

# Precision calculations for future $e^+e^-$ colliders: targets and tools

Jun 7 – 17, 2022

CERN

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## Overview

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TH secretariat

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The main goal is to identify clear theoretical and computational targets for high-precision predictions of relevance to the programme of future  $e^+e^-$  colliders. The workshop will be divided into two parts, of one week each, as follows:

- **Week 1** (7th to 10th of June): select key physics questions and observables
- **Week 2** (13th to 17th of June): current status and advancements in multi-loop calculations required to match the precision goals

During **Week 1**, we will involve both theorists and experimentalists to identify key physics objectives and a set of important observables, to provide clear reference targets for the theory community.

During **Week 2**, we will dive deeper into the technical aspects of such calculations, with an overview of the status of current computational techniques (with a focus on multi-loop calculations), and discussions on the required future steps.

The workshop will have a limited number of invited talks occupying only a small fraction of the days, with ample time for discussions and collaborative work. We aim at promoting the development of an active community and foster new collaborations that will target these important questions.

## Theory and experiment at the Z pole

### □ Tasks for theory

- ◆ Identify observables/parameters that contain sensitivity to new phenomena
  - Via loops in  $\gamma$ , Z, W propagators (flavour universal), e.g., S, T, U @ LEP/SLC
  - Via boxes and vertices (flavour dependent), e.g.,  $\delta_b$  @ LEP/SLC
  - Via direct long distance propagator effects (universality violation): e.g., new Z'
  - Via mixing with known particles, e.g., Z'/Z mixing,  $\nu/N$  mixing, ...
- ◆ Develop high-precision SM procedures to extract these parameters from measurements
  - Precise (maybe not universal?) QED/QCD Monte Carlo / radiator for ISR/FSR/IFI, ...
- ◆ Perform high-precision calculations of these observables/parameters in the SM
  - Precise multi-loop calculations with, e.g.,  $m_Z$ ,  $G_F$ ,  $\alpha_{\text{QED}}(0)$  as basic inputs
    - Also requires high-precision theory to extract ancillary quantities from experimental measurements  $\alpha_{\text{QED}}(m_Z)$ ,  $\alpha_S(m_Z)$ ,  $m_{\text{top}}$ ,  $m_b$ ,  $m_H$ , etc. to reduce parametric uncertainties
- ◆ Develop sophisticated MC event generators, for direct tests of the theoretical prediction
  - Also needed to remove detector acceptance and selection inefficiencies

# Z/W physics

P. Janot

## Summary: Theory inputs for Z lineshape observables

Numbers are given here for FCC-ee (best prospects)

Observables	Present value	FCC-ee stat.	FCC-ee current syst.	FCC-ee ultimate syst.	Theory input (not exhaustive)
$m_Z$ (keV)	$91187500 \pm 2100$	4	100	10 ?	Lineshape QED unfolding Relation to measured quantities
$\Gamma_Z$ (keV)	$2495500 \pm 2300$ [*]	4	25	5 ?	Lineshape QED unfolding Relation to measured quantities
$\sigma_{\text{had}}^0$ (pb)	$41480.2 \pm 32.5$ [*]	0.04	4	0.8	Bhabha cross section to 0.01% $e^+e^- \rightarrow \gamma\gamma$ cross section to 0.002%
$N_V (\times 10^3)$ from $\sigma_{\text{had}}$	$2996.3 \pm 7.4$	0.007	1	0.2	Lineshape QED unfolding ( $\Gamma_{\nu\nu}/\Gamma_{\ell\ell}$ ) <sub>SM</sub>
$R_\ell (\times 10^3)$	$20766.6 \pm 24.7$	0.04	1	0.2 ?	Lepton angular distribution (QED ISR/FSR/IFI, EW corrections)
$\alpha_s(m_Z) (\times 10^4)$ from $R_\ell$	$1196 \pm 30$	0.1	1.5	0.4 ?	Higher order QCD corrections for $\Gamma_{\text{had}}$
$R_b (\times 10^6)$	$216290 \pm 660$	0.3	?	< 60 ?	QCD (gluon radiation, gluon splitting, fragmentation, decays, ...)

- And also sophisticated and state of the art MC generators (signal and backgrounds)
  - Plus, maybe, redefined EW Precision Parameters (EWPP) and extraction procedures ?

# Z/W physics

P. Janot

## Summary: Theory inputs for asymmetries

Observables	Present value ( $\times 10^4$ )	TeraZ / GigaZ stat.	TeraZ / GigaZ current syst.	Theory input (not exhaustive)
$A_e$ from $P_\tau$ (FCC-ee)	<b>1514 <math>\pm</math> 19</b>	<b>0.07</b>	<b>0.20</b>	SM relation to measured quantities
$A_e$ from $A_{LR}$ (ILC)		<b>0.15</b>	<b>0.80</b>	
$A_\mu$ from $A_{FB}$ (FCC-ee)	<b>1456 <math>\pm</math> 91</b>	<b>0.23</b>	<b>0.22</b>	Accurate QED (ISR, IFI, FSR)
$A_\mu$ from $A_{FB}^{pol}$ (ILC)		<b>0.30</b>	<b>0.80</b>	
$A_\tau$ from $P_\tau$ (FCC-ee)	<b>1449 <math>\pm</math> 40</b>	<b>0.05</b>	<b>2.00</b>	Prediction for non- $\tau$ backgrounds
$A_\tau$ from $A_{FB}$ (FCC-ee)		<b>0.23</b>	<b>1.30</b>	
$A_\tau$ from $A_{FB}^{pol}$ (ILC)		<b>0.30</b>	<b>0.80</b>	
$A_b$ from $A_{FB}$ (FCC-ee)	<b>8990 <math>\pm</math> 130</b>	<b>0.24</b>	<b>2.10</b>	QCD calculations
$A_b$ from $A_{FB}^{pol}$ (ILC)		<b>0.90</b>	<b>5.00</b>	
$A_c$ from $A_{FB}$ (FCC-ee)	<b>65400 <math>\pm</math> 210</b>	<b>2.00</b>	<b>1.50</b>	
$A_c$ from $A_{FB}^{pol}$ (ILC)		<b>2.00</b>	<b>3.70</b>	

- ◆ And also sophisticated and state of the art MC generators (signal and backgrounds)
  - Plus, maybe, redefined EW Precision Parameters (EWPP) and extraction procedures ?

Projections from <https://www.overleaf.com/read/xnfpvbdgfgqpr>

# Z/W physics

A. Freitas

## Z lineshape

5/28

- Deconvolution of initial-state QED radiation:

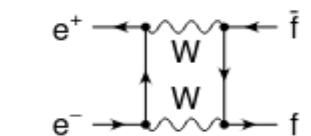
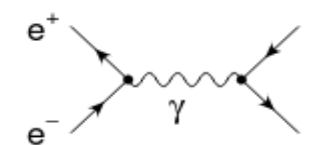
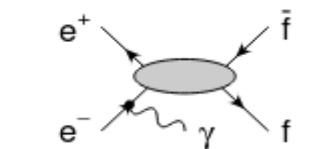
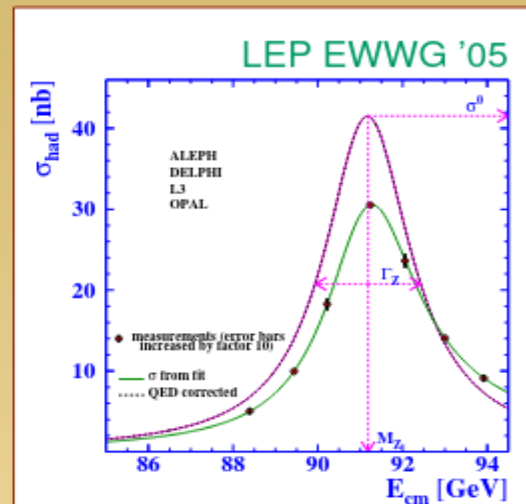
$$\sigma[e^+e^- \rightarrow f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of  $\gamma$ -exchange,  $\gamma$ -Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \underbrace{\sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}}_{\text{computed in SM}}$$

- Z-pole contribution:

$$\sigma_Z = \frac{R}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} + \sigma_{\text{non-res}}$$



# Z/W physics

A. Freitas

## Pole expansion

11/28

Expand amplitude for  $e^+e^- \rightarrow f\bar{f}$  about **complex pole**  $s_0 \equiv \overline{M}_Z^2 + i\overline{M}_Z\overline{\Gamma}_Z$ :

$$\mathcal{M}_{ij} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \dots \quad (i, j = V, A)$$

**Current state of art:**  $R$  @ NNLO + leading higher orders

$S$  @ NLO

$S'$  @ (N)LO

**For future ee colliders:** (at least) one order more!

→ also matching to Monte-Carlo for QED/QCD ISR/FSR/IFI

# Z/W physics

A. Freitas

## Future theory and parametric uncertainties

19/28

	CEPC	FCC-ee	perturb. error with 3-loop <sup>†</sup>	Freitas, Heinemeyer, et al. '19	
				Param. error scen. 1*	scen. 2*
$M_W$ [MeV]	0.5	0.4	1	2.1	0.6
$\Gamma_Z$ [MeV]	0.025	0.025	0.15	0.15	0.1
$R_b$ [ $10^{-5}$ ]	4.3	6	5	< 1	< 1
$\sin^2 \theta_{\text{eff}}^\ell$ [ $10^{-5}$ ]	< 1	0.5	1.5	2	1

<sup>†</sup> **Theory scenario:**  $\mathcal{O}(\alpha\alpha_s^2)$ ,  $\mathcal{O}(N_f\alpha^2\alpha_s)$ ,  $\mathcal{O}(N_f^2\alpha^2\alpha_s)$ , leading 4-loop  
( $N_f^n$  = at least  $n$  closed fermion loops)

Parametric inputs:

**\*Scenario 1:**  $\delta m_t = 600$  MeV,  $\delta\alpha_s = 0.0002$ ,  $\delta M_Z = 0.5$  MeV,  
 $\delta(\Delta\alpha) = 5 \times 10^{-5}$

**\*Scenario 2:**  $\delta m_t = 50$  MeV,  $\delta\alpha_s = 0.0002$ ,  $\delta M_Z = 0.5$  MeV,  
 $\delta(\Delta\alpha) = 3 \times 10^{-5}$

# Z/W physics

- Measurement of WW threshold: [P. Azzurri]  
 $\Delta\Gamma_W = 0.96 \text{ MeV}$  and  $\Delta m_W = 0.41 \text{ MeV}$
- Measurements in WW continuum: [P. Azzurri, G. Wilson]  
W mass with  $\sim 2 \text{ MeV}$ , precise W decay BRs  
→ good control of ISR and final-state jet dynamics
- Need NLL and NNLL QED PDFs for simulation of QED radiation [S. Frixione]



## Shopping list

	f.o.	resummation	soft corrections
$e^+e^- \rightarrow 3 \text{ jets}$	N3LO?	beyond N3LL?	y dependent
$e^+e^- \rightarrow 4 \text{ jets}$	NNLO?	MC N(N?)LL PS	possible? MC
$e^+e^- \rightarrow 5 \text{ jets}$	NNLO?	MC N(N?)LL PS	possible? MC
$e^+e^- \rightarrow \text{hadrons}$		beyond NMMLA?	
$e^+e^- \rightarrow \gamma+n \text{ jets}$	NLO EW		
$e^+e^- \rightarrow Q\bar{Q}g$	NNLO	NNLL? N(N?)LL MC PS?	
Z/W EWPO	$\alpha_s^5, \alpha^2, \alpha^3, \alpha\alpha_s^2, \alpha^2\alpha_s$		
$\tau$ had. decays	$\alpha_s^5?$ FOPT/CIPT <sup>RS</sup>		OPE vs DV models



# QCD

- Non-perturbative aspects important for strong coupling determination [P. Nason]
  - new ideas to test and reconcile MC and analytic methods
  - move to higher cm energies to reduce NP QCD
- Definition of new observables to reduce NP QCD [A. Banfi]

## Summary

I  $e^+e^- \rightarrow t\bar{t}X$  cross section near threshold now computed at NNNLO in (PNR)QCD + top-Yukawa effects

- Sizeable 3rd order corrections and reduction of theoretical uncertainty to about  $\pm 3\%$ .


II Realistic predictions for  $e^+e^- \rightarrow W^+W^-b\bar{b}$  near top-pair threshold

- NNLO available, including cuts invariant mass cuts.

III Parameter dependences  $(m_t, \Gamma_t, y_t, \alpha_s)$  can be studied.

- $(m_t, \Gamma_t)$  with unrivaled accuracy.
- $y_t$  with 20% accuracy from threshold already challenging.

IV Further requirements:

- ISR / QED PDF's for  $x \rightarrow 1$  with NLL evolution
- N4LO QCD would be reassuring, but appears prohibitive.  needed for  $\Delta m_t < 40$  MeV

<https://www.hepforge.org/downloads/qqbathreshold/>

# Higgs

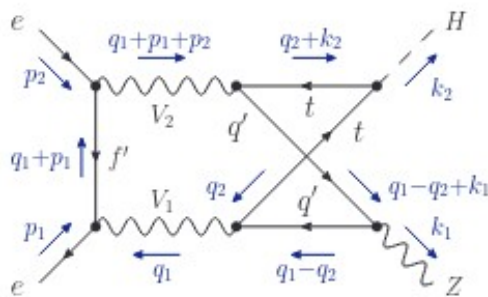
L. Yang

## Towards two-loop EW corrections to ZH

A must to match the  $\sim 0.3\%$  experimental accuracy

A rather challenging task:  $\sim 20000$  diagrams, a lot of physical scales [Li, Wang, Wu: 2012.12513](#)

Evaluation of a class of double boxes with a top quark loop [Song, Freitas: 2101.00308](#)



Further development of computational techniques required!

→ Talks covering both analytic and numeric methods

e.g.: Canonical differential equations in both GPL sectors and elliptic sectors

Numeric solutions (pySecDec, DiffExp, AMFlow, ...)

Perhaps some kind of approximate result is good enough

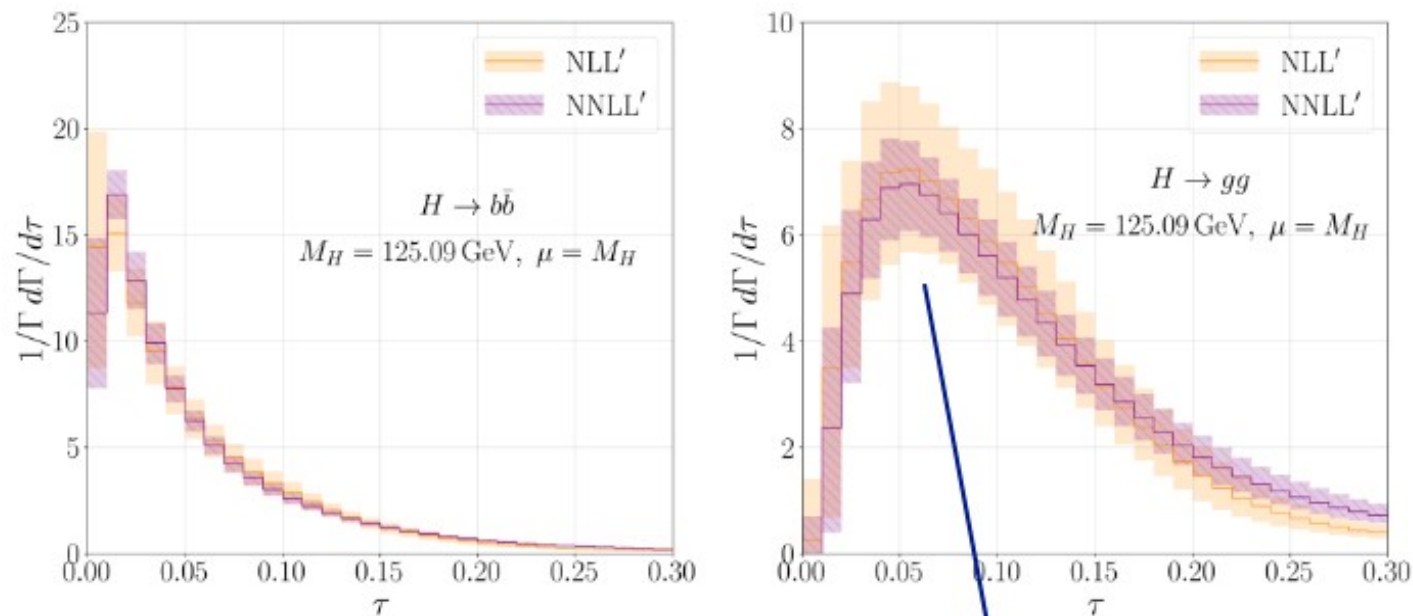
→ Vague thought: asymptotic expansion in the limit  $m^2_{\text{everything}} \ll s, m_t^2$  ?

# Higgs

L. Yang

## Resummed predictions

Alioli et al.: 2009.13533



Large uncertainties in the gluon channel; N<sup>3</sup>LL or N<sup>3</sup>LL' needed?

# Techniques for EW calc.

- **Challenge:** many parameters (masses, kin. scales)
- Common approach: reduction of large # of integrals to small # of *master integrals* (IBP red.)  
[T. Peraro, V. Sotnikov]
  - Huge expressions when many scales/masses
  - Partially overcome with finite-field methods (reconstruct analytical from numerical) and syzygies (find easily solvable relations)
  - Or fully numerically...?
- Solve MIs with differential equations
  - Special functions [A. v. Manteuffel, S. Weinzierl]
  - Numerical integration [L. Chen]
  - Series expansions [N. Rana, M. Hidding, X. Liu]

(IBP red. needed but can be simplified in some cases)

# Techniques for EW calc.

- Direct numerical integration

- Sector decomposition

[V. Magerya]

- Mellin-Barns representations

[J. Gluza]

Slow convergence of multi-dim. Integrals, but can be improved with QMC integration, GPUs

- Local unitarity (loop-tree duality)

[V. Hirschi]

- integrate loop and real radiation together

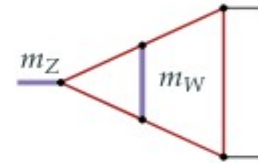
- need to avoid/subtract spurious and physical singularities

- (related discussion by [C. Anastasiou](#))

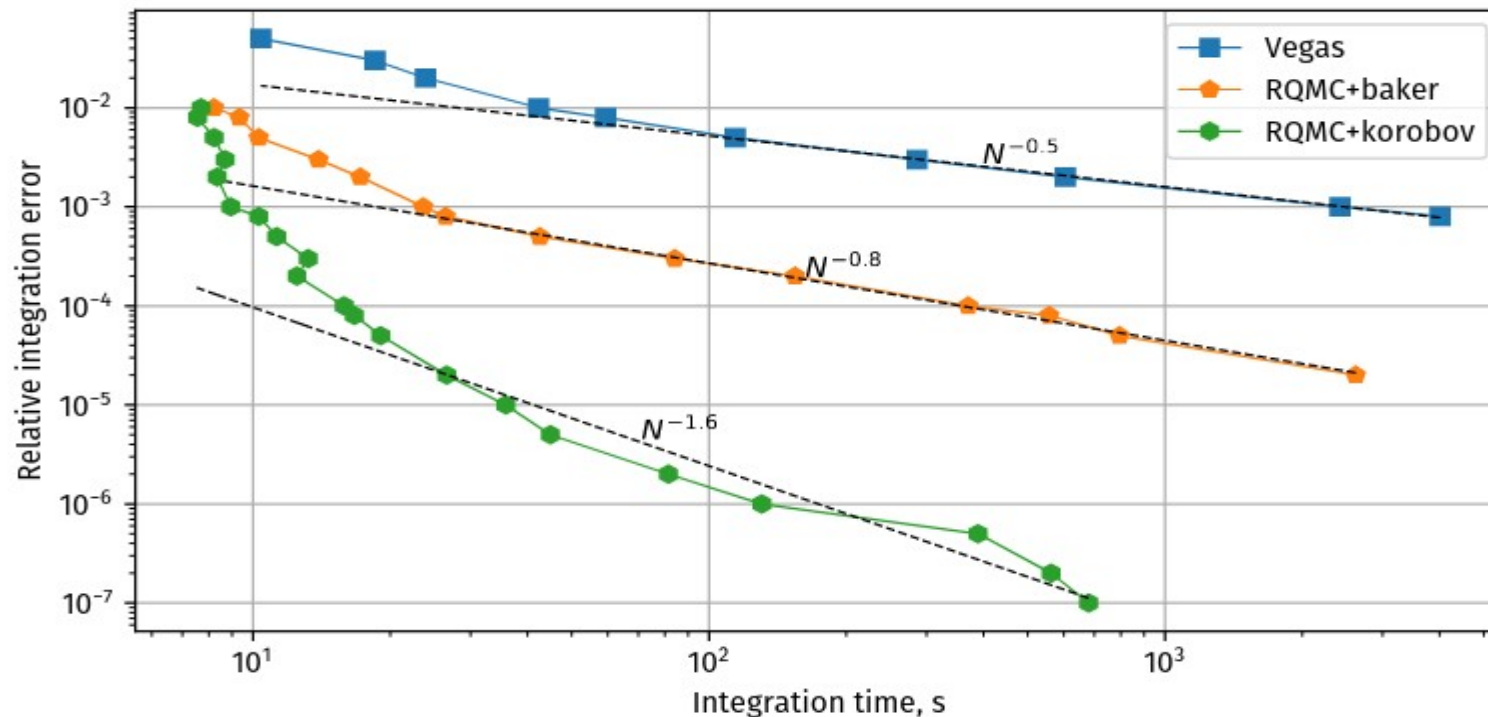
# Techniques for EW calc.

## Monte Carlo vs RQMC

Integration time scaling for Monte Carlo (VEGAS) vs Randomized Quasi Monte Carlo (QMC).<sup>1</sup>



V. Magerya



<sup>1</sup>pySecDEC v1.5.3 on NVidia A100 GPU.