Multi-Harmonic RF system for final cooling of muon beams

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Outline

- Introduction and motivation
- Example of low frequency RF cavities: 40 and 80 MHz RF for CERN PS
- An idea: multi-harmonic RF system for final cooling
- Conclusions

Example of previous design of final cooling, 1

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MUON COLLIDER FINAL COOLING IN 30-50 T SOLENOIDS*

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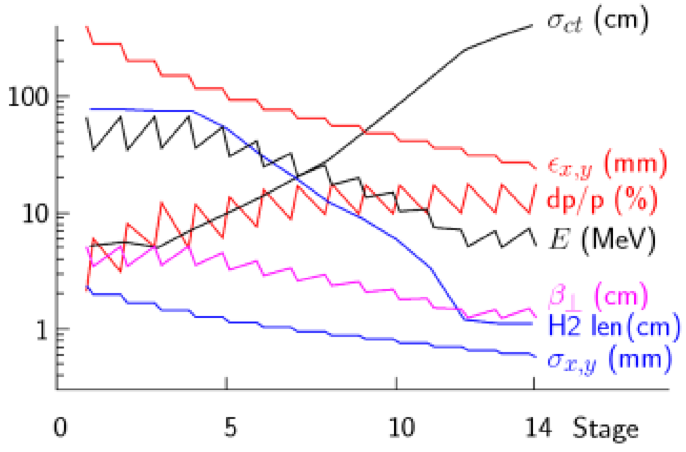

Figure 6: Some parameters vs. stage for the 40 T sequence.

Table 1: Rf Parameters of 40 T Example

	E1 MeV	E2 MeV	freq MHz	grad MV/m	acc L m
NCRF	34.6	66.6	201	15.5	2.1
NCRF	34.8	66.9	201	15.5	2.1
NCRF	36.0	67.1	201	15.5	2.0
NCRF	36.0	54.5	153	11.1	1.7
NCRF	30.6	41.3	110	7.4	1.5
NCRF	24.9	32.4	77	4.7	1.6
NCRF	20.7	25.7	53	2.9	1.7
NCRF	17.4	20.0	31	1.5	1.7
Induction	13.6	15.0	18	1.0	1.4
Induction	10.3	10.7	10	1.0	0.4
Induction	7.5	7.2	6	1.0	0.7
Induction	5.1	7.0	5	1.0	1.8
Induction	5.1	7.4	4	1.0	2.3

- In each stage, RF re-accelerates and phase-rotates the muons providing energy and energy-spread required for the next stage
- To keep energy spread low the bunch length is increased from 5 cm in the 1st stage to 400 cm in the last one
- The RF frequency must follow this trend to make sure that the bucket is large enough and
  - the RF curvature in accelerating mode is not too large compared to energy spread
  - RF slop is linear enough in the phase rotation mode
- The RF curvature is probably more critical leading to criteria for RF frequency choice: $\sigma_{ct} < \lambda/20$ for bunches shorter than 75 cm
- The lowest frequency is 4 MHz assuming induction linac
- The lowest NCRF cavity frequency is 31 MHz

Example of previous design of final cooling, 2

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High field – low energy muon ionization cooling channel

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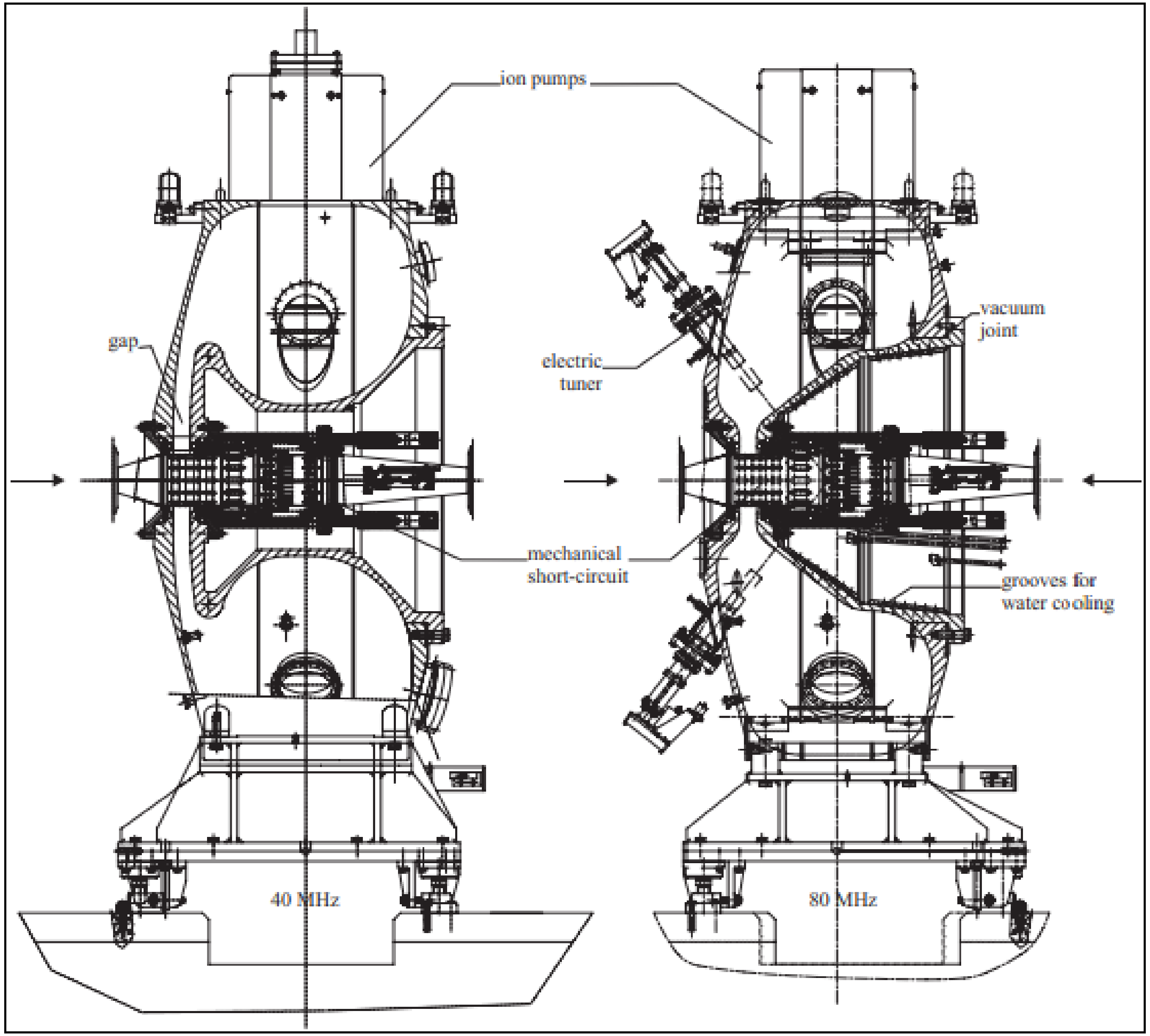
TABLE II. Parameters of the high-field low-energy cooling channel.

Stage [N]	P [MeV/c]	Energy spread σ_E [MeV]	LH_2 thickness [cm]	Drift length [m]	rf length [m]	rf frequency [MHz]	Field flip
1	135.0	2.29	65	0.434	2.25	325	Yes
2	130.0	2.48	60	0.459	2.25	250	Yes
3	129.0	2.78	60	0.450	2.5	220	No
4	129.0	3.10	59	0.458	2.5	201	No
5	122.0	3.60	57	1.629	5.0	201	Yes
6	124.0	4.90	53	2.22	4.5	180	No
7	116.0	3.40	42	2.21	3.25	150	No
8	111.0	3.90	40	2.0	3.5	150	No
9	106.0	3.50	40	3.13	5.0	125	Yes
10	98.0	3.07	35	3.13	5.0	120	No
11	89.4	3.11	20	3.12	5.0	110	No
12	87.9	2.76	20	3.1	8.0	100	No
13	85.9	2.67	20	3.0	7.5	100	Yes
14	79.7	3.08	15	2.7	7.0	70	No
15	71.1	4.0	15	2.6	6.0	50	No
16	71.0	3.80	13	2.5	6.0	20	No
17	70.0	3.80	10	...	...	20	...

- More recent paper, less details on RF though
- Same criteria is used for RF frequency choice: $\sigma_{ct} < \lambda/20$
- General trend: Higher energies -> shorter bunches -> higher RF frequencies
- The lowest RF frequency is 20 MHz, which might still require induction linac
- There is strong motivation to avoid induction linac due to its rather low gradient: ~1MV/m

CERN PS RF cavities

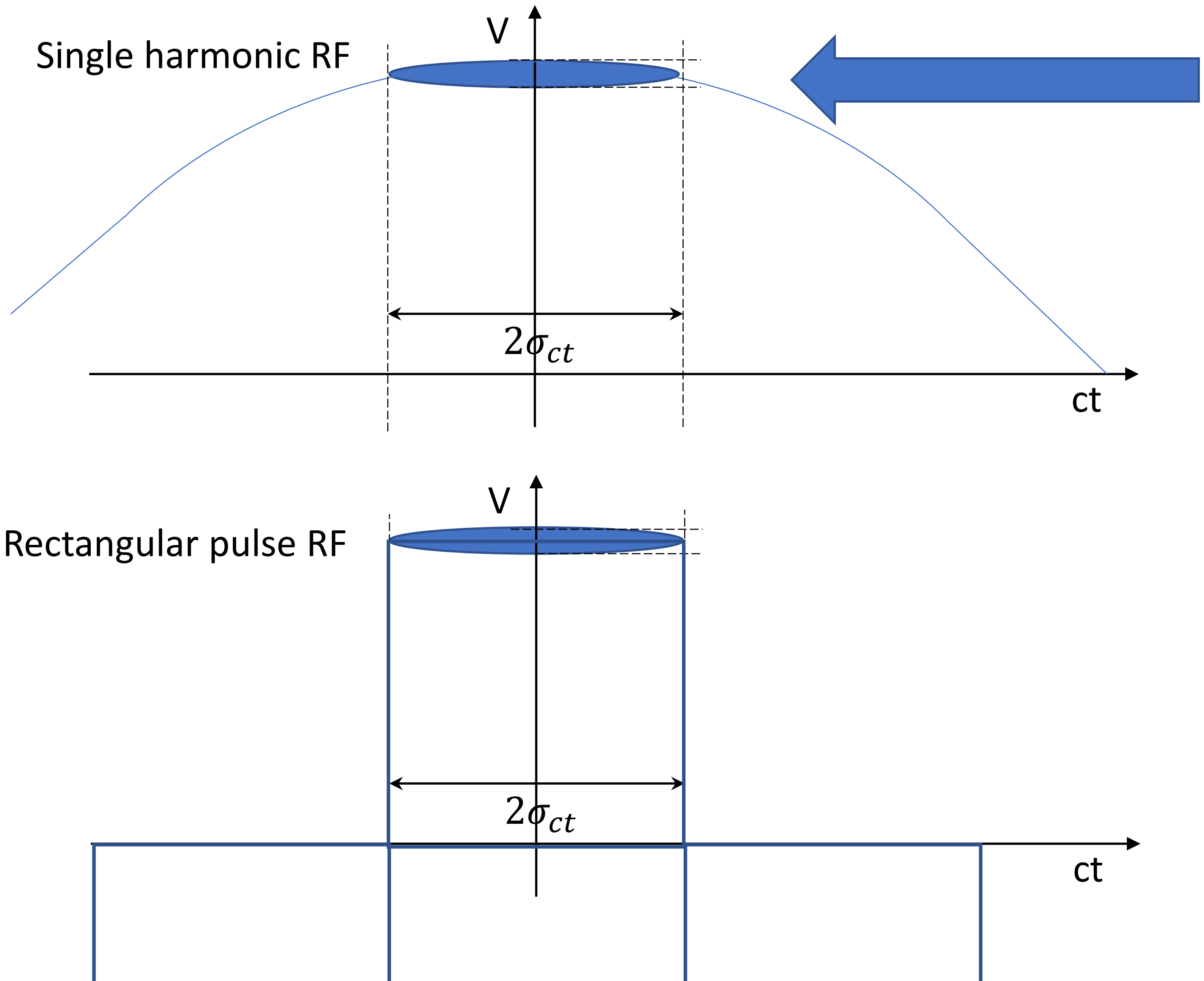
- 1.6m diameter, 1m long
- Far from pillbox
- Gap is very small: 5cm, so even for reasonable electric field in the gap of 10MV/m@CW voltage and real estate gradient are very low: 0.5MV and 0.5MV/m, respectively.
- It can potentially be increased in pulsed operation mode…
- => Strong motivation to increase lowest RF frequency required for final cooling



LHC TDR v3, p59

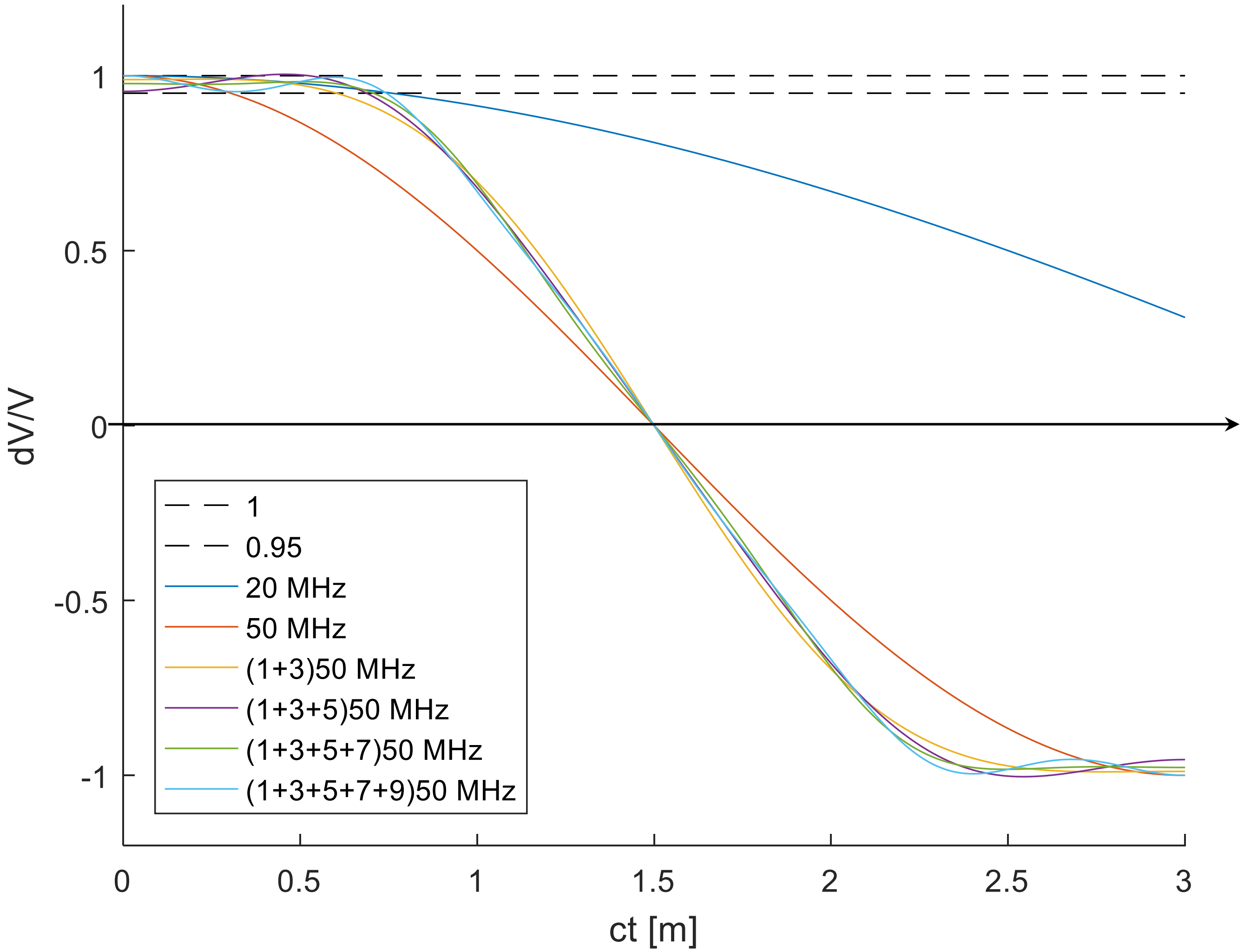
Figure 8.3: Longitudinal section of the 40 MHz (left) and 80 MHz cavities (right).

Idea: Multi-harmonic RF for acceleration



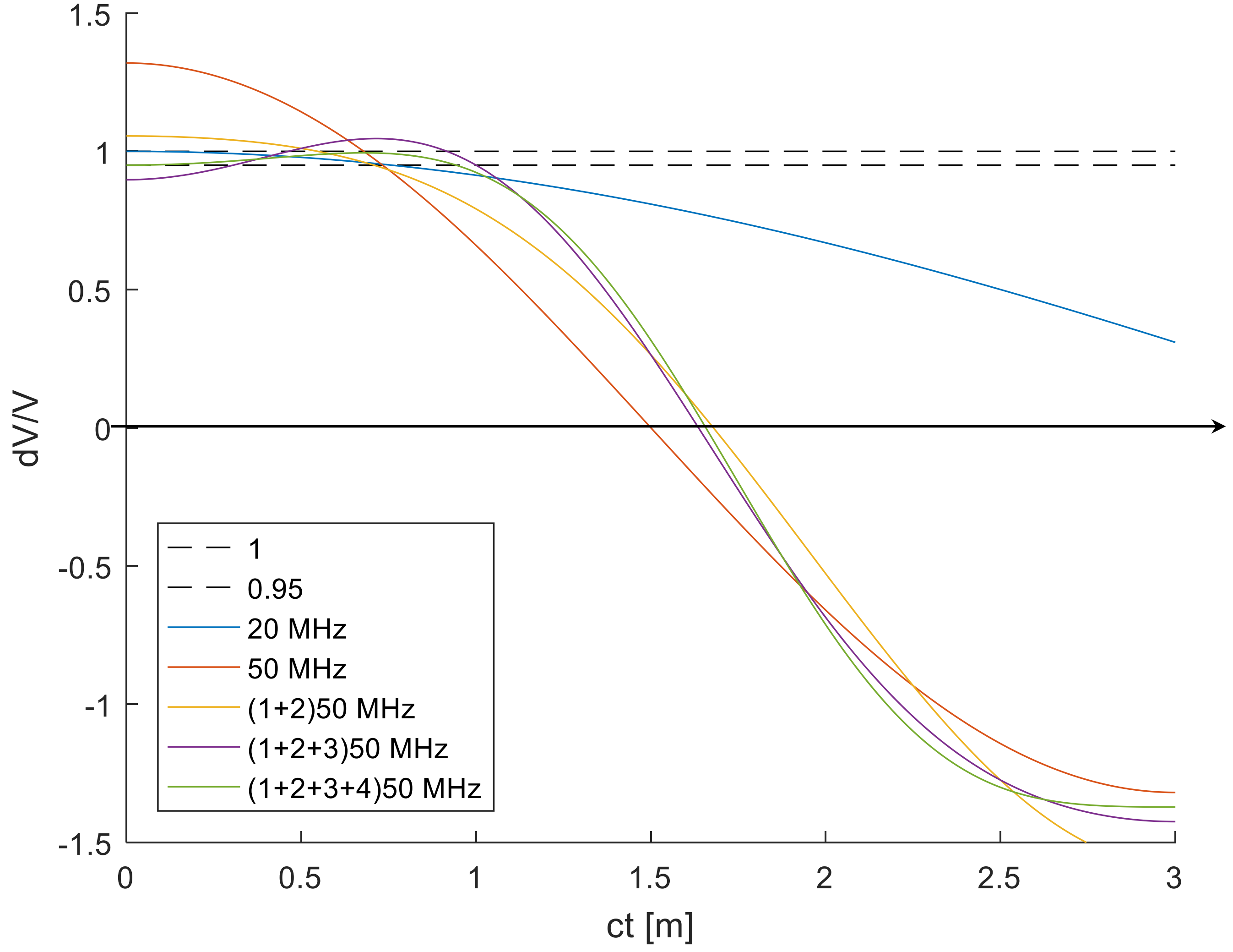
- RF frequency criteria: $\sigma_{ct} < \lambda/20$ for $\sigma_{ct} < 0.75$m related to energy spread introduced by the RF curvature
- For total bunch length of $2\sigma_{ct}$: dV/V = $cos\,\frac{2\pi\sigma_{ct}}{\lambda} \approx \cos\left(\frac{2\pi}{20}\right) \approx$ **5%**
- So, for σ_{ct}=0.75m
  - single harmonic RF λmax= $20\sigma_{ct} = 15$m -> fmin=**20 MHz**
  - Rectangular meander RF with period of $4\sigma_{ct} = 3$m –> fmin=**100 MHz**
- But **infinite number of higher** harmonics is necessary to approach the ideal pulse shape of rectangular meander

An example: odd harmonic



- In practice **finite number of harmonics** can be used only
- To maintain energy spread of dV/V <5% over σ_{ct} <0.75m, several harmonics of 50 MHz have been used
- 50 MHz provides dV/V <5% over σ_{ct} <0.3m only
- 50, 150, 250 MHz extend the range up to ~0.6m
- 50, 150, 250, 350, 450 MHz make it up to 0.75m satisfying the specs
- Using only odd harmonics keeps voltage +,- symmetric
- Using higher harmonics is less and less effective

An example: all harmonic



- Using all harmonics makes voltage +,- non-symmetric
- Is it an issue for the beam?
- 50, 100, 150, 200 MHz case meets the specs up to σ_{ct} =1m
- Using all harmonics is more effective than using just odd harmonics
- By using all harmonics, minimum frequency can be increased from 50 MHz probably up to 100 MHz

Conclusions

- Multi-harmonic RF system for final cooling allow significant increase of the minimum RF frequency required for minimization of the RF curvature induced energy spread
- For the example from PRAB2015, where 20 MHz is required for the last stage, using multi-harmonic RF system with 5 odd harmonics from 50 MHz and above provides the same energy spread as in single harmonic 20 MHz RF system
- If all harmonics can be used with +,- non-symmetric voltage, the minimum frequency can further be increased possibly up to 100 MHz.
- This approach opens the way to potentially avoid using induction linacs and very low frequency RF cavities
- It also can be applied to any stage in order to reduce the number of low frequency RF cavities
- Using several harmonic frequencies does not have any impact on the total number of RF frequencies since they are used anyway at the earlier stages
- However, It will probably have an impact on the total number of cavities, but the higher harmonics have much lower amplitude and can be excited by significantly more compact cavities