

# Research Overview: Scattering Amplitudes for the LHC

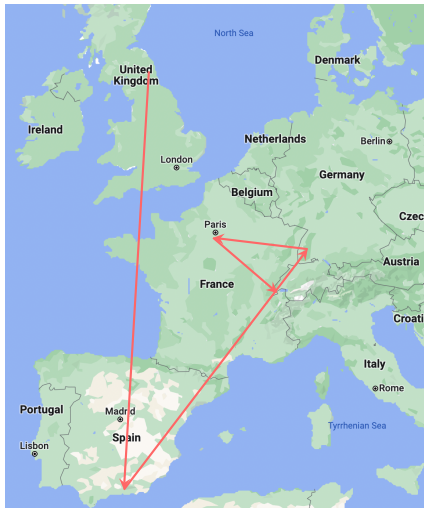
Ben Page

CERN Theoretical Physics Department



Theory Retreat  
9<sup>th</sup> November 2022

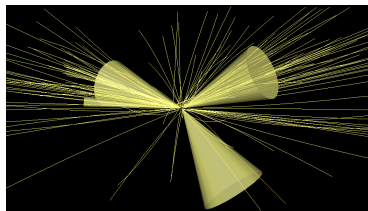
# Career Path



- [’08–’12] Student: Durham, UK<sup>a</sup>.  
<sup>a</sup>CERN Summer Student 2011.
- [’12–’15] Masters/PhD student<sup>b</sup>:  
Granada, Spain.  
<sup>b</sup>CERN Visitor 2014–2015.
- [’15–’18] Post-doctoral researcher:  
University of Freiburg, Germany.
- [’18–’21] Post-doctoral researcher:  
IPhT Saclay, France.
- [’21–’23] Senior Fellow<sup>c</sup>:  
CERN, Switzerland.  
<sup>c</sup>Marie Curie fellowship.

# Physics Objective: Enable Precise Predictions

- Entering “**precision era**”. Many processes experimental uncertainty projected to reach **1% level**.
- LHC physics program **driven** by theory/experiment comparison.
- **NNLO precision** theory prediction demanded to understand data.



To make the most of **LHC physics potential**, we need NNLO precise predictions for many complex **scattering processes**.

# Performing Predictions

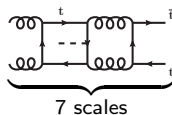
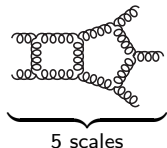
- Theory prediction built from **phase space** integrals of **amplitudes**.

$$\text{cross section} \sim \int_{\text{phase space}} d\Phi \left[ \underbrace{|\mathcal{A}|^2}_{\text{amplitude}} \right]$$

- For NNLO precision, need **“two-loop” amplitudes**, e.g. for  $pp \rightarrow 3 \text{ jets}$ .

$$\mathcal{A}^{(2)} = \text{[diagram of a two-loop amplitude]} + \dots = \text{8-fold integral}$$

- Amplitudes are functions of many variables: **mass/momentum scales**.

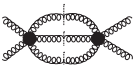


For precise LHC physics involving multiple (heavy) particles, we need **two-loop amplitudes depending on many scales**.

Methods for/Computations of Feynman **Integrals/Amplitudes** [w/ Abreu].

$$A^{(2)} = \sum_k \underbrace{C_k}_{\text{rational functions}} \times \underbrace{I_k}_{\text{integrals}} .$$

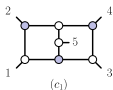
- Methods:

<p>Numerical Unitarity</p> <p><math>\text{Disc}[A_4] \sim</math></p> 	<p>Differential Equations + Ansatz</p> <p><math>d\mathcal{I}_j = \epsilon \mathbf{M}_{jk} \mathcal{I}_k \pmod p</math></p>
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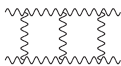
- Integrals: Two-loop **five-point** massless and one-mass.

- Amplitudes:

$$\mathcal{N} = 4 / \mathcal{N} = 8$$



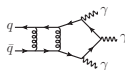
4-graviton



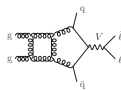
$pp \rightarrow 3j$



$pp \rightarrow 3\gamma$

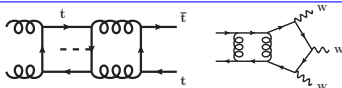


$pp \rightarrow W + 2j$



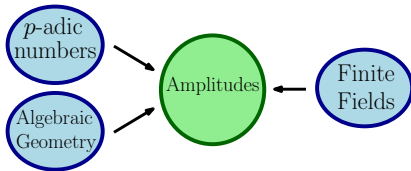
# The Next Frontier

Amplitudes with **multiple massive particles** are currently out of reach.



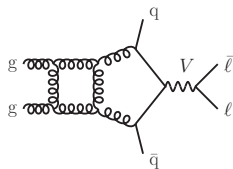
- New problems arising in multi-scale amplitude building blocks:
  - 1 **Integrals:**  $\mathcal{I}_k$  with massive fields involve new special functions.
  - 2 **Coefficients:** Rational functions becoming **gigantic**.

What physical/mathematical **structures** control multi-scale amplitudes?



# Research Interest Summary

- Multi-scale amplitudes for precise **LHC** predictions.



- Understanding of **multi-loop** integrals/amplitudes.

$$\text{Diagram} \rightarrow p_2[\mu^2] = \frac{1}{\epsilon} \int_0^1 dx \text{Diagram}^{xp_1, [1-x]p_1} + [1 \leftrightarrow 2] + \mathcal{O}(\epsilon).$$

- Algebraic geometry** in pQFT.

$$\mathbb{F}[\lambda_1^\alpha, \tilde{\lambda}_1^{\dot{\alpha}}, \dots, \lambda_n^\alpha, \tilde{\lambda}_n^{\dot{\alpha}}] / J_{\Lambda_n} \leftrightarrow$$



## Two-Loop Amplitudes.

- Feynman Integrals: more scales, more points, [masses](#). [Abreu, Monni]
- [LHC scattering](#): sub-leading color,  $Z/H + 2$ -jets, ... [Abreu et al].

## Structural studies:

- [Evanescent integrals](#) from local subtraction. [Georgoudis]
- Algebraic geometry and real radiation. [Kosower]
- Integral relations in [four-dimensional limit](#). [Usovitsch]