FCC note, Physics Experiments and Detectors, EW group
PRELIMINARY

Impact on EWPOs of the $\alpha_{\mathrm{QED}}(m_{\mathrm{Z}}^2)$ uncertainty; possible mitigation

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Contribution to the MiniWorkshop: parametric uncertainties: $\alpha_{QED}(m_{\mathrm{Z}}^2)$

Abstract. We consider the uncertainties on the prediction of most important Electroweak quantities at the FCC-ee upon the Electromagnetic coupling constant $\alpha_{\mathrm{QED}}(m_{\mathrm{Z}}^2)$. A direct and precise measurement of $\alpha_{\mathrm{QED}}(m_{\mathrm{Z}}^2)$ is possible at FCC-ee using the variation of the forward-backward muon pair asymmetry across the Z resonance peak, which is sensitive to the Z-γ interference; However the achievable precision leads to uncertainties that exceed the target experimental precision for the prediction of the most important EW observables, e.g. the effective weak mixing angle, the W mass, the Z width and the Z leptonic patial width. It is shown however that this part of the prediction uncertainties could be reduced to a more acceptable level by substituting the measured effective weak mixing angle to $\alpha_{\mathrm{QED}}(m_{\mathrm{Z}}^2)$ as alternative input for the prediction of the other observables.

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1 Introduction

With statistics of 2.5 10^{12} Z bosons and 210^{8} W pairs in each of two to four experiments, the electroweak precision programme is extremely powerful. The statistical uncertainties should be reduced with respect to LEP by a factor 100 (for W measurements) to 500 (at the Z), or even more in some cases where significant improvements in detector technology (e.g. vertex detector) or accelerator operation (e.g. for the center-of-mass energy calibration, alignments etc.). will allow This all requires a proactive attitude on detector construction, alignment, stability, luminosity measurement; at the same time a considerable effort is ongoing to prepare theoretical predictions that can match the experimental capacities, in order to take full advantage of the achievable precision.

Precise predictions of electroweak quantities require a set of very precise experimental inputs (also called 'renormalisation scheme'). The main inputs since LEP times have been fixed to be the Z mass, $\mathrm{m_Z}$, the Fermi constant extracted from the muon life time, $\mathrm{G_F}$, and the fine-structure constant α_{QED} determined by quantum Hall effect (or equivalently by the magnetic anomaly of the electron).

2 Uncertainties related to $\alpha_{QED}(m_{\mathrm{Z}}^2)$

The main Standard Model relations are listed in a practical way in e.g. section 1.4 of the LEPEWWG 2005 report [1]. The most notable relations connect the W mass and the effective weak mixing angle to the reference quantities.

$$(1-\frac{m_{\mathrm{W}}^2}{m_{\mathrm{Z}}^2}).\frac{m_{\mathrm{W}}^2}{m_{\mathrm{Z}}^2} = \frac{\pi\alpha_{\mathrm{QED}}(0)}{\sqrt{2}G_{\mathrm{F}}m_{\mathrm{Z}}^2}\times\frac{1}{1-\Delta r} \tag{1}$$

$$\sin^2\theta_{\mathrm{W}}^{\mathrm{eff}}\cos^2\theta_{\mathrm{W}}^{\mathrm{eff}} = \frac{\pi\alpha_{\mathrm{QED}}(0)}{\sqrt{2}G_{\mathrm{F}}m_{\mathrm{Z}}^2}\times\frac{1}{1-\Delta r'}, \tag{2}$$

where $\Delta r = \Delta\alpha + \Delta r_w$ and $\Delta r' = \Delta\alpha + \Delta r'_w$. The quantity $\Delta\alpha = 0.059$ represents the running of the electromagnetic constant from the low Q^2 of the corresponding measurements to the Z mass and is common.

$$\alpha_{\mathrm{QED}}(m_{\mathrm{Z}}^2) = \frac{\alpha_{\mathrm{QED}}(0)}{1-\Delta\alpha} \tag{3}$$

The quantities Δr_w and $\Delta r'_w$ contain electroweak effects including, in the SM the sensitivity to the top and Higgs boson masses. In addition, in the case of the electroweak mixing angle, they include flavour-dependent vertex corrections, which might also reveal the additional effect of particular types of new physics.

At first order, the particular parametric uncertainty related to the running of α_{QED} is contained in the term $\Delta\alpha$. The impact of this uncertainty on higher order corrections will be reduced by typically two orders of magnitude and can safely be neglected in the present discussion. As examples: the final state photon emission correction to the leptonic partial width $1 + \frac{3}{4}.\frac{\alpha}{\pi}$ would lead to a relative uncertainty of $5\ 10^{-8}$; similarly, the leading top quark correction to the rho-parameter, $\Delta\rho_{\mathrm{top}} = \frac{\alpha}{\pi}\frac{m_{\mathrm{top}}^2}{m_{\mathrm{w}}^2} \simeq 1\%$ would have a parametric uncertainty of 310^{-7} from $\Delta\alpha$, which is small in comparison from that arising from e.g. a top quark mass error of 20 MeV, which would amount to 2.510^{-6} ; both are one to two orders of magnitude smaller than the primary uncertainty from on $\alpha_{\mathrm{QED}}(m_{\mathrm{Z}}^2)$.

The couplings of the Z to fermion pairs follow the general equation $\mathrm{g}^f_{L,R} = \sqrt{\rho^f}[I^f_{L,R} - Q^f\kappa^f sin^2\theta^{\mathrm{eff}}_{\mathrm{W}}]$. (These κ factors are similar but different from the $\mathcal{K}^f$ factors which relate the couplings to the quantity $(1 - \frac{m_{\mathrm{W}}^2}{m_{\mathrm{Z}}^2})$, often used as a definition of the weak mixing angle). Both ρ^f and κ^f are dominated by universal corrections, with accurately calculable fermion dependent corrections in the SM, which are generally very small, with the notable exception of the b-quark final state.

The effective weak mixing angle $\sin^2\theta^{\mathrm{eff}}_{\mathrm{W}}$ and the universal ρ parameter are defined for the charged lepton channel by setting $\kappa^\ell = 1$; and $\rho^\ell = (1 + \Delta\rho)$ where $\Delta\rho$ is often noted ϵ_1 or αT, sometimes after subtraction of known SM effects. Other quantities are related to the effective weak mixing angle by relations that do not depend on $\Delta\alpha$, such as :

$$\sin^2\theta^{\mathrm{eff}}_{\mathrm{W}} = \mathcal{K}_w(1 - \frac{m_{\mathrm{W}}^2}{m_{\mathrm{Z}}^2}) \tag{4}$$

where $\mathcal{K}_w$ contains sensitivity to the top mass, Higgs mass, the known masses of light quarks and leptons, and new physics, but not to $\Delta\alpha$. In a similar way, the electroweak corrections coefficients ρ^f and κ^f contain no first order sensitivity to $\Delta\alpha$.

As a consequence of the above, two things can be done

1. One can straightforwardly calculate the sensitivity to $\Delta\alpha$ of the W mass and Z pole EW observables. (The neutrinos being neutral the Z neutrino partial width has zero sensitivity to $\Delta\alpha$). This was done in column 4 of table 3, using the expected precision on $\Delta\alpha$ from the direct measurement [2] – other methods of estimating this important quantity give different uncertainties but of similar magnitude. It is notable that these parametric uncertainties are significantly larger than the expected statistical errors of FCC-ee, in essentially all cases.
2. One can also observe that one can use the equations above relating all definitions of the weak mixing angle at the weak scale, as a way to predict all the EWPOs, at the exception of the effective weak mixing angle itself, obviously, once it has been measured with a precision exceeding that obtained from eq. (2) above. This is effectively the case at FCC-ee with the envisaged run plan and scan of the Z line shape: the expected statistical precision on $\sin^2\theta^{\mathrm{eff}}_{\mathrm{W}}$, is 7 times more precise than the parametric error arising from $\Delta\alpha$. As can be seen in column 5 of table 3, this allows a reduction of the corresponding parametric uncertainty for all other observables.

3 Conclusions

Certainly an accurate prediction of the effective weak mixing angle is of great importance, given its great sensitivity to BSM physics both arising at the Electroweak scale and in possibly in muon decay. A dominant parametric uncertainty comes from the uncertainty in $\Delta\alpha$, the running of the Electromagnetic coupling constant up to the Z mass. All efforts should be made to reduce uncertainties in this quantity. Nevertheless is is quite possible that the effective weak mixing angle will be measured better than it is predicted.

Although some of the experimental sensitivity would be lost in this case, it should be remembered that the loss need not be incurred by all electroweak quantities. This note has shown that the measured value of $\sin^2\theta^{\mathrm{eff}}_{\mathrm{W}}$ can be used as an alternative input parameter for the prediction of other quantities, thereby eliminating a significant fraction of the corresponding parametric uncertainty.

Table 1. Dependence of selected precision measurements at FCC-ee upon the uncertainty on $\alpha_{\rm QED}(\rm m_Z^2)$, or on $\sin^2\theta_{\rm W}^{\rm eff}$. Experimental data have been compiled from [3].

Observable	present value ± error	FCC-ee **Stat.**	from $\boldsymbol{\alpha_{\rm QED}(\rm m_Z^2)}$	from $\boldsymbol{\sin^2\theta_{\rm W}^{\rm eff}}$	Comments
$\rm m_Z$ (keV)	91186700 ± 2200	**4**	N.A.	N.A.	Input
$\rm G_F(\times 10^{-5})$	1.166378 ± 0.000006	N.A	N.A.	N.A	Input
$1/\alpha_{\rm QED}(\rm m_Z^2)(\times 10^3)$	128952 ± 14	**3**	N.A.	N.A.	Input; from $\rm A_{FB}^{\mu\mu}$ off peak
$\sin^2\theta_{\rm W}^{\rm eff}(\times 10^6)$	231480 ± 160	**1.5**	**10.5**	N.A.	from $\rm A_{FB}^{\mu\mu}$ and $\rm A_{FB}^{pol,\tau}$ at Z peak **Possible alternative input**
$\rm m_W$ (MeV)	80350 ± 15	**0.250**	**0.547**	**0.078**	
Γ_ℓ (keV)	83985 ± 86	**0.2**	**0.53**	**0.076**	stat. based on muon pair statistics ρ parameter
$\rm R_\ell^Z$ $(\times 10^3)$	20767 ± 25	**0.06**	**0.17**	**0.025**	ratio of hadrons to leptons quark and lepton universality determination of $\alpha_{\rm QCD}(m_{\rm Z}^2$
$\rm R_b$ $(\times 10^6)$	216290 ± 660	**0.3**	**0.42**	**0.06**	ratio of $\rm b\bar{b}$ to hadrons N.P. coupled to 3d generation
$\Gamma_{\rm Z}$ (keV)	2495200 ± 2300	**4**	**27**	**4**	From Z line shape scan Beam energy calibration

References

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