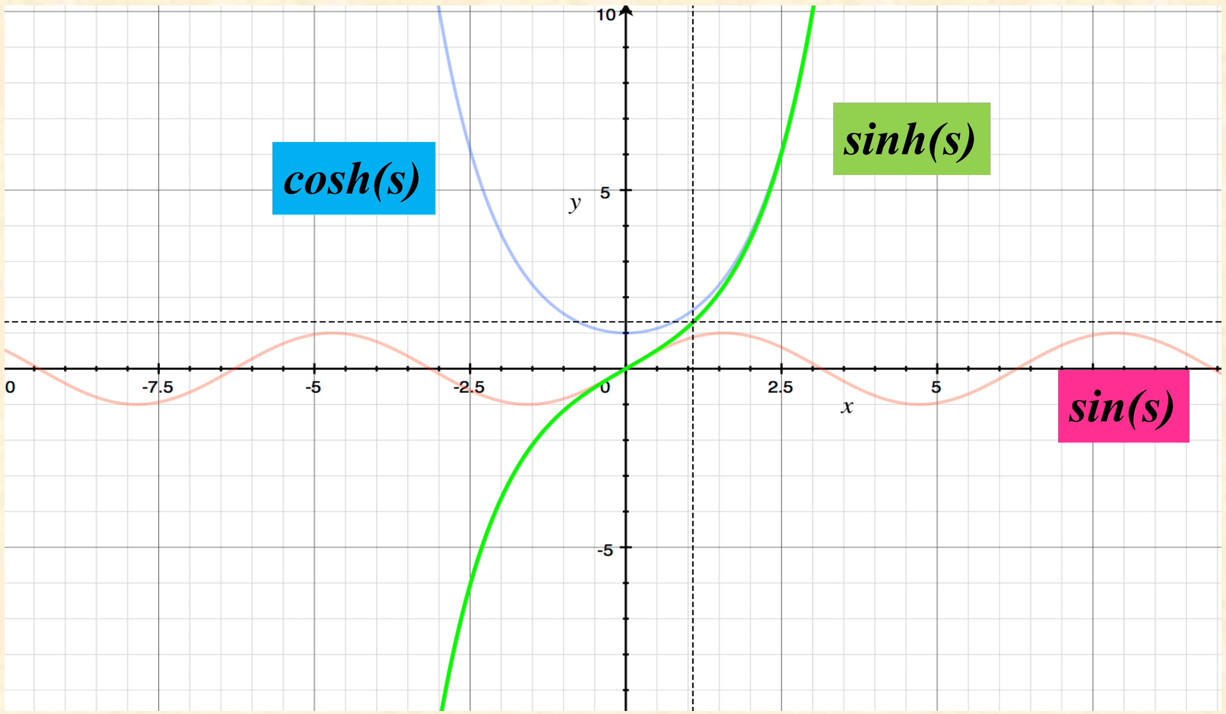
$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

$f(s) = sin(s)$ $\quad$ $f(s) = cos(s)$

$f(s) = sinh(s)$ $\quad$ $f(s) = cosh(s)$

Ansatz für die Teilchenbewegung im defokusierenden Fall:

$$x(s) = a_1 \cdot \cosh(\omega\, s) + a_2 \cdot \sinh(\omega\, s)$$

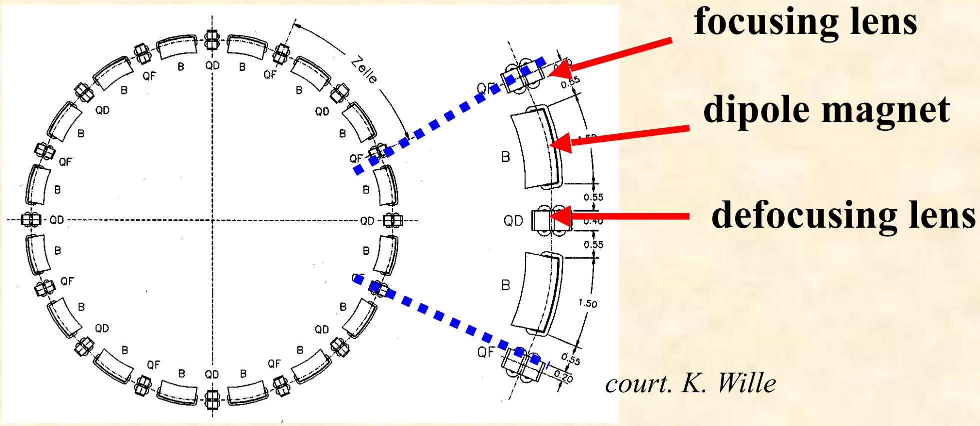


Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

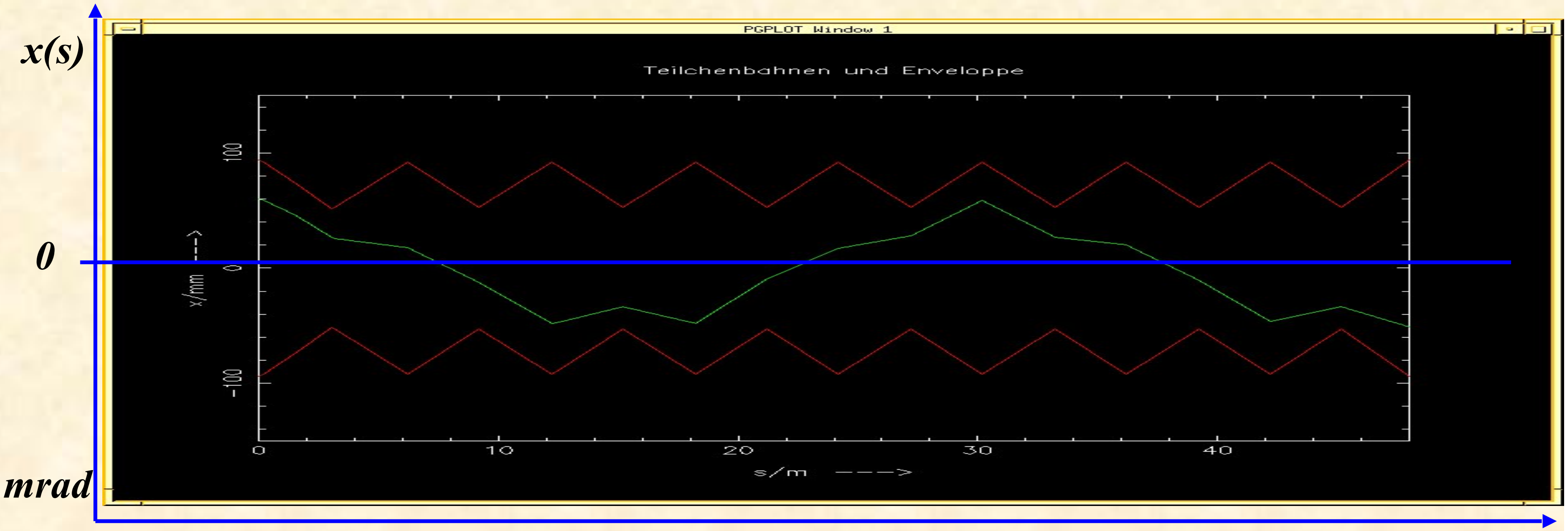
$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*}.....$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_2} = M(s_2, s_1) \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_{s_1}$$



court. K. Wille

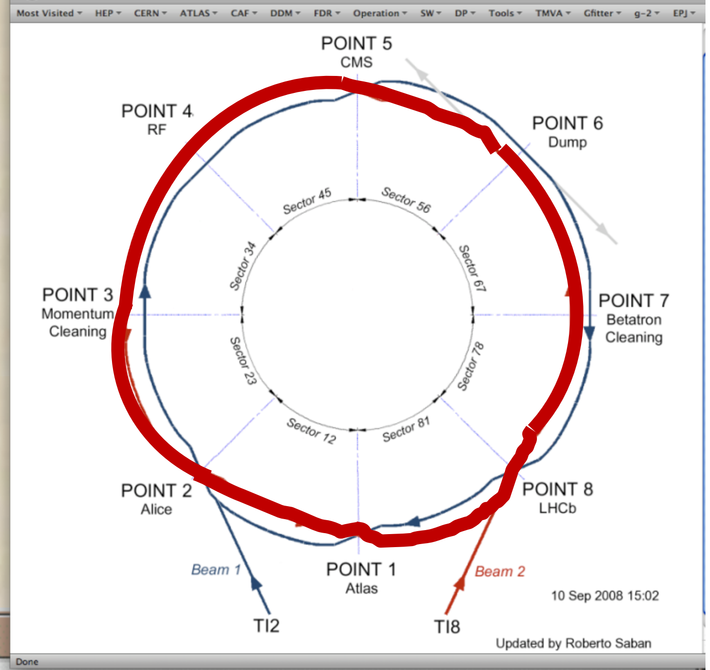
in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



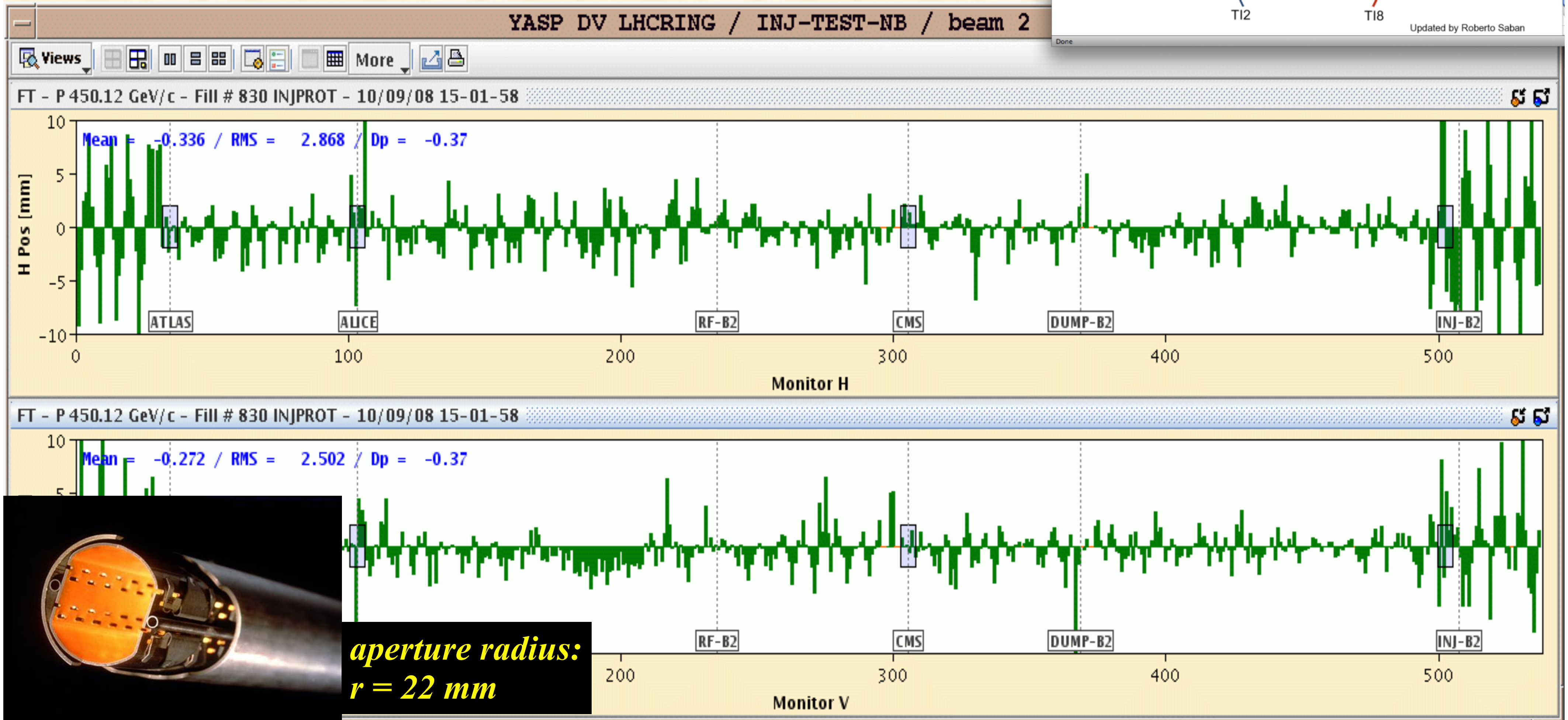
typical values in a strong foc. machine:
$x \approx mm$, $x' \leq mrad$

LHC Operation: Beam Commissioning

The transverse focusing fields create a harmonic oscillation of the particles with a well defined "Eigenfrequency" which is called tune



First turn steering "by sector:"



aperture radius:
r = 22 mm

“Once more unto the breach, dear friends, once more”
(W. Shakespeare, Henry 5)

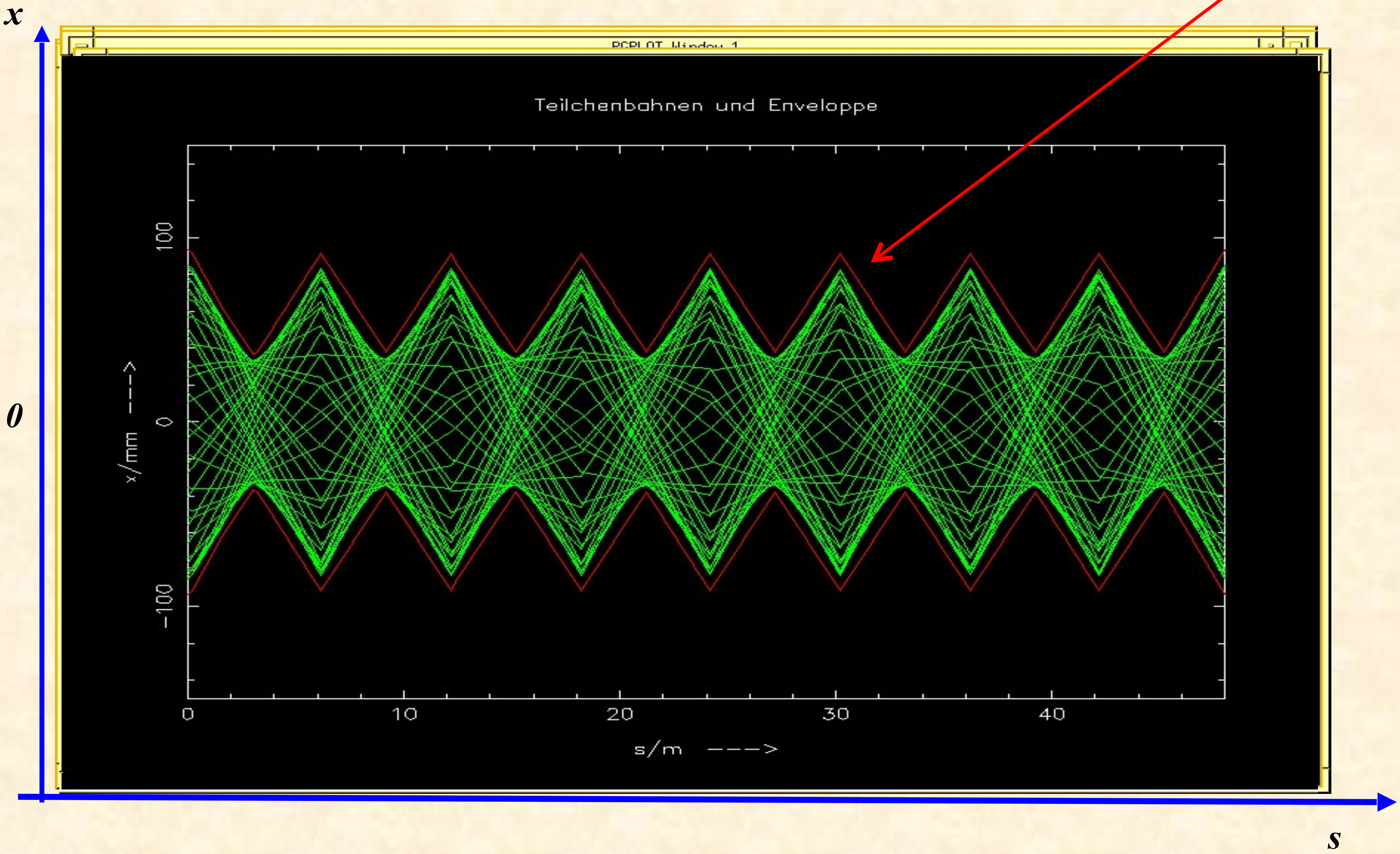
“Do they actually drop ?”

Answer: No

Question: ***what will happen, if the particle performs a second turn ?***

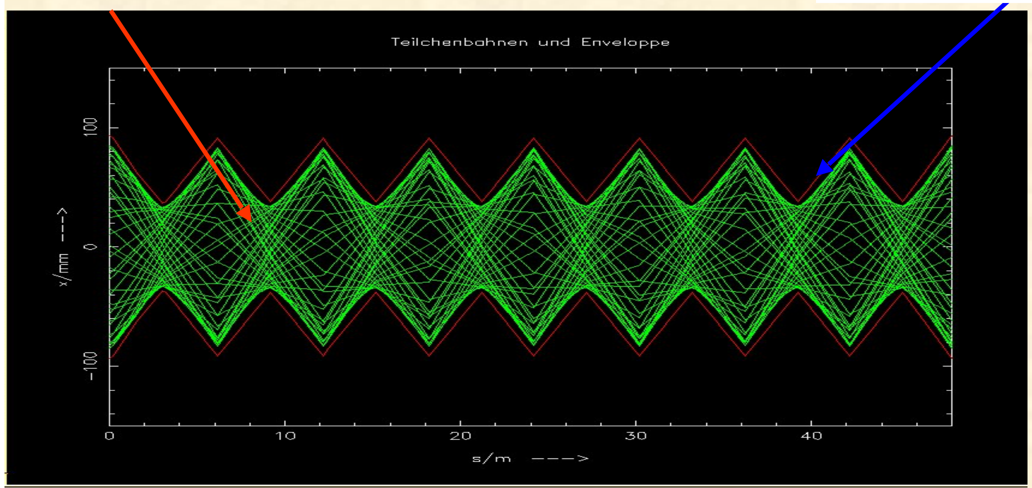
... or a third one or ... 10^{10} turns

$$\sigma = \sqrt{\varepsilon\beta}$$



Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \qquad \hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$$



single particle trajectories, $N \approx 10^{11}$ per bunch

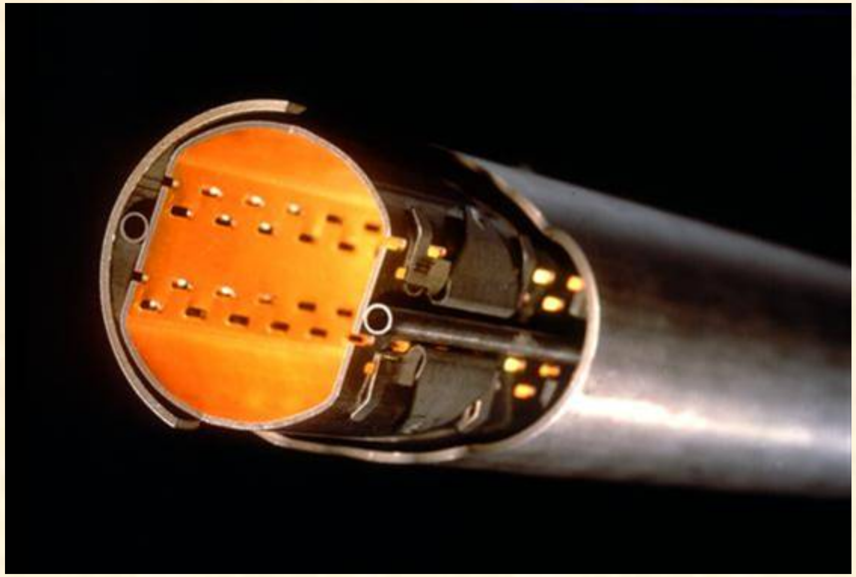
Gauß Particle Distribution:

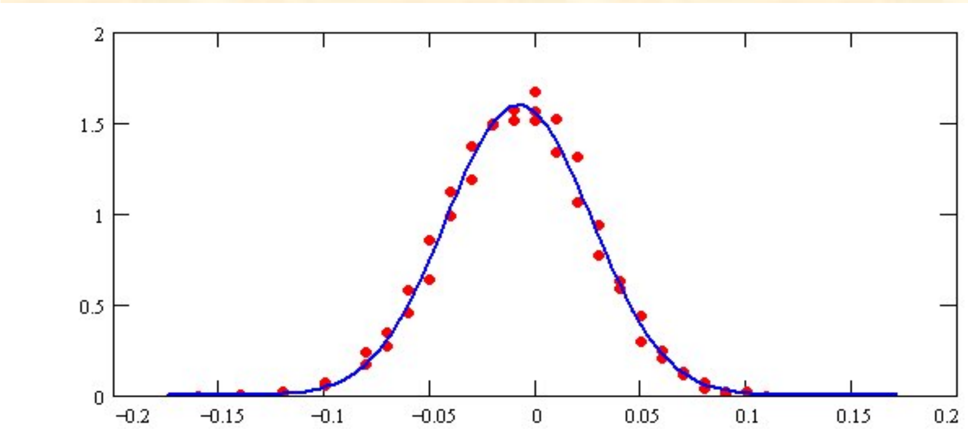
$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\,\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

LHC:

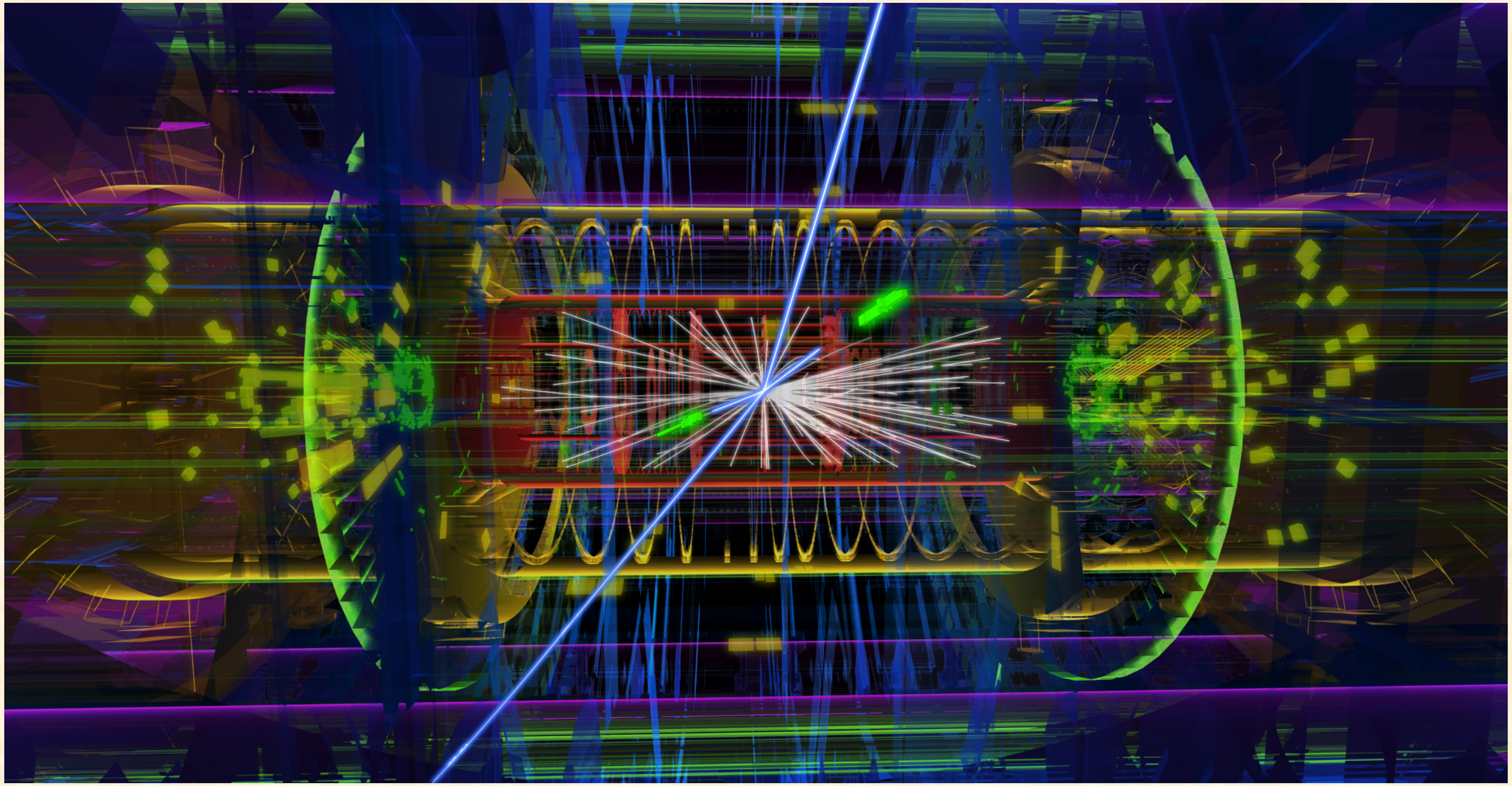
$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5*10^{-10}\,m * 180\,m} = 0.3\,mm$$





aperture requirements: $r_0 = 17 * \sigma$

Collisions



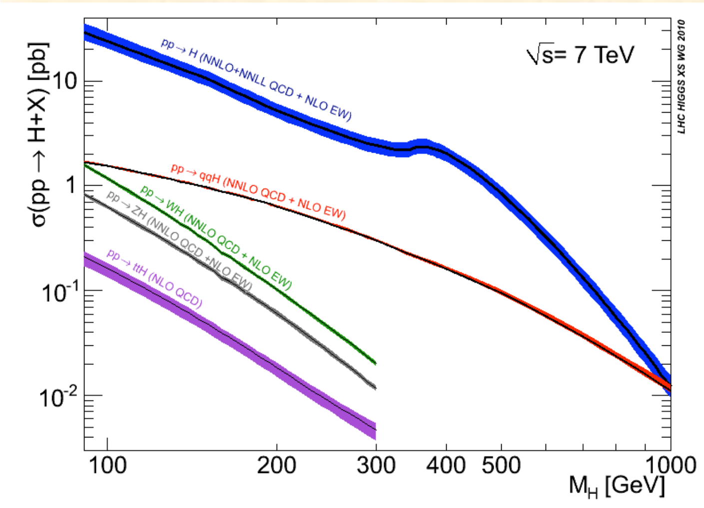
ATLAS event display: Higgs => two electrons & two muons

$$E = m_0c^2 = m_{e1} + m_{e2} + m_{\mu 1} + m_{\mu 2} = 125.4\ GeV$$

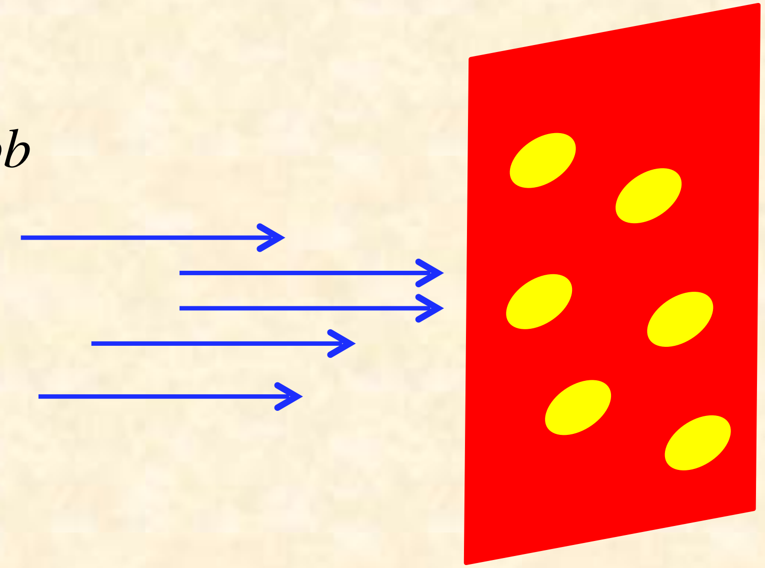
Problem: Our particles are VERY small !!

man trifft nicht so häufig.

Overall cross section of the Higgs:



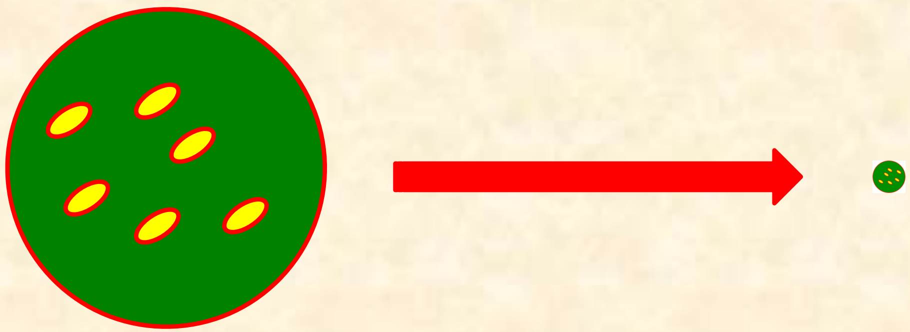
$$\Sigma_{react} \approx 1pb$$



$$1b = 10^{-24} cm^2 = \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{10000} \quad mm^2 \longrightarrow \boxed{1pb = 10^{-12} b \approx ZERO}$$

The particles are "very small"

The only chance we have:
compress the transverse beam size … at the IP

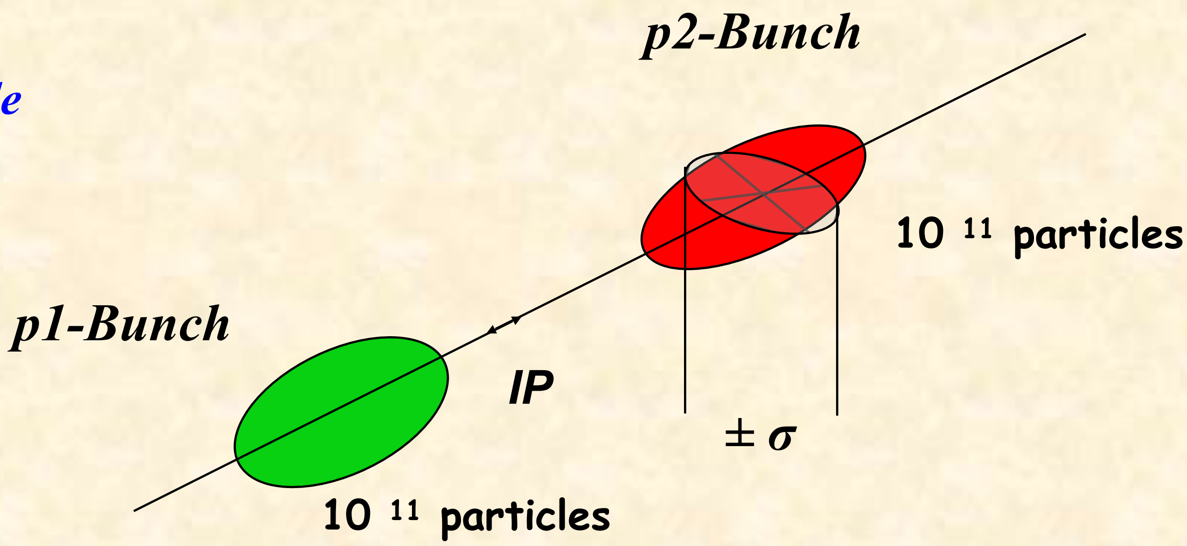


LHC typical → 16 µm

5.) Luminosity

Ereignis Rate: "Physik" pro Sekunde

$$R = L \cdot \Sigma_{react}$$

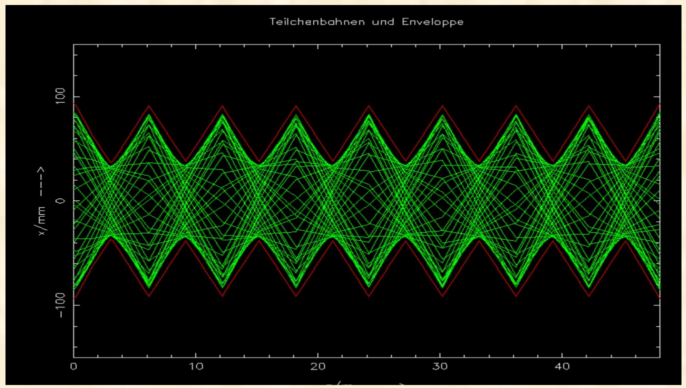


Example: Luminosity run at LHC

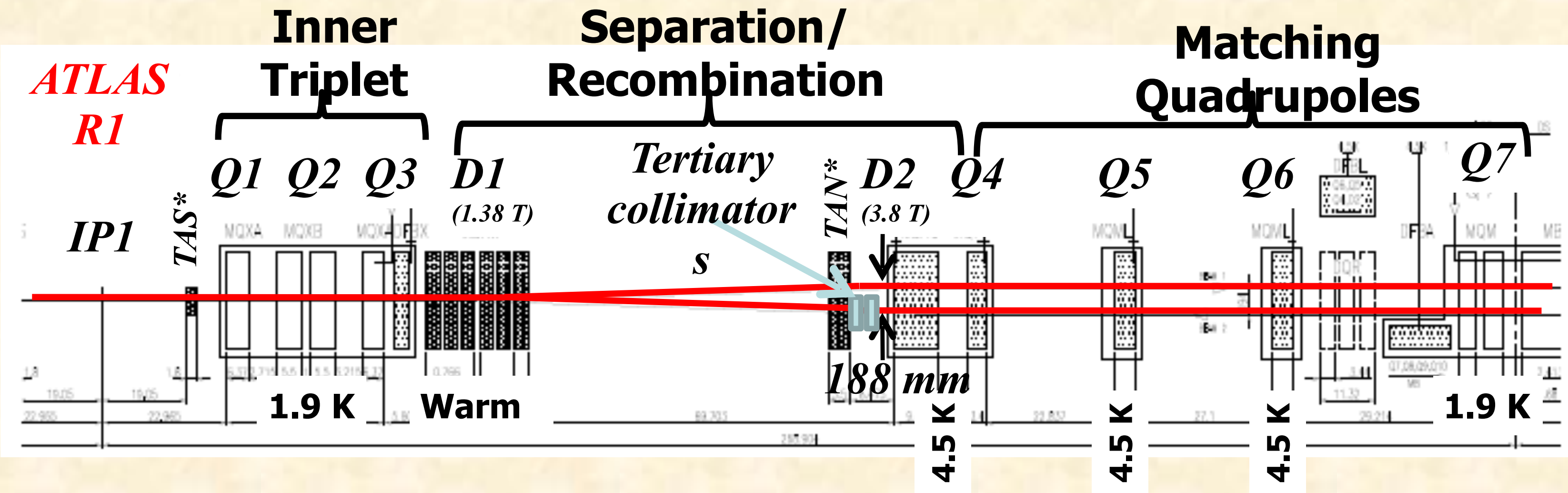
$\sigma_x = \sigma_y = 16\mu m$	*Strahlgröße am IP*
$f_0 = 11.245\ kHz$	*Umlaufsfrequenz*
$n_b = 2808$	*Zahl der Bunche*
$N_p = 1.2 \cdot 10^{11}$	*Teilchen in einem Bunch*
$I_p = 584\ mA$	*Strahlstrom*

$$L = 1.0 * 10^{34}\ {}^{1}\!/_{cm^2 s}$$

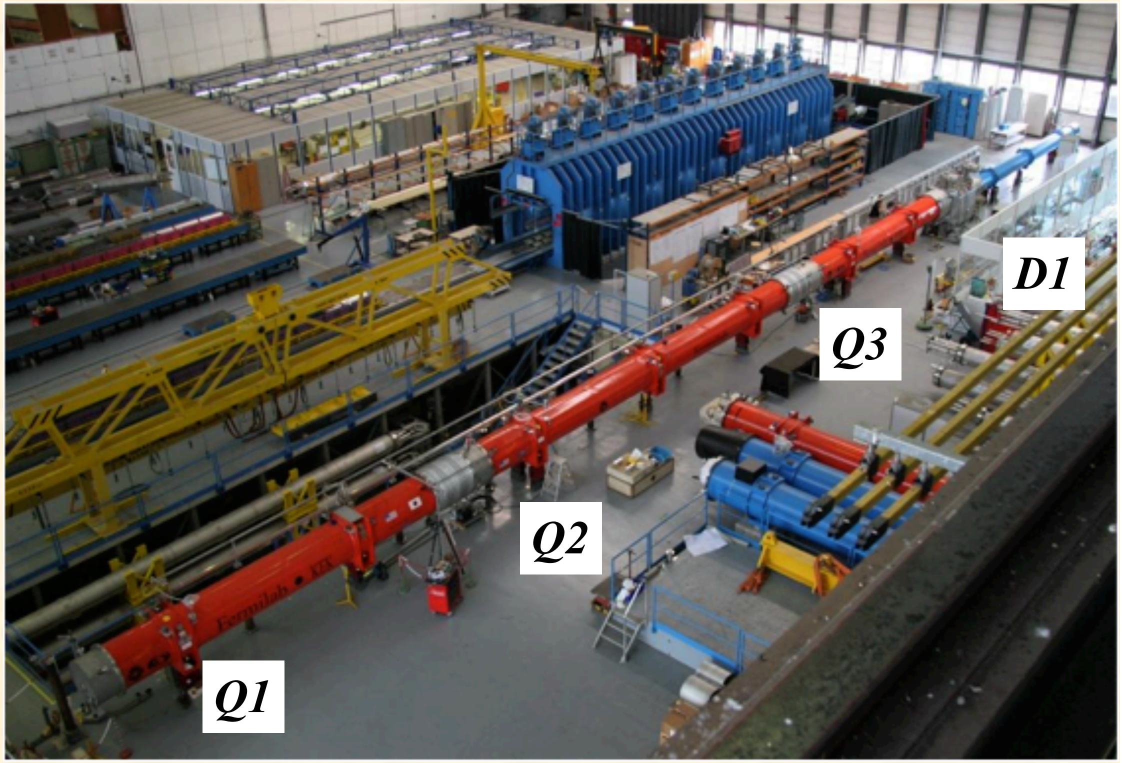
$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$



The LHC Mini-Beta-Insertions

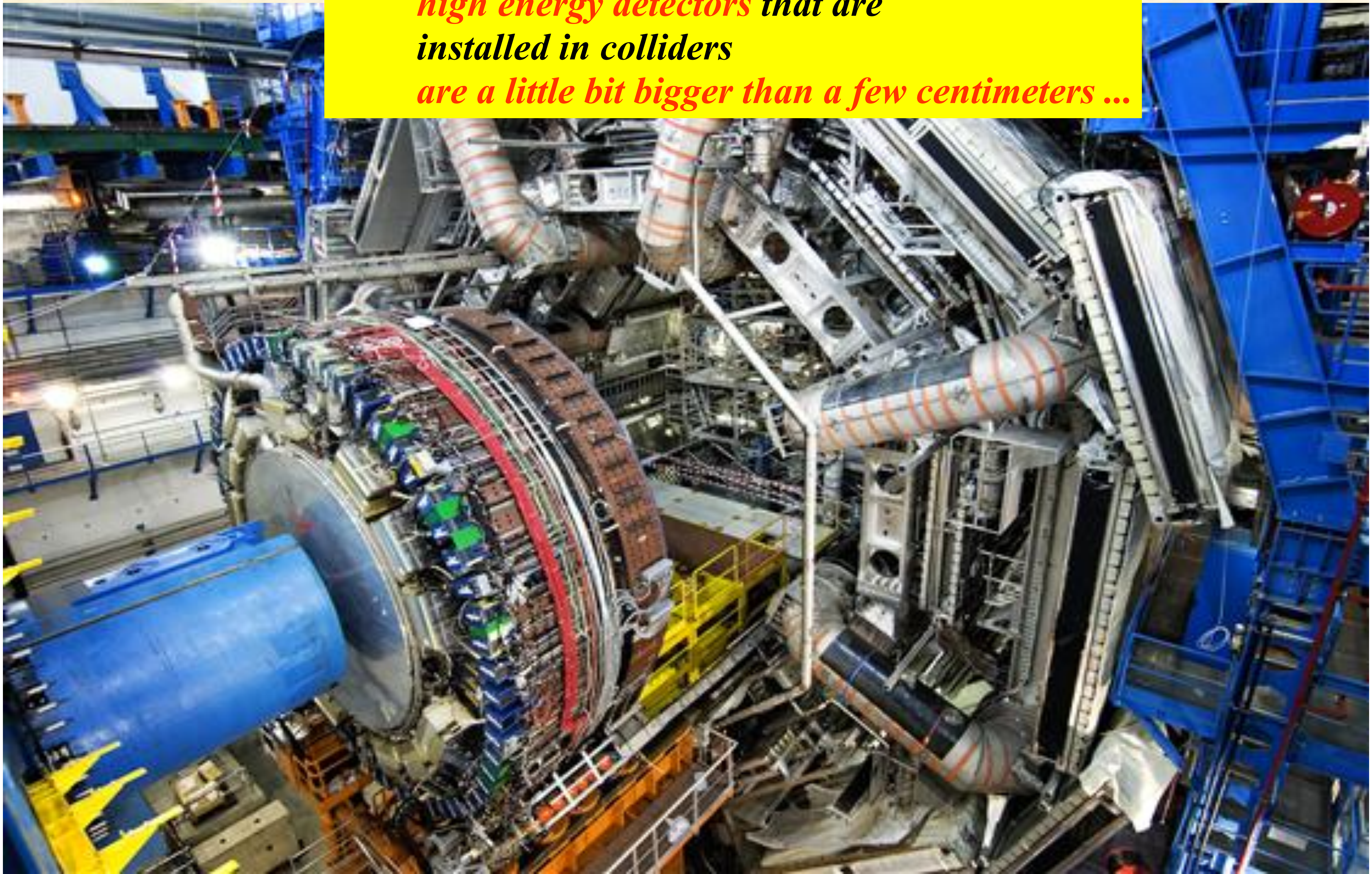


Extrem starke Fokussierung (in beiden Ebenen) für beide Strahlen, um die Trajektorien der 10^{11} Teilchen auf micro Meter zu komprimieren.



... clearly there is another problem !!!

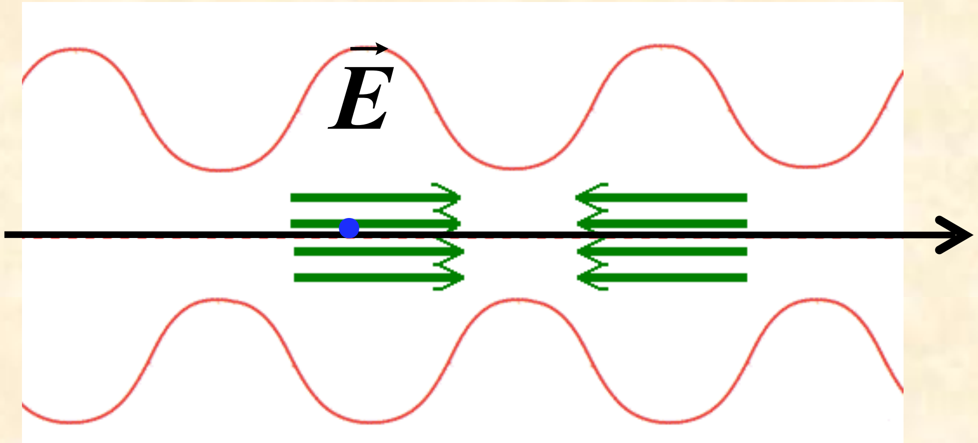
... unfortunately ... in general
high energy detectors that are
installed in colliders
are a little bit bigger than a few centimeters ...

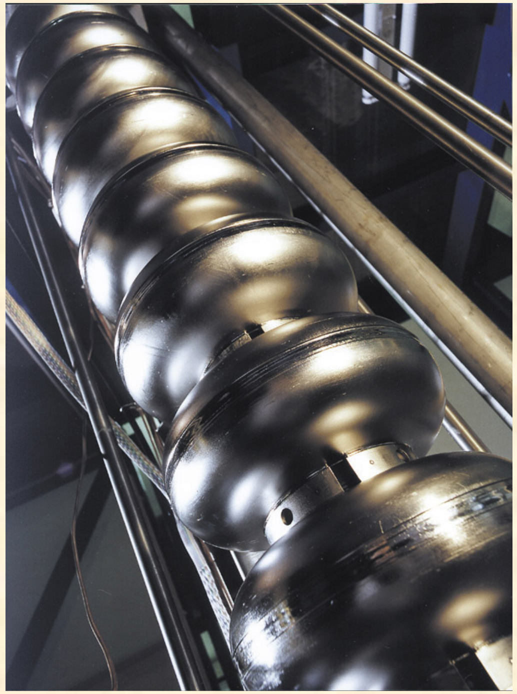


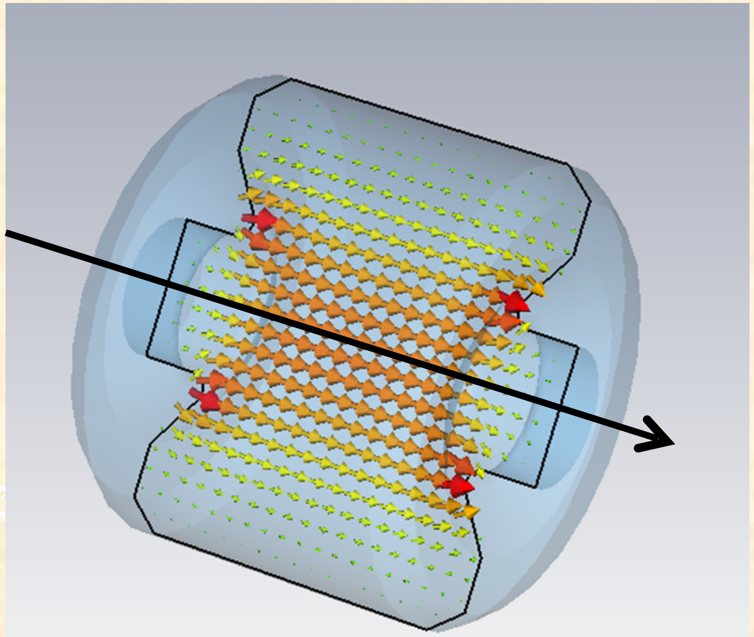
III. The Acceleration

Where is the acceleration?

Install an RF accelerating structure in the ring:







B. Salvant
N. Biancacci

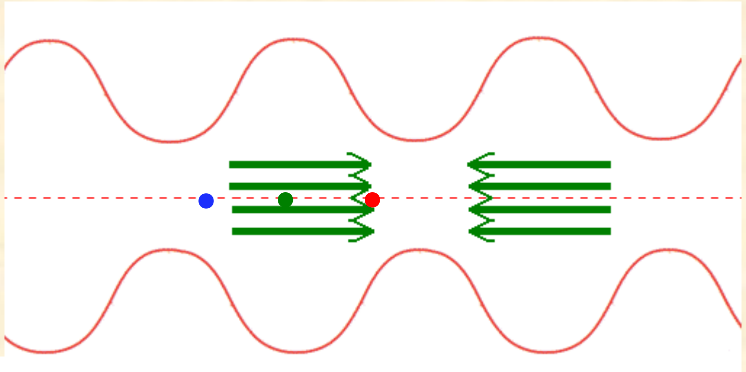
The Acceleration & "Phase Focusing"

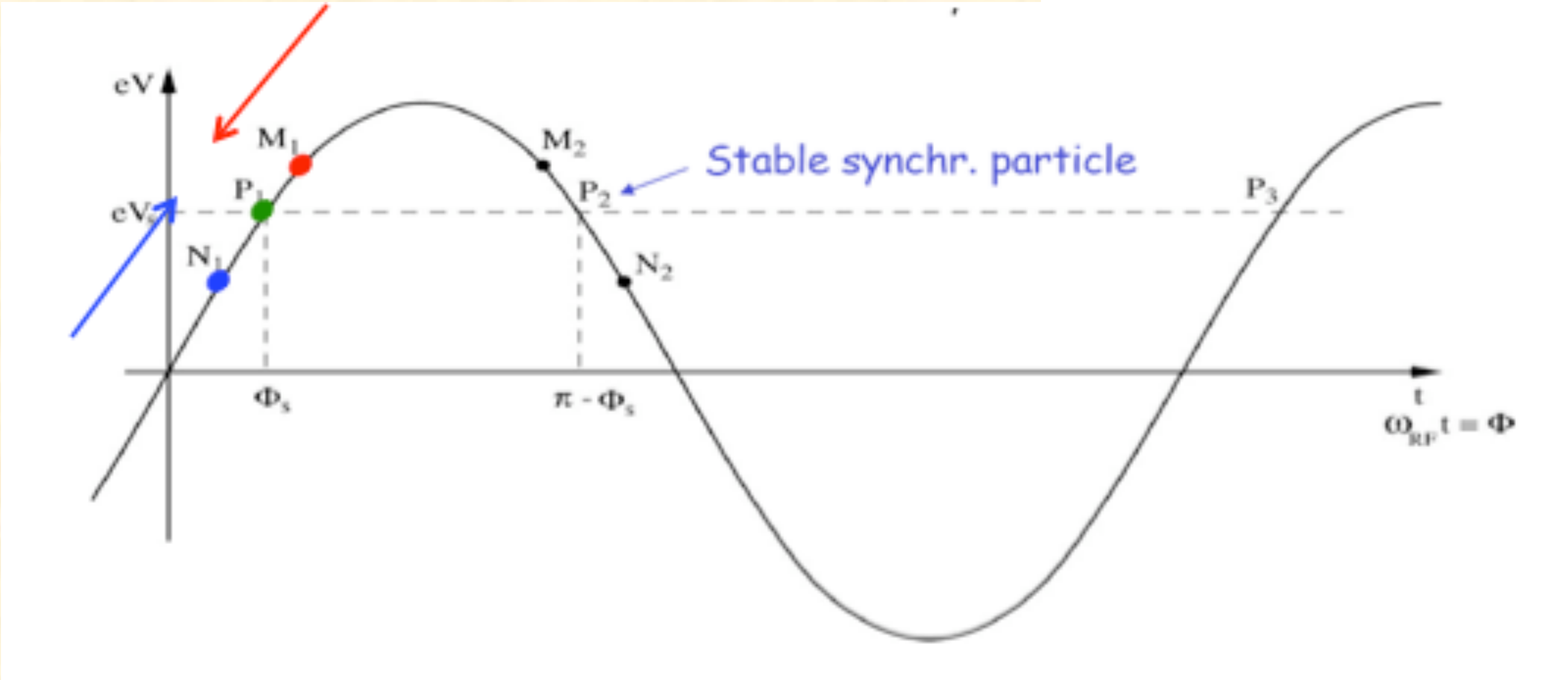
$\Delta p/p \neq 0$ below transition

ideal particle •

particle with $\Delta p/p > 0$ • *faster*

particle with $\Delta p/p < 0$ • *slower*



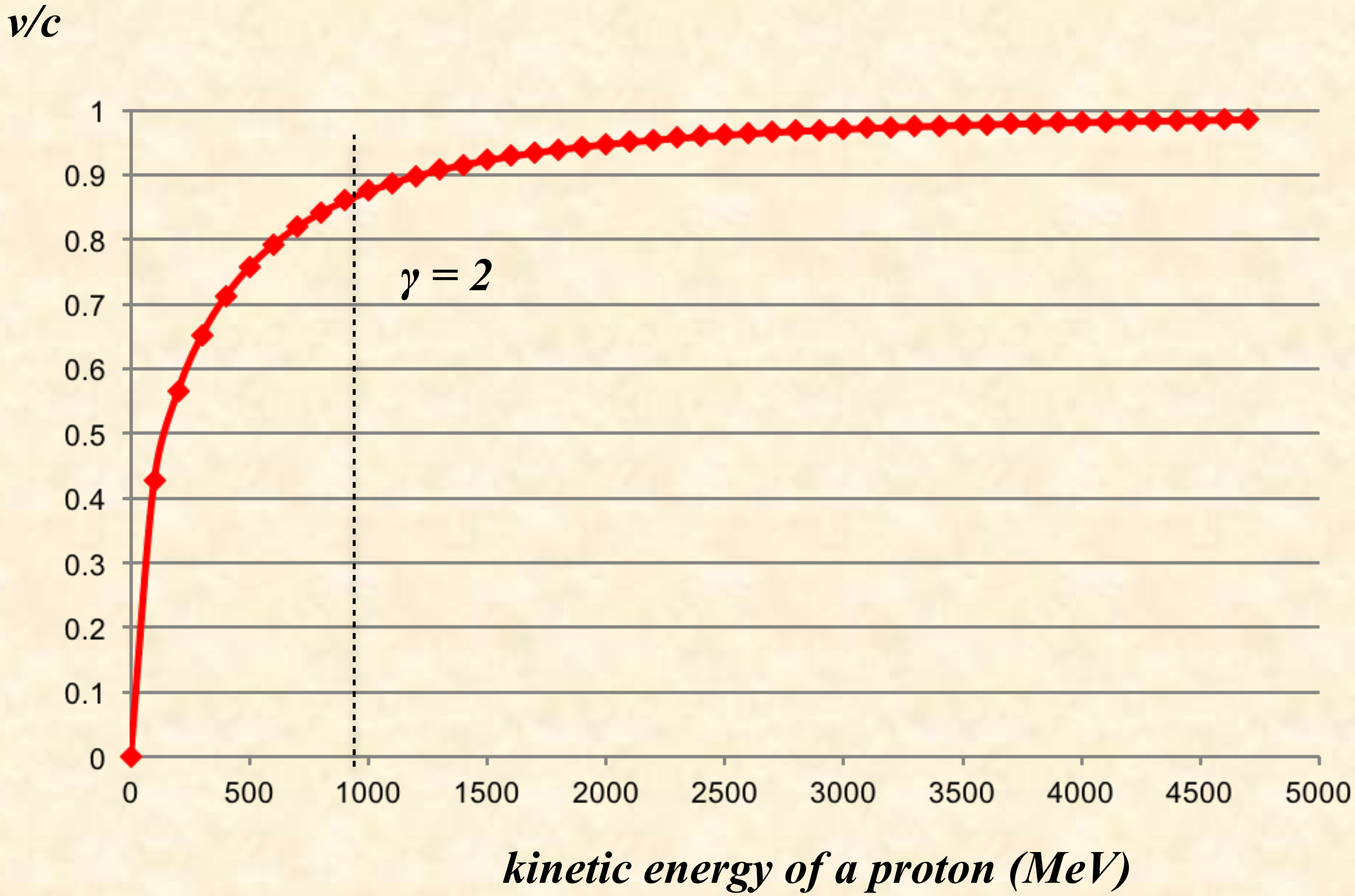


Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0 \cos\phi_s}{E_s}}$ *≈ some Hz*

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$



... some when the particles do not get faster anymore

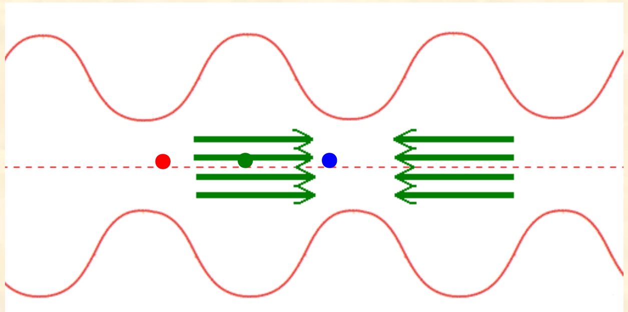
.... but heavier !

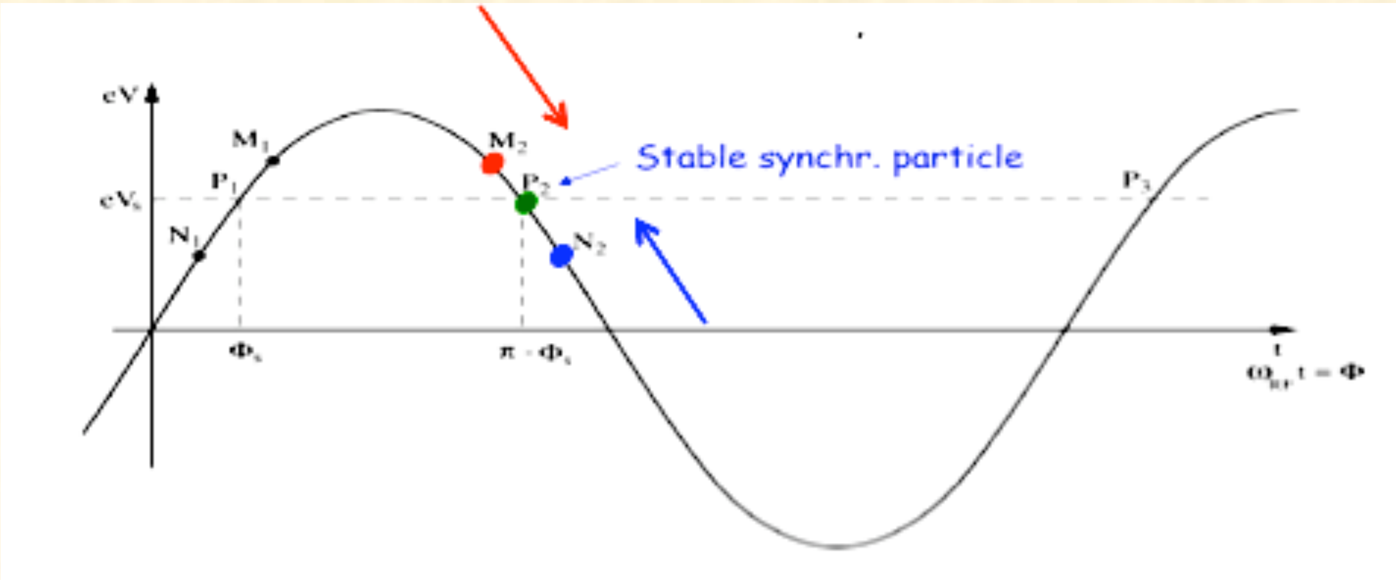
The Acceleration above transition

ideal particle

particle with $\Delta p/p > 0$ *heavier*

particle with $\Delta p/p < 0$ *lighter*



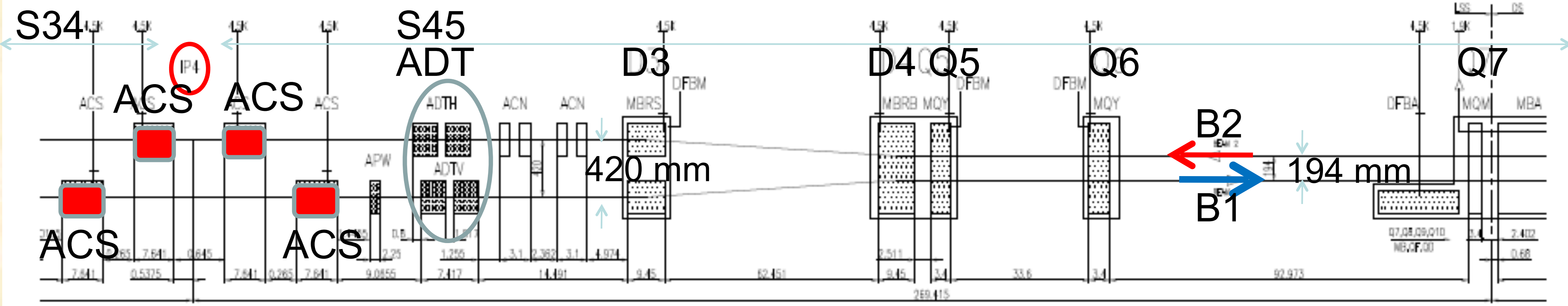


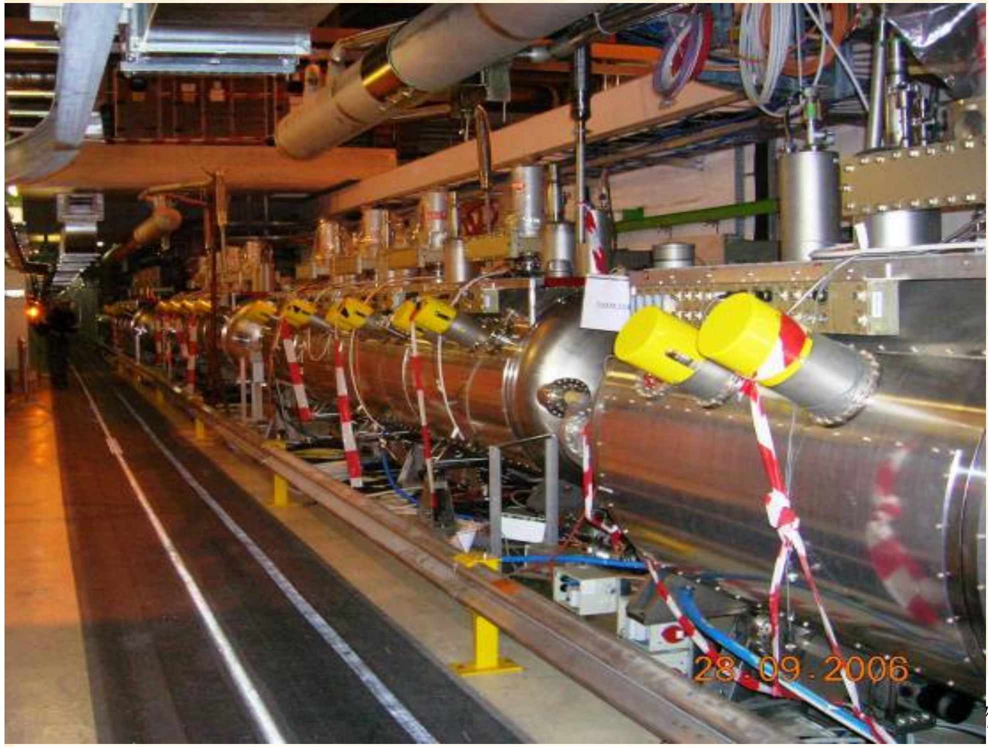
Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4





Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	*10^{-3}*	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*MHz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

1.) Where are we ?

**** Standard Model of HEP***
**** Higgs discovery***

Merci