Some highlights of Gabriele's research

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Gabriele turning 80

Plan of the talk

1. Introduction
2. After the Veneziano model (1969-1972)
3. What about the mesons?
4. The $U(1)$ problem
5. After the 1984 string revolution
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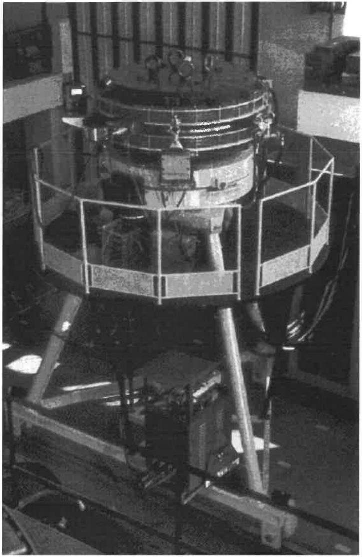
Introduction

- My master thesis advisor in Rome was Bruno Touschek.
- To use all energy available he proposed the construction of storage rings in alternative to fixed target accelerators.
- The first e^+e^- storage ring, called ADA, was constructed (early sixties) in Frascati and brought to Orsay to reach higher luminosity.
- As a thesis he asked me to compute the cross-section of double bremsstrahlung

$$e^- + e^+ \to e^- + e^+ + 2\gamma$$

- He was thinking of using it as a monitor for the luminosity of ADONE (big ADA) that was under construction in Frascati.
- After many months of work we (together with Mario Greco in 1966) finally finished this calculation publishing a paper on the total cross-section as a function of the frequencies of the two photons.
- Although all the group in Frascati was involved in the calculation of radiative corrections of various processes to be measured in ADONE, I decided to leave QED and move to S-matrix theory.



ADA (Anello Di Accumulazione), 1961-1964

- QFT did not seem to be very useful for strong interactions because of the large number of hadrons also with high spin and the pion-nucleon coupling constant was too big to use perturbation theory.
- Specially after the hard calculations in QED I was attracted by the idea of computing directly the S-matrix from the properties that it was supposed to satisfy (analyticity, crossing symmetry, unitarity, Regge behaviour)
- In June 1969 I heard in Naples a seminar by Sergio Fubini on how to compute the spectrum of hadrons within the newly constructed DRM (Dual Resonance Model).
- I was so excited that, after his seminar, I asked him if I could use my NATO fellowship at MIT.
- In December 1969 I started to work at MIT where I met Gabriele.
- Sergio had worked a lot on current algebra and had tried to compute the spectrum of hadrons by saturating the commutation relations of vector and axial vector currents.

- The work on current algebra and S-matrix theory in the sixties is summarised in a big book (874 pages) he wrote: De Alfaro, Fubini, Furlan and Rossetti, Currents in hadron physics, 1973].
- I learned a lot studying various parts of this book.
- When Fubini saw the Veneziano model and specially its extension to N external legs containing an infinite number of simple poles, it was immediately clear to him that from it one could extract the hadron (meson) spectrum.
- Look at the residue of each pole and check that, at the pole, the N-point amplitude factorises in a finite number of terms.
- Then each of these terms corresponds to a state of the meson spectrum that is exchanged in the process.
- Unitarity requires the norm of the exchanged state to be positive.
- A state with negative norm is called ghost and violates unitarity.
- In the next section I will describe the work of Sergio and Gabriele in checking factorisation and absence of ghosts that brought to string theory.
- Then I will be discussing some other highlights of Gabriele's research until today.

After the Veneziano model (1969-1972)

- The Veneziano model for four scalar particles was immediately generalised to the case of N scalar particles:

$$B_N = \int_{-\infty}^{\infty} \frac{\prod_1^N dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \prod_{i=1}^{N} \left[(z_i - z_{i+1})^{\alpha_0 - 1} \right] \prod_{j>i} (z_i - z_j)^{2\alpha' p_i \cdot p_j}$$

  where α_0 is the intercept of the Regge trajectory related to the mass of the scalar particle by $\alpha_0 + \alpha' m^2 = 0$.
- The questions are: how do we extract the meson spectrum from the N-point amplitude? Is the spectrum free of ghosts?
- But, before of this, let us describe the situation at that time.
- In USA the only somewhat older and established physicists who were very excited and actively working on these new developments were Fubini, Mandelstam and Nambu.
- Gell-Mann was very supportive but not personally engaged in them.

- Weinberg started to work in this direction, but then, after 't Hooft paper on renormalizability, went back to the electro-weak theory (see below). Went back to string theory after 1984.
- Others as F. Low were agnostic.
- The rest of the establishment was very strongly against them.
- L. Clavelli and P. Ramond were hired in the newly created theoretical group at NAL (Fermilab) in the fall 1969, but were dismissed already in the summer 1970.
- The explanation was that the interaction between them and the experimental physicists had not developed as fully as had been expected [L. Clavelli, in the Birth of String Theory, edited by A. Cappelli and al].
- Pierre went almost out of physics and only in the last moment he got one year at Yale.
- Gell-Mann writes: ... I set up at Caltech a nature reserve for endangered superstring theorists. I brought J. H. Schwarz and P. Ramond at Caltech and encouraged A. Neveu to visit. [Gell-Mann, in the Birth of String Theory, edited by A. Cappelli and al].

- In Europe the situation was much better, but I was also obliged to stop working on string theory if I wanted to get a job.
- Introducing an infinite set of harmonic oscillators and a position and momentum operator, Fubini and Veneziano constructed:

$$Q(z) = \hat{q} - 2i\alpha'\hat{p}\log z + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left(\frac{a_n}{\sqrt{n}}z^{-n} - \frac{a_n^\dagger}{\sqrt{n}}z^n\right)$$

with

$$[a_n^\mu, a_m^{\dagger\nu}] = \eta^{\mu\nu}\delta_{nm} \;\; ; \;\; [\hat{q}^\mu, \hat{p}^\nu] = i\eta^{\mu\nu} \;\; ; \;\; \eta^{\mu\nu} = (-1, 1, 1, 1)$$

- Then they introduced the vertex operator corresponding to the external scalar particle with momentum p:

$$V(z; p) =: e^{ip\cdot Q(z)} :$$

[Fubini, Gordon and Veneziano, Phys. Lett. **29** B (1969) 679]
[Nambu, Proc. Int. Conf. on Symm. and Quark models, Wayne Univ. 1969 (Gordon and Breach), 1970, pag. 269]
[Fubini and Veneziano, Nuovo Cimento A**67** (1970) 29]
[Fubini and Veneziano, Annals of Physics, **63** (1971) 12]

- For reasons that become clear later I consider the case $\alpha_0 = 1$:

$$B_N = \int_{-\infty}^{\infty} \frac{\prod_1^N dz_i \theta(z_i - z_{i+1})}{dV_{abc}} \langle 0| \prod_{i=1}^{N} V(z_i, p_i)|0\rangle$$

- B_N can be rewritten as a product of vertices and propagators

$$B_N = \langle 0, p_1| V(1, p_2) D V(1, p_3) \dots D V(1, p_{N-1})|0, p_N\rangle$$
$$= \langle p_{1,M}| \frac{1}{L_0 - 1} |p_{M+1,N}\rangle$$

where

$$D = \frac{1}{L_0 - 1} \ ; \ L_0 = \alpha' \hat{p}^2 + \sum_{m=1}^{\infty} m a_m^\dagger a_m$$

and

$$\langle p_{(1,M)}| = \langle 0, p_1| V(1, p_2) D V(1, p_3) \dots V(1, p_M)$$
$$|p_{(M+1,N)}\rangle = V(1, p_{M+1}) D \dots V(1, p_{N-1})|p_N, 0\rangle$$

- Introducing the quantity:

$$R = \sum_{m=1}^{\infty} m a_m^\dagger \cdot a_m$$

one can finally write the factorisation equation as follows:

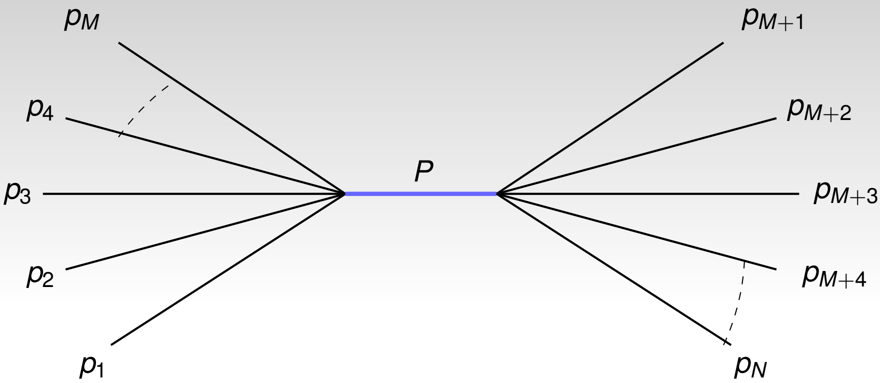
$$B_N = \sum_{\lambda} \langle p_{(1,M)} | \lambda, P \rangle \langle \lambda | \frac{1}{R - \alpha(P^2)} | \lambda \rangle \langle \lambda, P | p_{(M+1,N)} \rangle$$

where $\sum_\lambda$ runs over a complete and orthonormal set of eigenstates $|\lambda\rangle$ of R and

$$\alpha(P^2) = 1 - \alpha' P^2 \;\; ; \;\; P^\mu = -\sum_{i=1}^{M} p_i^\mu = \sum_{i=M+1}^{N} p_i^\mu$$

- B_N has a pole in the channel $(1, M)$ when $\alpha(P^2) = n = 0, 1, 2 \ldots$.
- The states $|\lambda\rangle$ that contribute to its residue are those satisfying the relation:

$$R|\lambda\rangle = n|\lambda\rangle$$



- Their number gives the degeneracy of states contributing to the pole at $\alpha(P^2) = n$.
- Unitarity requires that each state $|\lambda\rangle$ contributing to the pole must have positive norm $\langle\lambda|\lambda\rangle > 0$.

- For $n = 0$ the only state is $|0\rangle \Longrightarrow$ a scalar with mass $\alpha' m^2 = -1$.
- For $n = 1$ the only state is $a^\dagger_{1\mu}|0\rangle \Longrightarrow$ a massless vector.
- For $n = 2$ we have two states: $a^\dagger_{1\mu}a^\dagger_{1\nu}|0\rangle$ and $a^\dagger_{2\mu}|0\rangle$.
- For $n = 3$ we have: $a^\dagger_{1\mu}a^\dagger_{1\nu}a^\dagger_{1\rho}|0\rangle$, $a^\dagger_{2\mu}a^\dagger_{1\nu}|0\rangle$ and $a^\dagger_{3\mu}|0\rangle$.
- The degeneracy of states $T_D(n)$ at the level n in D dimensions can be obtained from the partition function

$$Tr(x^R) = \prod_{m=1}^{\infty} \frac{1}{(1-x^m)^D} = \sum_{n=0}^{\infty} T_D(n) x^n$$

- For $n \to \infty$ they got with great surprise

$$T_D(n) \sim e^{\frac{2\pi}{\sqrt{6}}\sqrt{Dn}} \sim e^{\frac{2\pi\sqrt{D\alpha'}}{\sqrt{6}} m} \quad ; \quad n \sim \alpha' m^2$$

[Fubini and Veneziano, Nuovo Cimento A **64** (1969) 811]

- Limiting Hagedorn temperature:

$$kT_0 = \frac{\sqrt{6}}{2\pi}\frac{1}{\sqrt{D\alpha'}} \Longrightarrow kT_0 = \frac{1}{4\pi\sqrt{\alpha'}}$$

- We will see that in the bosonic string $D \to D - 2 = 24$.

- Because of manifest relativistic invariance the space spanned by the complete system of states contains states with negative norm.
- They correspond to states having an odd number of oscillators in the time direction.
- This contradicts unitarity that requires the physical states to span a positive definite Hilbert space.
- There must exist a number of relations that decouple some of the states leaving with a positive definite physical Hilbert space.
- Such relation exists:

$$W_1 | p_{(M+1,N)} \rangle = 0 \ ; \ \ W_1 = L_1 - L_0$$

but it is not enough and therefore, for general α_0, there are ghosts [Fubini and Veneziano, Nuovo Cimento A **64** (1969) 811] [Bardakçi and Mandelstam, Phys. Rev. **184** (1969) 1640]
- If $\alpha_0 = 1$ Virasoro showed the existence of an infinite number of them:

$$W_m | p_{(M+1,N)} \rangle = 0 \ ; \ \ W_m = L_m - L_0 - (m-1) \ ; \ \ m = 1, 2 \ldots$$

- They prevent some of the states to contribute to the residue.
- Generalisation of what happens in QED at the photon pole:

$$B_N \sim \frac{A^{\mu}_{1,M}\eta_{\mu\nu}C^{\nu}_{M+1,N}}{k^2} \;\; ; \;\; k_{\mu}A^{\mu} = k_{\mu}C^{\mu} = 0$$

  where the two last conditions follow from gauge invariance.
- They cancel in the residue the components of A^{μ} and C^{μ} along the directions of k and $\bar{k}$ keeping only the two physical transverse polarisations.
- If $k^{\mu} = (k^0, 0, 0, k^3)$ then $\bar{k}^{\mu} = (k^0, 0, 0, -k^3)$.
- Gauge invariance makes Lorentz invariance to be consistent with unitarity.

- L_n in terms of oscillators:

$$L_n = \sqrt{2\alpha' n}\, \hat{p} \cdot a_n + \sum_{m=1}^{\infty} \sqrt{m(n+m)} a_{n+m} \cdot a_m^\dagger$$
$$+ \frac{1}{2} \sum_{m=1}^{n-1} \sqrt{m(n-m)} a_{n-m} \cdot a_m \quad ; n \geq 0 \quad L_n = L_n^\dagger$$

- From the action on $Q(z)$ Fubini and Veneziano constructed the generators L_n of the two-dim conformal algebra:

$$[L_n, Q(z)] = z^{n+1} \frac{dQ(z)}{dz} \Longrightarrow L_n = z^{n+1} \frac{d}{dz}$$

- They were the first to write the "Virasoro algebra":

$$[L_n, L_m] = (n-m) L_{n+m}$$

- The central charge was derived by J. Weis using the expression of L_n in terms of the oscillators.
- The algebra with only the operators L_0 and $L_{\pm 1}$ was already known from [F. Gliozzi, Lett. Nuovo Cimento **2** (1969) 846] [Chiu, Matsuda and Rebbi, Phys. Rev. Lett. **23** (1969) 1526]

- From the decoupling Virasoro conditions one can construct the equations that characterise the physical on-shell states $|\lambda\rangle$:

$$L_n|\lambda\rangle = (L_0 - 1)|\lambda\rangle = 0$$

[Del Giudice and Di Vecchia, Nuovo Cimento A **70** (1970) 90.]

- Using the vertex operator of the massless state at the level $n = 1$ an infinite number of transverse oscillators were constructed:

$$A_{i,n} = \frac{i}{\sqrt{2\alpha'}} \oint_0 dz \epsilon_i^{\mu} P_{\mu}(z) e^{ik\cdot Q(z)} \;\; ; \;\; [A_{n,i}, A_{m,j}] = n\delta_{ij}\delta_{n+m;0}$$

where the index i runs over the $D-2$ transverse directions, that are orthogonal to the momenta k and $\bar{k}$.

- They create physical states:

$$[L_n, A_{i,m}] = 0$$

- They are the so-called DDF operators [Del Giudice, Di Vecchia and Fubini, Ann. of Phys. **70** (1972) 378].

- Are they complete? Yes, if $D = 26$. But this became clear later.
- The study of the one-loop non-planar diagram showed the presence of unitarity violating cuts.
- Those cuts become perfectly allowed additional poles if $D = 26$ (corresponding to closed string states)
  [Lovelace, unpublished (1970) and Phys. Lett. **34** B (1971) 500].
- By the end of 1970 it was clear that the N-point amplitude was consistent with unitarity only if $\alpha_0 = 1$ and $D = 26$.
- No ghost at the tree level only if $\alpha_0 = 1$ and $D < 26$
  [Goddard and Thorn, Phys. Lett. B **40** (1972) 235]
  [Brower, Phys. Rev. D **6** (1972) 1655]
  but trouble with unitarity at one loop if $D \neq 26$.

- The vertex involving three arbitrary states was constructed by [Sciuto, Lett. Nuovo Cimento **2** (1969) 411] [Della Selva and Saito, Lett. Nuovo Cimento **4** (1970) 689]
- It was made completely symmetric under the exchange of the three states [Caneschi, Schwimmer and Veneziano, Phys. Lett. B **30** (1969) 356]
- Its generalisation to N legs, called N-Reggeon vertex, was computed and used for computing multiloop amplitudes by means of the sewing procedure [Alessandrini, Amati, Le Bellac and Olive, Phys. Rep. **1** (1971) 269]
- The sewing procedure produced functions as the abelian differentials, period matrix, prime form, Green functions that are well defined on genus g Riemann surface. Why?
- But the exact measure of integration over the moduli space was only determined after the BRST invariant formulation of the N-Reggeon vertex and of the propagator [Di Vecchia, Frau, Lerda and Sciuto, Phys. Lett. B **199** (1987) 49] [Petersen and Sidenius, Nucl. Phys. B bf 301 (1988) 247] [Mandelstam, Phys. Lett. B **277** (1992) 82]

- But what is the underlying theory?
- Because of the infinite number of oscillators, already in 1970, Nambu, Nielsen and Susskind proposed that the underlying theory was a string theory.
- The Nambu-Goto string action was written in 1970, but nobody knew how to extract from it the spectrum of states and their scattering amplitudes.
- Only at the end of 1972, by quantising the string action in the light-cone gauge, it became clear that the theory underlying the dual resonance model was a string theory [Goddard, Goldstone, Rebbi and Thorn, Nucl. Phys. B **56** (1973) 109.]
- Not a good theory for hadrons and in 1973 QCD takes over.

What about the mesons?

- The original Veneziano model was for the process $\pi\pi \to \pi\omega$.
- The leading Regge trajectory exchanged was that of the ρ meson.
- For massless pions $\alpha_0 = \frac{1}{2}$ and $\alpha' = \frac{1}{2m_\rho^2}$ $(\alpha(m_\rho^2) = 1)$.
- A realistic $\pi\pi$ amplitude: Lovelace-Shapiro model

$$A(s,t) = \beta \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(t)-\alpha(s))}$$

$$\alpha(s) = \alpha_0 + \alpha' s \ ; \ \alpha_0 = \frac{m_\rho^2 - 2m_\pi^2}{2(m_\rho^2 - m_\pi^2)} \ ; \ \alpha' = \frac{1}{2(m_\rho^2 - m_\pi^2)}$$

[Lovelace, Phys. Lett. B **28** (1968) 265]
[Shapiro, Phys. Rev. **179** (1969) 1345]

- Adler zero when one of the pions has zero momentum.
- If $\beta = -\frac{1}{2\alpha' F_\pi^2}$ it reduces for $\alpha' \to 0$ to the amplitude of the non-linear σ model (it gives a good description of low-energy QCD, see below).
- Ghosts if $D > 4$ but also if $m_\pi^2 \neq 0$
[Veneziano, Yankielowicz and Onofri, JHEP **04** (2017) 151]

- Generalising the LS model to the process $\pi A \to BC$ one gets that Regge trajectories with opposite normality ($P(-1)^J$) , that can be connected by pion emission, must have the same slope and their intercepts must differ by a half-odd integer (in agreement with experiments) [Ademollo, Veneziano and Weinberg, Phys. Rev. Lett. **22** (1969) 83]
- To get the full meson spectrum one needs the N pion amplitude.
- A N pion amplitude with Adler zeroes in all channels was constructed but it has ghosts
  [Neveu and Thorn, Phys. Rev. Lett. **27** (1971) 1758]
  [Schwarz, Phys. Rev. D **5** (1972) 886]
  [Bianchi, Consoli and Di Vecchia, JHEP **03** (2021) 119]
- To eliminate ghosts one must have $\alpha_0 = 1$, but then one gets the Neveu-Schwarz model that is part of the superstring.
- Gabriele started to work in a non-perturbative unitarisation of the DRM based on a topological reorganisation of the loop diagrams: planar topology, cilinder topology, higher genus, but could not find a clear mathematical way to justify it [Di Giacomo, Fubini, Sertorio and Veneziano, Phys. Lett. B **33** (1970) 171]

- After the work of 't Hooft in 1974 where he introduced the large N_c expansion in QCD with fixed $g^2 N_c$, Gabriele introduced first the large N_f expansion in the DRM/string
[Veneziano, Phys. Lett. **52** B (1974) 220]
and then the expansion in gauge theories where N_c, $N_f \to \infty$, while $g^2 N_c$ and $\frac{N_f}{N_c}$ are kept fixed
[Veneziano, Nucl. Phys. B **117** (1976) 519].
- The perturbative expansion is an expansion in powers of the gauge coupling constant.
- The large N_c expansion is instead an expansion in the topology of the Feynman diagrams, as in the case of string theory.
- It is non-perturbative.
- Therefore we expect that the mesons in large N_c QCD are described by some kind of string theory. Which one?
- What is the $\pi\pi$ amplitude that one gets in large N_c QCD?

The $U(1)$ problem

- Strong interactions are described by the QCD Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} + \sum_{i=1}^{N_f} \overline{\Psi}_i (i\gamma^\mu D_\mu - m_i)\Psi_i - \theta q$$

where q is the topological charge density:

$$q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \ , \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

- For $m_i = 0$, $\mathcal{L}_{QCD}$ possesses, at the classical level, a $U_R(N_f) \otimes U_L(N_f)$ chiral symmetry that is spontaneously broken to the diagonal vectorial subgroup $U_V(N_f)$.
- The pseudoscalar mesons are the corresponding massless (if $m_i = 0$) Goldstone bosons.
- For $N_f = 2$ they are the three pions and η.
- For $N_f = 3$ they are the three pions, the four K mesons, η and η'.

- At low energy they are described by the following effective Lagrangian:

$$L = \frac{1}{2}\mathrm{Tr}\left[\partial_\mu U \partial_\mu U^\dagger\right] + \frac{F_\pi}{2\sqrt{2}}\mathrm{Tr}\left[M(U+U^\dagger)\right]$$

  where U contains the fields of the pseudoscalar mesons, that are composite states of a quark and an antiquark.
- $F_\pi = 95$ MeV is the pion decay constant
- Assume diagonal and real mass matrices for both quarks and mesons:

$$m_{ij} = m_i \delta_{ij} \; , \qquad M_{ij} = \mu_i^2 \delta_{ij} \quad ; \quad i,j = 1, \dots, N_f$$

- They are related by the Gell-Mann, Oakes and Renner relation:

$$\mu_i^2 F_\pi^2 = -2 m_i < \overline{\Psi}_{R;i} \cdot \Psi_{L;i} >$$

- The ratio $\frac{m_i}{\mu_i^2}$ is independent of i since, for small masses, both F_π and the vacuum expectation value are flavour independent.

- The kinetic term of L is invariant under chiral transformations

$$U \to g_L U g_R^\dagger \; ; \qquad U^\dagger \to g_R U^\dagger g_L^\dagger \; ; \qquad g_L^{-1} = g_L^\dagger ; \qquad g_R^{-1} = g_R^\dagger$$

  while the mass term breaks explicitly this symmetry precisely as the quark mass matrix does in QCD.
- Chiral symmetry is spontaneously broken by imposing that the meson field satisfies the constraint:

$$UU^\dagger = \frac{F_\pi^2}{2} \Longrightarrow U(x) = \frac{F_\pi}{\sqrt{2}} e^{i\sqrt{2}\frac{\Phi(x)}{F_\pi}} \;\; ; \;\; \Phi(x) = \Pi^a T^a + \frac{S}{\sqrt{N_f}}$$

- In the case of a $U(3)$ flavour symmetry $\Pi^a(x)$ corresponds to the fields of the octet of the pseudoscalar mesons, while S is a $SU(3)$ singlet. In this case we get:

$$\Pi^a T^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0 + \eta_8/\sqrt{3} & \sqrt{2}\pi^+ & \sqrt{2}k^+ \\ \sqrt{2}\pi^- & -\pi^0 + \eta_8/\sqrt{3} & \sqrt{2}k^0 \\ \sqrt{2}k^- & \sqrt{2}\bar{k}^0 & -2\eta_8/\sqrt{3} \end{pmatrix}$$

- The measured masses of the pseudoscalar mesons are:

$$m_\pi \sim 139 MeV \; ; \; m_K \sim 494 MeV \; , \; m_\eta \sim 548 MeV \; , \; m_{\eta'} \sim 958 MeV$$

- The splitting between η' and the other mesons is too big to be explained by the mass term in L : $U(1)$ problem.

- At the quantum level the singlet axial vector current has an anomaly (axial $U(1)$ anomaly)

$$\partial^\mu \left(\sum_{i=1}^{N_f} \overline{\Psi}_i \gamma_\mu \gamma_5 \Psi_i \right) = 2N_f q(x) + 2i \sum_{i=1}^{N_f} m_i \overline{\Psi}_i \gamma_5 \Psi_i$$

- At large N the axial anomaly disappears ($g^2 N$ is kept fixed).
- The effect of the axial anomaly is included in the effective Lagrangian by adding to it

$$L_{anomaly} = \frac{i}{2} q(x) \mathrm{Tr} \left[\log U - \log U^\dagger \right]$$

- Under a singlet axial transformation $U \to e^{i\phi} U$ one gets:

$$L_{anomaly} \to L_{anomaly} - N_f \phi q(x)$$

- Once we included $q(x)$ in the Lagrangian we could also think of adding any power of q times a function of U and $U^\dagger$.
- Keeping only the leading terms in the large N colour expansion only one additional term is needed.
- In this way we arrive at the following Lagrangian for the mesons:

$$L = \frac{1}{2}\mathrm{Tr}\left[\partial_\mu U \partial_\mu U^\dagger\right] + \frac{F_\pi}{2\sqrt{2}}\mathrm{Tr}\left[M(U + U^\dagger)\right] + \frac{i}{2} q(x)\mathrm{Tr}\left[\log \frac{U}{U^\dagger}\right] + \frac{q(x)^2}{aF_\pi^2}$$

that is valid for large number of colours with the hope that $N = 3$ is not far away from $N = \infty$!
[Rosenzweig, Schechter and Trahern, Phys. Rev. D **21** (1980) 3388]
[Di Vecchia and Veneziano, Nucl. Phys. B **171** (1980) 253]
[Witten, Annals of Phys. **128** (1980) 363]

- The quadratic term obtained from L is:

$$L_2 = \frac{1}{2}\left((\partial_\mu \Pi^a)^2 - m_\pi^2 \Pi^2\right) + \frac{1}{2}\left((\partial_\mu S)^2 - m_\pi^2 S^2\right) + \frac{q^2}{aF_\pi^2} - \frac{\sqrt{2N_f}}{F_\pi} S q$$

where for simplicity we have taken $\mu_i^2 = m_\pi^2, i = 1 \dots N_f$.
- From L we get

$$\int d^4x\, e^{ipx} \langle S(x) S(0) \rangle = \frac{1}{p^2 - M_S^2} \quad ; \quad M_S^2 = m_\pi^2 + aN_f$$

- and

$$\int d^4x\, e^{ipx} \langle q(x) q(0) \rangle = \frac{aF_\pi^2}{2} \frac{p^2 - m_\pi^2}{p^2 - M_S^2}$$

$$\int d^4x\, e^{ipx} \langle q(x) S(0) \rangle = \frac{aF_\pi \sqrt{N_f}}{\sqrt{2}} \frac{1}{p^2 - M_S^2}$$

- Gabriele obtained the previous correlators by imposing the anomalous Ward identities.
[G. Veneziano, Nucl. Phys. B **159** (1979) 213.]
- At large N the anomaly disappears and $M_S^2 = m_\pi^2$ because $a \sim \frac{1}{N}$.

- In the case of pure Yang-Mills without quarks L has only the last contribution:

$$L_{YM} = \frac{q(x)^2}{aF_\pi^2} - q\theta - iJq$$

with the addition of a term wth θ and a source term with J.
- From the previous expression one can compute the partition function:

$$Z(\theta, J) = e^{iW(\theta,J)} \;\; ; \;\; W(\theta, J) = \frac{V_4(\theta + J)^2 aF_\pi^2}{4}$$

- From it we can extract the vacuum energy

$$E(\theta) = \frac{W(\theta, 0)}{V_4} = \frac{\theta^2 aF_\pi^2}{4} \Longrightarrow \frac{2N_f}{F_\pi^2}\frac{d^2 E(\theta)}{d\theta^2}|_{\theta=0} = aN_f = M_S^2$$

[E. Witten, Nuclear Physics B **156** (1979) 269]
- This is the Witten-Veneziano relation.
- All this for N large. What about the real world?

- In the approximation that $\mu_1, \mu_2 << \mu_3$ Gabriele has computed the masses of η and η':

$$M_{\pm}^2 = M_K^2 + \frac{3a}{2} \pm \frac{1}{2}\sqrt{(2m_K^2 - 2m_\pi^2 - a)^2 + 8a^2}$$

- It implies

$$m_\eta^2 + m_{\eta'}^2 - 2m_K^2 = 3a \Longrightarrow a = 0.24(GeV)^2$$

- Using this value of a and neglecting the square term in the square root he got:

$$m_\eta^2 \simeq m_K^2 + \frac{3 - 2\sqrt{2}}{2} = 0.27(GeV)^2 \;\; ; \;\; Exp.\ 0.30(GeV)^2$$

$$m_{\eta'}^2 \simeq m_K^2 + \frac{3 + 2\sqrt{2}}{2} = 0.95(GeV)^2 \;\; ; \;\; Exp.\ 0.92(GeV)^2$$

that gave the following value for the topological susceptibility in pure YM:

$$\chi_{YM} \equiv \int d^4x \langle q(x) q(0) \rangle = \frac{aF_\pi^2}{2} = (180\ MeV)^4$$

- The first attempt to compute χ_{YM} on the lattice goes back to the early eighties (in good agreement with the result above) [P. Di Vecchia, K. Fabricius, G.C. Rossi and G. Veneziano, Nucl. Phys. B **192** (1981) 392, Phys. Lett. **108** B (1982) 323] [K. Fabricius and G.C. Rossi, Phys. Lett. **127** B (1983) 229] but the renormalisation factor in front of the lattice definition of the topological charge density was forgotten [Campostrini, Di Giacomo and Panagopoulos, Phys. Lett. B **212** (1988) 206]
- More recent values have been obtained:

$$\frac{F_\pi^2}{2N_f} m_{\eta'}^2 = ((198 \pm 20) MeV)^4$$

[L. Giusti, M. Testa, G.C. Rossi and G. Veneziano, Nucl. Phys. B **628** (2002) 234] and

$$\frac{F_\pi^2}{2N_f} m_{\eta'}^2 = ((191 \pm 5) MeV)^4$$

[Del Debbio, Giusti and Pica, Phys. Rev. Lett. **94** (2005) 032003]

- The previous effective Lagrangian with the addition of the θ term was used to compute the dependence on θ of various physical observables
  [Di Vecchia and Veneziano, Nucl. Phys. B **171** (1980) 253]
  [Witten, Annals of Phys. **128** (1980) 363]
- One can also add the baryons, determine the CP violating pion-nucleon coupling constant and use it to get a bound on $\theta \leq 10^{-9}$ from the electric dipole moment of the neutron
  [Crewther, Di Vecchia, Veneziano and Witten, Phys. Lett. **88** B (1979) 123].
- Finally the previous Lagrangian has been extended to include a QCD axion and to discuss its properties as the axion potential
  [Di Vecchia and Sannino, Eur. Phys.J. Plus, **129** (2014) 252]
  [Di Vecchia, Rossi, Veneziano and Yankielowicz, JHEP **12** (2017) 104.]

- The previous results were strongly inspired by the two-dim CP^{N-1} model:

$$L = \overline{D_\mu z} D^\mu z + Fermions \;\; ; \;\; |z|^2 = \frac{1}{g} \;\; ; \;\; D_\mu z = \partial_\mu z + igA_\mu z$$

where $z^i = z^1 z^2 \dots z^N$.
- Conformal invariance at classical level.
- Dimensional transmutation.
- Linear potential and confinement at quantum level.
- Effective Lagrangian for the composite fields corresponding the mesons in QCD can be explicitly constructed in the large N expansion and turns out to be the same used previously [Di Vecchia, Phys. Lett.B **35** (1979) 357]
- The anomaly term is the same as in the case of QCD [F. Riva, Nuovo Cimento A **61** (1981) 69]
- The properties of this model were studied by [D'Adda, Di Vecchia and Lüscher, Nucl. Phys. B **146** (1978) 63 and Nucl. Phys. B **152** (1979) 125] [Witten, Nucl. Phys. B **149** (1979) 285]

After the 1984 string revolution

- Gabriele went back, at least part time, to string theory concentrating on basically two gravity related topics:
  1. String cosmology with Gasperini, Brustein, Buonanno, Damour, Giovannini, Maharana, Meissner......
  2. Gravitational scattering of strings with Amati and Ciafaloni

  I have time only for the second one.
- The scattering amplitude of four dilatons in the superstring in the Regge limit ($s >> -t$) is equal to

$$A(s,t) \sim \frac{32\pi G_N}{\alpha'} \frac{\Gamma(-\frac{\alpha'}{4}t)}{\Gamma(1+\frac{\alpha'}{4}t)} \left(\frac{\alpha'}{4}s\right)^{2+\frac{\alpha'}{2}t} \mathrm{e}^{-i\pi\frac{\alpha'}{4}t}$$

  that in the field theory limit ($\alpha' \to 0$) becomes:

$$A(s,t) = 8\pi G_N \frac{s^2}{(-t)}$$

- It is divergent at high energy and violates unitarity.

- Summing ladder diagrams in the massless case, the tree diagram exponentiates getting a result that is consistent with unitarity [Amati, Ciafaloni and Veneziano, Phys. Lett. B **197** (1987) 81] [Muzinich and Soldate, Phys. Rev. D**37** (1988) 359]
- It has been extended to the massive case by [Kabat and Ortiz, Nucl. Phys. B **388** (1992) 570]
- More precisely, by going to impact parameter space, the quantity that exponentiates is called the leading eikonal:

$$2\delta_0(s,b) = \int \frac{d^{D-2}q}{(2\pi)^{D-2}} e^{iqb} \frac{A(s,t=-q^2)}{2s} = -\frac{R_S\sqrt{s}}{\hbar}\log b$$

where $R_S = 2G\sqrt{s}$ is the Schwarzschild radius.

- It agrees with the phase shift for the scattering of a massless particle, seen as a test body, in the background metric of the other particle, the Aichelburg-Sexl metric ['t Hooft, Phys. Lett. B **198** (1987) 61]

- From the eikonal one gets the classical deflection angle

$$\chi = -\frac{2\hbar}{\sqrt{s}} \frac{\partial}{\partial b} 2\delta_0 = \frac{2R_S}{b}$$

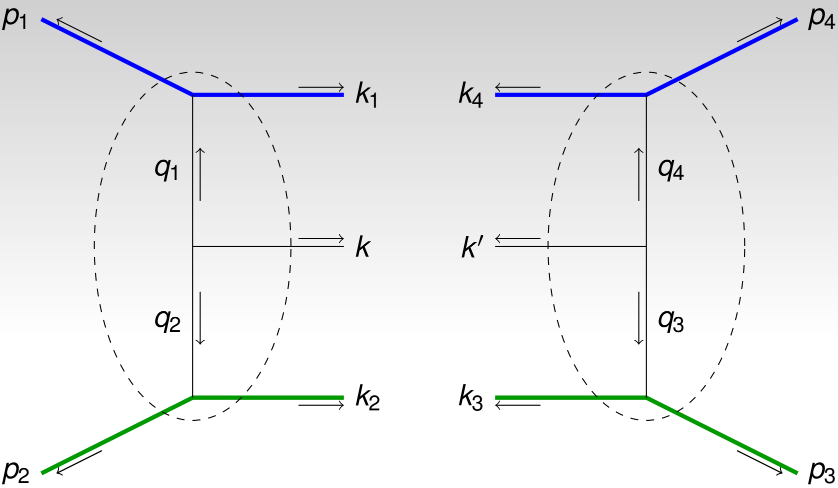
- Keeping the string corrections ($\alpha' \neq 0$) they computed the tidal excitations that are a consequence of the fact that a string is an extended object. [Amati, Ciafaloni and Veneziano, Phys. Lett. B **197** (1987) 81]
- In an impressive paper ACV showed that the first classical correction to the leading eikonal is zero ($\delta_1 = 0$) and then computed the next to the next leading eikonal

$$\text{Re} 2\delta_2 = \frac{4G_N^3 s^2}{\hbar b^2}$$

obtaining the classical deflection angle up to the order G^3

$$\sin\frac{\chi}{2} = -\frac{2\hbar}{\sqrt{s}} \frac{\partial}{\partial b} (2\delta_0 + 2\delta_2) = \frac{R}{b} + \frac{R^3}{b^3}$$

[Amati, Ciafaloni and Veneziano, Nucl. Phys. B **347** (1990) 550]



- They computed the three-particle cut from the unitarity relation getting $\mathrm{Im}\, 2\delta_2$.
- Then they used analyticity, crossing symmetry and eikonal exponentiation to get also $\mathrm{Re}\, 2\delta_2$.

- An easier system to study eikonal exponentiation and string effects is the scattering of a closed string on a stack of N maximally supersymmetric Dp-branes. [D'Appollonio, Di Vecchia, Russo and Veneziano, JHEP **11** (2010) 100]
- The simplification is that the background is given and is not produced by the other string as in the previous cases.
- For very large values of the impact parameter b, everything is as in the case of a point-particle and the eikonal is a c-number.
- For $b < b_D >> \sqrt{\alpha'}$, due to tidal excitations, inelastic channels open and become more important than the elastic one.
- The eikonal becomes an operator including all inelastic processes, again in agreement with unitarity.
- For $b \sim \sqrt{\alpha'}$ unitarity is again violated.
- To restore it one must also include the possibility that the closed string is absorbed by the branes decaying in any number of open strings. [D'Appollonio, Di Vecchia, Russo and Veneziano, JHEP **03** (2016) 030]

- In an impressive calculation more than three years ago the conservative part of the elastic scattering, involving two scalar particles with mass m_1 and m_2, was computed at two-loop order (3PM) by [Bern, Cheung, Roiban, Shen, Solon, Phys. Rev. Lett. **122** (2019) 20, 201603]
- They extracted the deflection angle that turned out to be divergent at high energy ($s \to \infty$).
- This was in contradiction with the results of ACV90.
- After infinite discussions on what was the origin of this problem, only an explicit calculation in massive $\mathcal{N} = 8$ supergravity convinced everybody that the problem disappears if one adds to the conservative piece, computed by [Parra-Martinez, Ruf and Zeng, JHEP **11** (2020) 023], also the contribution of radiation reaction.

- This was done by approximating the integrals in the soft rather than in the potential region
[Di Vecchia, Heissenberg, Russo, Veneziano, Phys. Lett. B **811** (2020) 135924].
- But then how to compute this extra piece in GR if the total classical integrand of the two-loop amplitude was not known?
- From the loss of angular momentum T. Damour computed the radiation reaction contribution to the deflection angle in GR that, added to the conservative part, eliminated the problem with ACV90 also in GR, [Damour, Phys. Rev. **102** (2020) 12].
- The same result was obtained by keeping in the five-point amplitude only the leading soft graviton contribution (divergent for $\omega \to 0$) and by using unitarity and real analyticity
[Di Vecchia, Heissenberg, Russo, Veneziano, Phys. Lett. B **818** (2021) 136379].

- Putting together the conservative part and the radiation reaction one obtained ($s = m_1^2 + m_2^2 + 2m_1 m_2 \sigma$):

$$\text{Re}\, 2\delta_2^{(gr)} = \frac{4G^3 m_1^2 m_2^2}{b^2} \Bigg\{ \frac{(2\sigma^2-1)^2(8-5\sigma^2)}{6(\sigma^2-1)^2} - \frac{\sigma(14\sigma^2+25)}{3\sqrt{\sigma^2-1}} + \frac{s(12\sigma^4-10\sigma^2+1)}{2m_1 m_2(\sigma^2-1)^{\frac{3}{2}}} + \cosh^{-1}\sigma \times \left[\frac{\sigma(2\sigma^2-1)^2(2\sigma^2-3)}{2(\sigma^2-1)^{\frac{5}{2}}} + \frac{-4\sigma^4+12\sigma^2+3}{\sigma^2-1} \right] \Bigg\} \Longrightarrow \frac{4G^3 s^2}{b^2}$$

in agreement with ACV90 at high energy!

- A similar problem appears now at 4PM (three loops) [Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon and Zeng, Phys. Rev. Lett. **128** (2022) 16, 161103] [Dlapa, Kälin, Liu and Porto, Phys. Rev. Lett. **128** (2022) 16, 161104].
- What is its solution?

Conclusions

- I have presented some of the highlights of Gabriele's research
- There is much more that I could not cover and that I hope will be covered by Thibault and others.
- Here is a small list
  - Supersymmetric instantons
  - Spin of the proton
  - Multiquark states
  - Supersymmetric effective Lagrangian
  - Perturbative QCD
  - Supersymmetry on the lattice
  - A lot on cosmology
- To conclude I wish

HAPPY BIRTHDAY, GABRIELE!