QCD Corrections for Inclusive B Decays

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CV

- PhD: Zurich & Padova with M. Passera, T. Gehrmann
- Postdoc:
  - Bern (CH)
  - Siegen (D)
  - Karlsruhe (D)
- MSCA Fellow at CERN:
  Phenomenology of B-meson semileptonic inclusive decays:
  from the Standard Model to the New Physics

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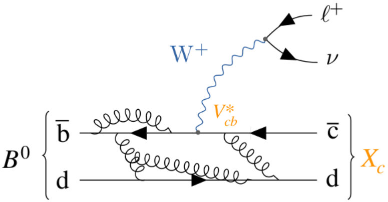
  Phenomenology of B-meson semileptonic inclusive decays: from the Standard Model to the New Physics
- Secret project:

Carloni Calame, Passera, Trentadue, Venanzoni, PLB 2015;
Abbiendi et al. EPJ C77 (2017) 139



$$\Gamma_{\rm sl} = \Gamma_{\rm free} + \Gamma_{\mu_\pi}\frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \Gamma_{\rho_D}\frac{\rho_D^3}{m_b^3} + \Gamma_{\rho_{LS}}\frac{\rho_{LS}^3}{m_b^3} + \ldots$$

Reviews:
Benson, Bigi, Mannel, Uraltsev, Nucl.Phys. B665 (2003) 367;
Dingfelder, Mannel, Rev.Mod.Phys. 88 (2016) 035008.

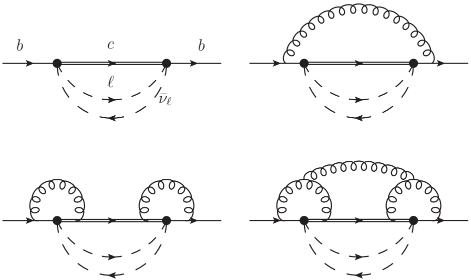
- Γ_i are computed in **perturbative QCD**.
- The HQE parameters:
  $\mu_\pi, \mu_G, \rho_D, \rho_{LS} \sim \langle B| \, \mathcal{O}_i^{\bar{b}b} \, |B\rangle$
- HQE parameters are **extracted from data**.

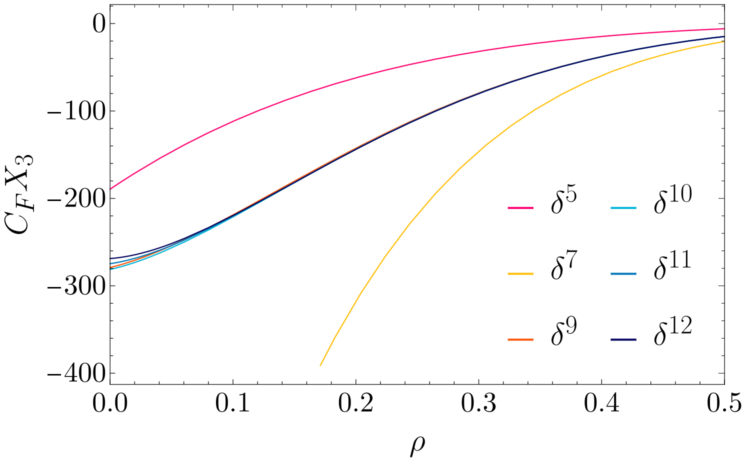
$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Third order corrections for $b \to X_c \ell \bar{\nu}_\ell$

$$\Gamma_{\text{free}} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \sum_{n \geq 0} X_n(\rho) \left(\frac{\alpha_s}{\pi}\right)^n$$

- GOAL: $X_3(\rho)$
- Expand in $\boldsymbol{\rho = m_c/m_b \simeq 0.3}$
  too complicated at $O(\alpha_s^3)$
- Expand in $\boldsymbol{\delta = 1 - m_c/m_b \simeq 0.7}$
  Crucial factorization Feynman integrals!





$$C_F X_3(\rho = 0.28) = -91.2 \pm 0.4 \quad (0.4\%)$$

MF, Schönwald, Steinhauser, Phys.Rev.D 104 (2021) 1, 016003

Short-distance mass scheme

$$\Gamma_{\text{free}} = \frac{G_F^2 |V_{cb}|^2 (m_b^{\text{OS}})^5}{192\pi^3} f(0.28) \left[1 - 1.72 \left(\frac{\alpha_s}{\pi}\right) - 13.1 \left(\frac{\alpha_s}{\pi}\right)^2 - 163.3 \left(\frac{\alpha_s}{\pi}\right)^3 \right]$$

The kinetic mass

$$m_b = M_B - \overline{\Lambda} - \frac{\mu_\pi^2}{2m_b} + \dots$$

- $\overline{\Lambda}$: the B-meson binding energy.
- μ_π: the kinetic energy induced by the residual motion of the heavy quark.
- In pQCD, we can make a short-distance mass definition by identifying:

Bigi, Shifman, Uraltsev, Vainshtein, PRD 56 (1997) 4017.

$$m_b(\mu) \to m_b^{\mathrm{kin}}(\mu) \qquad M_B \to m_b^{\mathrm{OS}}$$

$$\overline{\Lambda}(\mu) \to [\overline{\Lambda}(\mu)]_{\mathrm{pert}} \qquad [\mu_\pi^2(\mu)] \to [\mu_\pi^2(\mu)]_{\mathrm{pert}}$$

The kinetic mass up to $O(\alpha_s^3)$

$$\frac{m^{\text{kin}}}{m^{\text{OS}}} = 1 - \frac{\alpha_s^{(n_l)}}{\pi} C_F \left(\frac{4}{3} \frac{\mu}{m^{\text{OS}}} + \frac{1}{2} \frac{\mu^2}{(m^{\text{OS}})^2} \right) + \left(\frac{\alpha_s^{(n_l)}}{\pi} \right)^2 C_F \left\{ \frac{\mu}{m^{\text{OS}}} \left[C_A \left(-\frac{215}{27} + \frac{2\pi^2}{9} + \frac{22}{9} l_\mu \right) + n_l T_F \left(\frac{64}{27} - \frac{8}{9} l_\mu \right) \right] \right.$$
$$+ \frac{\mu^2}{(m^{\text{OS}})^2} \left[C_A \left(-\frac{91}{36} + \frac{\pi^2}{12} + \frac{11}{12} l_\mu \right) + n_l T_F \left(\frac{13}{18} - \frac{1}{3} l_\mu \right) \right] \Bigg\} + \left(\frac{\alpha_s^{(n_l)}}{\pi} \right)^3 C_F \left\{ \frac{\mu}{m^{\text{OS}}} \left[C_A^2 \left(-\frac{130867}{1944} \right. \right. \right.$$
$$+ \frac{511\pi^2}{162} + \frac{19\zeta_3}{2} - \frac{\pi^4}{18} + \left(\frac{2518}{81} - \frac{22\pi^2}{27} \right) l_\mu - \frac{121}{27} l_\mu^2 \Bigg) + C_A n_l T_F \left(\frac{19453}{486} - \frac{104\pi^2}{81} - 2\zeta_3 \right.$$
$$+ \left(-\frac{1654}{81} + \frac{8\pi^2}{27} \right) l_\mu + \frac{88}{27} l_\mu^2 \Bigg) + C_F n_l T_F \left(\frac{11}{4} - \frac{4\zeta_3}{3} - \frac{2}{3} l_\mu \right) + n_l^2 T_F^2 \left(-\frac{1292}{243} + \frac{8\pi^2}{81} + \frac{256}{81} l_\mu - \frac{16}{27} l_\mu^2 \right) \Bigg]$$
$$+ \frac{\mu^2}{(m^{\text{OS}})^2} \left[C_A^2 \left(-\frac{96295}{5184} + \frac{445\pi^2}{432} + \frac{57\zeta_3}{16} - \frac{\pi^4}{48} + \left(\frac{2155}{216} - \frac{11\pi^2}{36} \right) l_\mu - \frac{121}{72} l_\mu^2 \right) + C_A n_l T_F \left(\frac{13699}{1296} - \frac{23\pi^2}{54} \right. \right.$$
$$\left. \left. \left. - \frac{3\zeta_3}{4} + \left(-\frac{695}{108} + \frac{\pi^2}{9} \right) l_\mu + \frac{11}{9} l_\mu^2 \right) + C_F n_l T_F \left(\frac{29}{32} - \frac{\zeta_3}{2} - \frac{1}{4} l_\mu \right) + n_l^2 T_F^2 \left(-\frac{209}{162} + \frac{\pi^2}{27} + \frac{26}{27} l_\mu - \frac{2}{9} l_\mu^2 \right) \right] \right\}, \quad (4)$$

MF, Schönwald, Steinhauser, PRL 125 (2020) 052003

Implications for $\overline{B} \to X_c \ell \bar{\nu}_\ell$

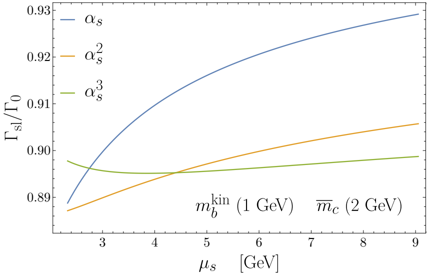
$$\Gamma_{\text{free}} = \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3} f(\rho) \left[1 + \sum_n Y_n \left(\frac{\alpha_s}{\pi}\right)^n \right]$$

with $\alpha_s \equiv \alpha_s^{(4)}(m_b)$.

n=1 Jezabek, Kühn, Jezabek, Kuhn, NPB 314 (1989) 1

n=2 Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015.

n=3 Fael, Schönwald, Steinhauser, PRD 104 (2021) 1, 016003



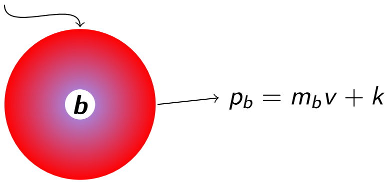
$$m_b^{\text{OS}} : m_c^{\text{OS}} \quad 1 - 1.78\left(\frac{\alpha_s}{\pi}\right) - 13.1\left(\frac{\alpha_s}{\pi}\right)^2 - 163.3\left(\frac{\alpha_s}{\pi}\right)^3$$

$$m_b^{\text{kin}}(1\text{ GeV}) : \overline{m}_c(2\text{ GeV}) \quad 1 - 1.24\left(\frac{\alpha_s}{\pi}\right) - 3.65\left(\frac{\alpha_s}{\pi}\right)^2 - 1.0\left(\frac{\alpha_s}{\pi}\right)^3$$

$$m_b^{\text{1S}} : m_c \text{ via HQET} \quad 1 - 1.38\left(\frac{\alpha_s}{\pi}\right) - 6.32\left(\frac{\alpha_s}{\pi}\right)^2 - 33.1\left(\frac{\alpha_s}{\pi}\right)^3$$

The Heavy-Quark Expansion

$$\Gamma_{\rm sl} = \Gamma_{\rm free} + \Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + \Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \dots$$



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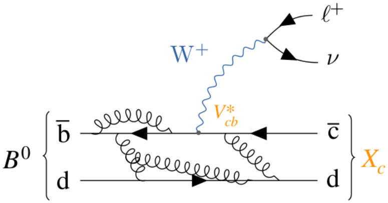
Dingfelder, Mannel, Rev.Mod.Phys. 88 (2016) 035008.

- HQE parameters are **extracted from data**.
  Problem: too many HQE parameters!
  - **4** up to $1/m_b^3$
  - **13** up to $1/m_b^4$
  - **31** up to $1/m_b^5$



A new method for measuring $|V_{cb}|$

- Di-lepton invariant mass (q^2) is invariant under Reparametrization

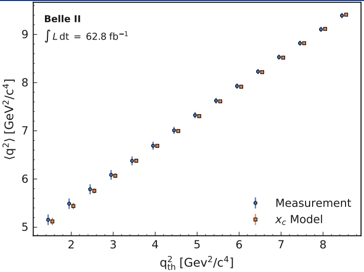
$$v \to v + \delta v$$

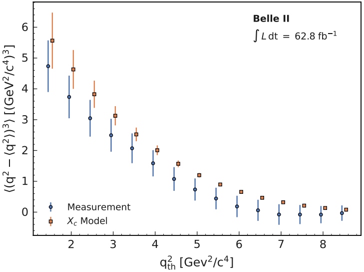
- Reduced set of HQE parameters!

$$\langle (q^2)^n \rangle_{\rm cut} = \int_{q^2 > q^2_{\rm cut}} dq^2 \, (q^2)^n \, \frac{d\Gamma}{dq^2}$$

- $|V_{cb}|$ from q^2 moments with the HQE up to $1/m_b^4$!

MF, Vos, Mannel, JHEP 02 (2019) 177





Belle, hep-ex/2109.01685 Belle II, hep-ex/2205.06372

New $|V_{cb}|$ determinations!

- Impact of new N3LO calculations in global fits of E_e and M_X moments.
- 34% improvement:

$$|V_{cb}|^{\mathrm{inc}} = (42.16 \pm 51) \times 10^{-3}$$

Bordone, Capdevila, Gambino, PLB 822 (2021) 136679

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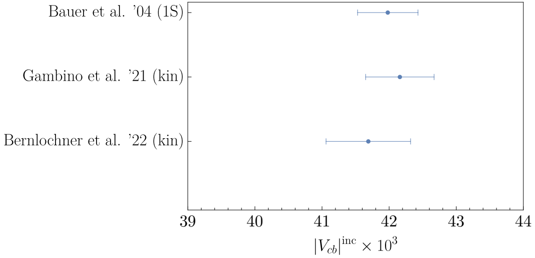
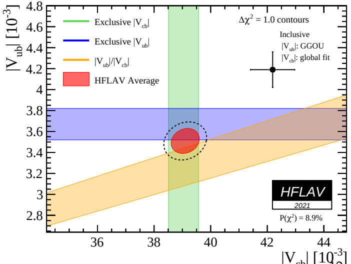
Bordone, Capdevila, Gambino, PLB 822 (2021) 136679

- First extraction from q^2 moments:

$$|V_{cb}|^{\text{inc}} = (41.69 \pm 0.63) \times 10^{-3}$$

MF, Bernlochner,Olschewsky, Persson, van Tonder,
Vos, Welsch, JHEP 10 (2022) 068





Outlook

- Experiments:
  - Upcoming new data from Belle II.
  - LHCb: Inclusive B_s decays.
- Improvements HQE:
  - NNLO differential distributions.
  - N3LO for q^2 spectrum doable?
  - $B \to X_u \ell \nu$ to $O(\alpha_s^3)$.
- Phenomenological analysis:
  - Global fit with NP operator effects.
  - Exclusive vs. inclusive B decays.
  - Interplay with lattice calculations.