Research of Ze Long Liu

CERN TH retreat @ Les Houches, November 9, 2022

My Research Career

- 07/2016, Ph.D, Peking University, China

- Postdoctoral Career

Mainz	LANL	Bern	CERN
10/2016	11/2019	09/2021	11/2022

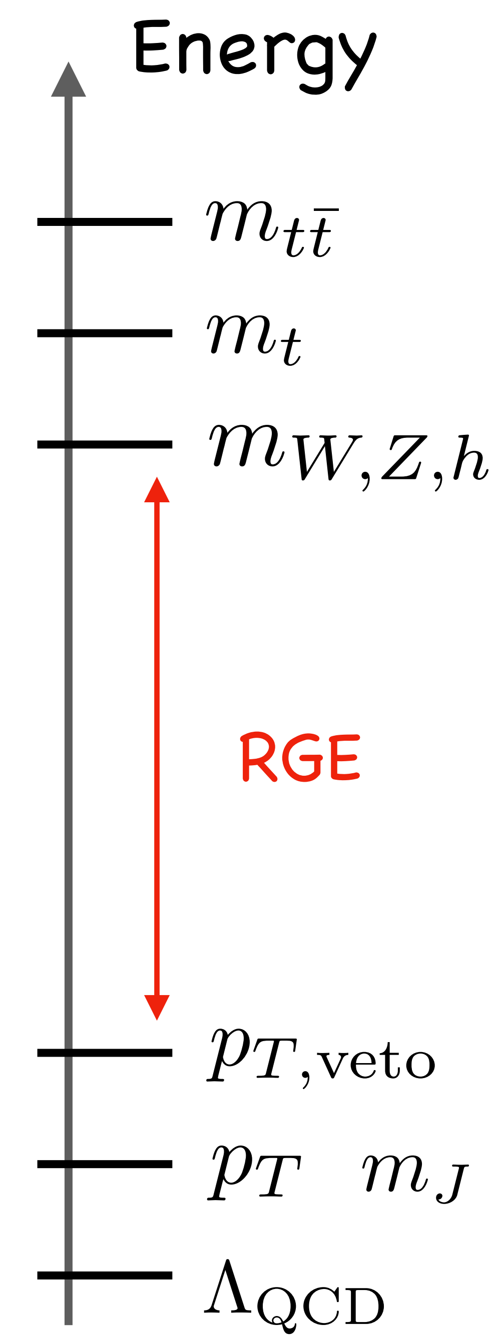
Precision Collider Observables

- Multi scales are involved in measurements
  - Fixed-order results are invalid due to large logarithms of scale ratios
  - Large logs need to be resummed to all orders in α_s
  - Renormalization-group evolutions are governed by anomalous dimensions

Soft-collinear effective theory (SCET) is powerful for scale separation and factorization

$$\sigma \sim H \otimes B_{a/N} \otimes B_{b/N} \otimes S \otimes \prod_i J$$

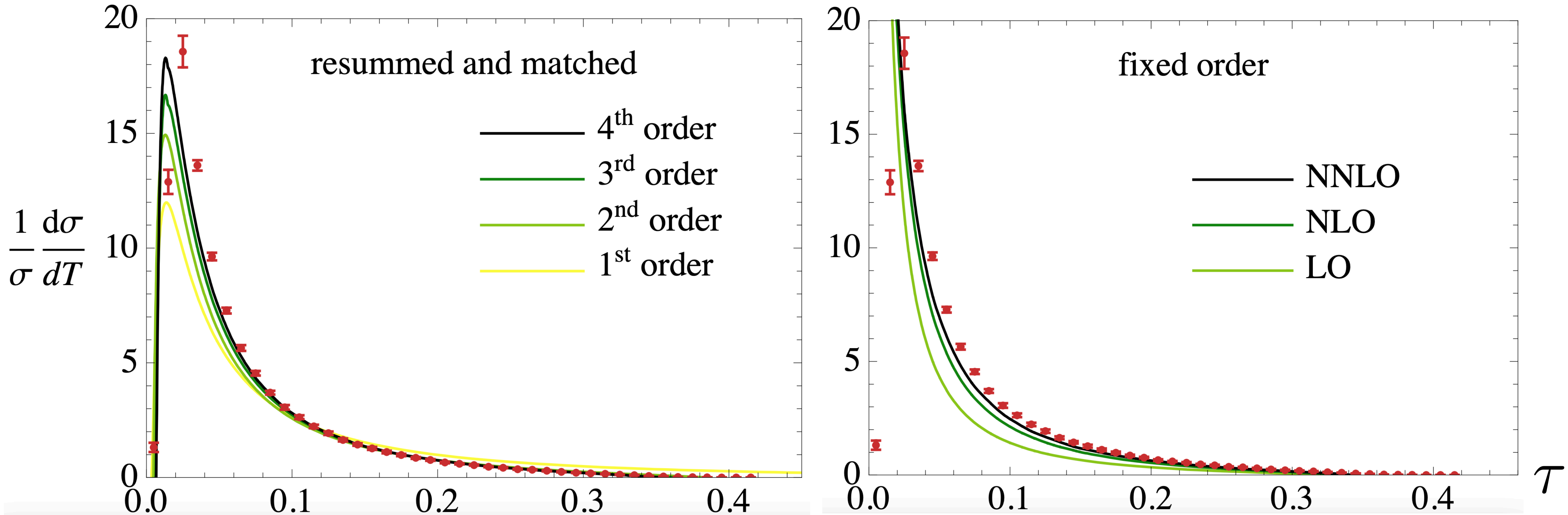
IR poles (H) UV poles in low energy matrix elements ($B_{a/N}$, $B_{b/N}$, S, J)



Precision Collider Observables

Thrust in e+e- collision: $T \equiv 1 - \tau = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$

Becher & Schwartz '11



$$
\begin{aligned}
\sigma \sim\ & \alpha_s L^2 + \alpha_s L + \alpha_s \\
& \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 \\
& \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \cdots \\
& \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots
\end{aligned}
$$

$\alpha_s L \sim 1 \qquad L = \ln \tau$

Expansion in α_s is not convergent,

but expansion in 1/L is !

Soft-Collinear Factorization

For n-jet amplitudes:

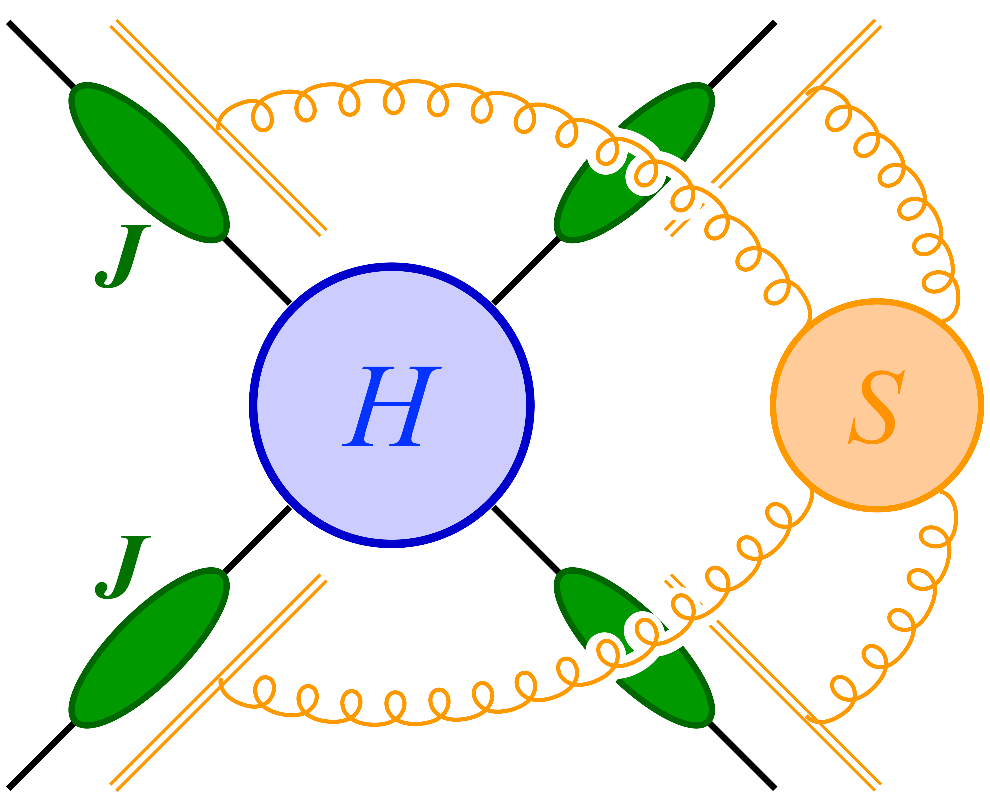
Matrix in color space

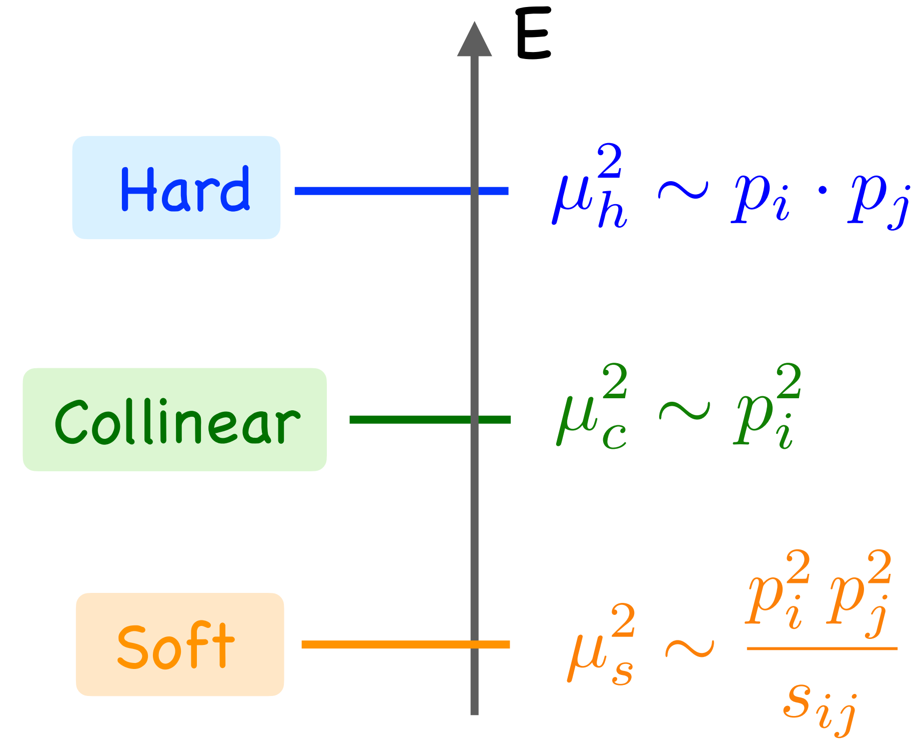
$$\underline{\text{Off-Shell Green's function}} = \boldsymbol{S}(\{\underline{\beta}\},\epsilon)\prod_i J(L_i^2,\epsilon)\,|\mathcal{M}(\{\underline{s}\},\epsilon)\rangle$$

UV renormalized
free of IR poles

$$\beta_{ij} = \ln\frac{(-s_{ij})\,\mu^2}{(-p_i^2)(-p_j^2)} \qquad \ln\frac{\mu^2}{-p_i^2} \qquad s_{ij} = \pm 2p_i\cdot p_j$$

cusp angles





Construct Soft Anomalous Dimensions

- Kinematic dependences
  - Linearly depends on cusp angles - soft-collinear factorization

$$\beta_{ij} = L_i + L_j - \ln \frac{\mu^2}{-s_{ij}} \qquad \beta_{Ij} = L_j - \ln \frac{m_I \mu}{-s_{Ij}} \qquad \beta_{IJ} = \cosh^{-1}\left(\frac{-s_{IJ}}{2 m_I m_J}\right)$$

  - Conformal cross ratios - independent on collinear scales

Purely massless: $\beta_{ijkl} = \beta_{ij} + \beta_{kl} - \beta_{ik} - \beta_{jl}$ — Gardi, Magnea '09; Becher, Neubert '09

One massive: $r_{ijI} \equiv \dfrac{v_I^2\,(n_i \cdot n_j)}{2\,(v_I \cdot n_i)(v_I \cdot n_j)}$ — ZLL, Schalch '22

- Only color connected - Non-Abelian Exponentiation Theorem

color dipole

Sum over all the permutations of i,j,k,...

$\boldsymbol{T}_i^a \boldsymbol{T}_j^a$ ✓ $\qquad \mathcal{T}_{ijk} \equiv i f^{abc} \left(\boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c\right)_+$ ✓ $\qquad \mathcal{T}_{ijkl} \equiv f^{ade} f^{bce} \left(\boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c \boldsymbol{T}_l^d\right)_+$ ✓

~~$\mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_k^a \mathbf{T}_l^b$~~

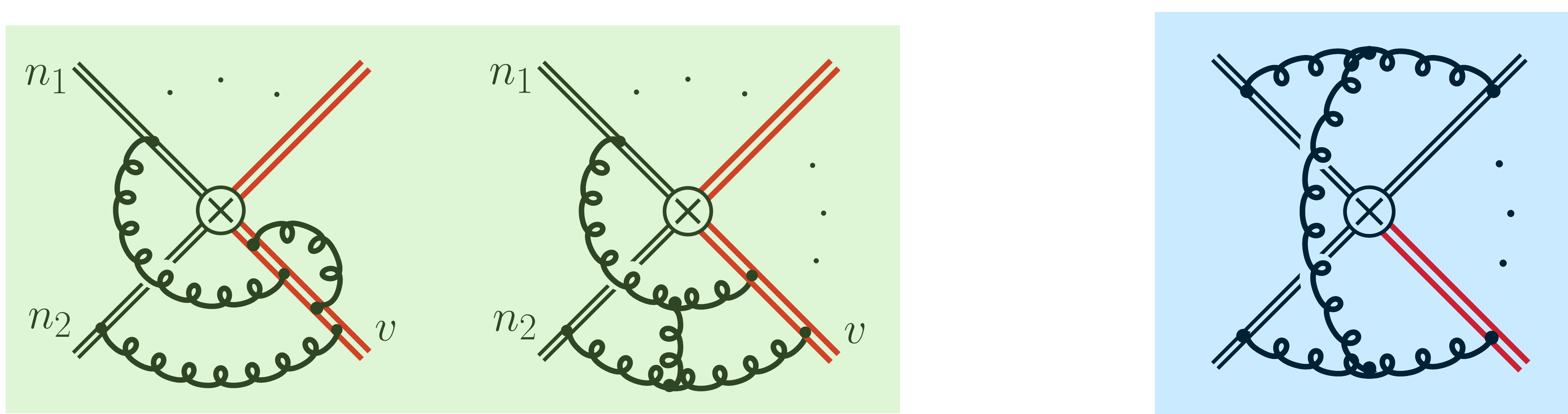
General Structure of Anomalous Dimension

Renormalization factor in $\overline{\mathrm{MS}}$ scheme

$$\boldsymbol{Z}(\epsilon, \{\underline{p}\}, \mu) = \mathbf{P} \exp \left[\int_\mu^\infty \frac{d\mu'}{\mu'} \boldsymbol{\Gamma}(\{\underline{p}\}, \mu') \right]$$

Non-dipole correlations involving one massive parton is give by

$$\boldsymbol{\Gamma}(\{\underline{p}\}, \mu)\big|_{\text{non-dipole}} = \sum_I \sum_{(i,j)} \mathcal{T}_{ijII} F_{\text{h2}}(r_{ijI}, \alpha_s) + \sum_I \sum_{(i,j,k)} \mathcal{T}_{ijkI} F_{\text{h3}}(r_{ijI}, r_{ikI}, r_{jkI}, \alpha_s)$$

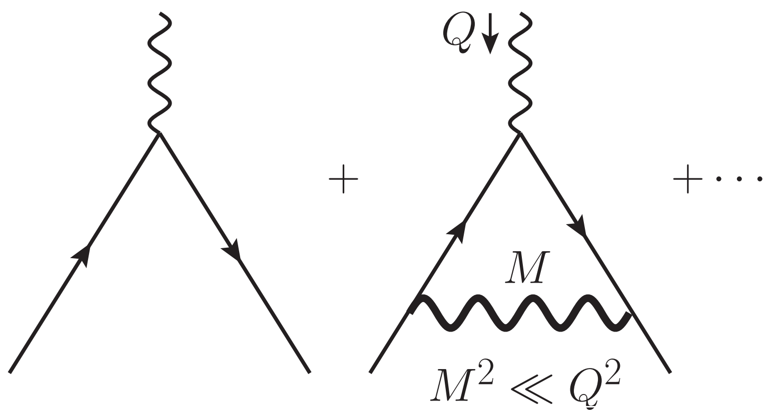


The analytical result has been obtained at 3 loops!

ZLL, Schalch, '22

Factorization at Subleading Power

Leading Power



$$L = \ln \frac{-Q^2}{M^2}$$

- Factorization is local

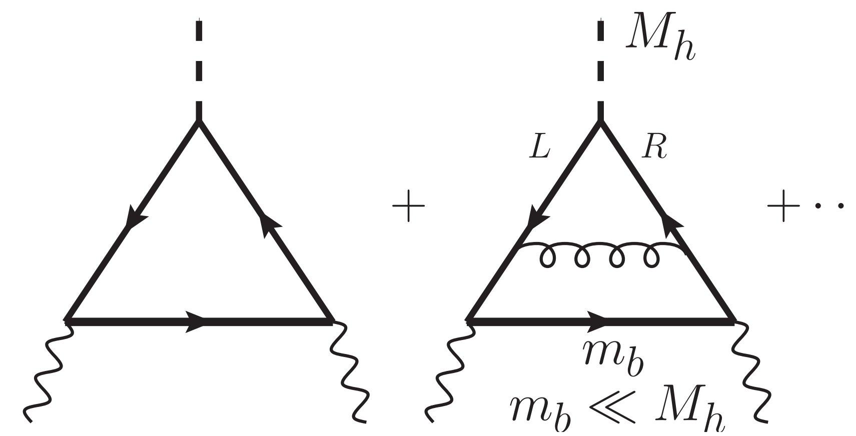
$$H(Q^2,\mu) J_n(M,\mu,\nu/Q) J_{\bar{n}}(M,\mu,\nu/Q) S(M,\mu,\nu/M)$$

$$\sim 1 + \frac{\alpha}{4\pi}\left(-L^2 + 4L + c_0\right) + \mathcal{O}(\alpha^2)$$

- No convolution, no endpoint

- rapidity divergence

Subleading Power



$$L = \ln \frac{-M_h^2}{m_b^2}$$

- Factorization is non-local

$$H_1\, O_1 + H_2 \otimes O_2 + H_3 \otimes J \otimes J \otimes S$$

$$\sim y_b \boxed{m_b} \left\{ \left(\frac{L^2}{2} - 2 \right) + \frac{C_F \alpha_s}{4\pi} \left[-\frac{L^4}{12} - L^3 + \cdots \right] \right\} + \mathcal{O}(\lambda^2)$$

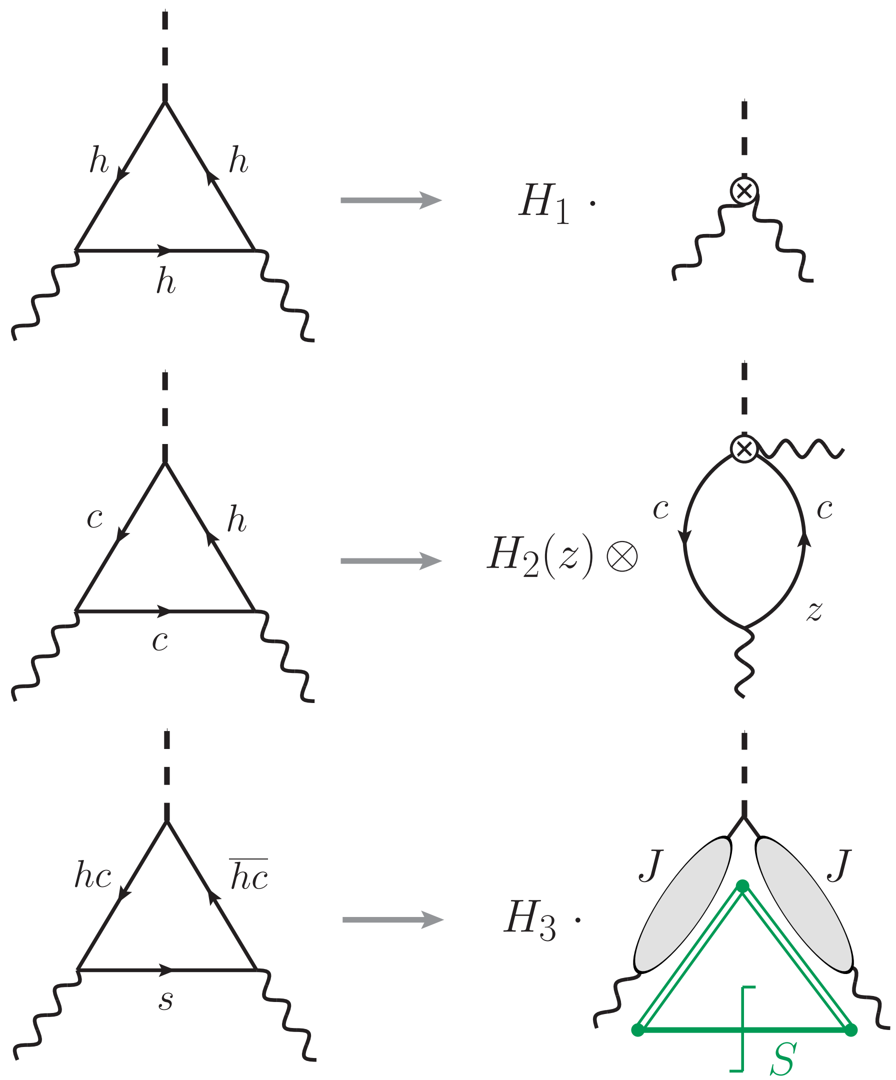
- Endpoint-divergent convolutions

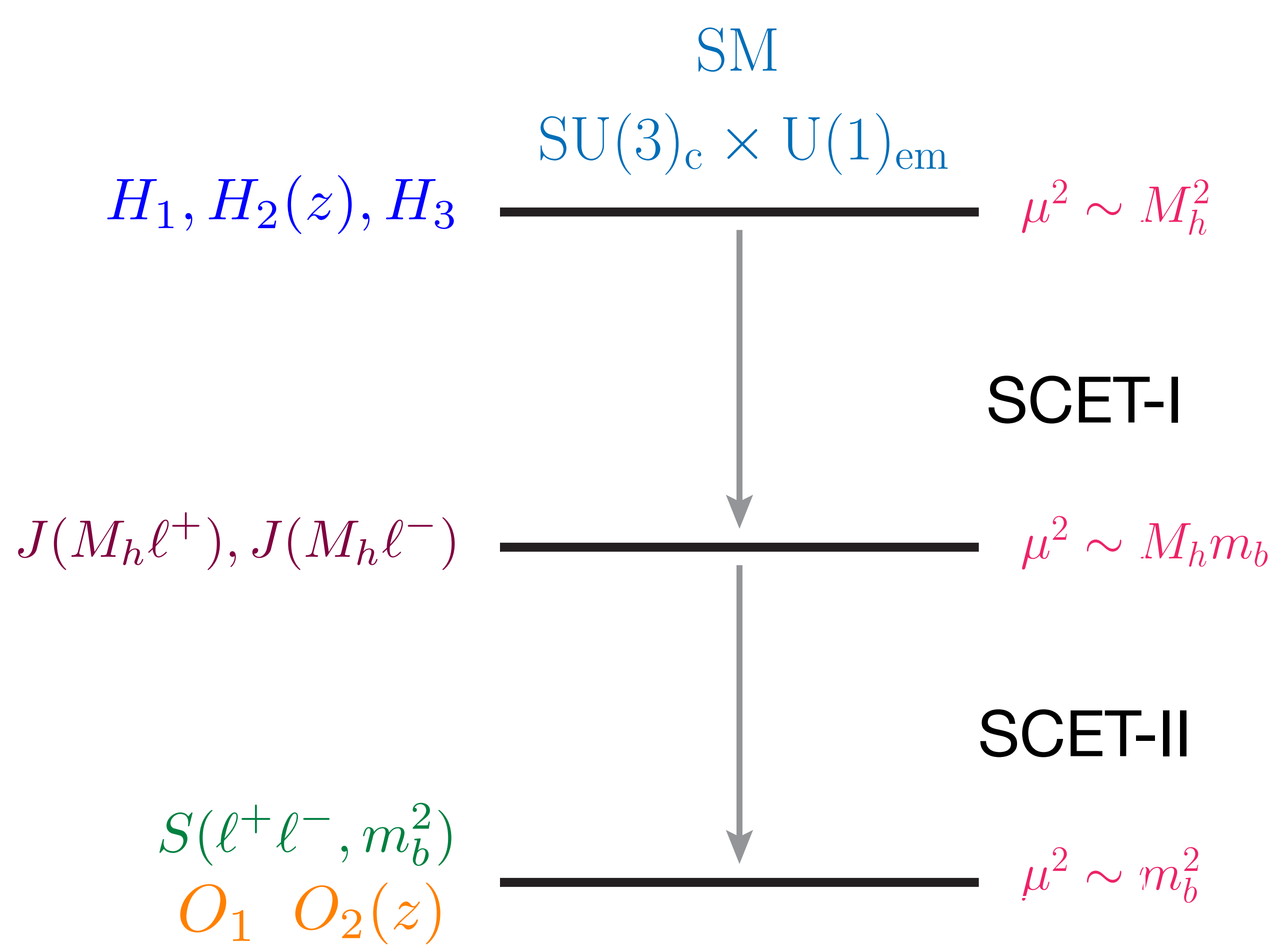
- rapidity divergence

Factorization at Subleading Power

- b-quark induced Higgs to diphoton decay ZLL, Neubert, '19

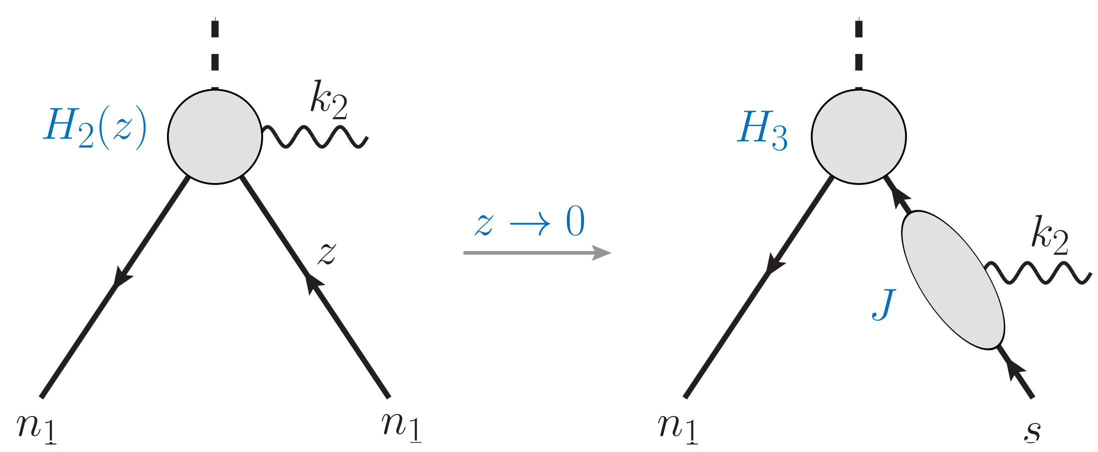
$$\ell^\mu = (\bar{n}\cdot\ell)\frac{n^\mu}{2} + (n\cdot\ell)\frac{\bar{n}^\mu}{2} + \ell_\perp^\mu \equiv \ell_-\frac{n^\mu}{2} + \ell_+\frac{\bar{n}^\mu}{2} + \ell_\perp^\mu$$



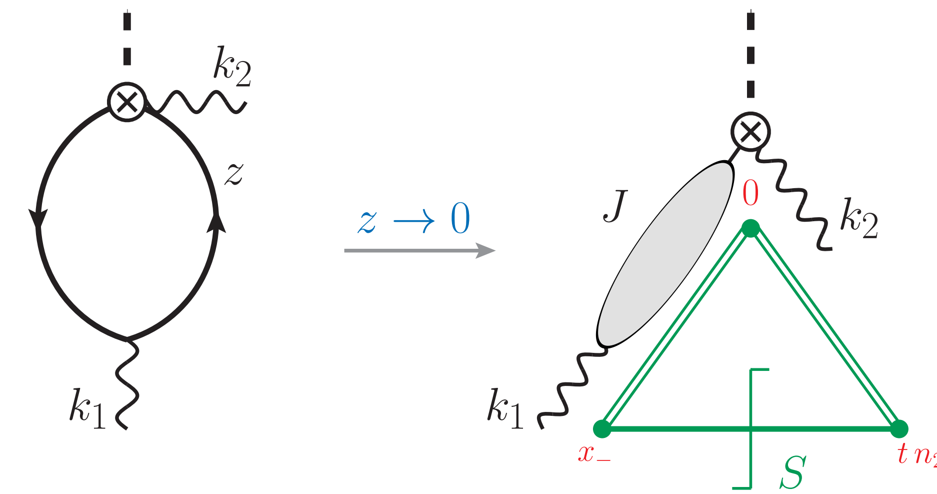


- factorization violated at $z \ll 1\,,\quad \ell^-/\ell^+ \gg 1 \quad \ell^+, \ell^- \sim M_h$

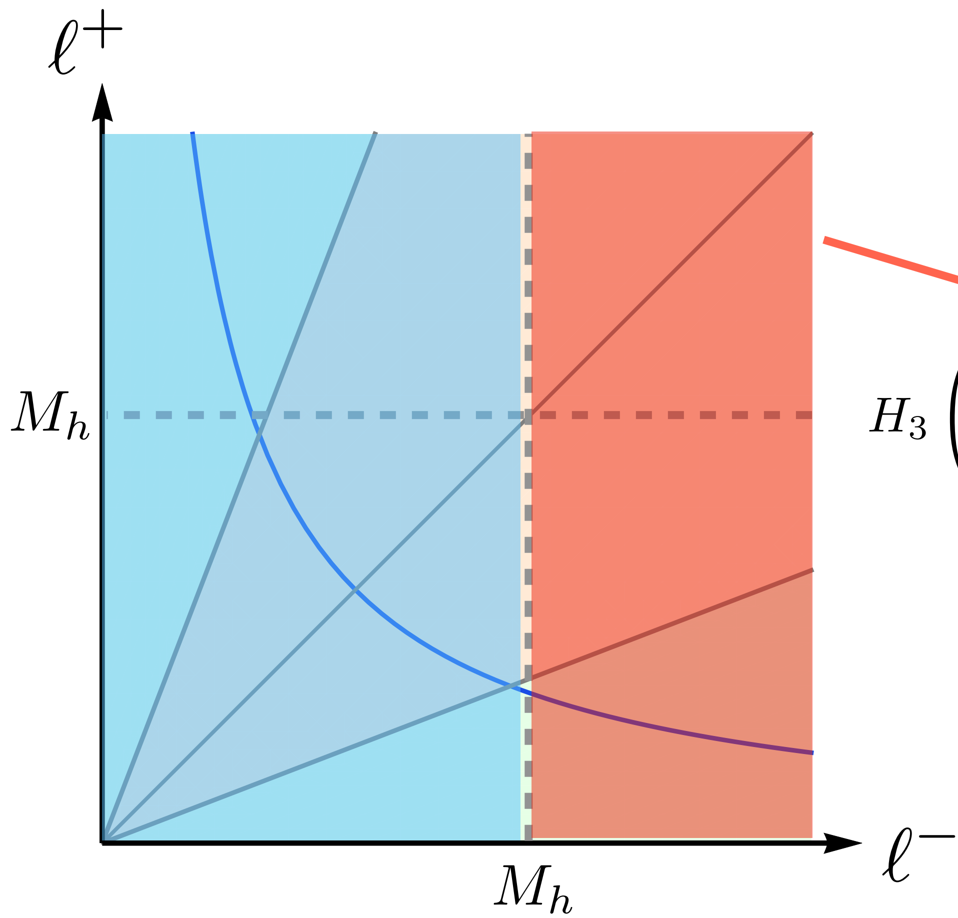
Refactorization near the endpoint



$$[\![H_2^{(0)}(z)]\!] = \frac{[\![\bar{H}_2^{(0)}(z)]\!]}{z} = -\frac{H_3^{(0)}}{z}\, J(zM_h^2)$$



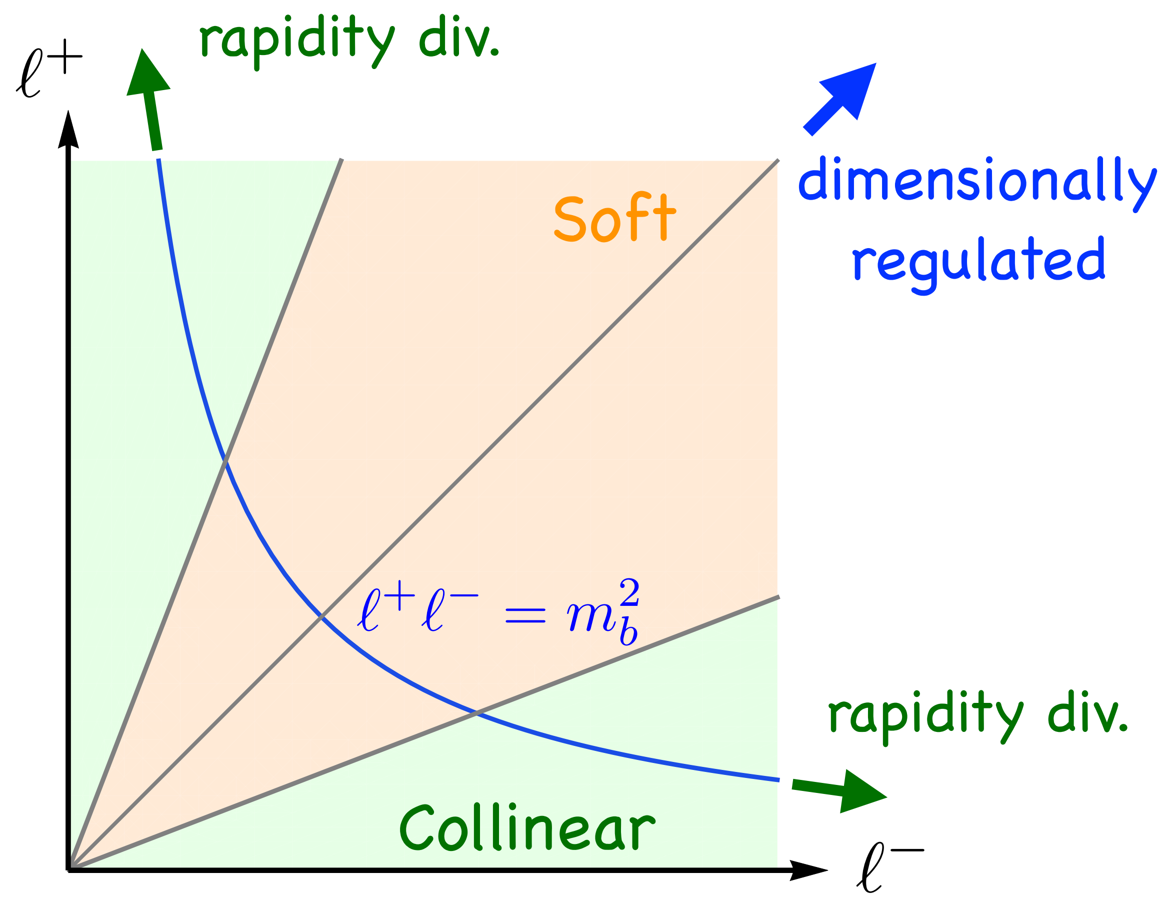
$$[\![\langle\gamma\gamma|\, O_2^{(0)}(z)\, |h\rangle]\!] = -\frac{g_\perp^{\mu\nu}}{2}\int_0^\infty \frac{d\ell_+}{\ell_+}\, J^{(0)}(-M_h\ell_+)\, S^{(0)}(zM_h\ell_+)$$

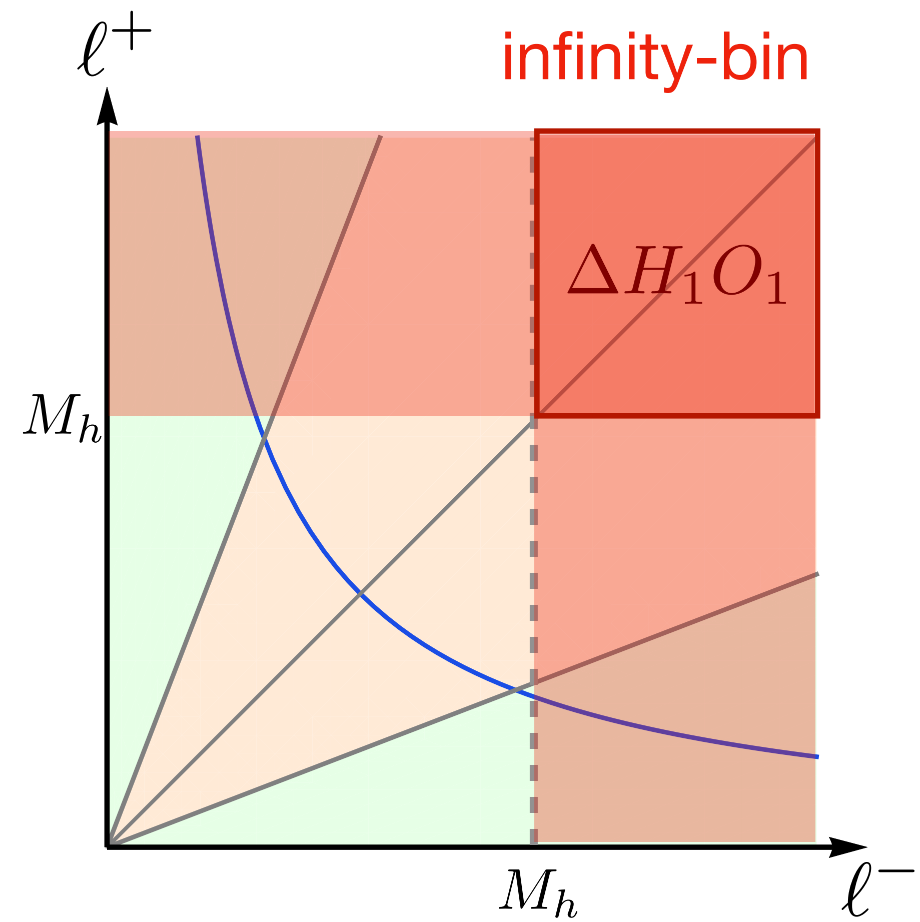


$$H_3\left(\int_{M_h}^\infty \frac{d\ell^-}{\ell^-}\int_0^\infty \frac{d\ell^+}{\ell^+} + \int_0^{M_h} \frac{d\ell^-}{\ell^-}\int_0^\infty \frac{d\ell^+}{\ell^+}\right) J(M_h\ell^-)J(M_h\ell^+)S(\ell^+\ell^-) = 0$$

$$\int_0^1 dz [\![H_2(z)]\!][\![O_2(z)]\!]$$

Endpoint Divergences





- Endpoint divergences are removed by rearrangement of phase space

$$\mathcal{M}(h \to \gamma\gamma) = (H_1 + \Delta H_1)\, O_1 \quad \text{infinity-bin subtraction}$$

$$+\, 4 \lim_{\delta \to 0} \int_\delta^1 dz \left[H_2(z)\, O_2(z) - [\![H_2(z)]\!]\, [\![O_2(z)]\!] \right]$$

$$+\, H_3 \lim_{\sigma \to -1} \int_0^{M_h} \frac{d\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{d\ell_+}{\ell_+}\, J(M_h \ell_-)\, J(-M_h \ell_+)\, S(\ell_+ \ell_-) + (\ell_+ \leftrightarrow \ell_-)$$

Renormalized Factorization Theorem

ZLL, Neubert, Mecaj, Wang '20

- Mismatch is a big problem!

$$T_3 = H_3 \int_0^{M_h} \frac{d\ell_-}{\ell_-} \int_0^{M_h} \frac{d\ell_+}{\ell_+} J(M_h\ell_-) J(-M_h\ell_+) S(\ell_+\ell_-) \qquad \text{bare}$$

$$\neq H_3(\mu) \int_0^{M_h} \frac{d\ell_-}{\ell_-} \int_0^{M_h} \frac{d\ell_+}{\ell_+} J(M_h\ell_-,\mu) J(-M_h\ell_+,\mu) S(\ell_+\ell_-,\mu) \qquad \text{renormalized}$$

Renormalization of jet and soft functions are non-local!

- Final renormalized factorization theorem

$$\mathcal{M}_b = H_1(\mu)\,\langle O_1(\mu)\rangle \qquad T_1$$

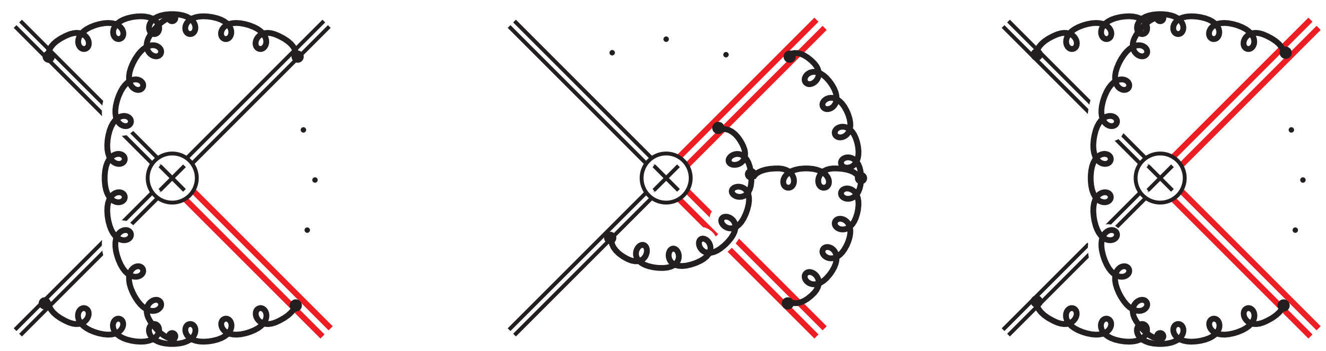
$$+\,2\int_0^1 dz \left[H_2(z,\mu)\,\langle O_2(z,\mu)\rangle - \frac{[\![\bar{H}_2(z,\mu)]\!]}{z} [\![\langle O_2(z,\mu)\rangle]\!] - \frac{[\![\bar{H}_2(\bar{z},\mu)]\!]}{\bar{z}} [\![\langle O_2(\bar{z},\mu)\rangle]\!] \right] \qquad T_2$$

$$+\,g_\perp^{\mu\nu}\, H_3(\mu) \lim_{\sigma\to -1} \int_0^{M_h} \frac{d\ell_-}{\ell_-} \int_0^{\sigma M_h} \frac{d\ell_+}{\ell_+}\, J(M_h\ell_-,\mu)\, J(-M_h\ell_+,\mu)\, S(\ell_+\ell_-,\mu)\Big|_{\text{leading power}} \qquad T_3$$

Using refactorization conditions, the sum of mismatch in T2 and T3 is purely hard, absorbed to T1

For the Future

- Multi-leg QCD amplitudes with massive partons



- Higher-order calculations of soft/collinear matrix elements
  - Hemisphere soft function at 3 loops - precision thrust distribution
  - 2-loop jet function for photon isolation
  - Develop methods to calculate phase space integration with step functions

Thanks for your attention!