

ECC 2022 Tutorial – Solution

1. Thermal conduction through sensor wires

a)

For Copper, you just need to use the given values for the thermal conductivity integral, as often found in tables. Please remember that a four-wire circuit is used.

$$\dot{Q}_{Cu} = 4 \frac{A}{L} \left[\int_{4K}^{77K} \lambda(T) dT + \int_{77K}^{300K} \lambda(T) dT \right] = 1.1 mW$$

In the case of Manganin, you need integrate the polynomial given for thermal conductivity yourself.

$$\int_{4K}^{300K} \lambda(T) dT \approx 8766 \frac{W}{m}$$

$$\dot{Q}_{Manganin} = 4 \frac{A}{L} \int_{4K}^{300K} \lambda(T) dT = 0.6 mW$$

b)

When the wires are (ideally) anchored to a thermal anchor at 77 K, only the heat flow from 77 K to 4 K need to be taken into account. Assuming that the anchor is located at half the wire length, this gives for Copper:

$$\dot{Q}_{Cu} = 4 \frac{A}{L} \int_{4K}^{77K} \lambda(T) dT \approx 0.93 mW$$

For Manganin, the thermal conductivity integral needs to be calculated again:

$$\int_{4K}^{77K} \lambda(T) dT \approx 1088 \frac{W}{m}$$

$$\dot{Q}_{Manganin} = 4 \frac{A}{L} \int_{4K}^{77K} \lambda(T) dT \approx 0.15 mW$$

2. Cryogenic vacuum seal

a)

Using values for thermal contractions according to the diagrams given in the problem, the pieces shrink by (only considering the 4 mm length of the seal):

$$\Delta l_{SS} = \frac{\Delta l}{l} \cdot l = -3 \cdot 10^{-3} \cdot 4 \text{ mm} = -0.012 \text{ mm}$$

$$\Delta l_{Al} = \frac{\Delta l}{l} \cdot l = -4.2 \cdot 10^{-3} \cdot 4 \text{ mm} = -0.0168 \text{ mm}$$

Hence, the following gap will occur at 4.2 K:

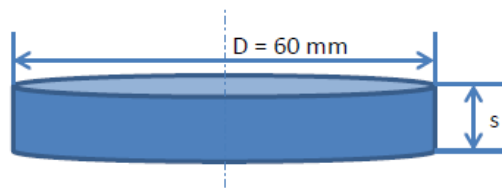
$$s = \Delta l_{SS} - \Delta l_{Al} = 4.8 \cdot 10^{-3} \text{ mm}$$

Alternatively, you can directly calculate s as:

$$s = (4.2 \cdot 10^{-3} - 3 \cdot 10^{-3}) \cdot 4 \text{ mm} = 4.8 \cdot 10^{-3} \text{ mm}$$

The cross-sectional area of the gap (leakage) is calculated as follows:

$$A = \pi D \cdot s = 188.5 \text{ mm} \cdot 4.8 \cdot 10^{-3} \text{ mm} = 0.9 \text{ mm}^2$$



b)

a. Working at 77 K instead of 4 K

$$s = (4.0 \cdot 10^{-3} - 2.9 \cdot 10^{-3}) \cdot 4 \text{ mm} = 4.4 \cdot 10^{-3} \text{ mm}$$

b. Copper seal instead of Aluminum seal

$$s = (3.2 \cdot 10^{-3} - 2.0 \cdot 10^{-3}) \cdot 4 \text{ mm} = 8 \cdot 10^{-4} \text{ mm}$$

c. Seal height reduced to 2 mm

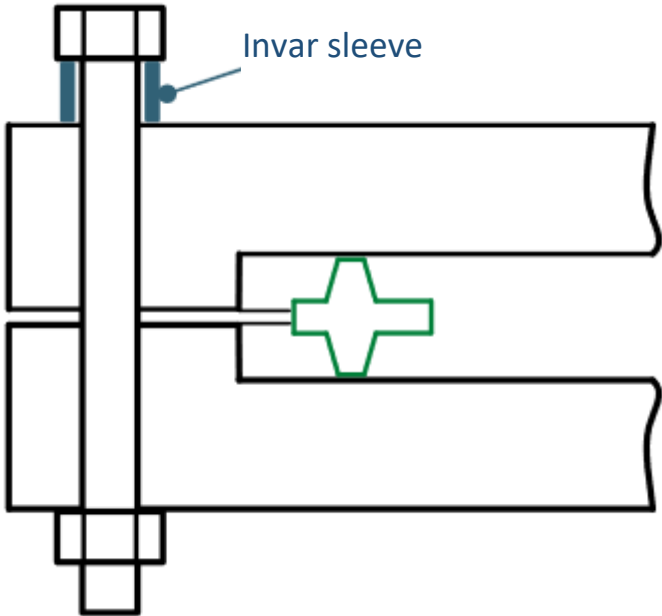
$$s = (4.2 \cdot 10^{-3} - 3 \cdot 10^{-3}) \cdot 2 \text{ mm} = 2.4 \cdot 10^{-3} \text{ mm}$$

c)

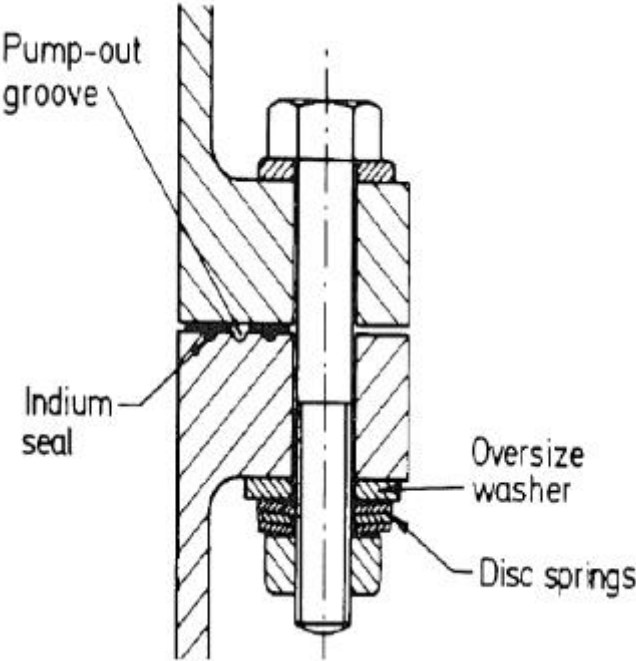
As you can see, none of the above approaches can solve the problem in principal.

Here are some proposals for solving the problem.

You could design the assembly such that the flanges **do not touch each other** and the differential contraction of the stainless steel parts (with higher contraction) and the seal (with lower contraction) is compensated by an **invar sleeve** (which as close to no contraction).



Similarly, you could use **disc springs** (e.g. Belleville springs) to introduce some elasticity into the flange assembly which can also compensate the differential shrinking of the parts.



As a third solution, you could use **C-ring seals** which introduce elasticity on the side of the seal. The seal can now extend its height in order to compensate for differential contraction. Like this, even with the flanges touching each other there won't be any leakage gap.

