# PY410 / 505
# Computational Physics 1

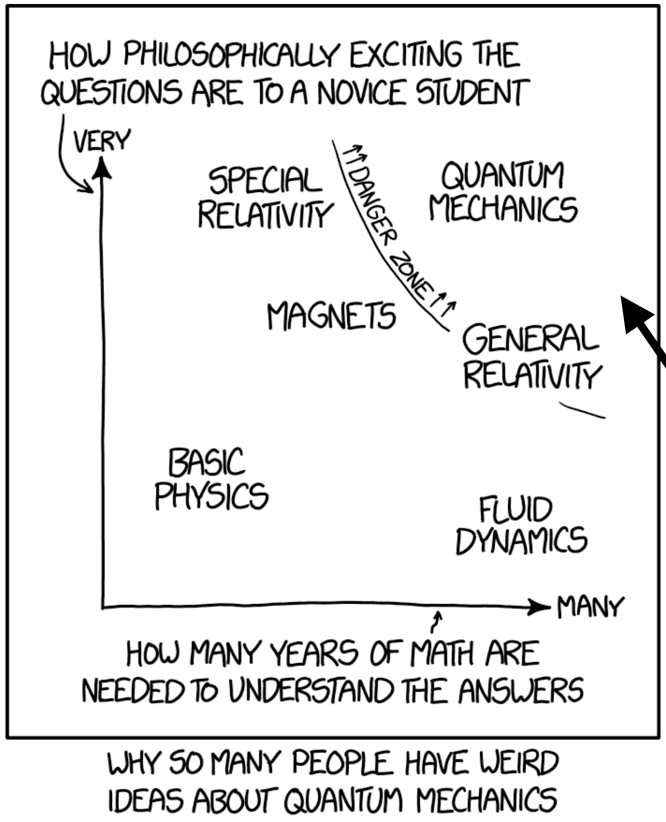
**Salvatore Rappoccio**

Many slides from Prof. Tim Thomay, Module from IBM qiskit examples

# Quantum Computing

- We are now using Jupyter, python, and qiskit to demonstrate the most controversial (important, profound) quantum experiment, Bell's inequality
- The "quantum weirdness" that people become enamored with will be explained
- We'll walk through a tiny bit of quantum mechanics for those who have not seen it before


HTTPS://XKCD.COM/1861



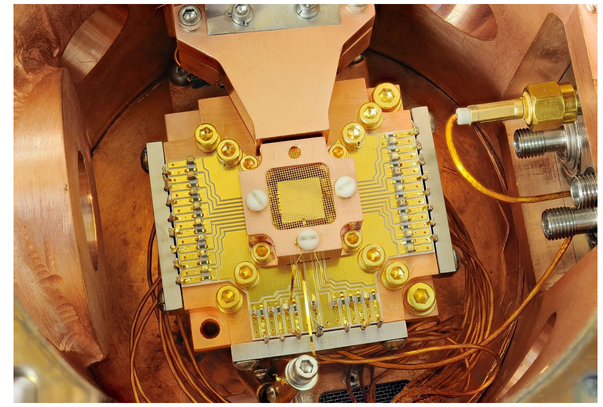

Incidentally, all will be covered in this class and the next semester

# What is a Quantum Computer

- Is a computer that uses quantum mechanical principles to operate
- In practice that means, non-classical properties such as
  - Superposition
  - Entanglement

# How Do Quantum Computers Work?

- Array of Quantum Gates
- Adiabatic / Quantum Annealing
- One way Quantum Computing
- Topological Quantum Computers
- Boson Samplers



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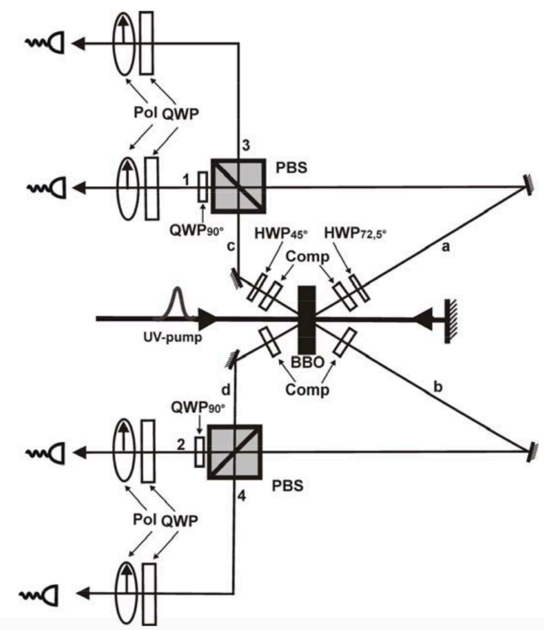


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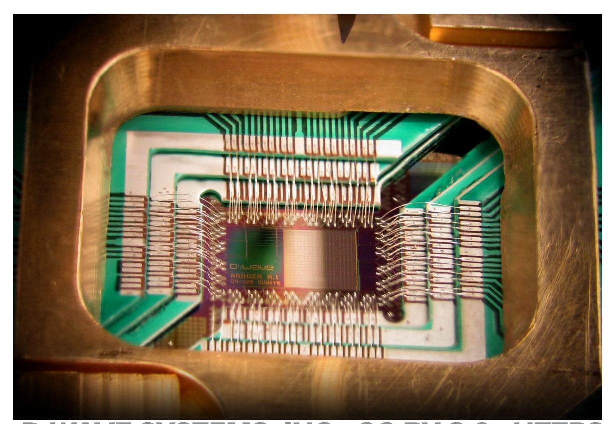

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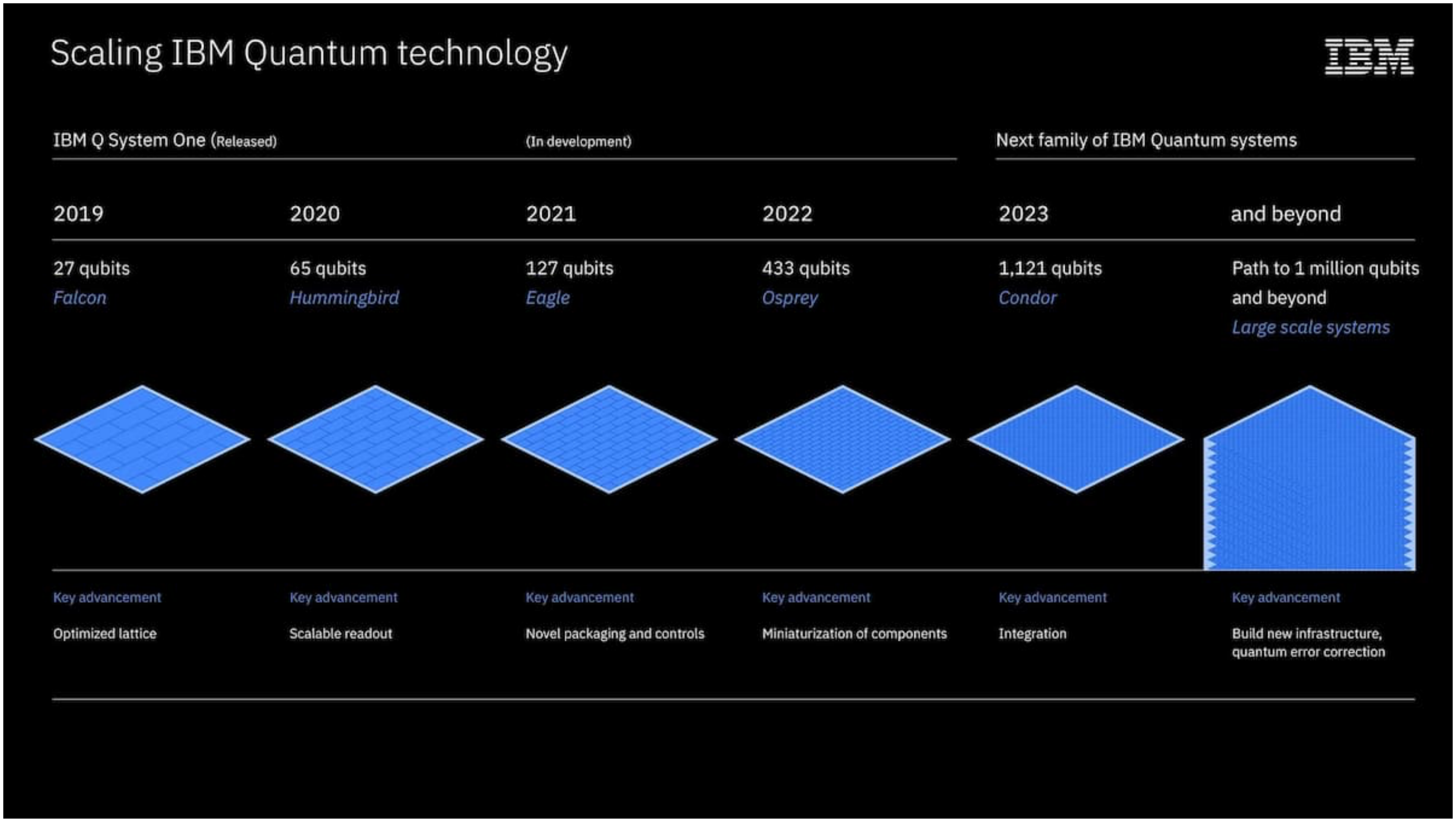
**WE DO NOT ANSWER THIS QUESTION IN THIS CLASS**

**WILL INSTEAD DISCUSS HOW TO PROGRAM THEM**

# WHY DO WE CARE?



# The three known types of quantum computing and their applications, generality, and computational power.

A very specialized form of quantum computing with unproven advantages over other specialized forms of conventional computing.

DIFFICULTY LEVEL

The most likely form of quantum computing that will first show true quantum speedup over conventional computing. This could happen within the next five years.

DIFFICULTY LEVEL

The true grand challenge in quantum computing. It offers the potential to be exponentially faster than traditional computers for a number of important applications for science and businesses.

DIFFICULTY LEVEL

## Quantum Annealer

The quantum annealer is least powerful and most restrictive form of quantum computers. It is the easiest to build, yet can only perform one specific function. The consensus of the scientific community is that a quantum annealer has no known advantages over conventional computing.

**APPLICATION**
Optimization Problems

**GENERALITY**
Restrictive

**COMPUTATIONAL POWER**
Same as traditional computers

## Analog Quantum

The analog quantum computer will be able to simulate complex quantum interactions that are intractable for any known conventional machine, or combinations of these machines. It is conjectured that the analog quantum computer will contain somewhere between 50 to 100 qubits.

**APPLICATIONS**
Quantum Chemistry
Material Science
Optimization Problems
Sampling
Quantum Dynamics

**GENERALITY**
Partial

**COMPUTATIONAL POWER**
High

## Universal Quantum

The universal quantum computer is the most powerful, the most general, and the hardest to build, posing a number of difficult technical challenges. Current estimates indicate that this machine will comprise more than 100,000 physical qubits.

**APPLICATIONS**
Secure computing
Machine Learning
Cryptography
Quantum Chemistry
Material Science
Optimization Problems
Sampling
Quantum Dynamics
Searching

**GENERALITY**
Complete with known speed up

**COMPUTATIONAL POWER**
Very High

IBM Research

Infographic by Carl De Torres for IBM Research

**IBM, 2015**

# Universal Quantum Computer

- Can solve any quantum algorithm
- Applications:
  - Nature (physics, chemistry, biology, astronomy, …)
  - Social (Finance, trade, …)
  - Computing

# Quantum Advantage

- When is a quantum computer solve problems that a classical computer cannot solve
  - Cannot solve in time t
  - Cannot solve
- Google announced QA in 2019 (https://www.youtube.com/watch?v=-ZNEzzDcllU)
  - 3min QC - 10,000 years CC
  - IBM: Our super computer can do it in 2.5 days …
- University of Science and Technology of China (USTC) announced 2020 QA for 10 billion larger Hilbert space as Google QC: 200s QC - 2.5 billion years CC

# What is the "weirdness" everyone obsesses about?

- Uncertainty principle:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

You cannot simultaneously know the position and momentum of a state with infinite precision

- Coherence and Decoherence:

$$\tau_c \geq \frac{\hbar}{\Delta p}$$

Systems with fewer momentum components decohere more slowly

# What is the “weirdness” everyone obsesses about?

- Schroedinger’s equation is conservation of energy

$$E = \frac{p^2}{2m} + V(x)$$

with the substitutions:

$$\vec{p} \rightarrow -i\hbar\vec{\nabla} \qquad E \rightarrow i\hbar\frac{\partial}{\partial t}$$

- In 1d we get:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t)$$

Will be returning to this later in the semester
and solving in intricate detail

# What is the “weirdness” everyone obsesses about?

- Solutions to the S.E. can be expressed as a sum of infinitely many sine and cosine waves (or complex exponentials). In time-independent cases,

$$\psi(x) = \sum_{i=0}^{\infty} A_n e^{ip_k x}$$

- Or in the continuum limit:

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} F(p) e^{ipx/\hbar} dp$$

## But! This is a Fourier transform!

(We will also cover FTs in this class)

# The Fourier Transform

- Quantum mechanics and the uncertainty principle are "really" the same as Fourier transforms:

Fourier coefficients

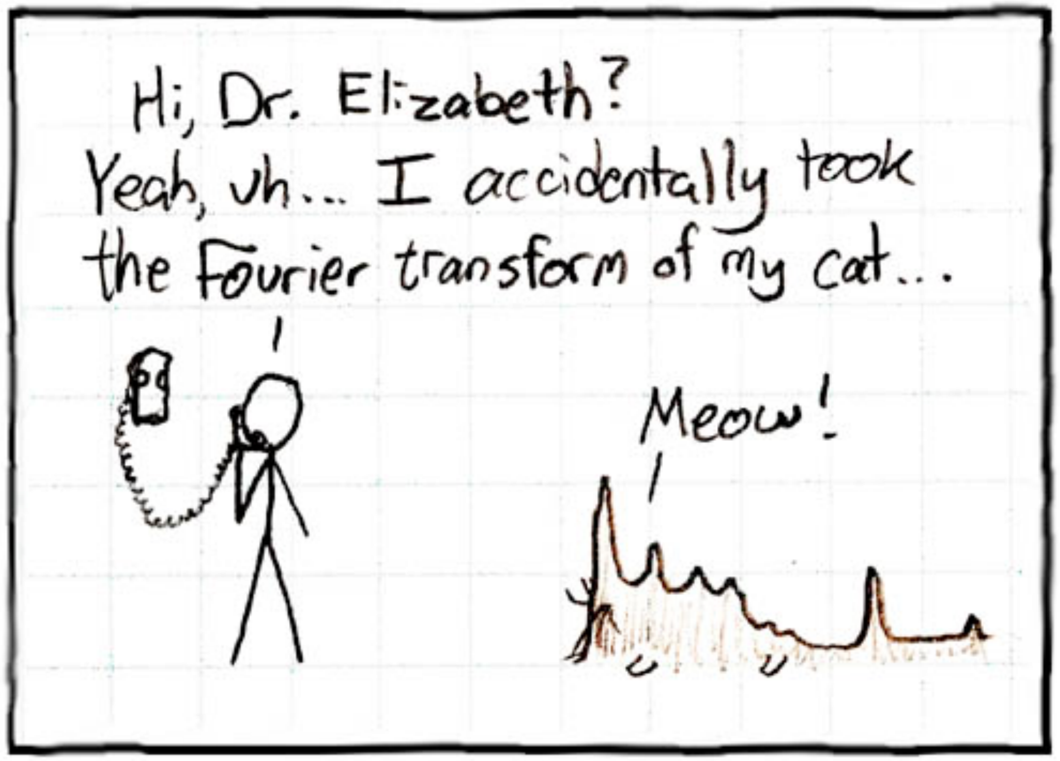
Function

Sine waves
(Complex exponential)

$$f(x) = \int_{-\infty}^{+\infty} F(k) e^{2\pi i k x} dk$$

$$F(k) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi i k x} dx$$

Can solve for Fourier coefficients



# The Fourier Transform

- Every function can be expressed as an infinite sum of sine waves (or complex exponentials)

Fourier series are the amplitudes of those sine waves at each frequency

# The Fourier Transform

- Example 1: Single frequency

$$
\begin{aligned}
f(x) &= e^{2\pi i k_0 x} \\
F(k) &= \int_{-\infty}^{+\infty} e^{2\pi i k_0 x} e^{-2\pi i k x} dx \\
F(k) &= \int_{-\infty}^{+\infty} e^{2\pi i (k_0 - k) x} dx \\
F(k) &= \delta\left(k - k_0\right)
\end{aligned}
$$

The Fourier transform of a single frequency is a delta function at that frequency

# The Fourier Transform

- Example 2: Gaussian distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$F(k) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\infty} e^{-\frac{x^2}{2\sigma^2}} e^{2\pi i k x} dx$$

$$F(k) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{+\infty} e^{-\frac{1}{2\sigma^2}\left(x^2 - 2(2\pi i \sigma^2 k)x + (2\pi i \sigma^2 k)^2 - (2\pi i \sigma^2 k)^2\right)} dx$$

$$F(k) = \frac{1}{\sqrt{2\pi}\sigma} \left[ e^{\frac{-4\pi^2\sigma^4 k^2}{2\sigma^2}} \right] \left[ \int_{-\infty}^{+\infty} e^{-\frac{1}{2\sigma^2}\left(x - 2\pi i \sigma^2 k\right)^2} dx \right]$$

$$F(k) = \frac{1}{\sqrt{2\pi}\sigma} \left[ e^{\frac{-4\pi^2\sigma^4 k^2}{2\sigma^2}} \right] \left[ \sqrt{2\pi}\sigma \right]$$

$$F(k) = e^{-2\pi^2\sigma^2 k^2}$$

**The Fourier transform of a Gaussian with width ~sigma is another Gaussian with width ~1/sigma**

See derivation here:

https://warwick.ac.uk/fac/sci/mathsys/courses/msc/ma934/resources/notes8.pdf

# Uncertainty principle

- In terms of wavefunctions, considering a wave packet that is Gaussian in x with width $\Delta x$, the momentum distribution will have a width $\Delta p$ that is inversely proportionate, with a minimum of $\hbar/2$:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

- This is a consequence of wave / particle duality

# Coherence and Decoherence:

- Coherence is mathematically related to the correlation of two time series (or functions) [wiki]
- Given two time series a(t) and b(t), their correlation is defined as

$$(a * b)(\tau) = \int_{-\infty}^{+\infty} \overline{a(t)} b(t+\tau) dt$$

where the bar over a(t) denotes complex conjugation (Note, the autocorrelation is when a=b)

- If A and B are the Fourier transforms of a and b, from the convolution theorem,

$$(a * b)(t) = \mathcal{F}^{-1}(A(f) \cdot B(f))$$

Where $\mathcal{F}^{-1}$ is the inverse Fourier transform and $A \cdot B$ is point-wise multiplication

Convolving numbers in the time domain is the same as multiplying numbers in the frequency domain and taking the inverse F.T.

# Coherence and Decoherence

- So given the spectral "density" distributions from the Fourier transform, we can define the coherence function as

$$\gamma^2_{ab}(f) = \frac{|S_{AB}(f)|^2}{S_{AA}(f)S_{BB}(f)}$$

- The decoherence time is a time interval, so given a frequency band $\Delta f$, the decoherence time is also inversely proportional to the spread in frequency (bandwidth):

$$\tau_c \Delta f \geq 1$$

# What is the “weirdness” everyone obsesses about?

- Uncertainty principle:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

You cannot simultaneously know the position and momentum of a state with infinite precision

- Coherence and Decoherence:

$$\tau_c \geq \frac{\hbar}{\Delta p}$$

Systems with fewer momentum components decohere more slowly

Both are mathematical consequences of the wave nature of matter and follow from the Fourier decomposition

# What is the “weirdness” everyone obsesses about?

- In addition to position and momentum (or time and frequency), ANY operators that do not have trivial commutation will have the same behavior.

- Components of spin and angular momentum (Jx, Jy, Jz, or J and Jz)
  - Probably familiar from previous physics classes

- Mass and flavor eigenstates
  - Probably NOT familiar

# Quantum States

- Before we go to quantum computing, it is helpful to introduce the concept of a quantum "state" that you may not be familiar with
- Quantum state: a set of specified quantum numbers
- Use "bra-ket" notation:
  - This is technically a vector in Hilbert space

bra $< \alpha|$

ket $|\alpha >$

- Operators ($\mathcal{O}$) can be applied to quantum states on the left or the right (similar to a matrix in Hilbert space)

$$< \alpha|\mathcal{O}$$

$$\mathcal{O}|\alpha >$$

$$< \alpha|\mathcal{O}|\alpha >$$

Also known as a "matrix element"

# Quantum States

- Concretely, let's consider quantum spin of fermions (spin-1/2 particles)
- In the base representation, there are two states
  - notations include up and down, or + and -, or +1/2 and -1/2 (all equivalent):

$$|\uparrow>, |\downarrow> \qquad |+>, |-> \qquad |+1/2>, |-1/2>$$

    - Sometimes the last one also denotes the total spin like:

$$|s, s_z> = |1/2, +1/2>, |1/2, -1/2>$$

- This is therefore really a two-component vector, we will write like:

$$|S_z+> = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|S_x\pm> = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ 1 \end{pmatrix} \quad |S_y\pm> = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

$$|S_z-> = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

# Quantum States

- The operators for spin in the x,y,z directions use the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

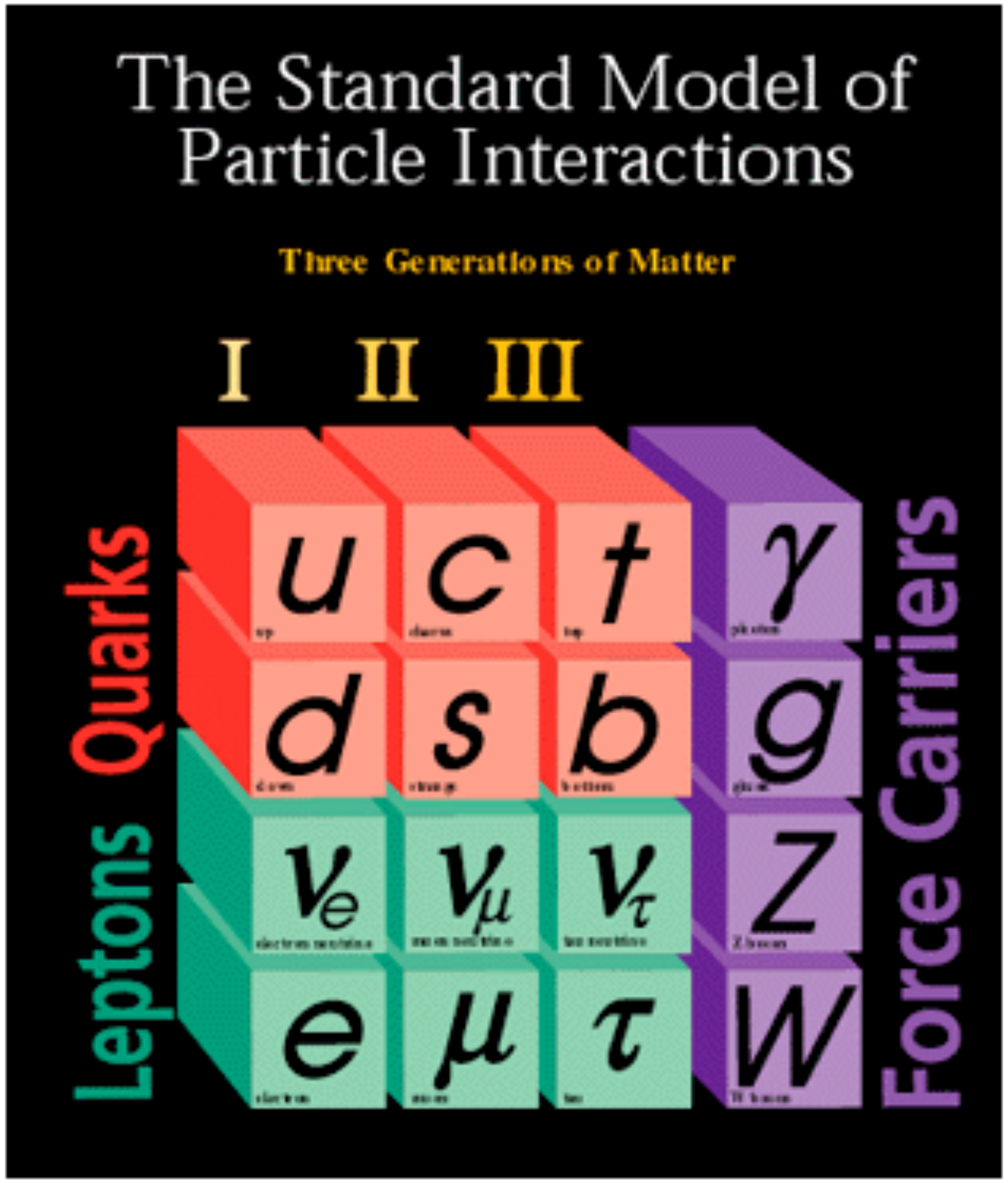
- And then the spin operators are

$$\hat{S}_{x,y,z} = \frac{\hbar}{2}\sigma_{x,y,z}$$

- These are NOT measurable simultaneously and also obey the uncertainty principle
- Can also measure along an arbitrary direction aside from x,y,z (more in a few slides)

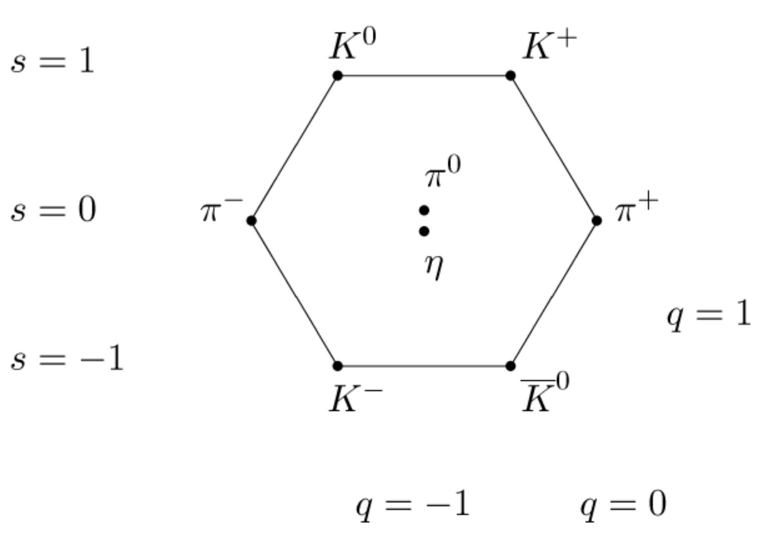
# Quantum States

- Another demonstration of a quantum state is the “flavor” of a particle versus its mass
- 6 quarks
  - Up / down
  - Charm / strange
  - Top / bottom
- 6 leptons
  - Electron / electron neutrino
  - Muon / muon neutrino
  - Tau / tau neutrino
- 4 force carries:
  - E+M: photon
  - Weak interaction: W+Z
  - Strong interaction: gluon

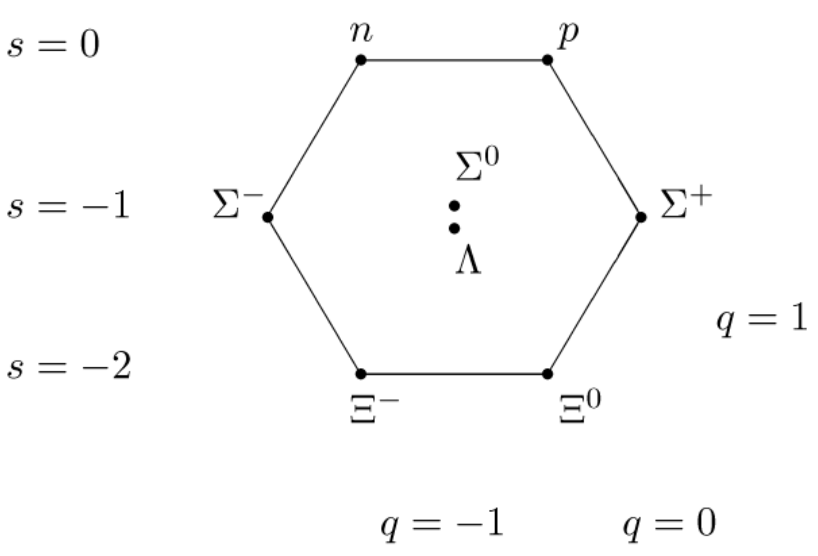


# Quantum States

- The composition of hadrons is a group of quarks (usually 2 or 3)
  - up/down/strange SU(3):



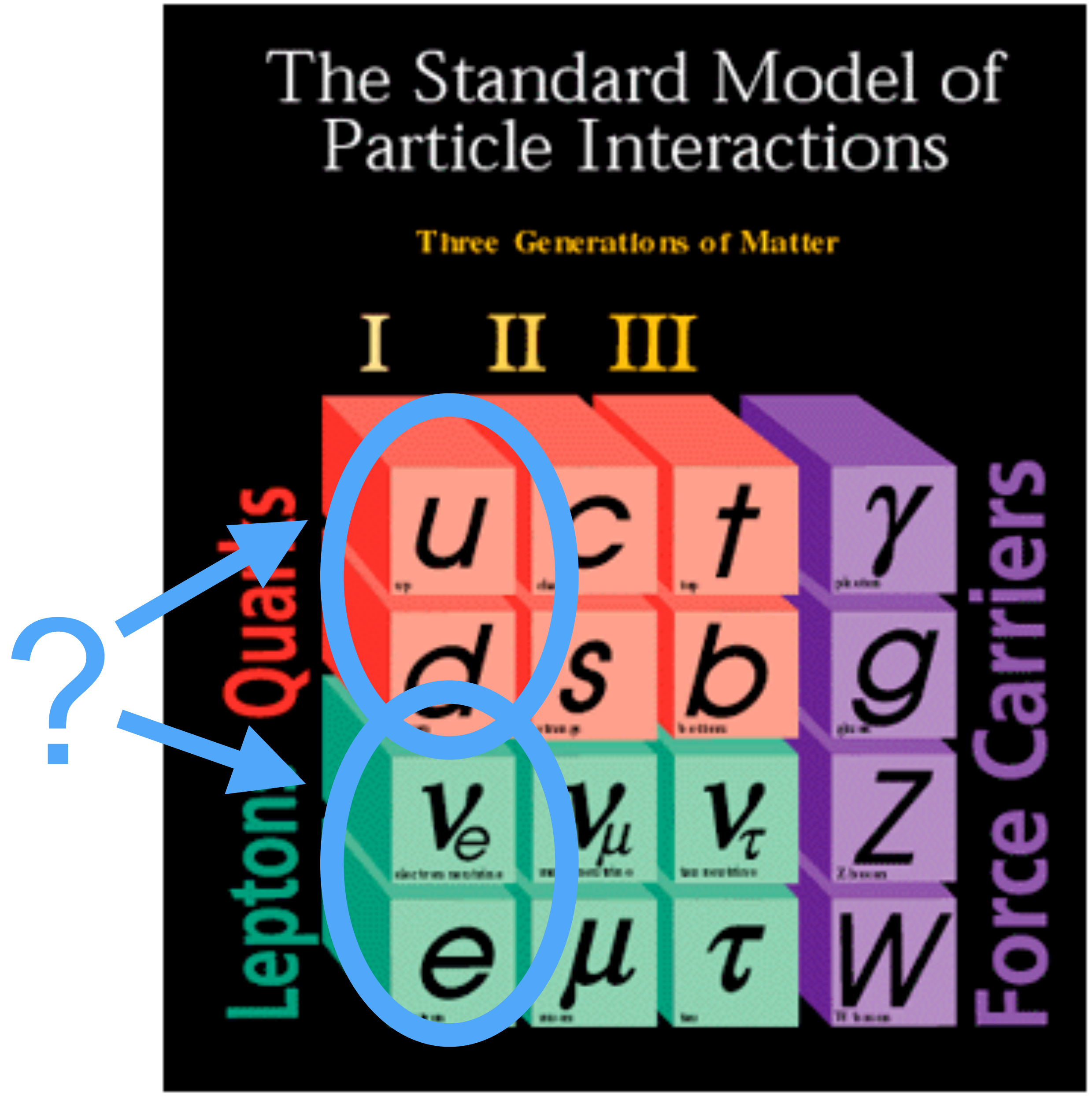

2 quarks in meson




3 quarks in baryon

# Weirdness with the Weak interaction

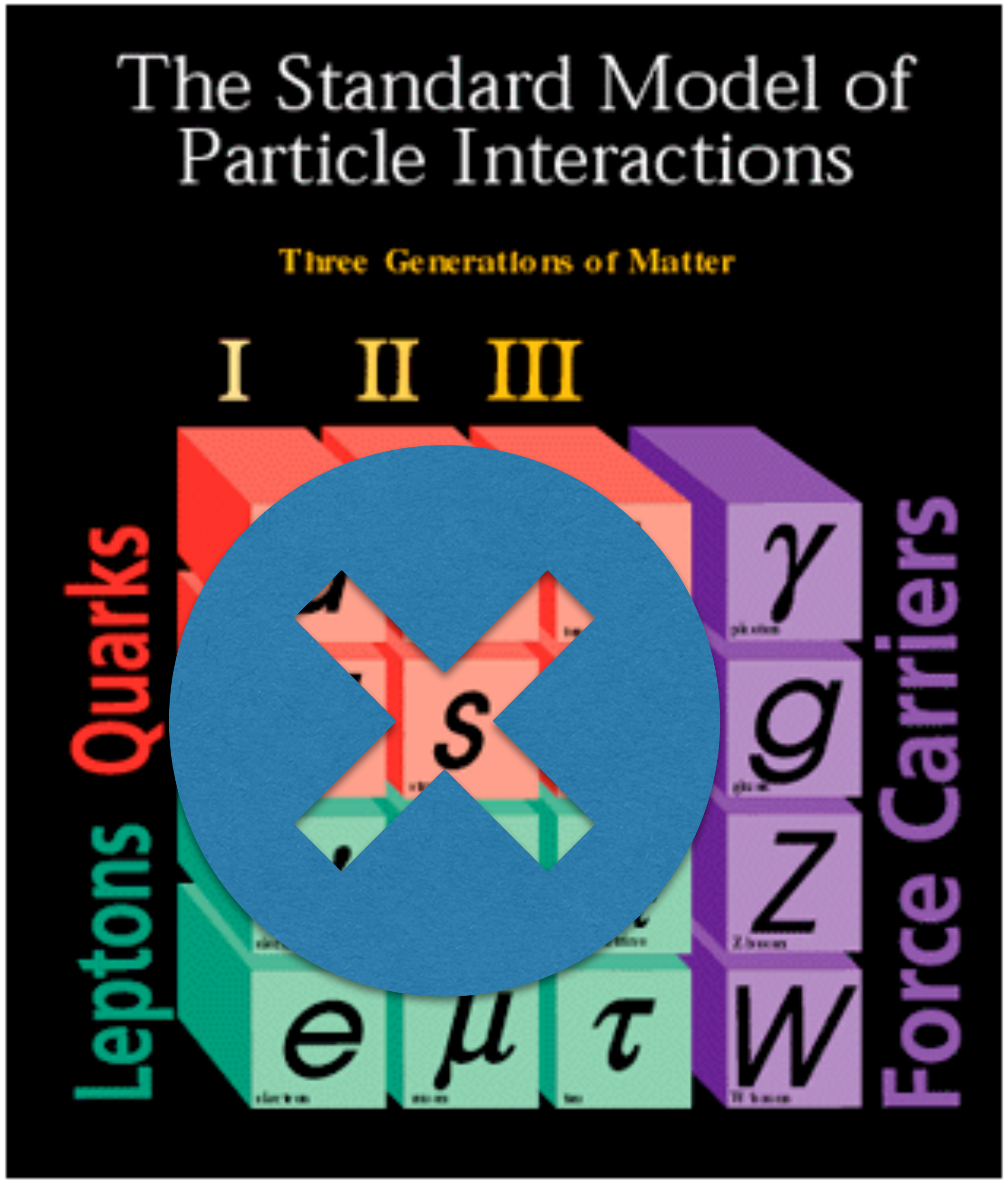
- The weak interaction couples 2 objects in an SU(2) "doublet" that works similarly to spin
  - u+d, $\nu_e$+ e?



# Weirdness with the Weak interaction

- The weak interaction couples 2 objects in an SU(2) "doublet" that works similarly to spin
  - u+d, $\nu_e$+ e?

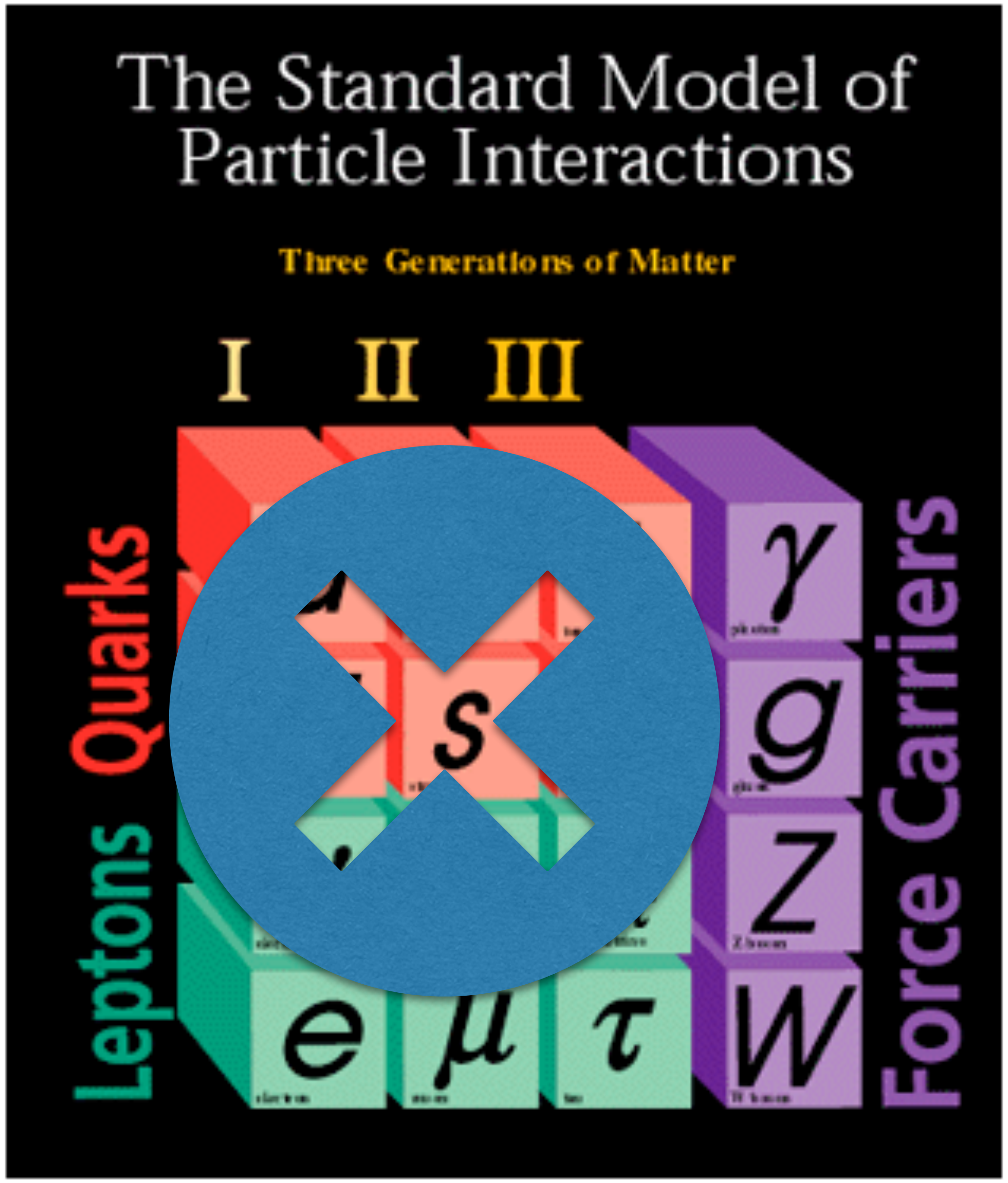
- Nope.



# Weirdness with the Weak interaction

- The eigenstates of the weak interaction are not the same as the particles themselves ("mass eigenstates") so there is a quantum-mechanical mixing that occurs between states
- Quarks: CKM matrix
- Neutrinos: PNMS matrix

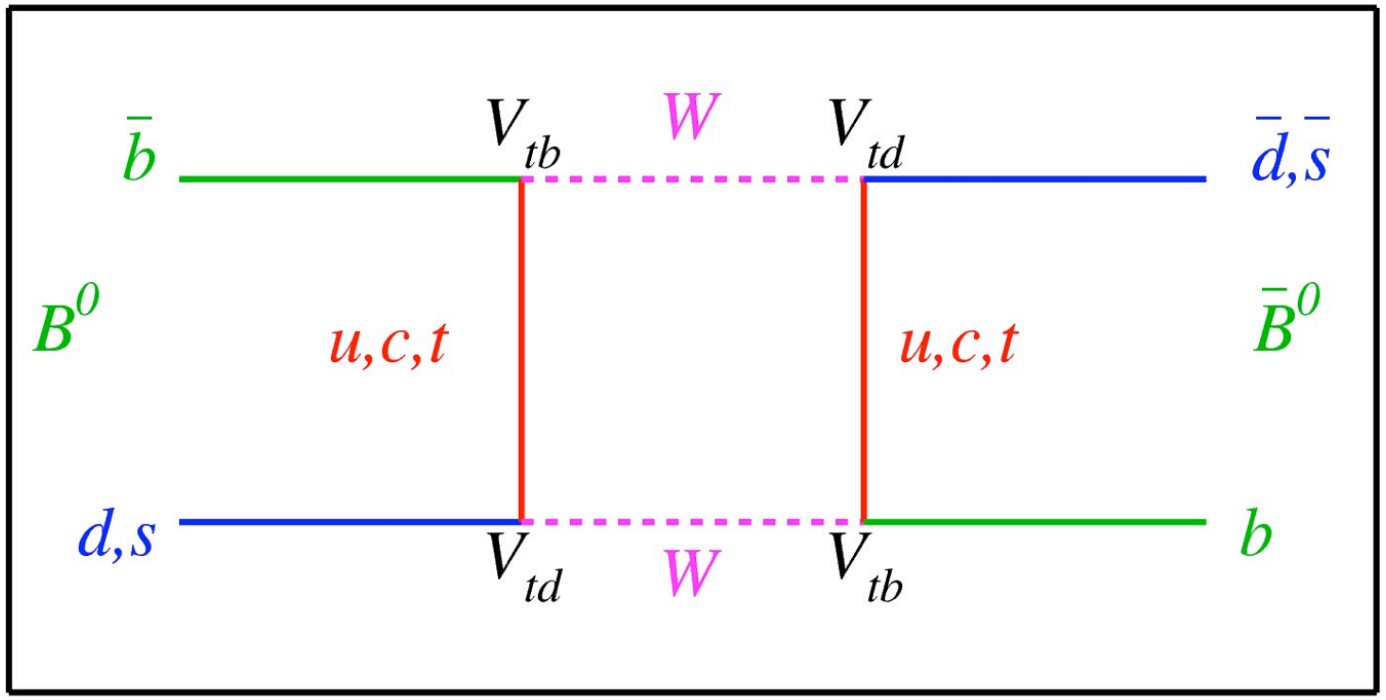
- Both quarks and neutrinos will mix and oscillate back and forth!



# Coherent states in the weak interaction

- Famous example: $B_0$ mixing and $\mathrm{B}_{0,s}$ mixing
- Spectacular measurement by the LHCb experiment to measure the oscillation frequency

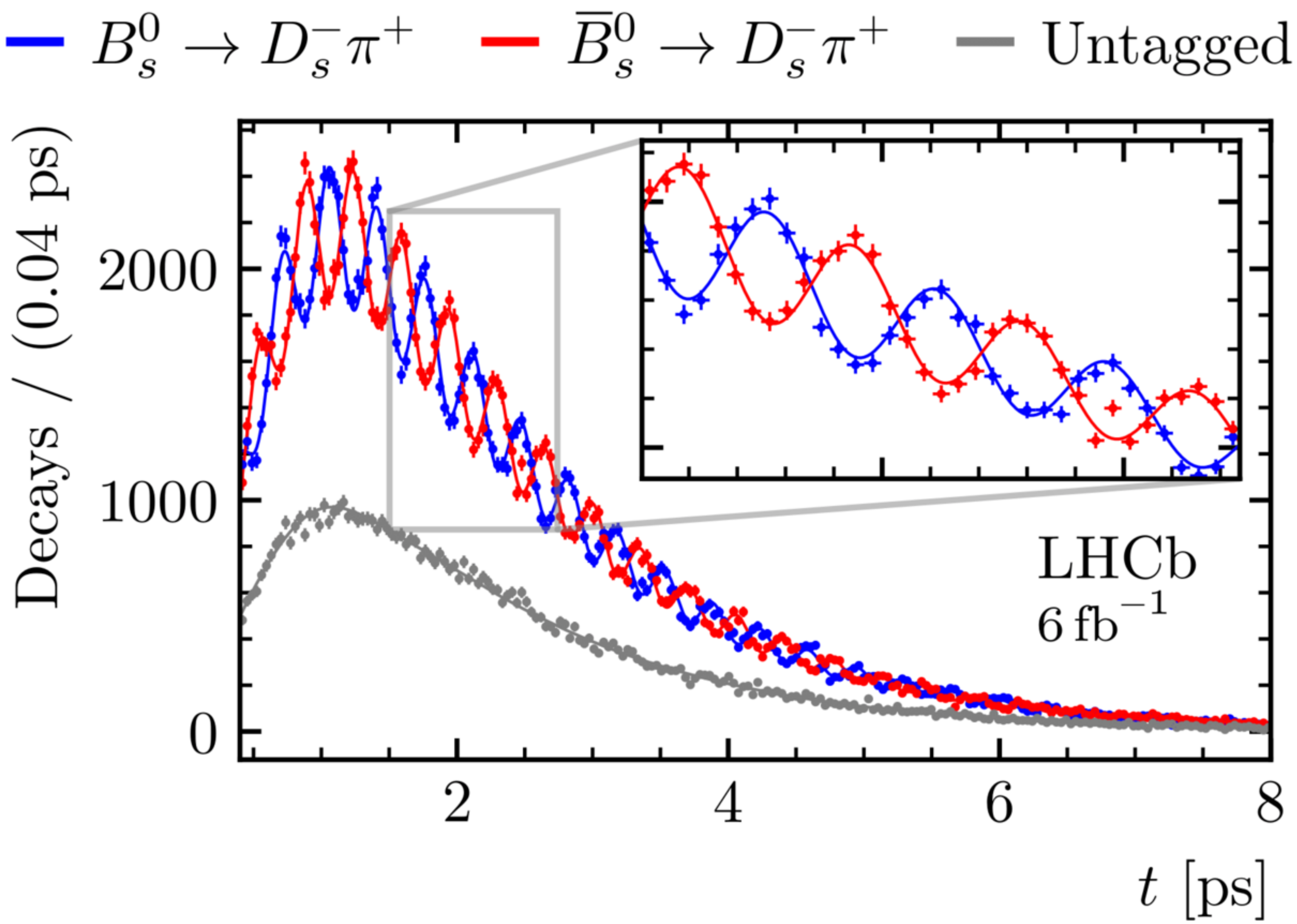
Coherent state




Coherent state

# Coherent states in the weak interaction

- Absolutely gorgeous demonstration of long-distance coherence oscillating back and forth!
- Because the number of states is very small (~2) the decoherence time is very long



# Computing with Coherent States

- The keys to quantum computing are to set up appropriate coherent states that are
  - Stable enough
  - Correctly readable

- Active area of research (also here at UB in the CM group!)

# EPR paradox

- Named after Einstein, Podolsky, Rosen
- “spukhafte Fernwirkung”, spooky interaction at a distance

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

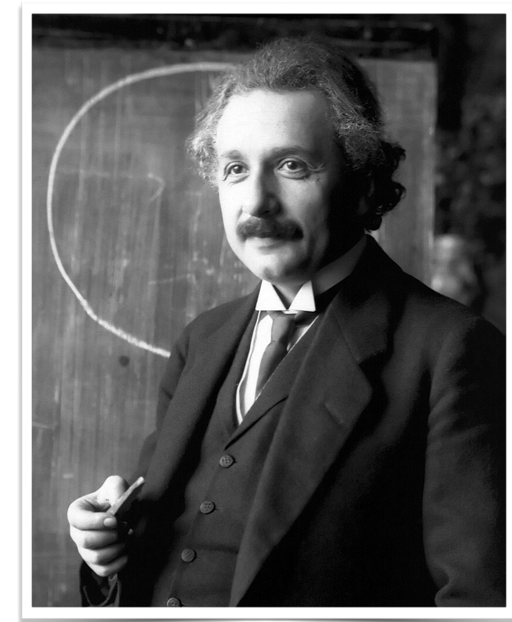
## Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

- How can we figure out if Quantum Mechanics is right, or we just don’t know everything?



**EINSTEIN ATTACKS QUANTUM THEORY**

Scientist and Two Colleagues Find It Is Not ‘Complete’ Even Though ‘Correct.’

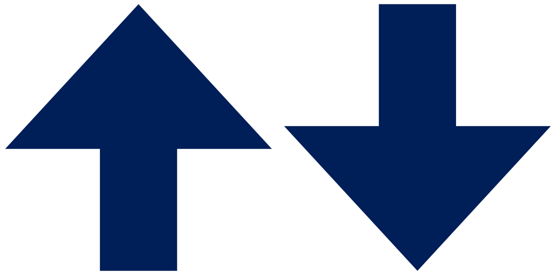
SEE FULLER ONE POSSIBLE

Believe a Whole Description of ‘the Physical Reality’ Can Be Provided Eventually.

NEW YORK TIMES, PUBLIC DOMAIN

# Gedankenexperiment

- 2 particles in an entangled state
  - Spin / mass / etc



# Gedankenexperiment

- 2 particles in an entangled state
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- They travel apart

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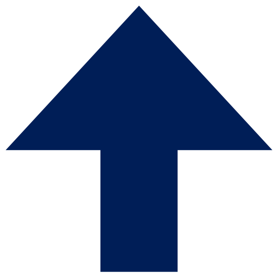
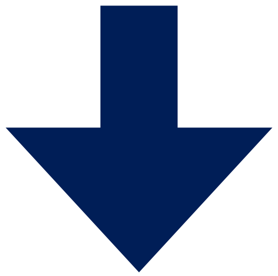


One is measured by “Alice”



One is measured by “Bob”

# Gedankenexperiment

- 2 particles in an entangled state
  - Spin / mass / etc
- They travel apart

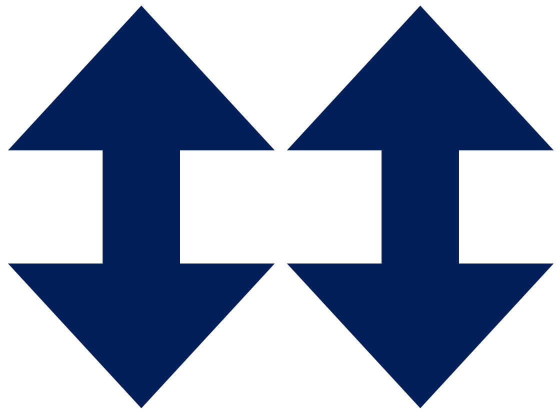
One is measured by “Alice”

One is measured by “Bob”

**But this picture is wrong!**

# Gedankenexperiment

- 2 particles in an entangled state
  - Spin / mass / etc



$$|A>=\frac{1}{\sqrt{2}}(|\uparrow> + |\downarrow>)$$

$$|B>=\frac{1}{\sqrt{2}}(|\downarrow> + |\uparrow>)$$

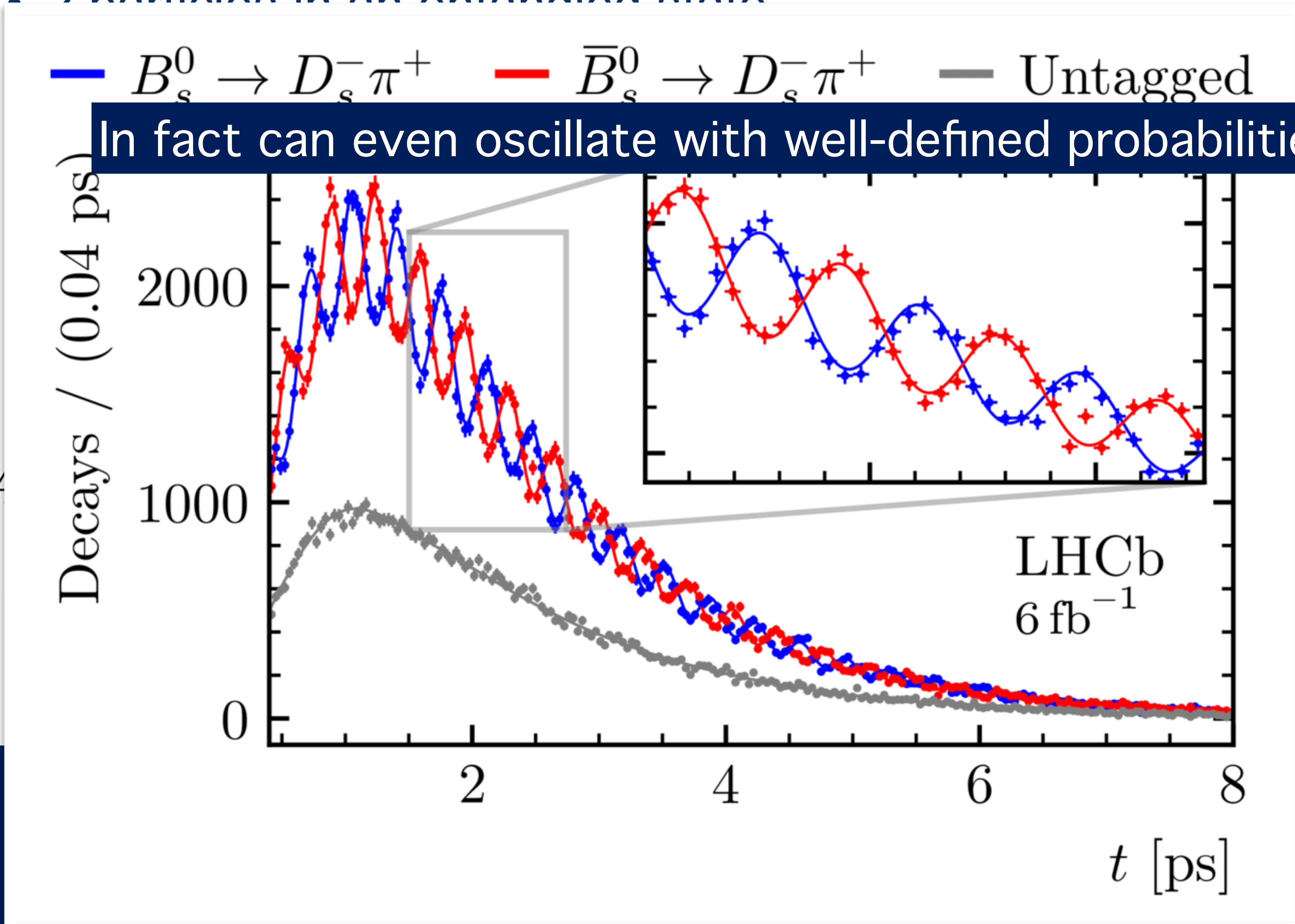
$$|AB>=\frac{1}{\sqrt{2}}(|\uparrow\downarrow> + |\downarrow\uparrow>)$$

Don’t know which is “up” and which is “down” to begin with. That’s not even a sensible question, they’re both in a superposition of up and down

# Gedankenexperiment

- 2 particles in an entangled state

**In fact can even oscillate with well-defined probabilities!**



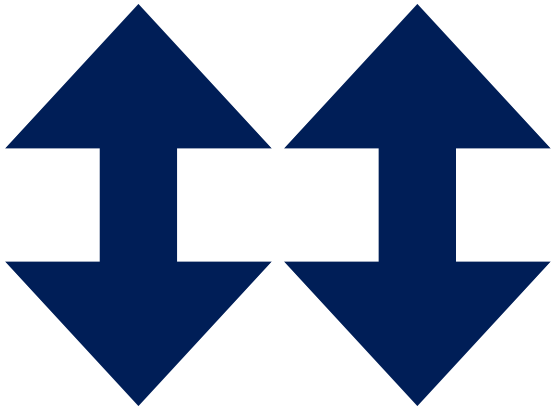

$|A$ … $)$

ith.

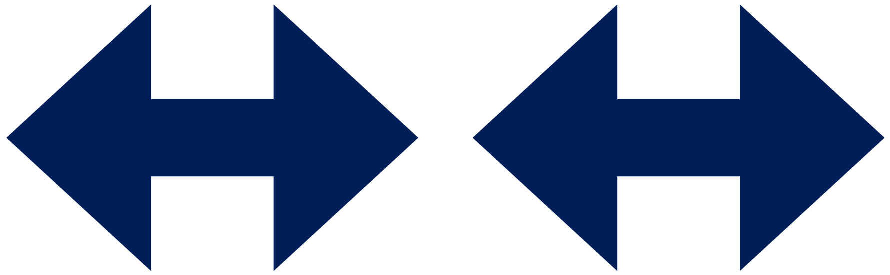
# Gedankenexperiment

- How to see this? Is there a difference, if the states were determined the whole time (via a hidden variable, “element of reality”)
- Or if the states are only determined at the time of measurement
- Gedankenexperiment:
  - Measure in different axis bases

Alice measures z-spin



Bob measures x-spin



These are incompatible observables, so QM predicts that “collapse” in z component does not affect x component!

# Gedankenexperiment

- So there is a difference if Alice measures in z basis and Bob measures in x basis
- Bob's (Alice's) state will not be determined in Quantum Mechanics, however it would be if there is a hidden variable, in particular if the decision in which basis the measurement will happened is done after the two particles are separated.
- Great, but how to do the real experiment?

VOLUME 49, NUMBER 2 PHYSICAL REVIEW LETTERS 12 JULY 1982

**Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities**

Alain Aspect, Philippe Grangier, and Gérard Roger
*Institut d'Optique Théorique et Appliquée, Laboratoire associé au Centre National de la Recherche Scientifique, Université Paris-Sud, F-91406 Orsay, France*
(Received 30 December 1981)

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers (i.e., optical analogs of Stern-Gerlach filters), is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved.

PACS numbers: 03.65.Bz, 35.80.+s

# Bell's inequality

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

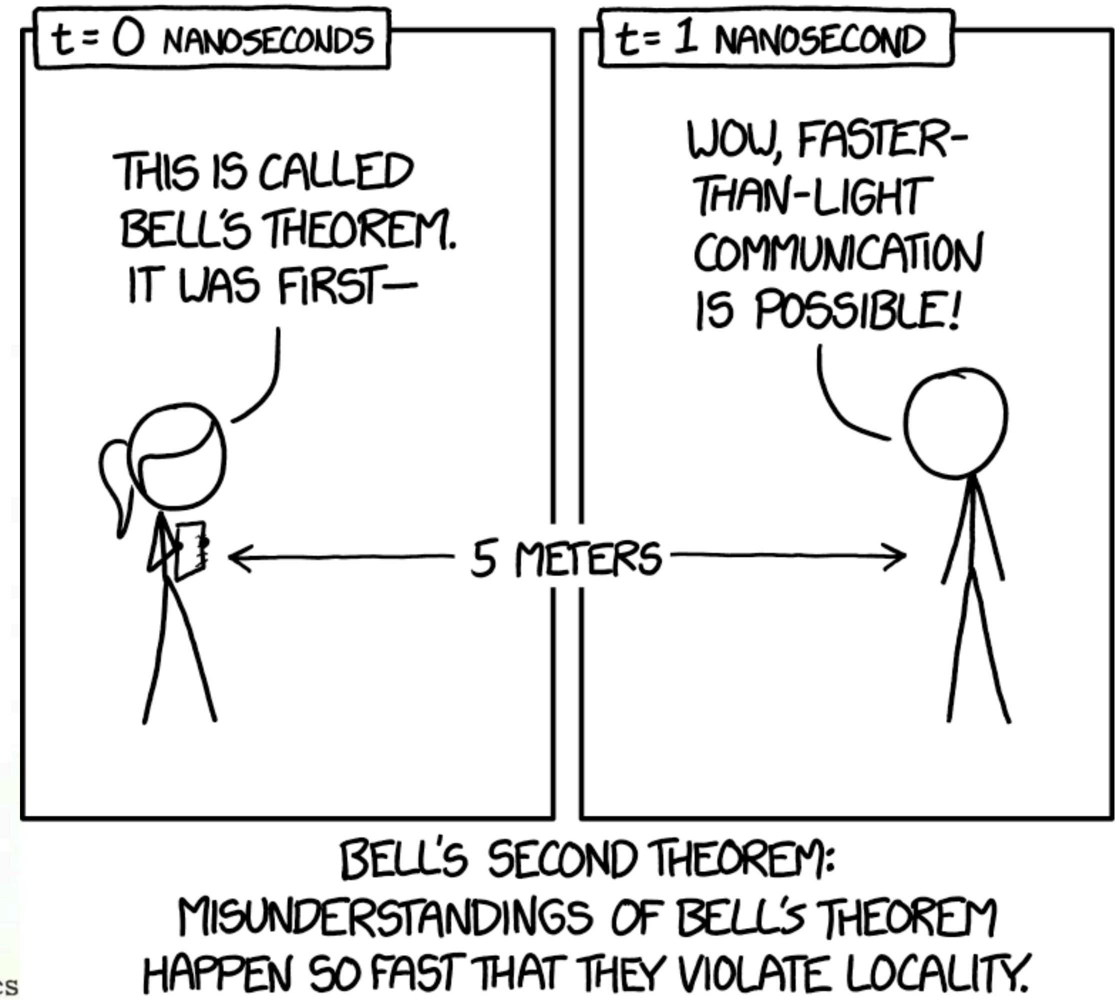
## ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
*Department of Physics, University of Wisconsin, Madison, Wisconsin*

(*Received* 4 *November* 1964)

### I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.



# Bell's inequality (CHSH version)

- Bell suggested to not use orthogonal but any kind of basis to do the measurements
  - We'll use Clauser, Horne, Shimony, and Holt (CHSH) version
- Eg:
  - Alice measures in z basis, Bob in z basis
  - Alice measures in z basis, Bob in z + pi/6 basis
  - Alice measures in z basis, Bob in z + 2(pi/6) basis
- Einstein: "das is einfach" (that's easy)
  - Independent from the basis we can calculate:
    - that if Alice measures in A and a basis
    - And Bob in B and b basis

$$\langle CHSH \rangle = \langle AB \rangle - \langle Ab \rangle + \langle aB \rangle + \langle ab \rangle \leq 2$$

    - Since: A,a,B,b can only yield ±1, so (B-b) and (B+b) can only be 0 or ±2 and thus A(B-b) and a(B+b) can only be ±2 and thus above inequality must be true for any case

# Hands on

- https://quantum-computing.ibm.com/jupyter/user/qiskit-textbook/content/ch-demos/chsh.ipynb
- https://qiskit.org/documentation/apidoc/tools_jupyter.html?highlight=get_provider

# Closing Bell Loopholes

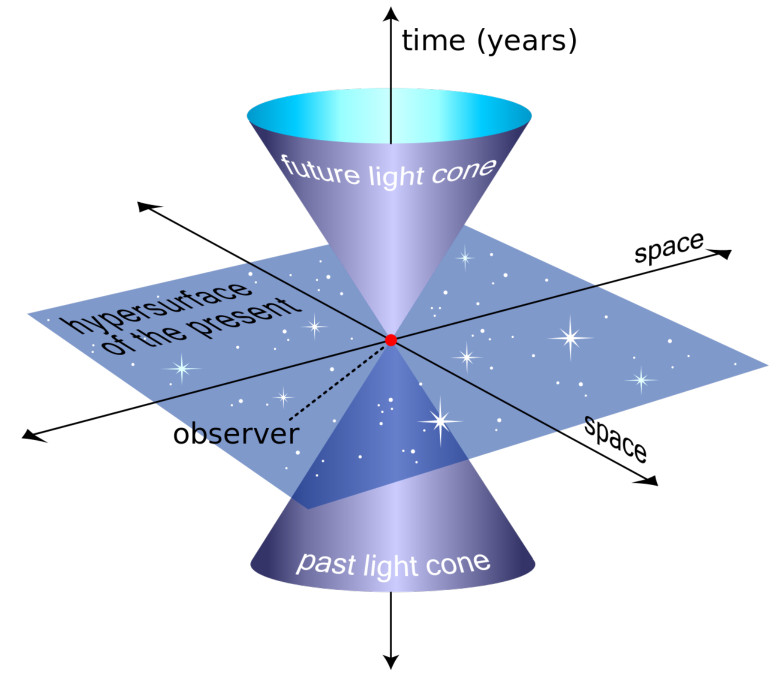
- Many Loopholes were suggested
  - Detection loophole (QC disproves that)
  - Locality loophole (QC still has that)
  - Freedom of Choice Loophole (https://arxiv.org/pdf/1805.04431.pdf)
  - …
- All are closed in the meantime (https://physics.aps.org/articles/v8/123)

**"Quantum Entanglement Documentary Atomic Physics and Reality", John Archibald Wheeler, John Stewart Bell, Alain Aspect, David Bohm, et.al. (1985)**

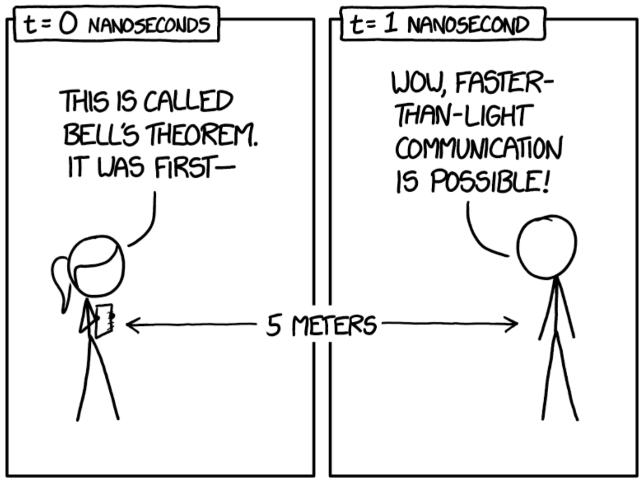
**https://www.youtube.com/watch?v=SEjte11wPeI**

# What have we learned?

- … how to run our first "real" python program that actually solves a physics question
- … that we can run code on a  QC that proofs Einstein wrong
- … how to read and understand code written by someone else
- … what is Realism and Nonlocality; in particular, a physical theory is:
  - real, if: the properties exist independent of a measurement (just disproved that for QM, bad choice of words here …)
  - nonlocal, if: two measurements apart from each other in space are correlated (nonlocal: Newton, Maxwell, QM, local: relativity)






BELL'S SECOND THEOREM: MISUNDERSTANDINGS OF BELL'S THEOREM HAPPEN SO FAST THAT THEY VIOLATE LOCALITY.

HTTPS://XKCD.COM/ABOUT/

# Quantum Computing Frameworks

- Qiskit (IBM) - we are using that
- Tensorflow Quantum / CIRQ (Google)
- Strawberry fields (Photonic quantum computing)
- Q# (Q sharp) (Microsoft)
- QuTiP (Quantum Toolbox in Python)

- So that’s a lot of programming languages? No, ALL of them can be used with python!

# Example: 1+1

- Let’s compute something with a quantum computer.
- You probably do not know how a classical computer does this either so we cover that on the way.

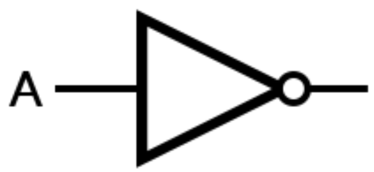
# Information storage (memory)

- We have to represent information. We do this in a (Qu)bit $q_0$
- Let's define (we do that slightly differently later): $|1\rangle := 1, \text{and}$
  $|0\rangle := 0$
- Now, we have the first problem since we "know" 1+1=2 ($q_0+q_1$) and we have no 2!
- Let's get another Qubit q for storing the "overflow"


$|0\rangle$, or $|1\rangle$ $\quad$ $|0\rangle$, or $|1\rangle$

- And represent 2 as the sum of $q_2$ and $q_3$

# Operations (Gates)

- But how do we operate on information?
- Gates: eg
  - NOT  / 
  - But this only acts on one Qubit
  - Reversible



| Input | Output |
|---|---|
| $q_0$ | $q_0$ |
| $\lvert 0\rangle$ | $\lvert 1\rangle$ |
| $\lvert 1\rangle$ | $\lvert 0\rangle$ |

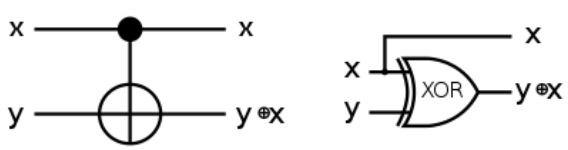
# Operations (Gates)

- Multi bit gates
- eg
  - XOR 
  - Not reversible

| Input | | Output |
|---|---|---|
| $q_0$ | $q_1$ | $q_2$ (sum bit) |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

# Operations (Gates)

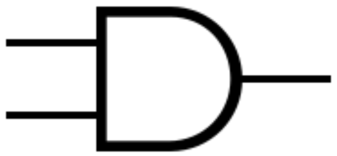
- Multi Qubit gates
- eg
  - CNOT 




  - reversible XOR

| Input | | Output | |
|---|---|---|---|
| $q_0$ | $q_1$ | $q_0$ | $q_1+q_0$ |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 |

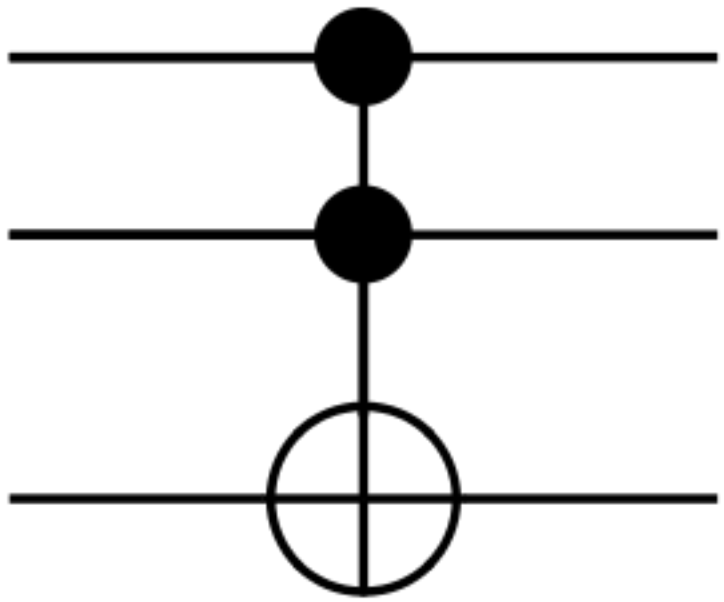
# Operations (Gates)

- Classic gate
- eg
  - AND 
  - Non reversible

| Input | | Output |
|---|---|---|
| $q_0$ | $q_1$ | $q_3$ (carry bit) |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

# Operations (Gates)

- Multi Qubit gates
- eg



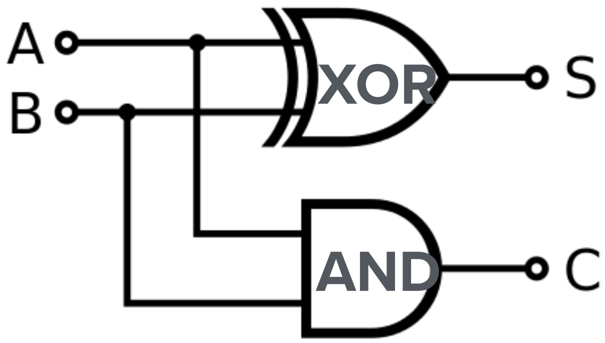
  - TOFFOLI
  - CCNOT gate
  - Reversible

| Input | | | Output | | |
|---|---|---|---|---|---|
| $q_0$ | $q_1$ | $q_3$ | $q_0$ | $q_1$ | $q_3$ |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |

# Half adder

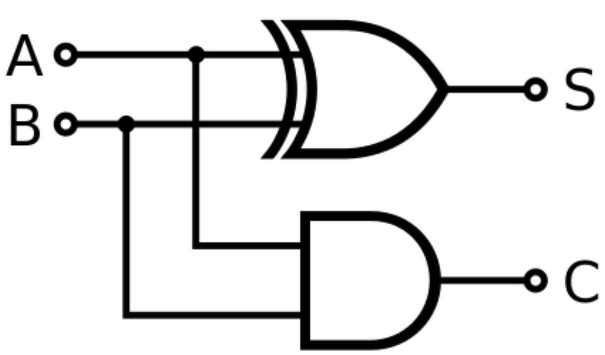
- If we combine an XOR (CNOT) and an AND (Tofoli) gate we can implement addition.

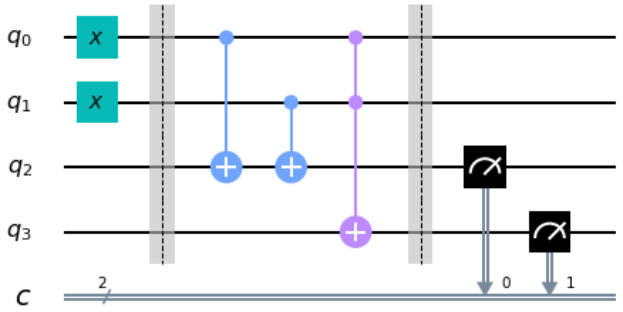
| $q_0$ | $q_1$ | $q_2$ sum | $q_3$ carry |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |



# What have we learned?

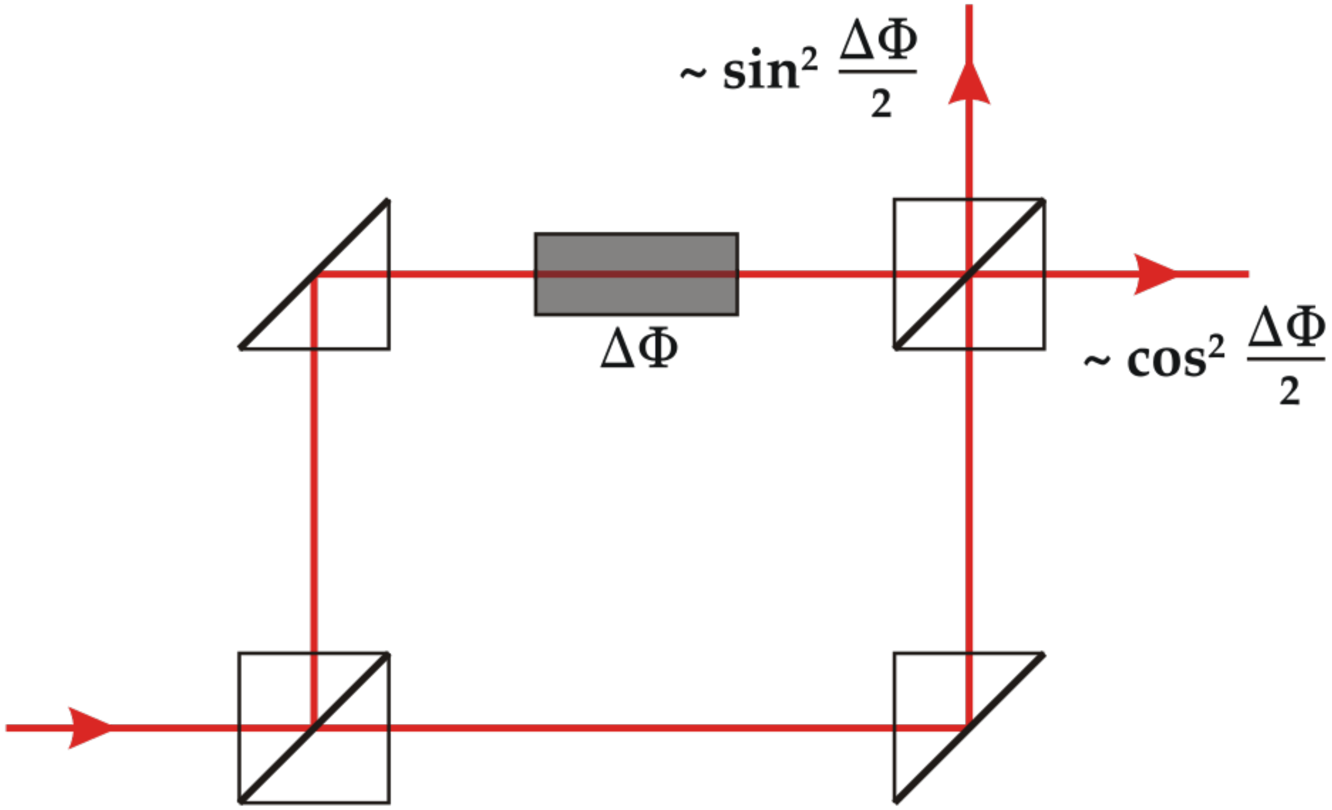
- … how to program a quantum computer! Yeah! Although for a classical example …
- … a QC is not fundamentally more complicated than a classical computer
- … that a QC probably shouldn't be used for classical computation
- … how to prepare and measure a quantum state
- … that we have to do a measurement!
- … that to add 2 Qubits we need actually 4 Qubits to store all information
- … that we need three Quantum gates (2xCNOT and 1xToffoli) to do one add operation
- … that this circuit is called a "half adder"



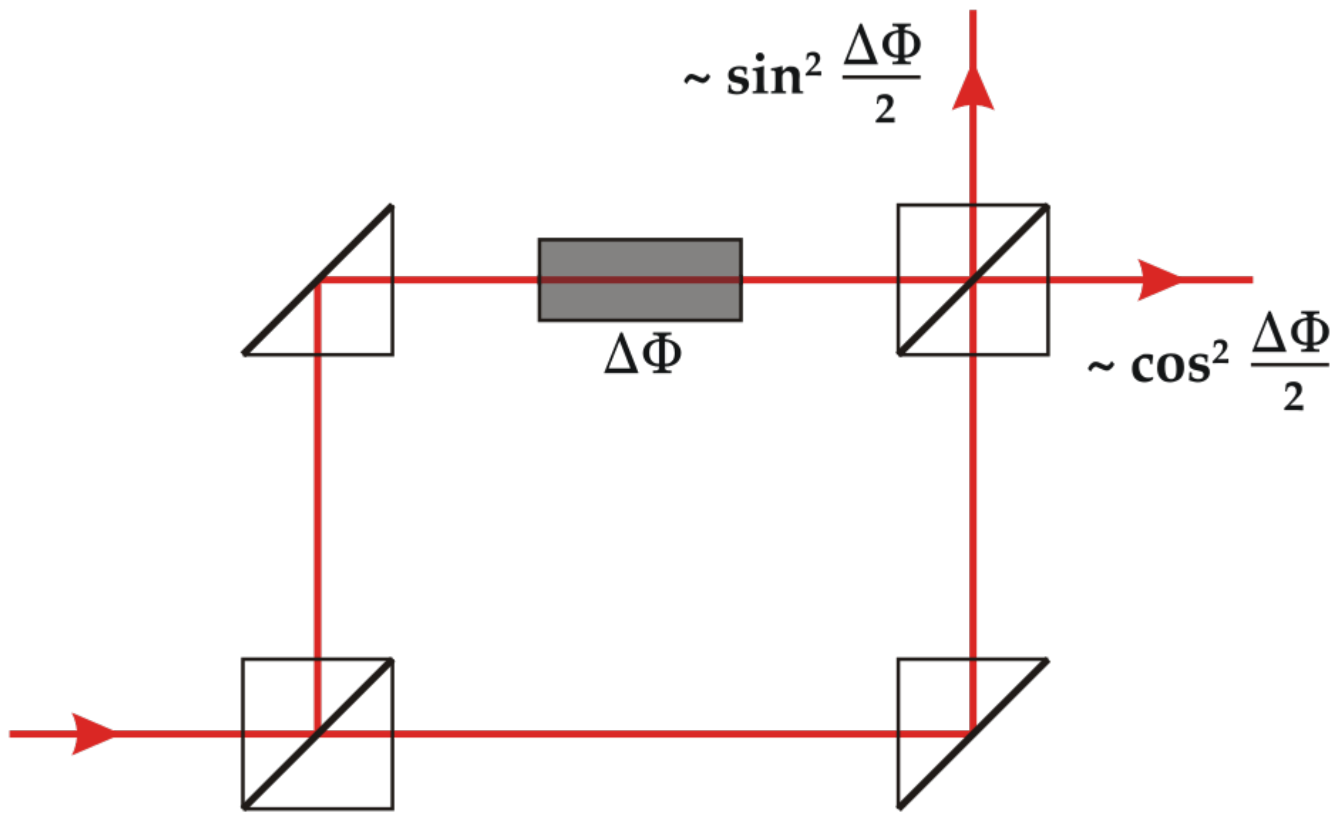



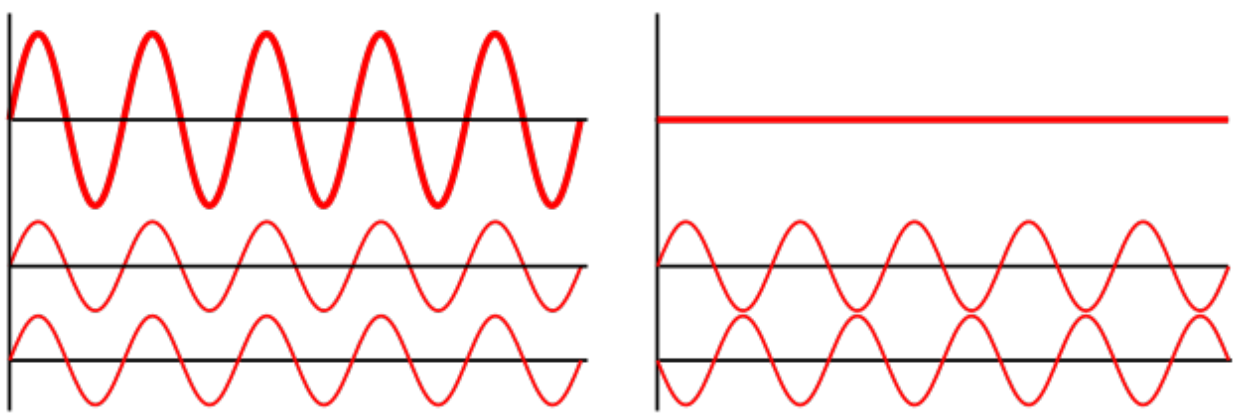
# Example: Mach-Zehnder Interferometer

- Let's take an easy optics example
- A MZ Interferometer consists of 2 equal Beamsplitters (BS)
- In the 1st BS the light is split equally
- In the 2nd BS the light is combined again
- Ignoring any phase shifts at interfaces it depends on the difference of optical path length how much light we detect at the two outputs of the 2. BS
- If the path difference is just half the wavelength we get constructive interference at one output and destructive at the other



# Example: Mach-Zehnder Interferometer



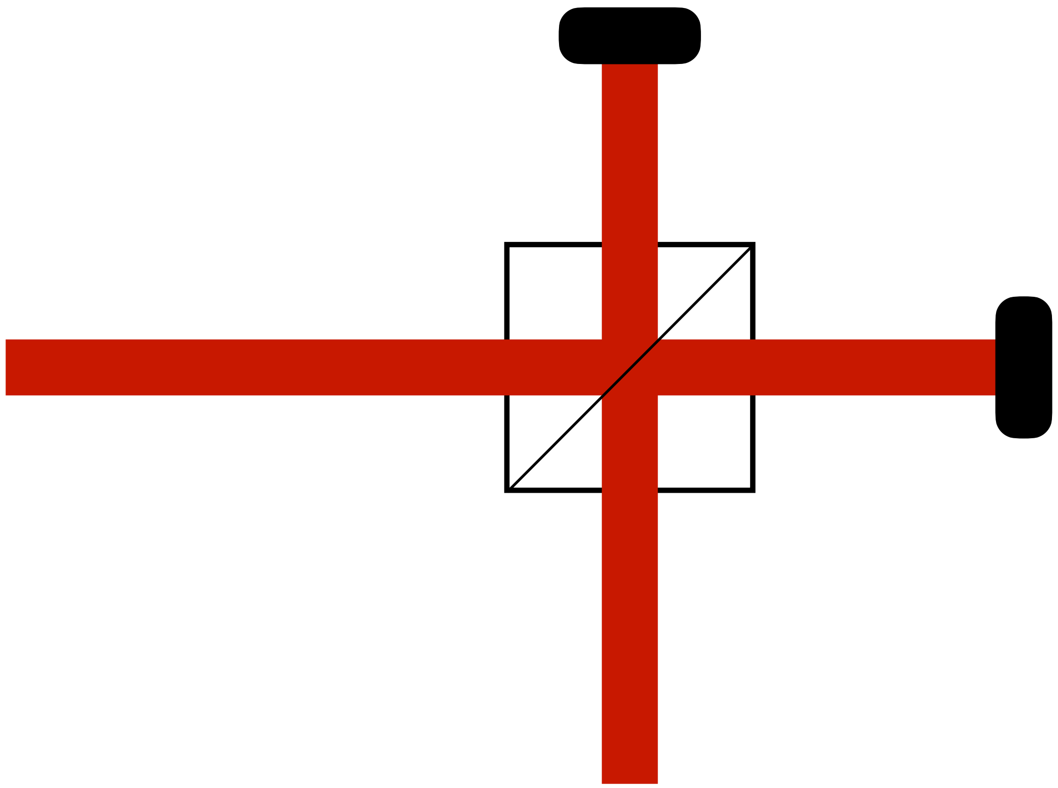


https://www.youtube.com/watch?v=M6y_igUpyCg
https://www.youtube.com/watch?v=os4rDtjrP4c

# Another kind of state

- Let’s have a laser pointer and a beam splitter
- Reversible

$\begin{pmatrix}1\\0\end{pmatrix} = |1\rangle$



$\begin{pmatrix}0\\1\end{pmatrix} = |0\rangle$

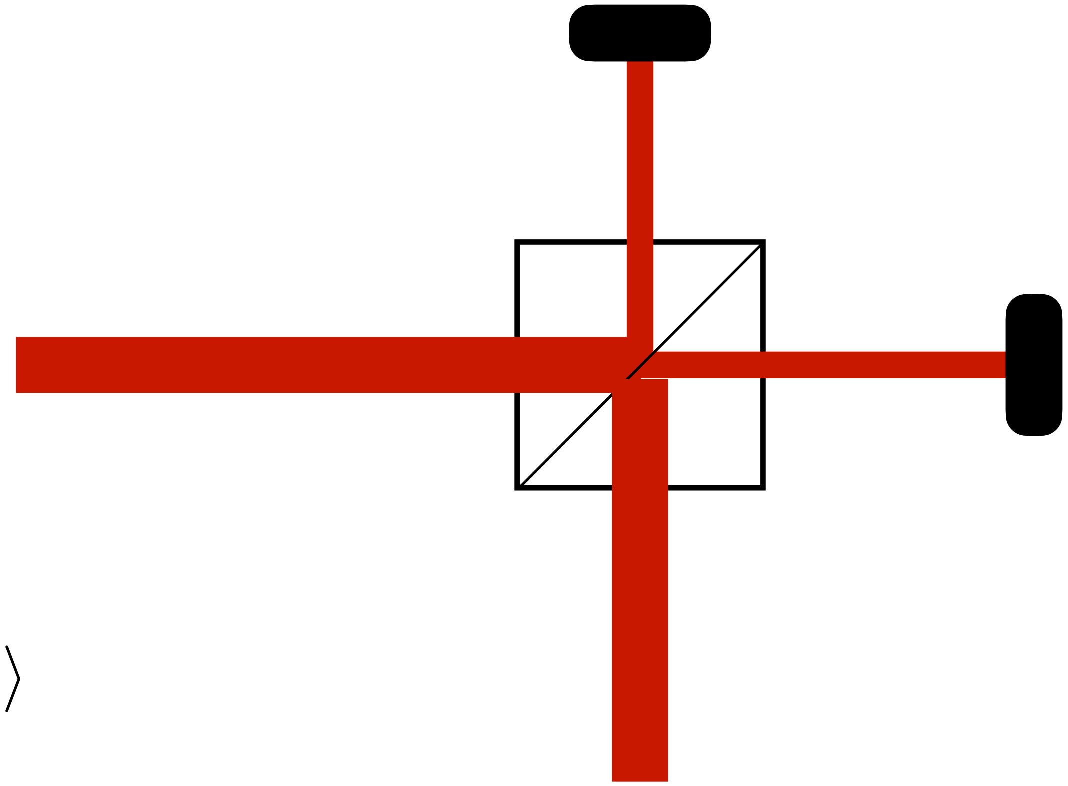
# Superposition

- Let's call that state a super position:
- And introduce an operator (gate) H for the BS so that:

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\begin{pmatrix}1\\0\end{pmatrix} = |1\rangle$$

**Or** $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = |+\rangle$



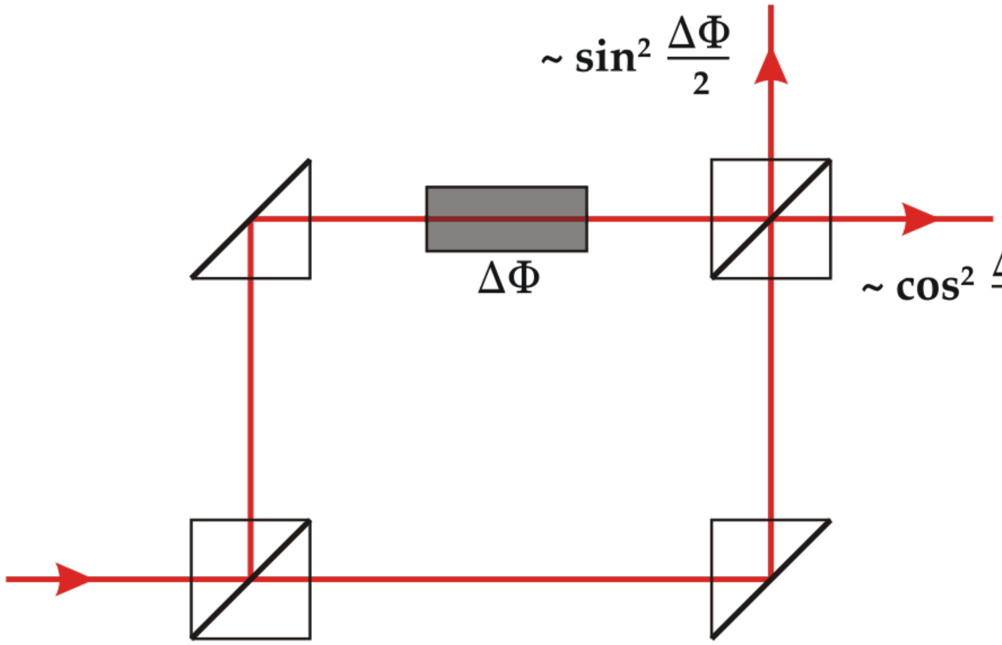
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

**Or** $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = |-\rangle$

$$\begin{pmatrix}0\\1\end{pmatrix} = |0\rangle$$

- If we add the second beamsplitter we can write that as the sum:

$$|+\rangle + |-\rangle = \frac{1}{\sqrt{2}} |0\rangle + |1\rangle + |0\rangle - |1\rangle = \frac{1}{2} |0\rangle + |0\rangle$$




- And we can write for the BS operation (Hadamard gate)

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

# Computational basis

- We can write an arbitrary quantum state $|\psi\rangle$ as a superposition of the basis vectors:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \qquad \text{with } \alpha,\ \beta \in \mathbb{C}^2, \text{ and } |\alpha|^2 + |\beta|^2 = 1$$

- With that we can use the H gate to transform the measurement basis for our measurement from

$$|0\rangle\,,\ |1\rangle \rightarrow |+\rangle\,,\ |-\rangle$$

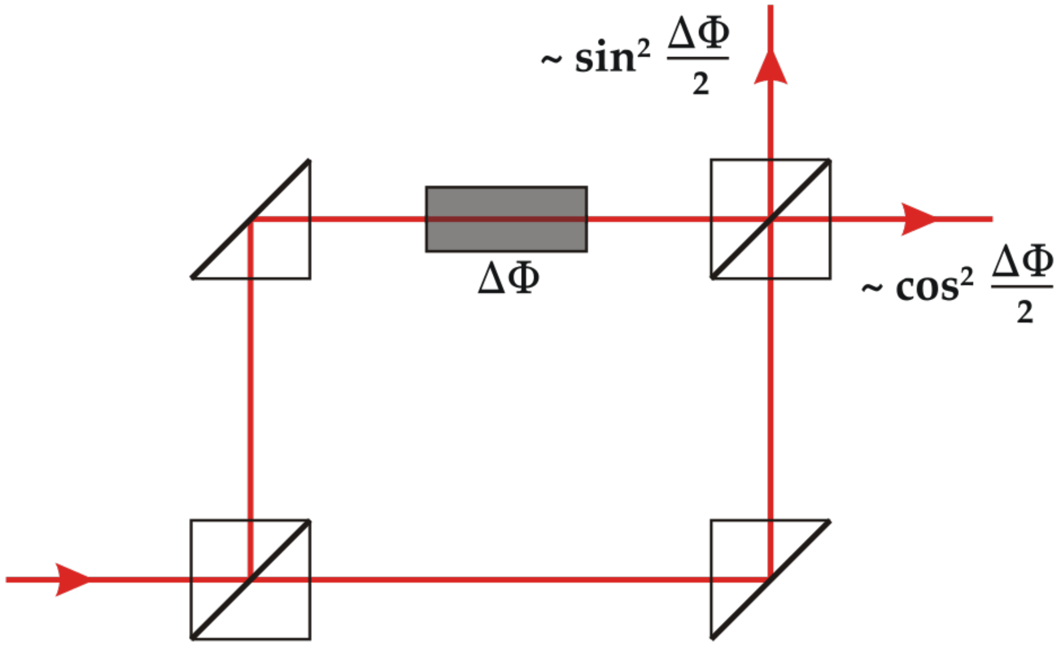
# What have we learned?

- … how to implement a quantum circuit on a QC
- … how to model a physics experiment on a QC
- … how to generate a superposition
- … that a superposition is not rolling dice
- … how to transform the measurement basis

# Phase Shifter

- The phase shifter is an operation that adds a phase shift lambda

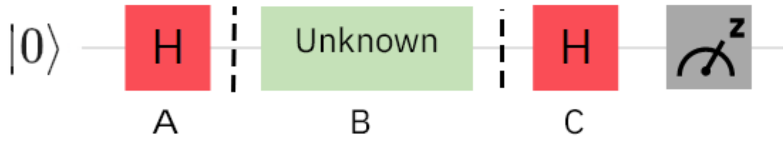
$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \qquad \begin{aligned} |0\rangle &\mapsto |0\rangle \\ |1\rangle &\mapsto e^{i\varphi}|1\rangle \end{aligned}$$




- On QC there usually many special gates with a specific value of phi used, eg:
  – Pauli Z pi: $$Z = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
  – S (SWAP) pi/2: $$S = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = \sqrt{Z}$$
  – T pi/4: $$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix} = \sqrt{S} = \sqrt[4]{Z}$$

# Example: Phase detector (Ramsey circuit)

- Let's find a computation (circuit) that can tell us the phase shift of an phase shifting element in a branch of the MZ interferometer



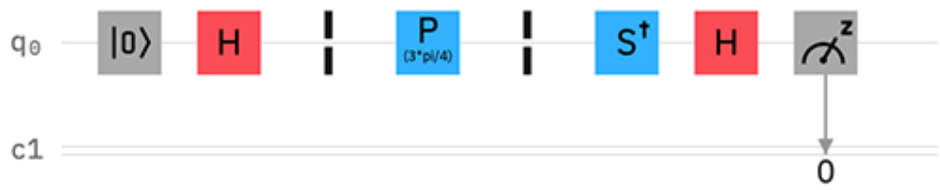

B. $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{j\varphi}|1\rangle)$

C. $|\psi\rangle = \frac{1}{2}[(1 + e^{j\varphi})|0\rangle + (1 - e^{j\varphi})|1\rangle]$

- Which gives us $p_0 = \frac{1}{2}[1 + \cos(\varphi)],\ p_1 = \frac{1}{2}[1 - \cos(\varphi)]$

- And thus the difference $d = p_0 - p_1 = \cos(\varphi)$
- Gives us the real part of phi $x = \mathrm{Re}[e^{j\varphi}]$

- For the imaginary part we need a second quantum circuit




- The S gate shift the imaginary plane onto the real plane so we can measure it

$$d = \sin(\varphi) \qquad y = \mathrm{Im}[e^{j\varphi}]$$

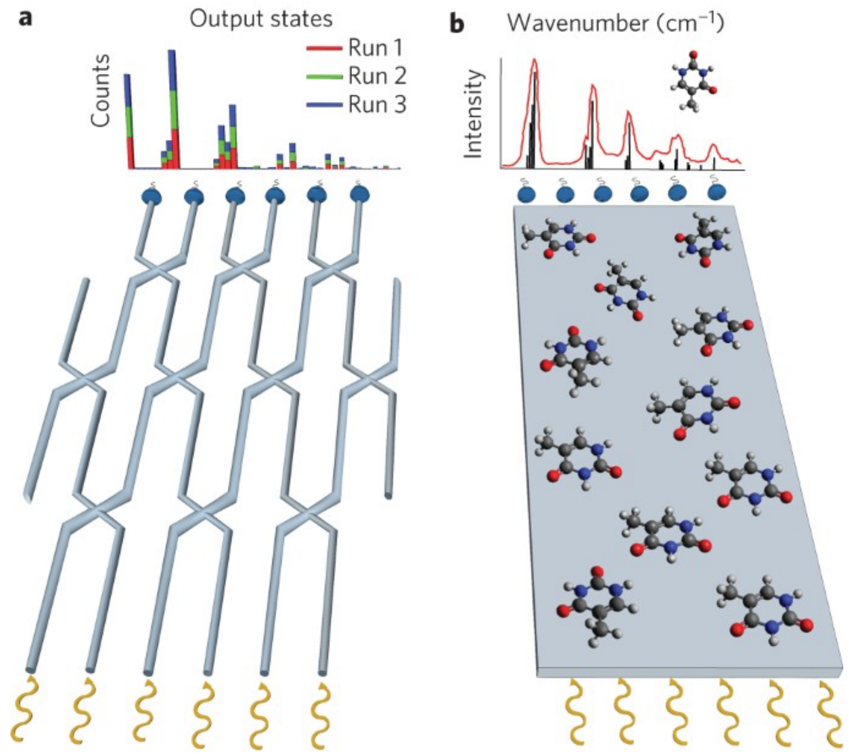
- Using atan2 (2-argument arctan: https://en.wikipedia.org/wiki/Atan2) we can calculate phi

# What have we learned?

- … how to do a non-classical computation
- … how to calculate a quantum property on a QC
- … do a quantum tomography experiment
- … that there is a global phase we cannot (and don’t need to) measure

# Boson Sampling

- Array of Beamsplitters (Mach-Zehnder)
- Indistinguishable photons at input
- Single photon counters at output
- Computes the determinant of a matrix (CC “hard P”)
- Problems:
  - For large matrices (>50) non-computable
  - What does a QC compute?






$$\mathrm{perm}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = aei + bfg + cdh + ceg + bdi + afh$$

# Quantum Gates

- Hadamard Gate:

$$|0\rangle \rightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|1\rangle \rightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

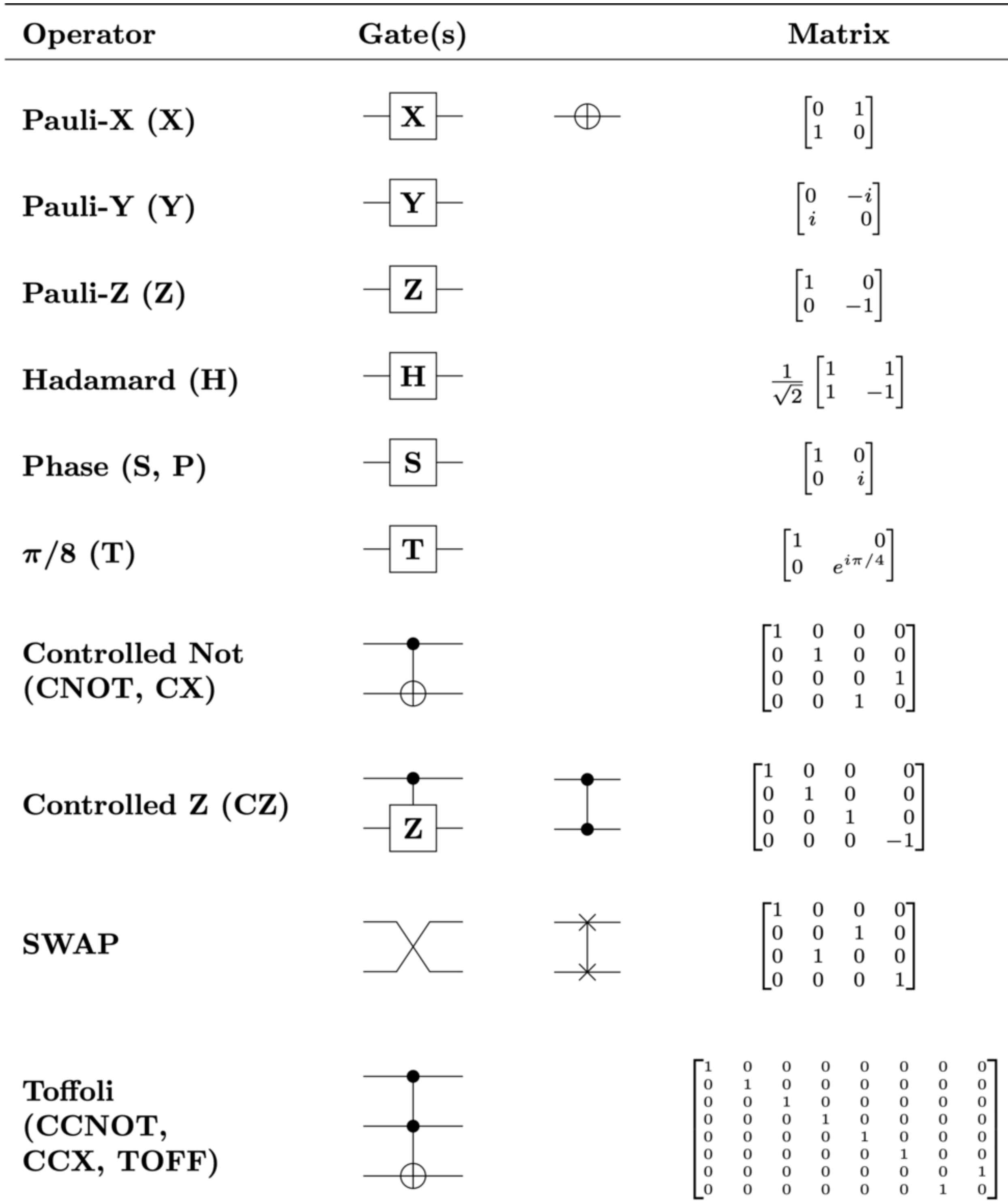
  – Superposition of a single qubit
  – $HH^\dagger = H^\dagger H = I$

- Pauli X gate (NOT gate)

$$|0> \rightarrow |1> \qquad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|1> \rightarrow |0>$$

- Pauli Y gate



| Operator | Gate(s) | Matrix |
| --- | --- | --- |
| Pauli-X (X) | X | $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ |
| Pauli-Y (Y) | Y | $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ |
| Pauli-Z (Z) | Z | $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ |
| Hadamard (H) | H | $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ |
| Phase (S, P) | S | $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ |
| $\pi/8$ (T) | T | $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$ |
| Controlled Not (CNOT, CX) | | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ |
| Controlled Z (CZ) | Z | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ |
| SWAP | | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| Toffoli (CCNOT, CCX, TOFF) | | $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ |

# Quantum Gates

- Phase shift Gates:

$$|0\rangle \mapsto |0\rangle \qquad |1\rangle \mapsto e^{i\phi}|1\rangle \qquad R_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$$

- Special cases of phi:
- Pauli Z pi:

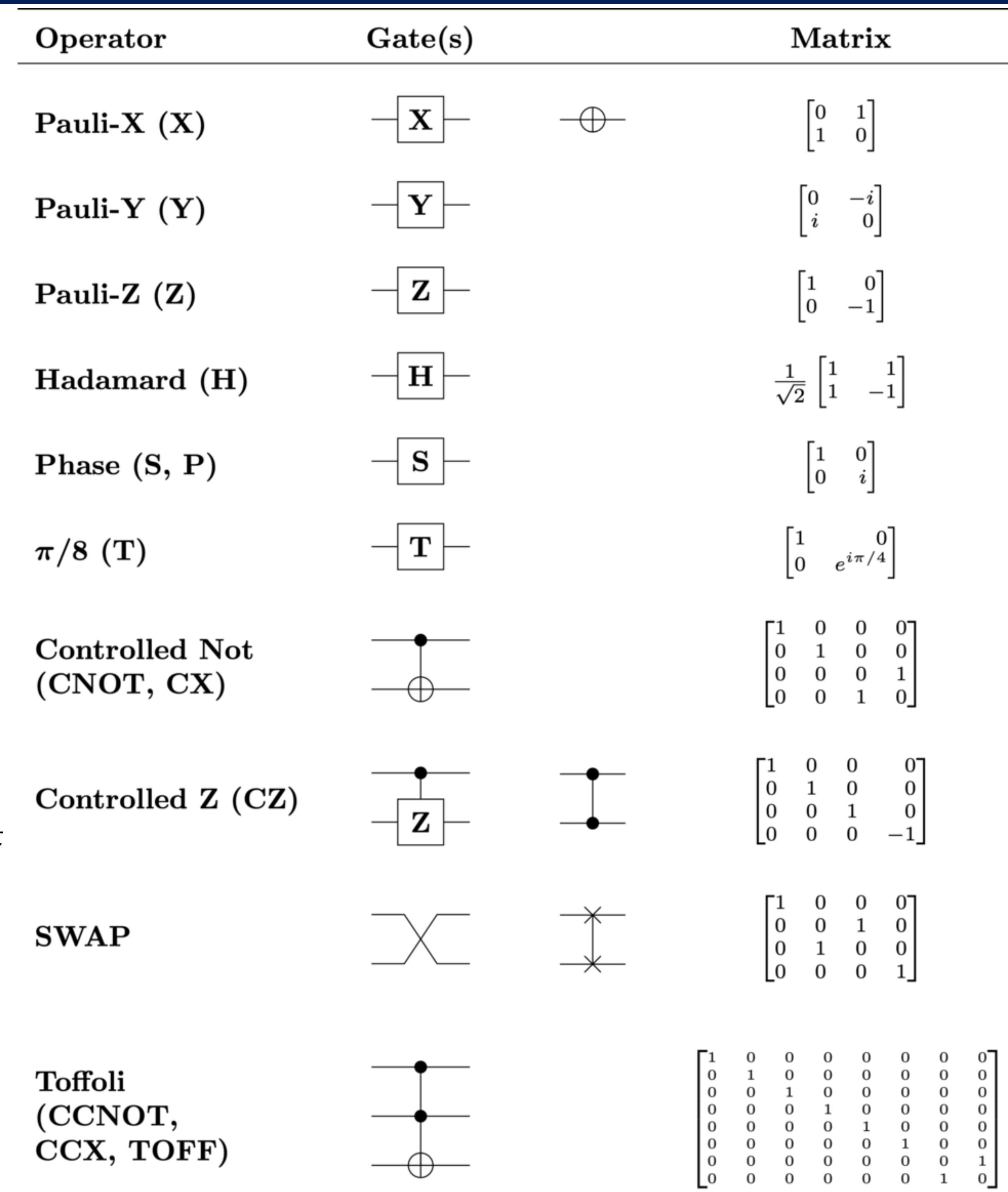
$$Z = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- S (swap) pi/2:

$$S = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = \sqrt{Z}$$

- T pi/4:

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{bmatrix} = \sqrt{S} = \sqrt[4]{Z}$$

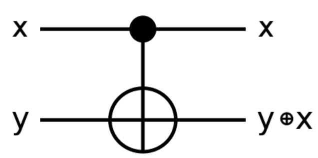


| Operator | Gate(s) | Matrix |
|---|---|---|
| Pauli-X (X) | X | $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ |
| Pauli-Y (Y) | Y | $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ |
| Pauli-Z (Z) | Z | $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ |
| Hadamard (H) | H | $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ |
| Phase (S, P) | S | $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ |
| $\pi/8$ (T) | T | $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$ |
| Controlled Not (CNOT, CX) | | $\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{bmatrix}$ |
| Controlled Z (CZ) | Z | $\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&-1 \end{bmatrix}$ |
| SWAP | | $\begin{bmatrix} 1&0&0&0 \\ 0&0&1&0 \\ 0&1&0&0 \\ 0&0&0&1 \end{bmatrix}$ |
| Toffoli (CCNOT, CCX, TOFF) | | $\begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{bmatrix}$ |

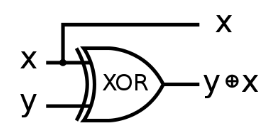
# Quantum Gates

- Control Gates (at least 2 qubits, multi control - one target)
  - CNOT (CX):

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$




| input x | input y | output x | output y+x |
|---|---|---|---|
| $\|0\rangle$ | $\|0\rangle$ | $\|0\rangle$ | $\|0\rangle$ |
| $\|0\rangle$ | $\|1\rangle$ | $\|0\rangle$ | $\|1\rangle$ |
| $\|1\rangle$ | $\|0\rangle$ | $\|1\rangle$ | $\|1\rangle$ |
| $\|1\rangle$ | $\|1\rangle$ | $\|1\rangle$ | $\|0\rangle$ |




| input x | input y | output x | output y+x |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 |

  - In Hadamard basis:

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

  - 2 qubit basis:

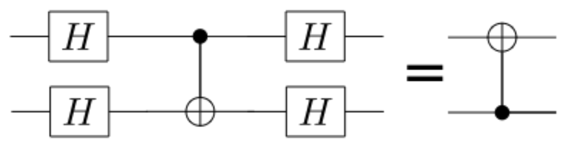
$$|++\rangle = |+\rangle \otimes |+\rangle = \frac{1}{2}(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

| Operator | Gate(s) | | Matrix |
|---|---|---|---|
| **Pauli-X (X)** | X | ⊕ | $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ |
| **Pauli-Y (Y)** | Y | | $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ |
| **Pauli-Z (Z)** | Z | | $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ |
| **Hadamard (H)** | H | | $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ |
| **Phase (S, P)** | S | | $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ |
| **$\pi/8$ (T)** | T | | $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$ |
| **Controlled Not (CNOT, CX)** | | | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ |
| **Controlled Z (CZ)** | Z | | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ |
| **SWAP** | | | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| **Toffoli (CCNOT, CCX, TOFF)** | | | $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ |

# Quantum Gates

- CNOT in Hadamard basis

$$|++\rangle \rightarrow C_{\text{NOT}} \rightarrow |++\rangle$$
$$|+-\rangle \rightarrow C_{\text{NOT}} \rightarrow |--\rangle$$
$$|-+\rangle \rightarrow C_{\text{NOT}} \rightarrow |-+\rangle$$
$$|--\rangle \rightarrow C_{\text{NOT}} \rightarrow |+-\rangle$$




- Equivalence of control and target bit
- Bell State (entangled state)

$$q0 : \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_A \quad \textbf{CNOT} \quad \rightarrow \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$q1 : |0\rangle_B$$

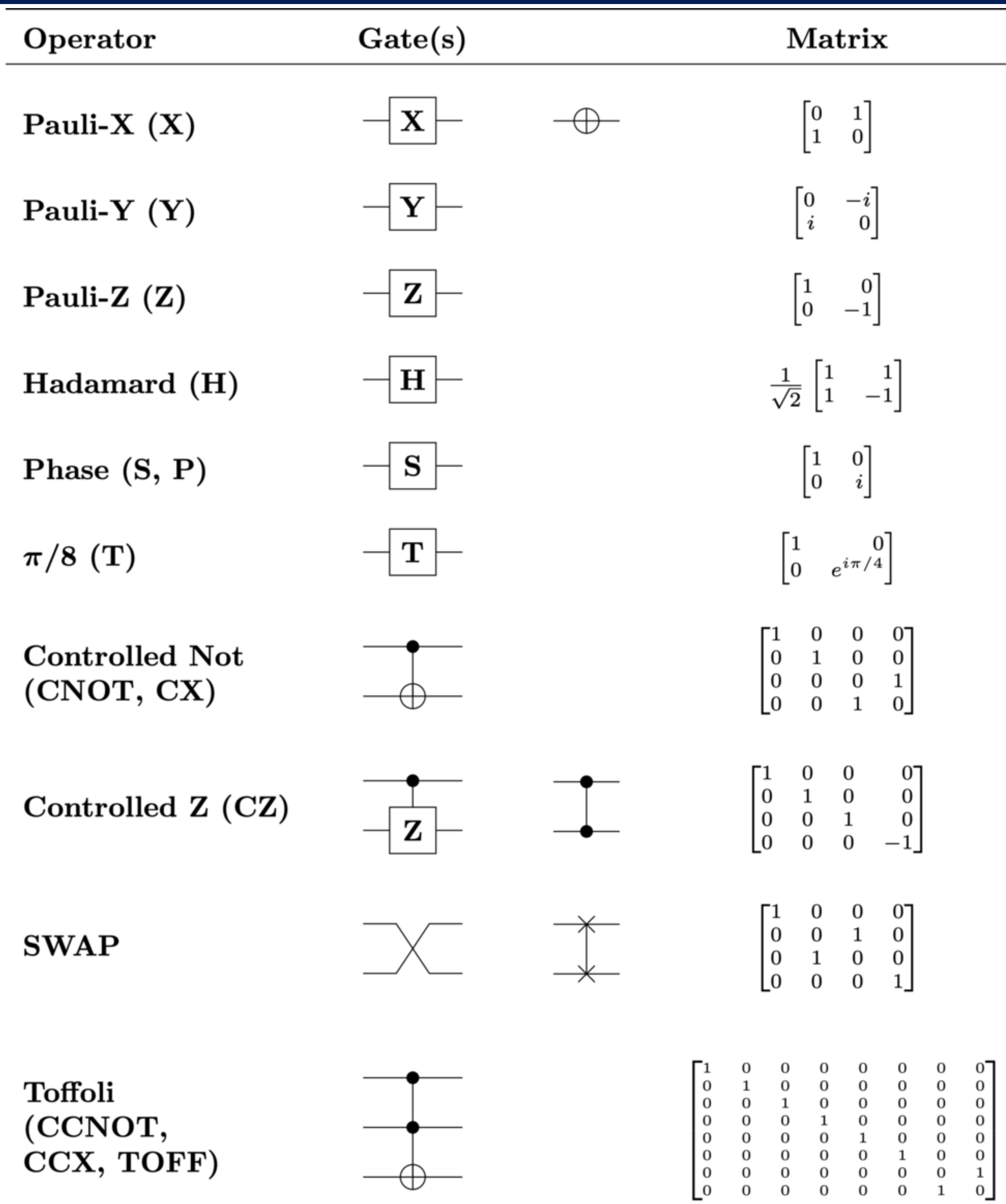
- A measurement in any basis will always give a 50:50 chance of each state. If A is measured B is given

| Operator | Gate(s) | Matrix |
|---|---|---|
| Pauli-X (X) | X | $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ |
| Pauli-Y (Y) | Y | $\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$ |
| Pauli-Z (Z) | Z | $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$ |
| Hadamard (H) | H | $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$ |
| Phase (S, P) | S | $\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$ |
| $\pi/8$ (T) | T | $\begin{bmatrix}1 & 0\\0 & e^{i\pi/4}\end{bmatrix}$ |
| Controlled Not (CNOT, CX) | | $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$ |
| Controlled Z (CZ) | Z | $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$ |
| SWAP | | $\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$ |
| Toffoli (CCNOT, CCX, TOFF) | | $\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$ |

# Quantum Gates

- CCNOT Gate (Toffoli Gate)
  - Reversible (like X)
  - Not universal for QC
- SWAP
- Ising Gate (X,Y,Z) (only ion trap QCs)

$$ZZ_\phi = \begin{bmatrix} e^{i\phi/2} & 0 & 0 & 0 \\ 0 & e^{-i\phi/2} & 0 & 0 \\ 0 & 0 & e^{-i\phi/2} & 0 \\ 0 & 0 & 0 & e^{i\phi/2} \end{bmatrix}$$



| Operator | Gate(s) | Matrix |
| --- | --- | --- |
| Pauli-X (X) | X | $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ |
| Pauli-Y (Y) | Y | $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ |
| Pauli-Z (Z) | Z | $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ |
| Hadamard (H) | H | $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ |
| Phase (S, P) | S | $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ |
| $\pi/8$ (T) | T | $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$ |
| Controlled Not (CNOT, CX) | | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ |
| Controlled Z (CZ) | Z | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ |
| SWAP | | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| Toffoli (CCNOT, CCX, TOFF) | | $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ |

# Quantum Gates

- Decompositions $H = X\sqrt{Y}$

  $H = Y\sqrt{X}$

  $H = Z\sqrt{Y}$

  $H = \sqrt{Y}Z$

- Square root gates eg: $\sqrt{X}$

$$|0\rangle \to \frac{(1+i)|0\rangle + (1-i)|1\rangle}{2}$$

$$|1\rangle \to \frac{(1-i)|0\rangle + (1+i)|1\rangle}{2}$$

$$\sqrt{X} = \sqrt{\mathrm{NOT}} = \frac{1}{2}\begin{bmatrix}1+i & 1-i\\ 1-i & 1+i\end{bmatrix}$$

$$X = (\sqrt{\mathrm{NOT}})^2 = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

| Operator | Gate(s) | | Matrix |
|---|---|---|---|
| **Pauli-X (X)** | X | ⊕ | $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ |
| **Pauli-Y (Y)** | Y | | $\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$ |
| **Pauli-Z (Z)** | Z | | $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ |
| **Hadamard (H)** | H | | $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$ |
| **Phase (S, P)** | S | | $\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$ |
| **$\pi/8$ (T)** | T | | $\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$ |
| **Controlled Not (CNOT, CX)** | | | $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$ |
| **Controlled Z (CZ)** | Z | | $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$ |
| **SWAP** | | | $\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$ |
| **Toffoli (CCNOT, CCX, TOFF)** | | | $\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$ |

# Physical gates

- All single acting Qubits can be written as

$$U(\theta,\phi,\lambda) = \begin{bmatrix} \cos(\theta/2) & -\exp(i\lambda)\sin(\theta/2) \\ \exp(i\lambda)\sin(\theta/2) & \exp(i\lambda + i\phi)\cos(\theta/2) \end{bmatrix}$$

- For the IBM QC all single Qubit operations will be decompiled as U1(0,0,lambda), U2(pi/2,phi,lambda) and U(3) operations
- For the IBM QC the only two Qubit operation is the CNOT gate. For all others it has to be decomposed in CNOT and single Qubit operations

# Quantum Decoherence

- Cannot be explained by a single measurement - needs statistics (QBism)
- Sources
  - quantum gates
  - lattice vibrations (phonons)
  - Scattering
  - background nuclear spin
  - Cosmic radiation (high energy)
  - Cosmic background radiation
- Solutions
  - Error correction
  - Cooling (30mK) limited resources of He3
  - Isotope pure materials