# Fundamentals of cryocoolers

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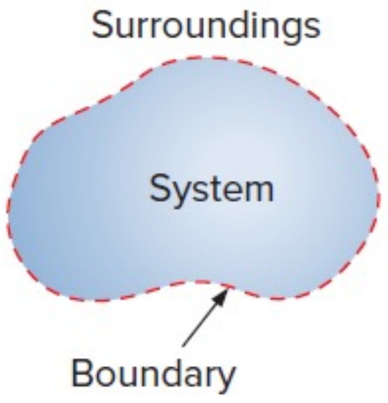
# Content

- Recap thermodynamic properties
- Recap first and second law
- Vapor compression cooler
- Linde Hampson cryocooler
- Generalization for cryocoolers
- Introduction to regenerative type

# Thermodynamic properties






★ Note: Thermodynamic laws and equations always refer to system variables.

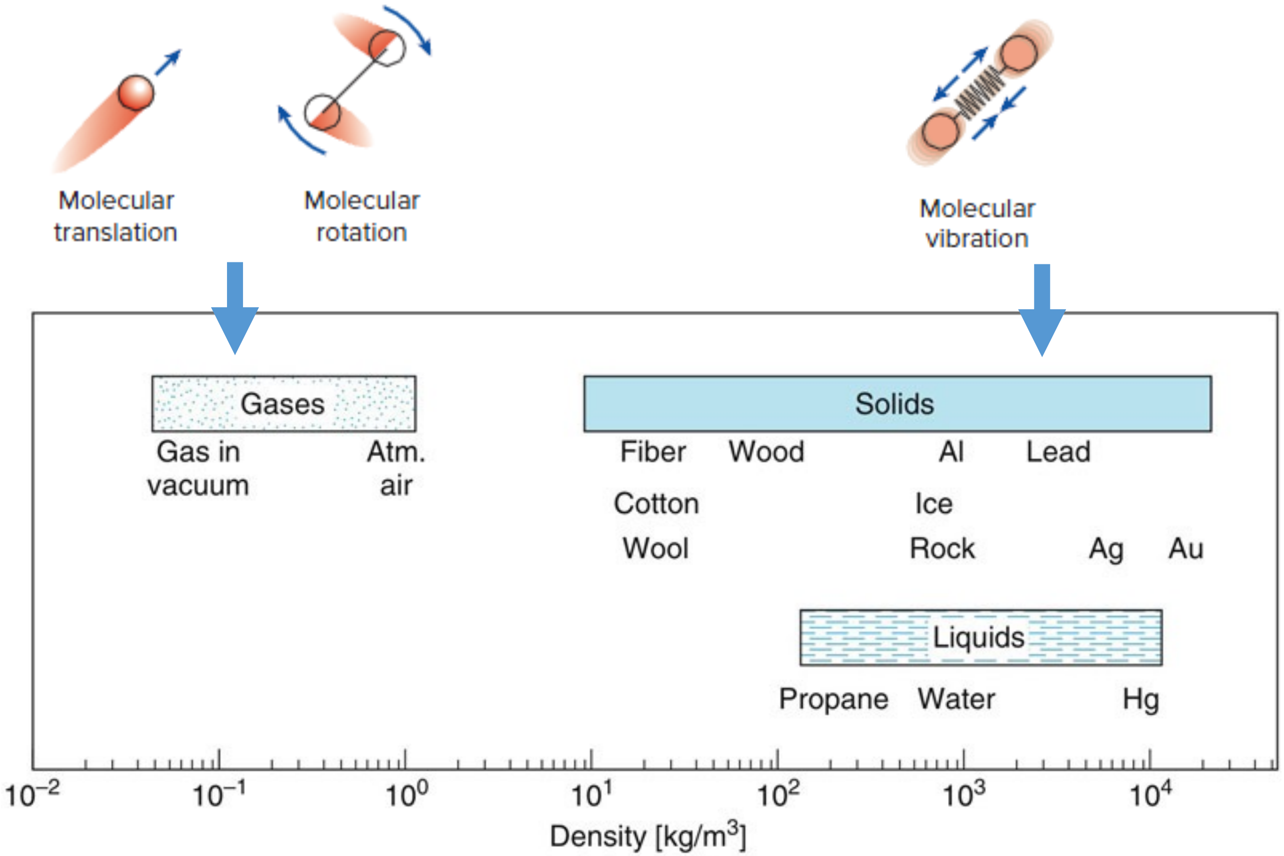
System or state variables

- Pressure, ***p***
- Temperature, ***T***
- Volume, ***V*** or density
- Internal Energy, ***U***
- Entropy, ***S***
- Enthalpy, ***H***
- Helmholtz free energy, ***F***
- Gibbs free energy, ***G***

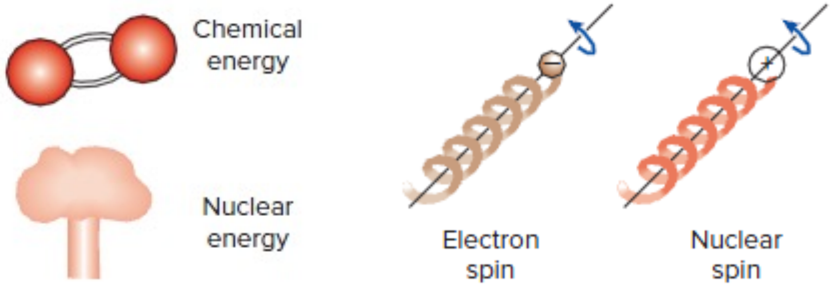
*and more depending on the domain, e.g. electrical system; charge and potential*

# Microscopic forms of energy: Internal energy, U

## Thermal

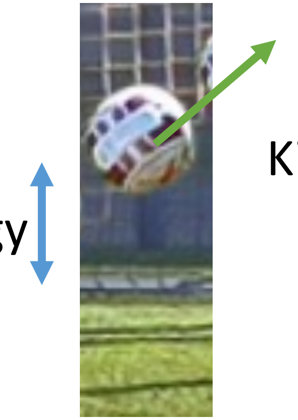



## Chemical + Nuclear



# Energy

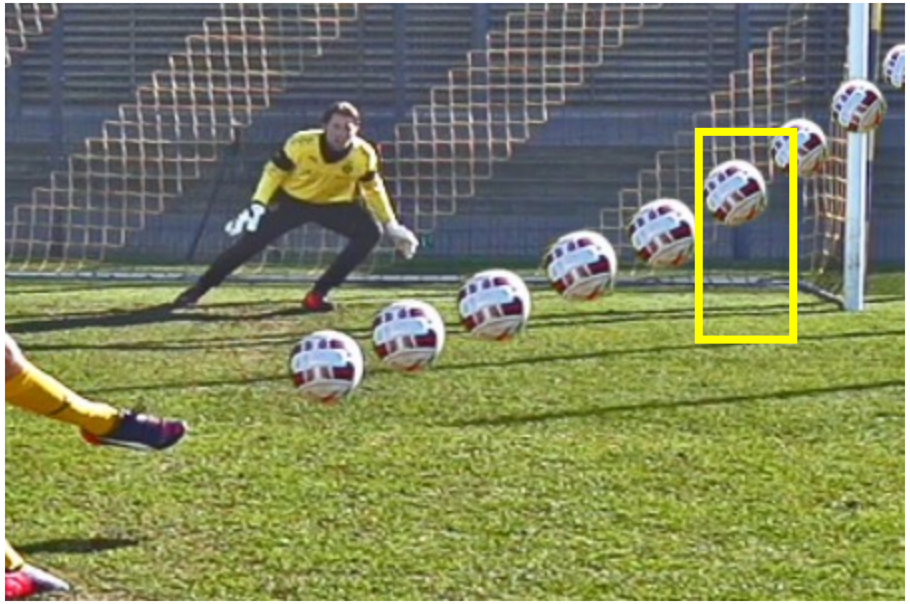
Let us consider the case of a football in action:



Potential energy

Kinetic energy

Internal energy



$$\boldsymbol{E = U + \text{potential energy} + \text{kinetic energy}}$$

# How much energy do you need to create an object?

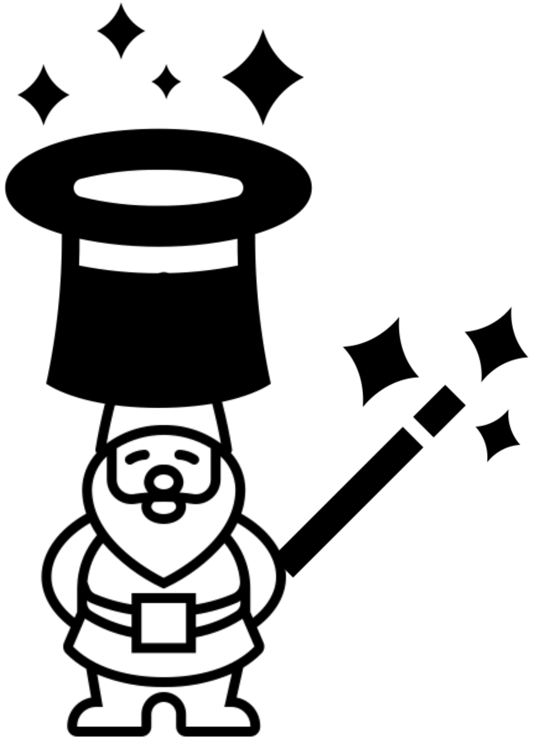
- Neglect energy of rubber
- Balloon is non-elastic
- Treat air as an ideal gas
- Diatomic gas, $f$ = 5


$p_{outside}$ = 1 bar

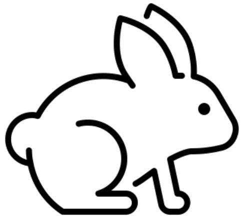
| | | |
|---|---|---|
| Volume | $V$=0 | $V$ = 1 liter |
| Internal energy | $U$=0 | $U$=? J 250 J |
| Work performed | | $W$=$pV$=? J 100 J |
| Total energy required to create the balloon | | $U$+$pV$ 350 J |

**Enthalpy, $H = U + pV$**

# Enthalpy – silly analogy

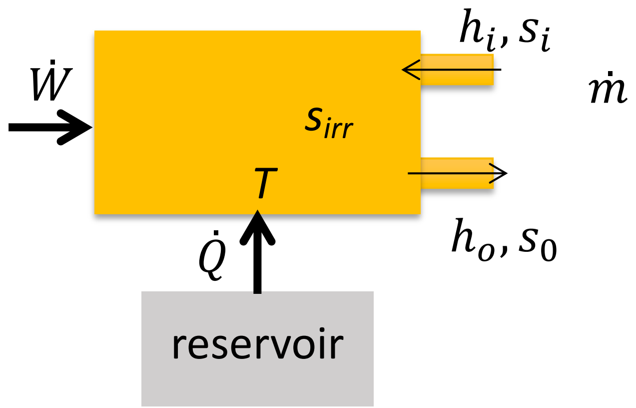


magician



rabbit

# Energy and Entropy balance in an open system




Energy balance (**steady case**)

$$\dot{Q} + \dot{W} = \dot{m}(h_o - h_i)$$

Entropy balance

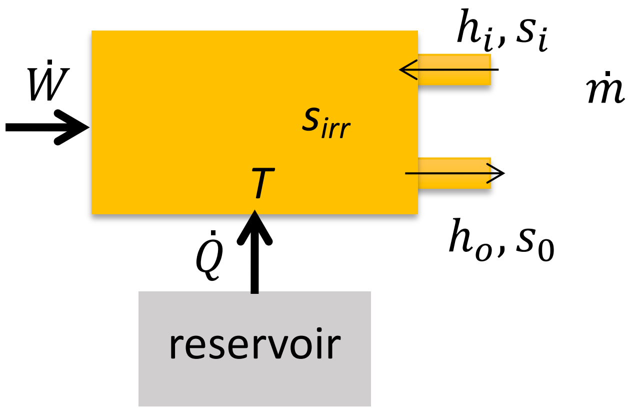
$$\frac{\dot{Q}}{T} + \dot{m}s_i + s_{irr} = \dot{m}s_o$$

What about **unsteady** case?

$$\dot{Q} + \dot{W} - \dot{m}h_o + \dot{m}h_i = \frac{dE}{dt}$$

$$\frac{\dot{Q}}{T} + \dot{m}s_i + s_{irr} - \dot{m}s_o = \frac{dS}{dt}$$

# Energy and Entropy balance in an open system (2)

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$$\dot{Q} + \dot{W} - \dot{m}h_o + \dot{m}h_i = \frac{dE}{dt}$$

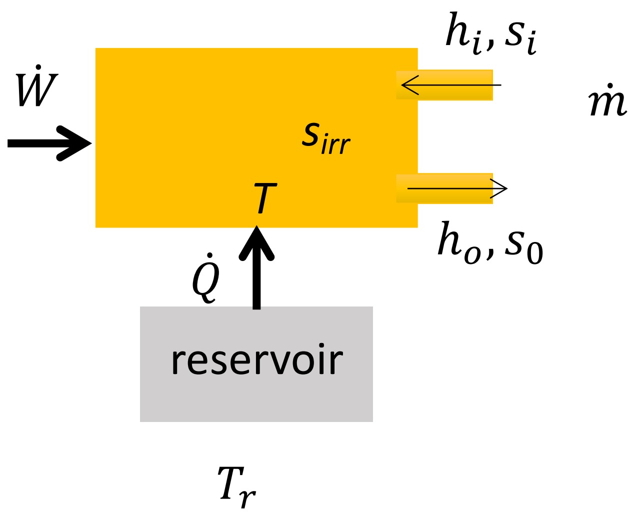
$$\frac{\dot{Q}}{T} + \dot{m}s_i + s_{irr} - \dot{m}s_o = \frac{dS}{dt}$$

**A question:** What happens when you open this valve?

Camping gas burner

a. Nothing, the gas escapes and that's a waste
b. The atmosphere gets a bit warmer (but you don't notice that)
c. The can gets colder (until water condenses)
d. The gas that escapes gets warmer (until you may have spontaneous combustion, risky!)

# Work performed by a compressor




Note: Surrounding temperature

Energy balance (**steady case**)

$$\dot{Q} + \dot{W} = \dot{m}(h_o - h_i)$$

Entropy balance

$$\frac{\dot{Q}}{T_r} + \dot{m}s_i + s_{irr} = \dot{m}s_o$$

$$\dot{W} = \dot{m}([h_o - T_r s_o] - [h_i - T_r s_i]) + T_r s_{irr}$$

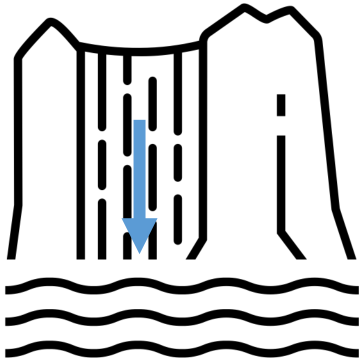
***Exergy, e = H-$T_{surr}$S***

$$\dot{W} = \dot{m}([h_o - T s_o] - [h_i - T s_i]) + T s_{irr}$$

***Gibbs free energy, G = H-TS***

Comment on these discoveries (generalize)

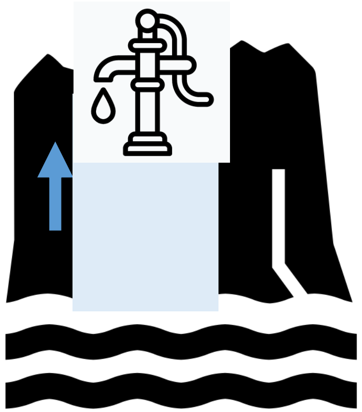
# Gibbs Free energy : silly analogy



**Spontaneous process**

**Spontaneous process**
**Work extraction**

⭐Maximum work

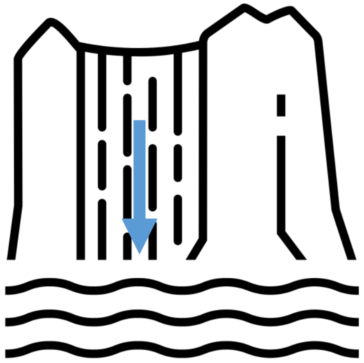


**Non-spontaneous process**
**Work input**

⭐Minimum work

**<u>Energy minimization……..</u>**

# Energy examples

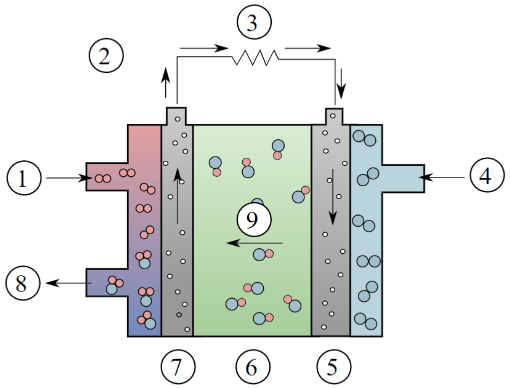


**Spontaneous process**

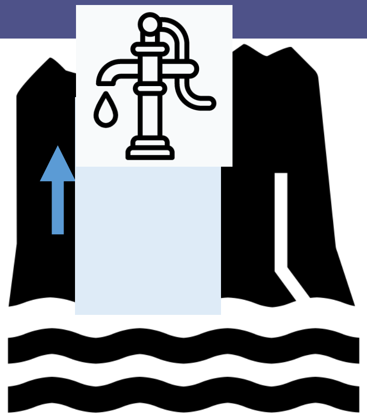
$H_2 + 0.5O_2 \rightarrow H_2O$

**Spontaneous process**
**Work extraction**

★Maximum work

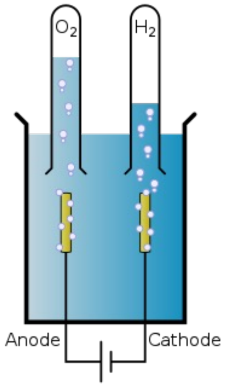



$H_2 + 0.5O_2 + 2e^- \rightarrow H_2O + 2e^-$



**Non-spontaneous process**
**Work input**

★ Minimum work




$H_2O + 2e^- \rightarrow H_2 + 0.5O_2 + 2e^-$

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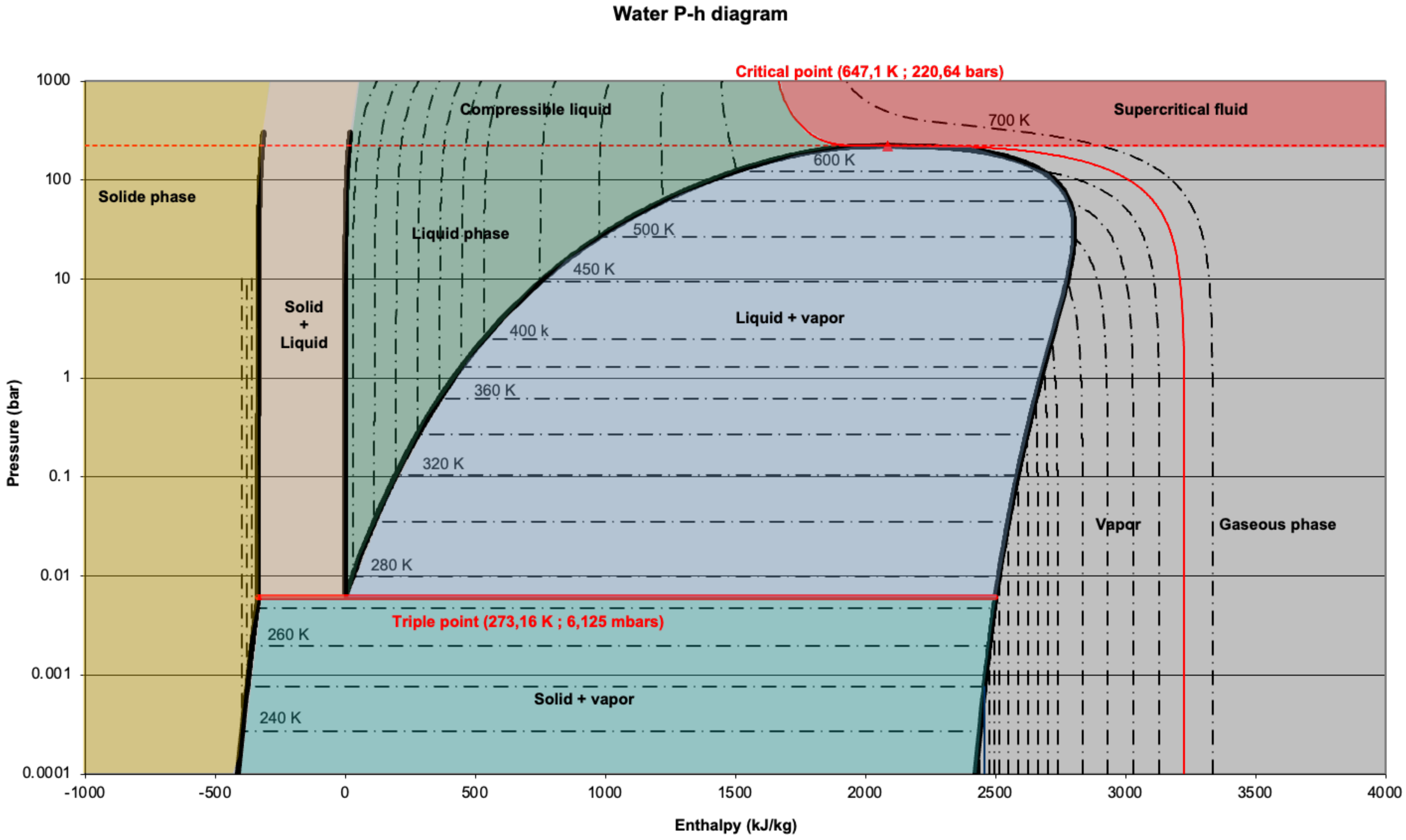
**Combustion** $\Delta G$ = -237.13 kJ

**Fuel cell** $\Delta G$ = -237.13 kJ

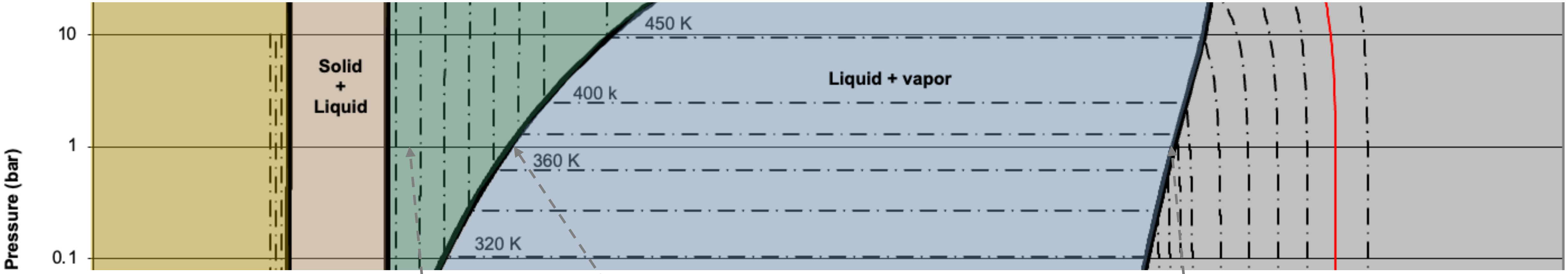
**Electrolyzer** $\Delta G$ = 237.13 kJ

# Phase diagram - Water




Question: deltaG in a phase change process, diamonds?

# Phase diagram - Water




298.15 K, 1 bar
h:104.92 kJ/kg
s :0.3672 kJ/kg/K
v :0.001003 $m^3$/kg

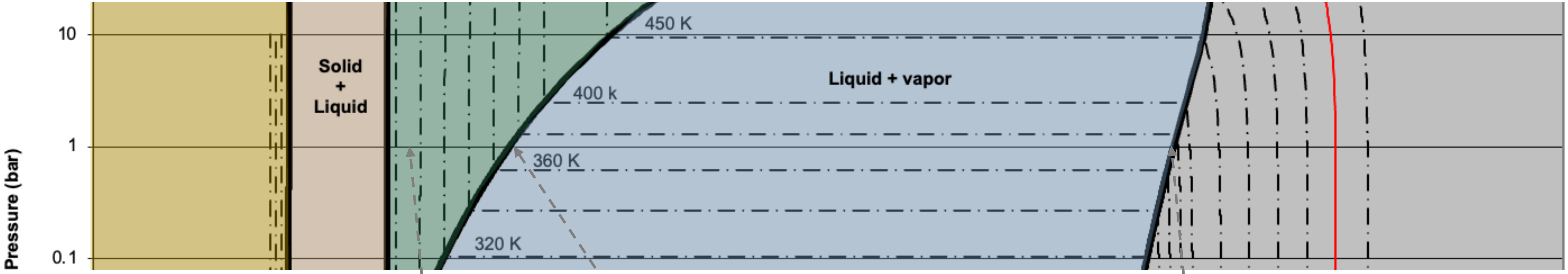
1 bar (sat liquid), 372.76 K
h: 417.50 kJ/kg
s: 1.3028 kJ/kg/K
v: 0.0010432 $m^3$/kg

1 bar (sat vapor), 372.76 K
h: 2674.9 kJ/kg
s: 7.3588 kJ/kg/K
v: 1.6939 $m^3$/kg

| | | |
|---|---|---|
| Change in enthalpy | 312.58 kJ/kg | 2257.4 kJ/kg |
| Change in entropy | 0.9356 kJ/kg/K | 6.056 kJ/kg/K |

Explain the meaning of internal energy, enthapy

# Phase diagram - Water

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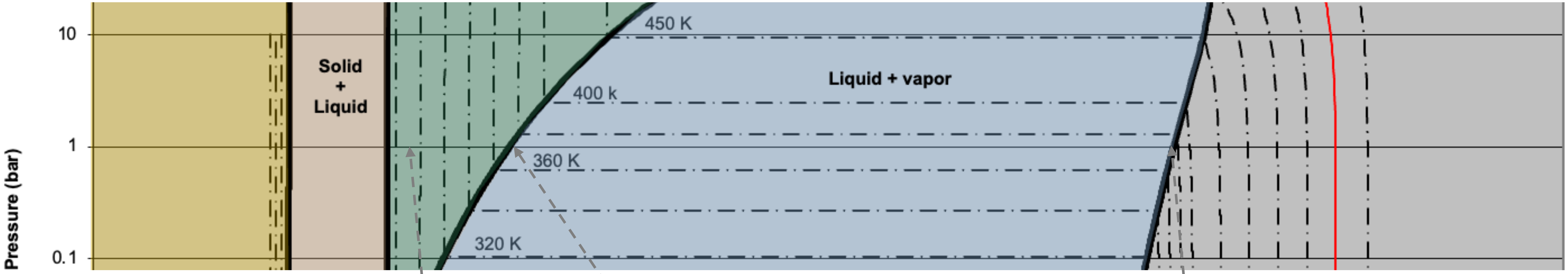
| pdV work? | | |

# Phase diagram - Water

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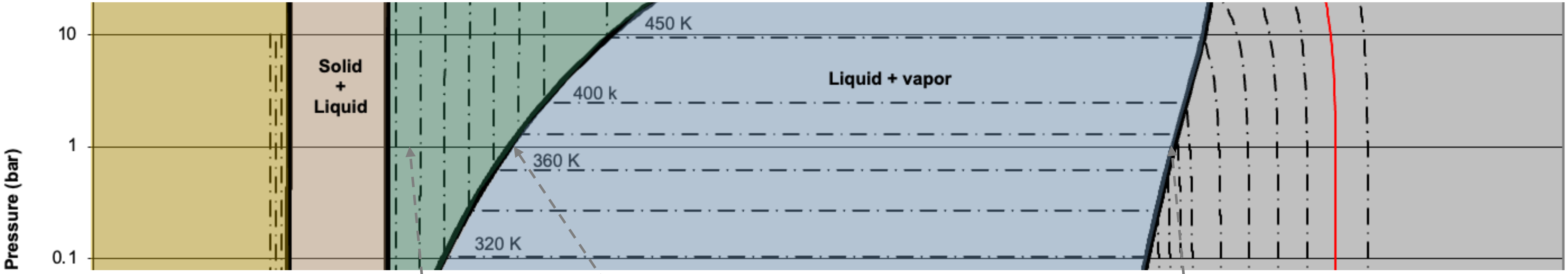
# Phase diagram - Water

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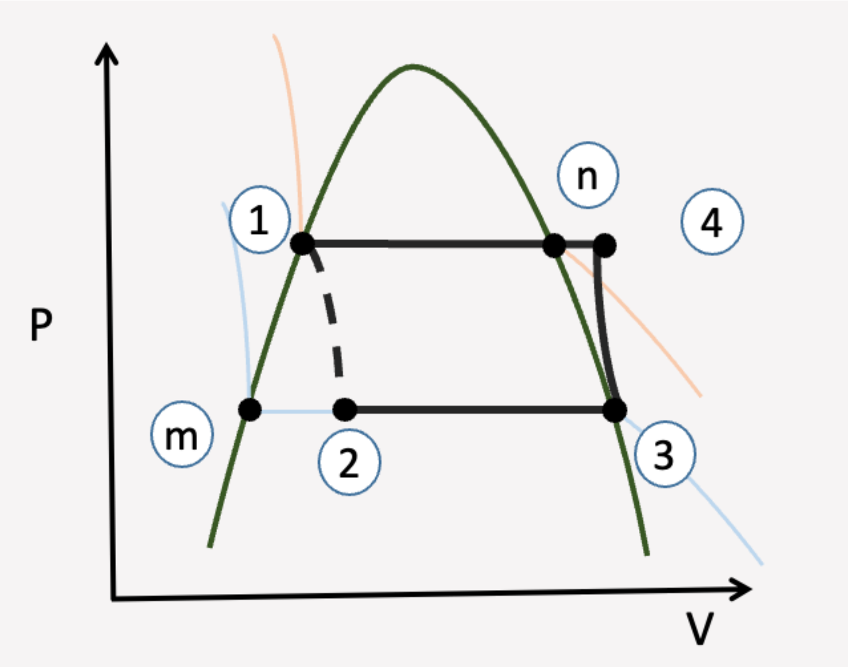
| -pdV work? | -0.00402 kJ/kg | -169.28568 kJ/kg |
| Change in internal energy | 312.58 kJ/kg | 2088.11 kJ/kg |

# Exercise I: Vapor compression (Freezer)




Fill the table with +, - or 0 (zero)

Counter clockwise

**34 (compressor):** adiabatic process
**41 (condenser):** heat from system to surroundings
constant pressure; **don't use $Q = mc_p\Delta T$**
**12 (throttle):** constant enthalpy process, why?
**23 (evaporator):** heat from surroundings to system (cooling)

[Steady operation; Sign convention: energy transfer to the system is positive]

| Process | W (J) {external work} | Q(J) | ΔH(J) | $\Delta S_{sys}$(J/K) |
|---|---|---|---|---|
| 1-> 2 | 0 | 0 | 0 | + |
| 2-> 3 | 0 | + | + | + |
| 3-> 4 | + | 0 | + | 0 |
| 4-> 1 | 0 | - | - | - |
| | | | | |
| Cycle | + | - | 0 | 0 |

# Cycle: Simple observations

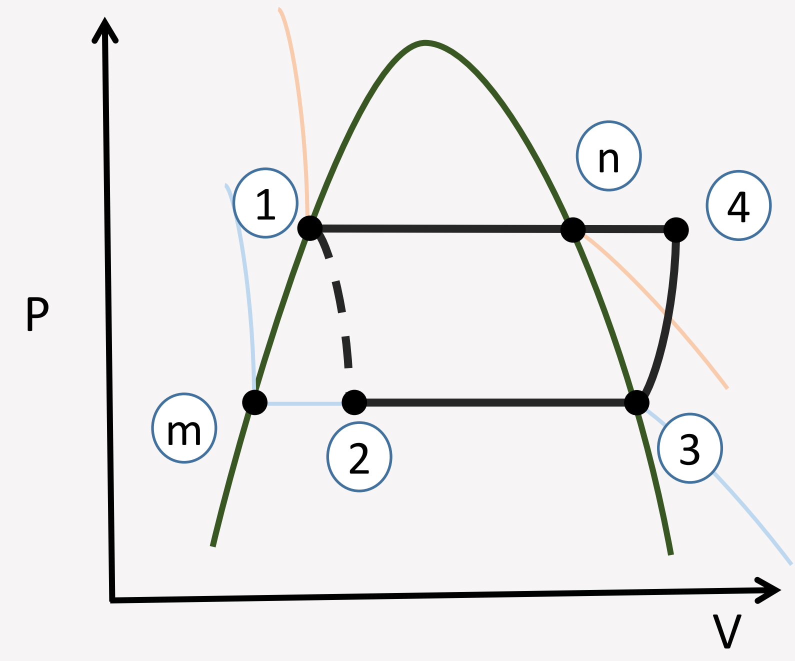
- The state variable change after one complete cycle is zero. For e.g.$\sum \Delta H = 0$; $\sum \Delta U = 0$; $\sum \Delta S = 0$; $\sum \Delta p = 0$; $\sum \Delta T = 0$ etc.

- System

$$0 = \sum Q + \sum W$$

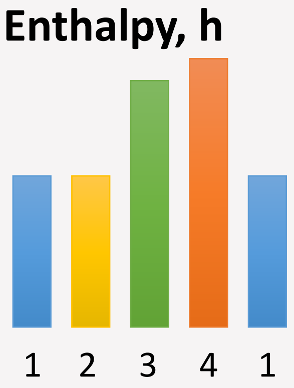
Note: this is a system landscape.
Surroundings variables are not represented.

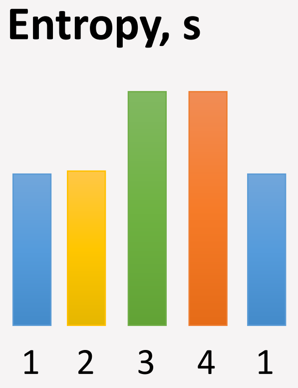
# Freezer

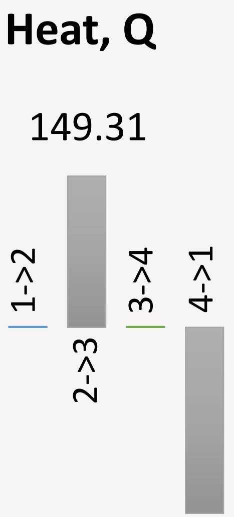


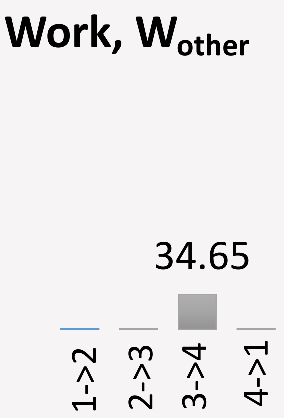

R134a

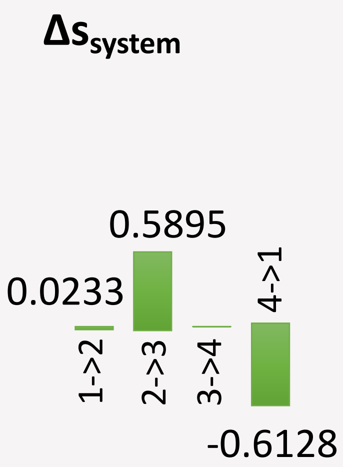
| | h (kJ/kg) | s(kJ/kg-K) |
|---|---|---|
| 1 | 237.19 | 1.1287 |
| 2 | 237.19 | 1.1517 |
| 3 | 386.50 | 1.7415 |
| 4 | 421.15 | 1.7415 |
| m | 173.44 | 0.9000 |
| n | 413.27 | 1.7156 |



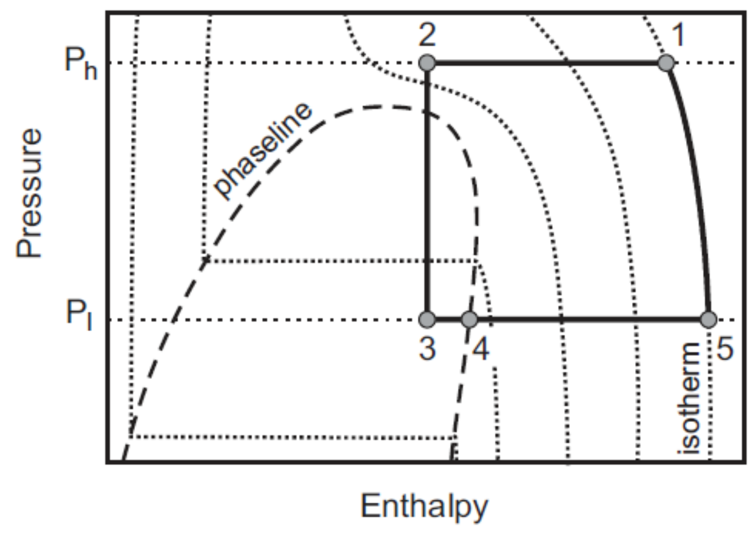



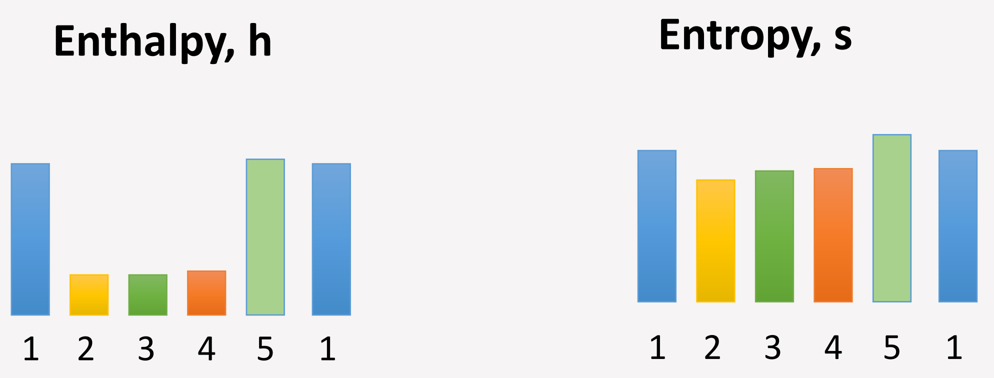



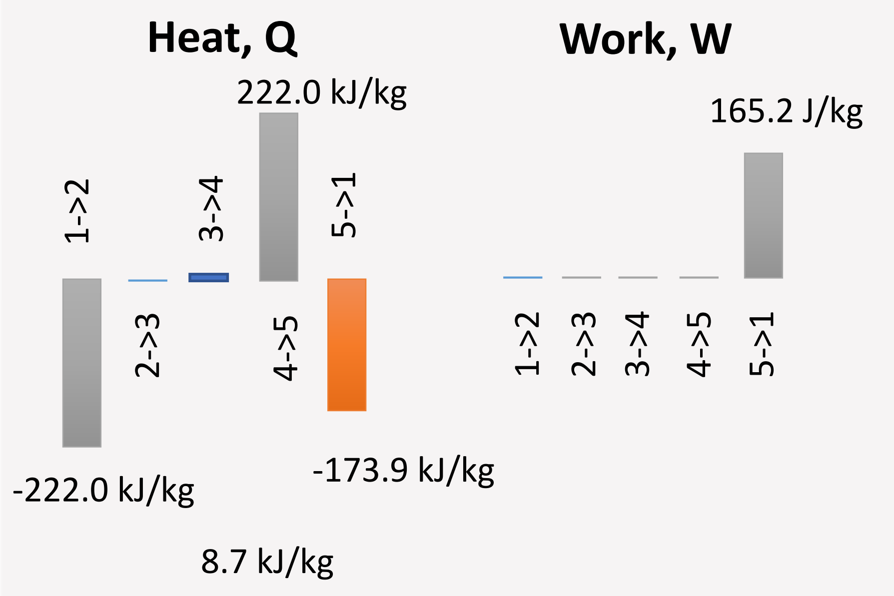
# Linde-Hampson cycle (analysis)

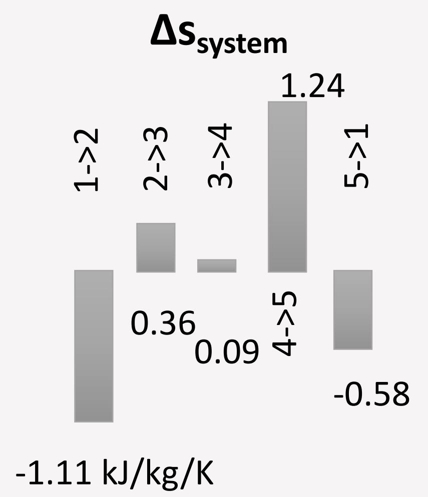



Nitrogen

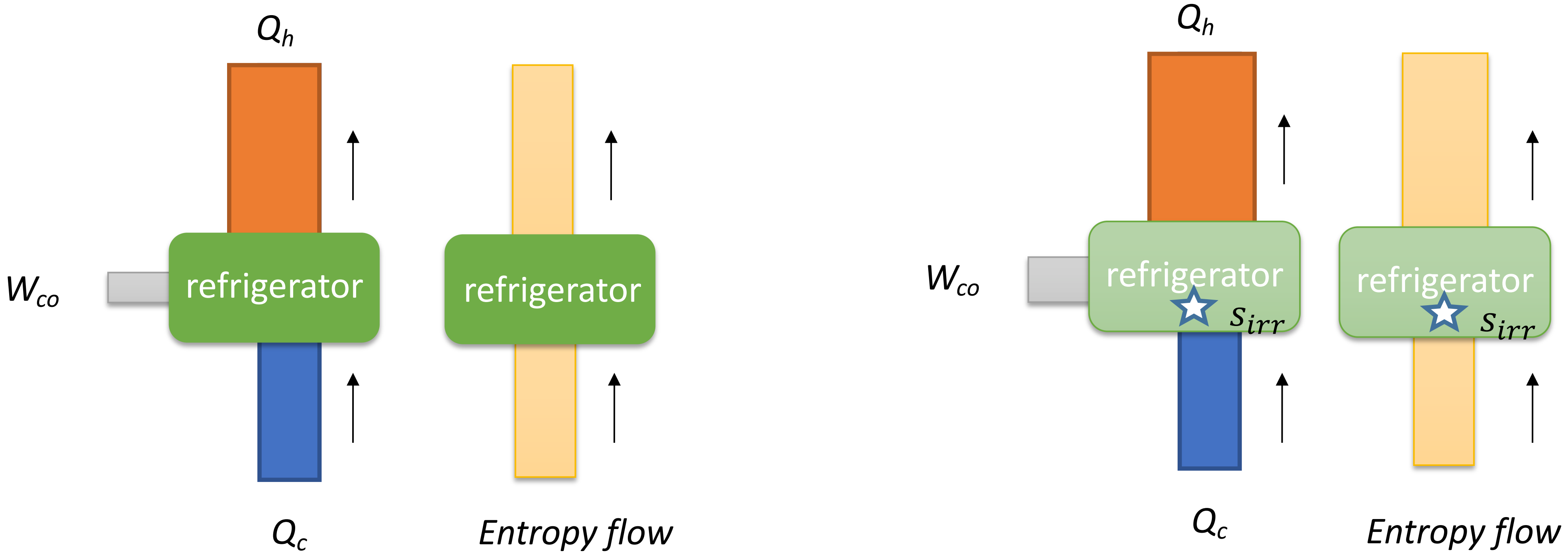
| State | P (bar) | T (K) | h (kJ/kg) | s(kJ/kgK) |
|---|---|---|---|---|
| 1 | 50.0 | 300 | 301.00 | 5.6522 |
| 2 | 50.0 | 142 | 79.10 | 4.5421 |
| 3 | 7.8 | 100 | 79.10 | 4.8991 |
| 4 | 7.8 | 100 | 87.80 | 4.9858 |
| 5 | 7.8 | 300 | 309.70 | 6.232 |



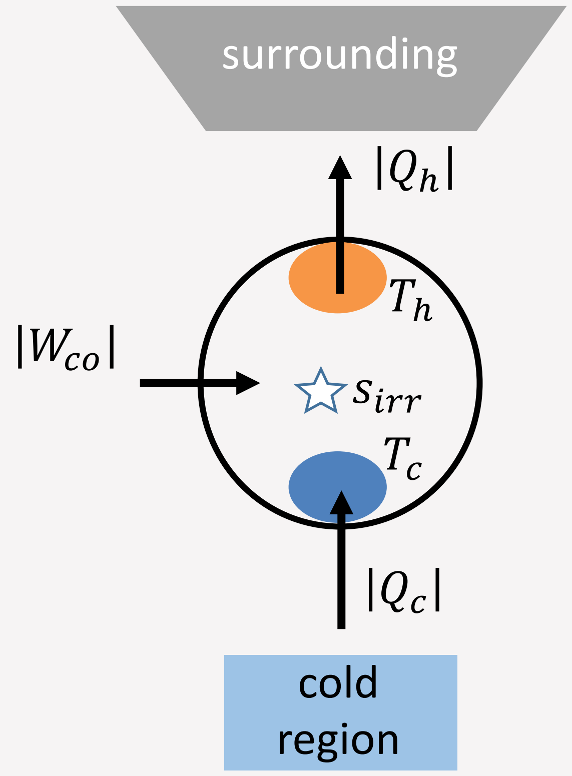



# Thermodynamic cycle



# Generic cooler (energy flows)




Energy balance (steady case) – for a cycle

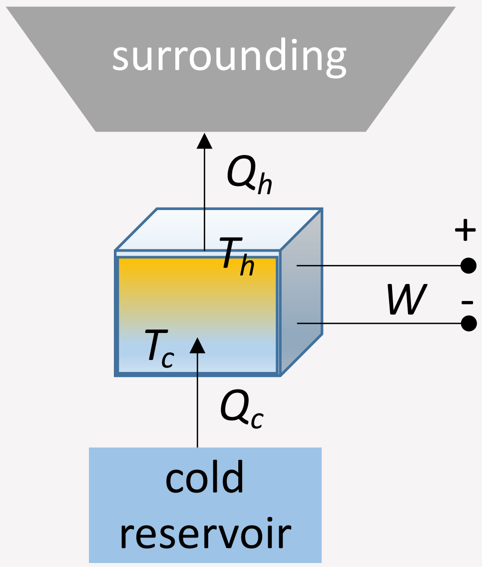
$$|Q_c| + |W_{co}| - |Q_h| = 0$$

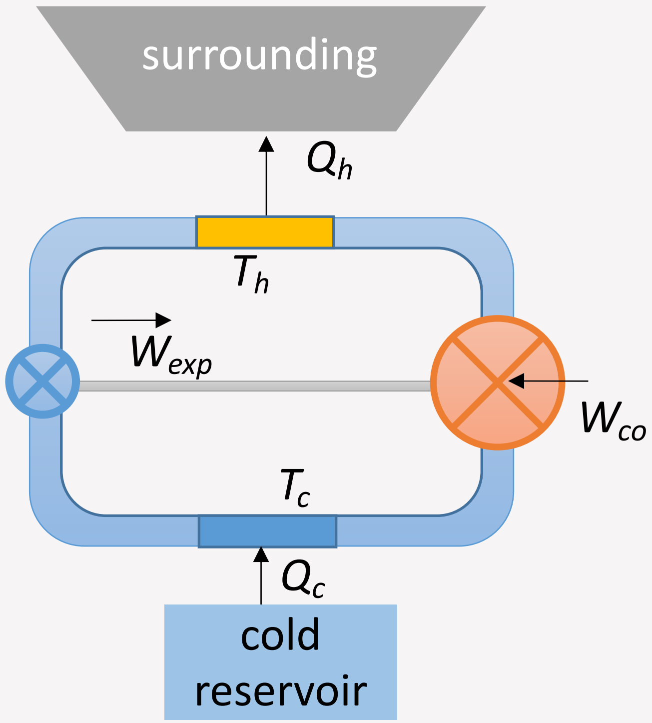
Entropy balance (steady case)

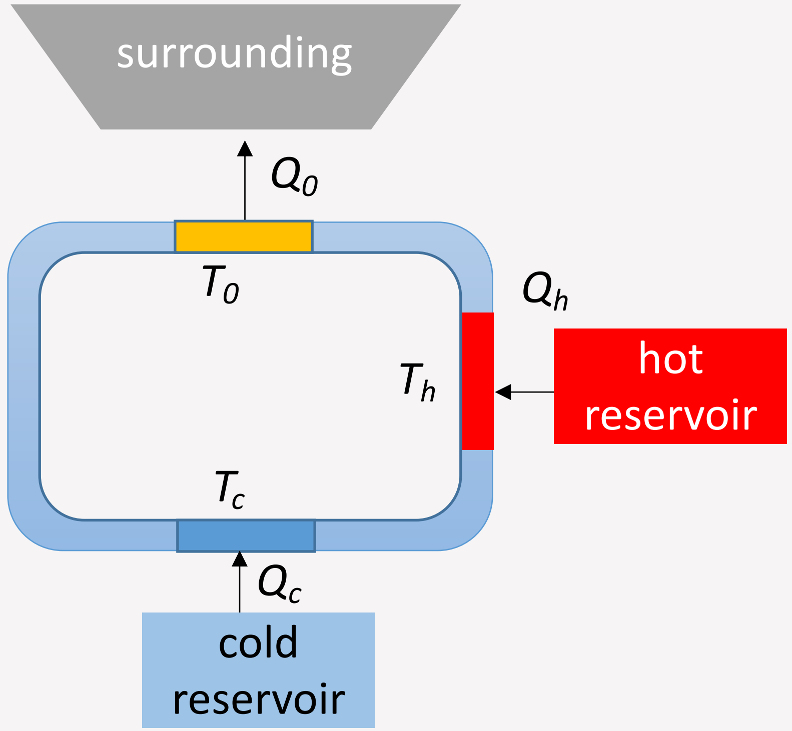
$$\frac{|Q_c|}{T_c} + s_{irr} - \frac{|Q_h|}{T_h} = 0$$

$$COP = \frac{|Q_c|}{|W_{co}|} = \frac{T_c}{T_h - T_c}\left[1 - \frac{T_h s_{irr}}{W_{co}}\right] = COP_{Carnot}\left[1 - \frac{T_0 \dot{S}_{irr}}{\dot{W}_{co}}\right]$$

# Apply first and second laws



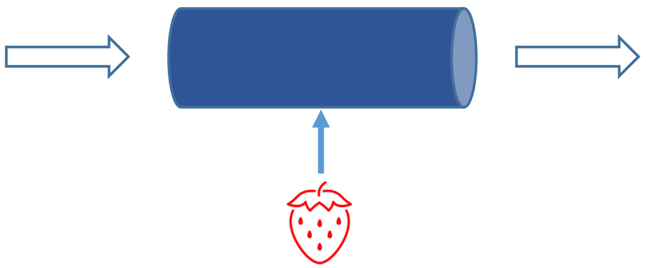
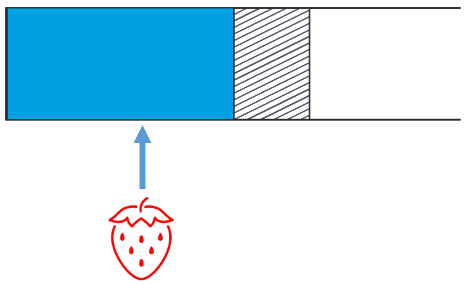




Note: absolute quantities

| | | |
|---|---|---|
| $|Q_c| + |W| - |Q_h| = 0$ | | |
| $\frac{|Q_c|}{T_c} - \frac{|Q_h|}{T_h} = 0$ | | |

# Examples of heat transfer and entropy change

| | | **Energy change** | **Entropy change** |
|---|---|---|---|
| Boiling liquid |  | | |
| Cold gas flow |  | | |
| Gas expansion |  | | |

# Examples of heat transfer and entropy change

**Energy change**

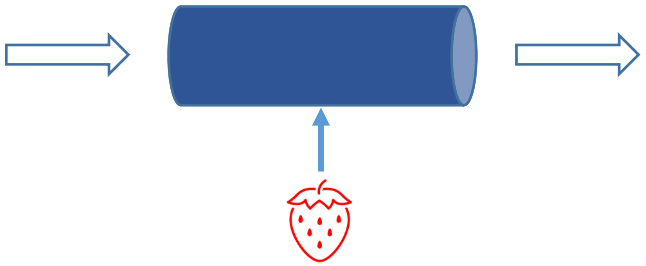
**Entropy change**

Boiling liquid

$$Q = \dot{m}\left(h_{vapor} - h_{liquid}\right)$$

$$\Delta S = \left(s_{vapor} - s_{liquid}\right)$$

Cold gas flow



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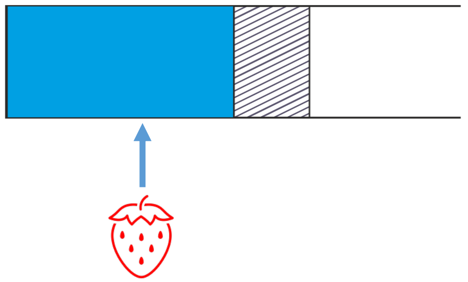

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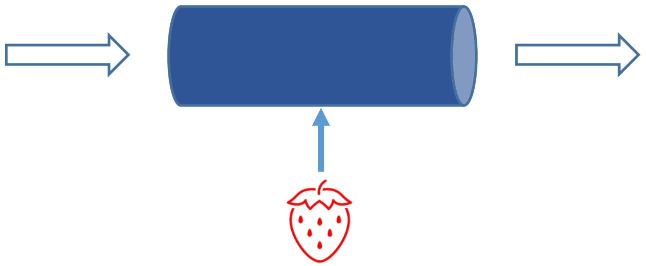
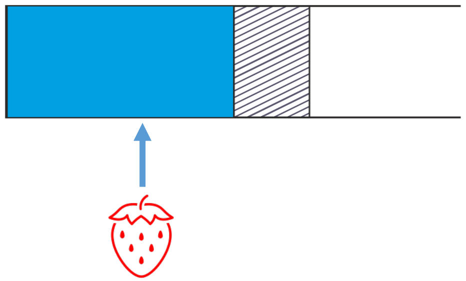
Cold gas flow



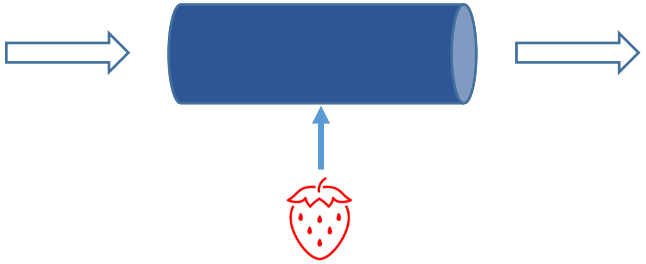
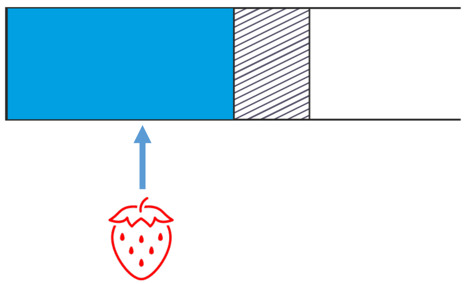
$$Q = \dot{m}\left(h_{gas,out} - h_{gas,in}\right)$$

$$\Delta S = \left(s_{gas,out} - s_{gas,in}\right)$$

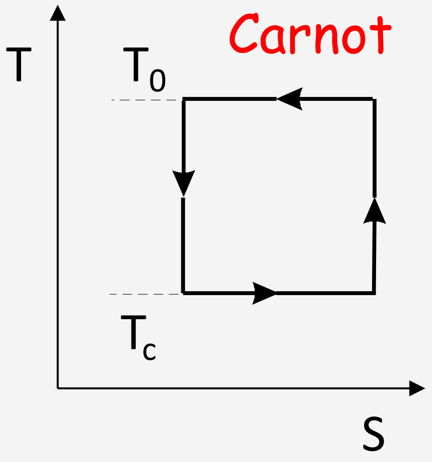
Gas expansion

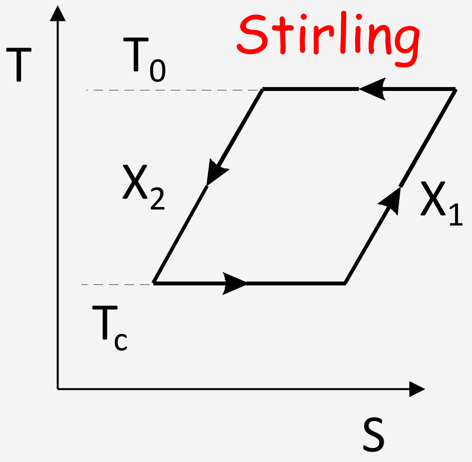


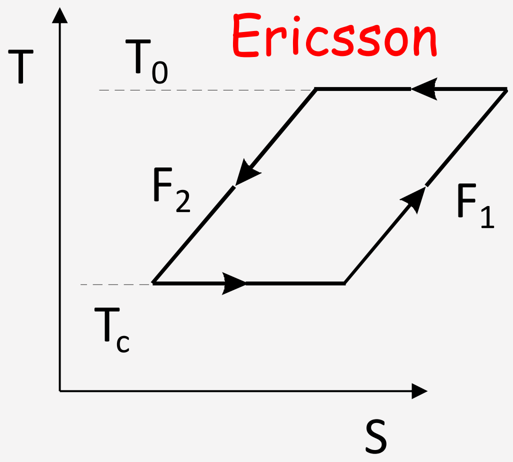
# Examples of heat transfer and entropy change

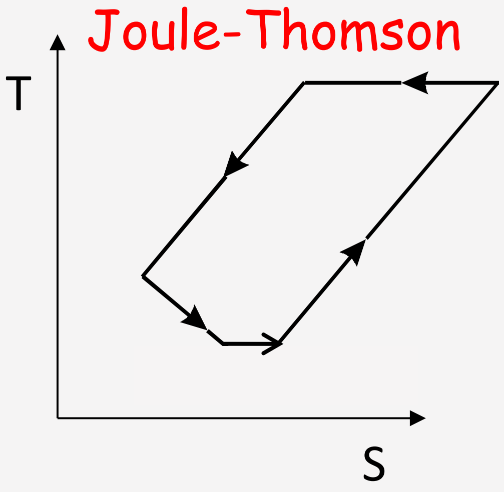
| | | **Energy change** | **Entropy change** |
|---|---|---|---|
| Boiling liquid |  | $Q = \dot{m}\left(h_{vapor} - h_{liquid}\right)$ | $\Delta S = \left(s_{vapor} - s_{liquid}\right)$ |
| Cold gas flow |  | $Q = \dot{m}\left(h_{gas,out} - h_{gas,in}\right)$ | $\Delta S = \left(s_{gas,out} - s_{gas,in}\right)$ |
| Gas expansion |  | $Q = -W$ | $\Delta S = \int_{V_1}^{V_2} \frac{pdV}{T} = \int_{V_1}^{V_2} \frac{RdV}{V} = Rln\frac{V_2}{V_1}$ <br> per mole |

# Cycles – key points




Heat transfer

**Steady state**

In a cycle system entropy change is zero

In a cooling process the entropy of the substance will increase

The rule of the game is to reduce the entropy of the substance in a cycle

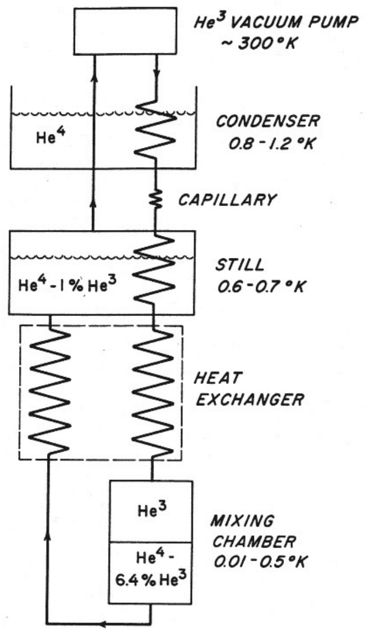
Any irreversibility in a cycle will result in larger heat transfer at the hot side. $S_{irr}$ increases and ….

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c} + s_{irr}$$

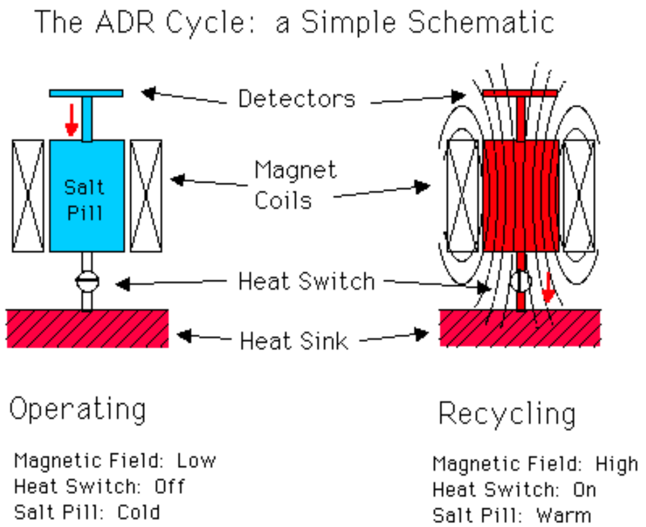
# Other types of cryocoolers

- Cooling principles other than gas {substance} – expansion

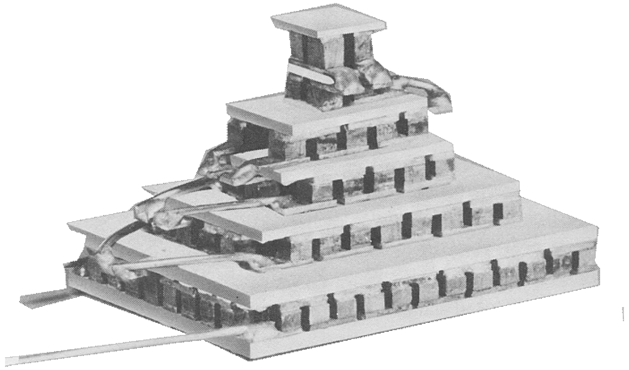
$^3$He-$^4$He Dilution Refrigerator




Magnetic Refrigerator




Thermoelectric



…

# Physical seperation of compression and expansion

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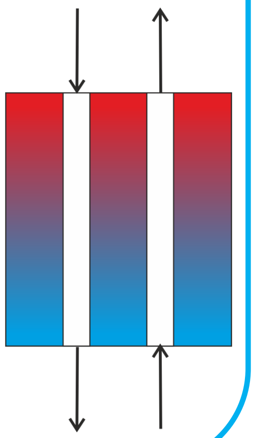
The gas/working substance should be located at the proper temperature reservoirs during isothermal heat rejection or in-take. How to acheive this ?

Use a heat exchanger in between the two temperature reservoirs

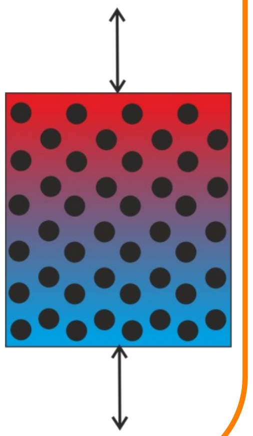
## Recuperative

Separate channels for hot and cold fluids.



## Regenerative

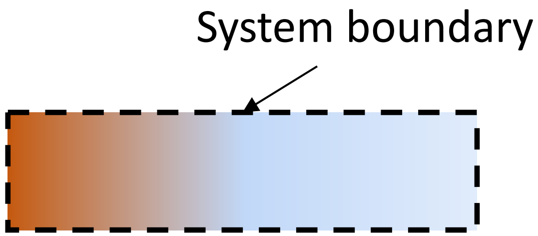
Single flow channel filled with porous material.



# Challenge

**irreversible**

$T_h = 400\ \mathrm{K}$




$T_c = 300\ \mathrm{K}$

$$\frac{kA}{l} = 1\ \ \mathrm{W/K}$$

**Estimate entropy generation rate**

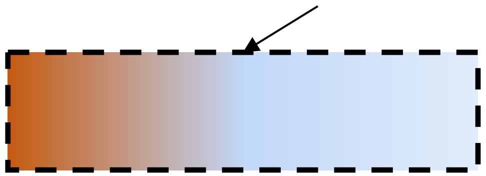
**reversible**

*Think of an ideal concept in which heat transfer over a finite difference will not increase total entropy*

# Challenge

**irreversible**

System boundary

$T_h = 400\ \mathrm{K}$  $T_c = 300\ \mathrm{K}$

$$\frac{kA}{l} = 1\ \ \mathrm{W/K}$$

**Estimate entropy generation rate**

$$Q = \frac{kA}{l}(T_h - T_c) = 100\ \ \mathrm{W}$$

$$\Delta S' = \frac{-Q}{T_h} + \frac{Q}{T_c} = \frac{-100}{400} + \frac{100}{300} = \frac{1}{12}\ \ \mathrm{W.K^{-1}}$$

| | |
|---|---|
| System | $\Delta S = 0$ |
| Surroundings | $\Delta S' > 0$ |
| Total | $\Delta S_{total} = \Delta S + \Delta S' > 0$ |

**reversible**

*Think of an ideal concept in which heat transfer over a finite difference will not increase total entropy*