CERN Academic Training 2023

The Physics of Music from Pythagoras to Microtones

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CONTENTS

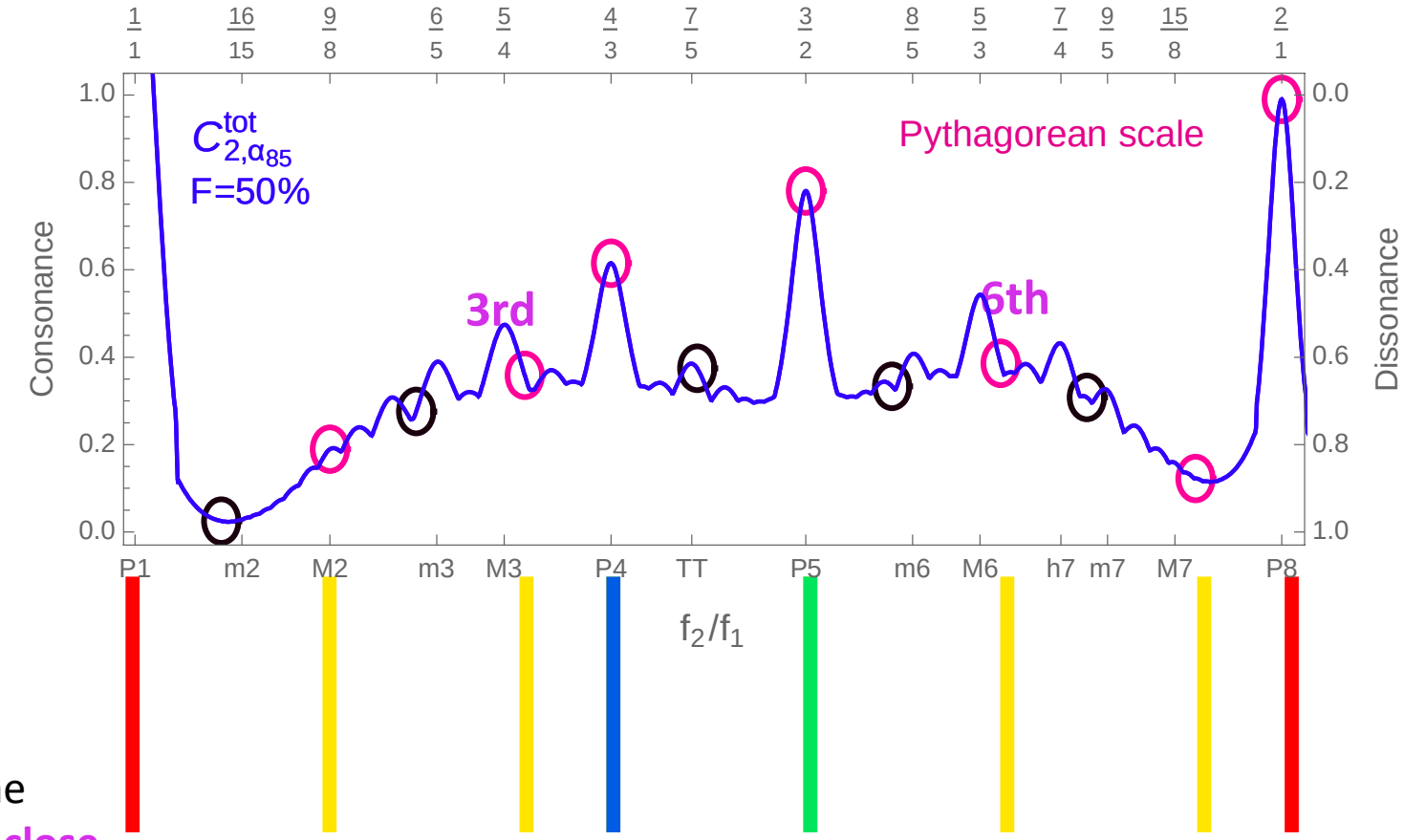
- Analysis of Temperaments from a Consonance perspective
  - Just Intonation, Meantone, Equal
- Going beyond 12 notes per octave with Microtones
- A *Gran Finale*: representations of the tonal space

Pythagorean Tuning Recap

DISADVANTAGE 1:

Thirds and sixths are not at their best

Counterpoint, polyphony and organs led to abandoning Pythagorian scale by upgrading 3rds and 6ths from dissonances to (imperfect) consonances.



DISADVANTAGE 2:

including chromatic scale (developed in the Middle Ages), the **circle of fifths does not close**

Tempered scales: meantone, equal, etc

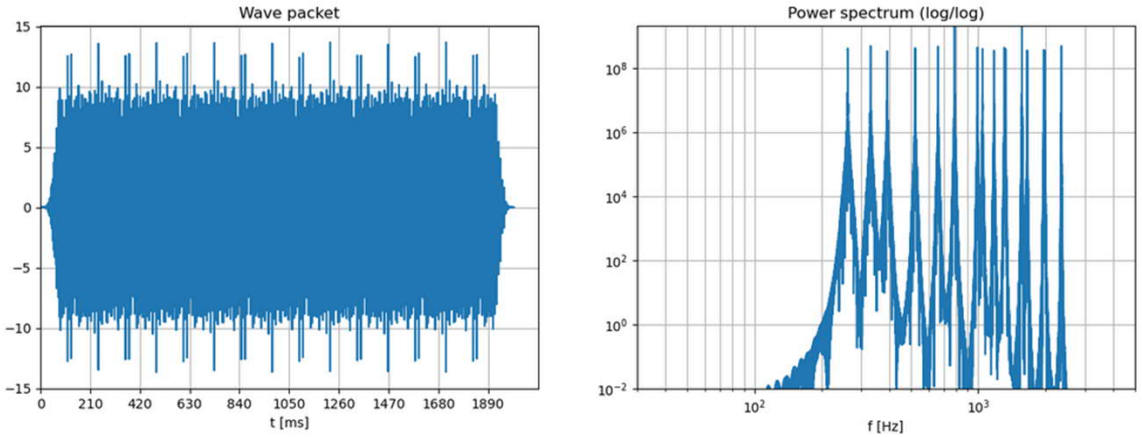
Pythagorean Tuning: an example

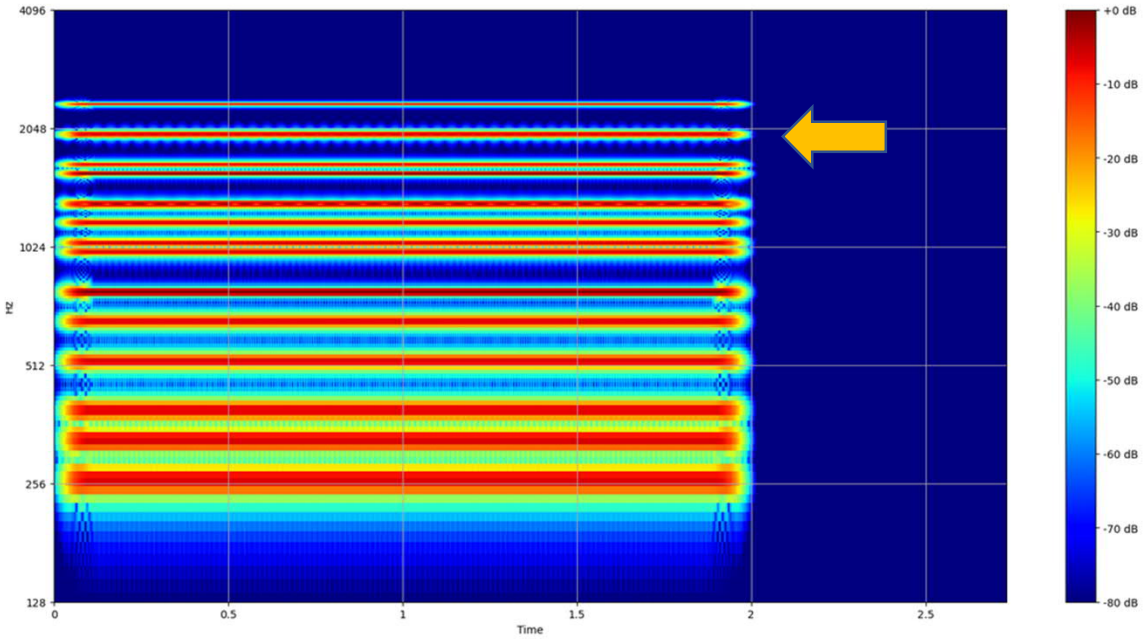
DISADVANTAGE 1:
Thirds and sixths are not at their best

For example, this is a C maj chord (C-E-G)

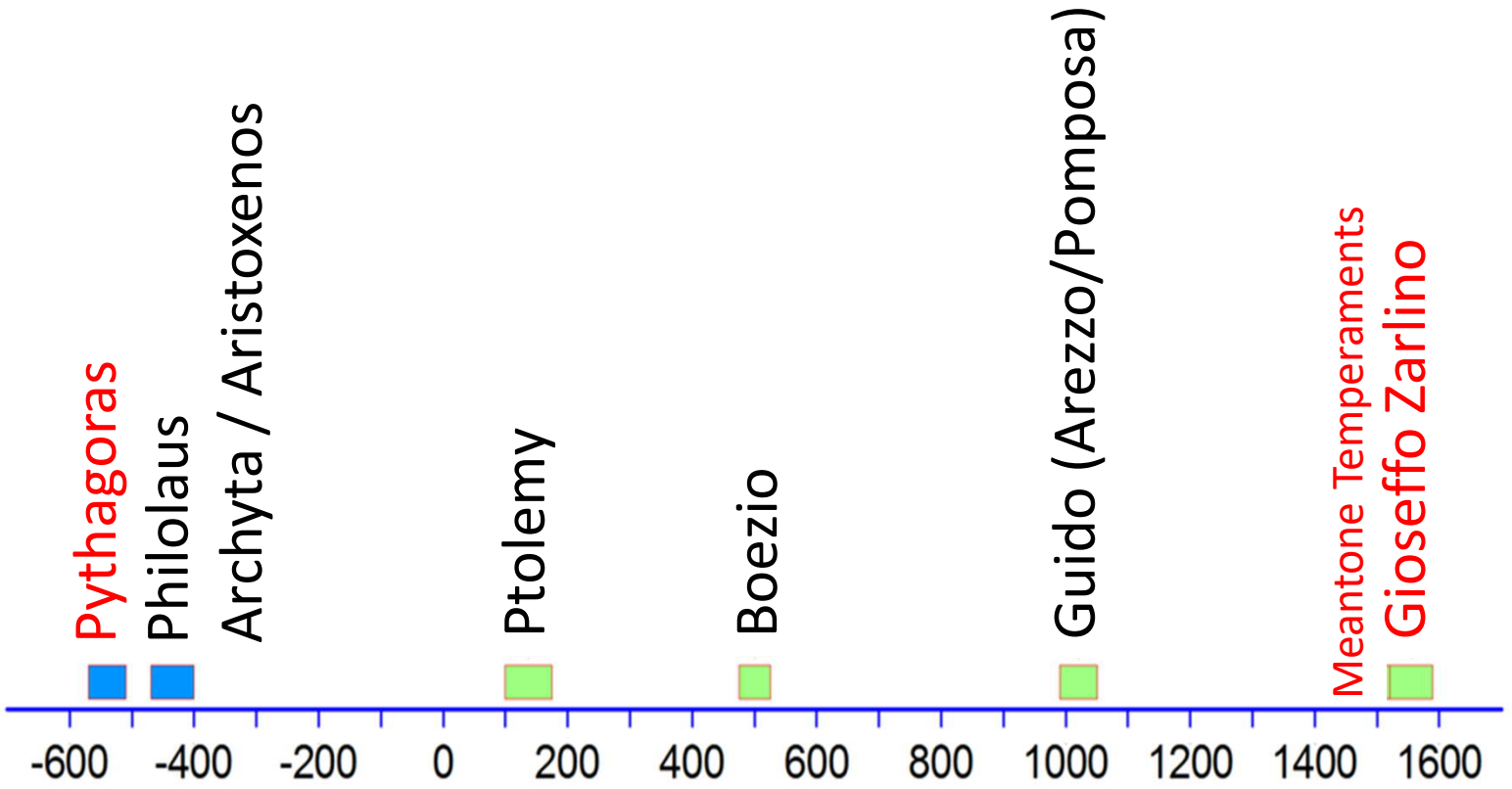
Similarly with a F maj chord (C-F-A)

Beatings: C's 5th harmonic does not match E's 4th harmonic





From Pythagoras to the XVII century...



Just or Natural Scale

G. Zarlino, *Istitutioni harmoniche* (1558):

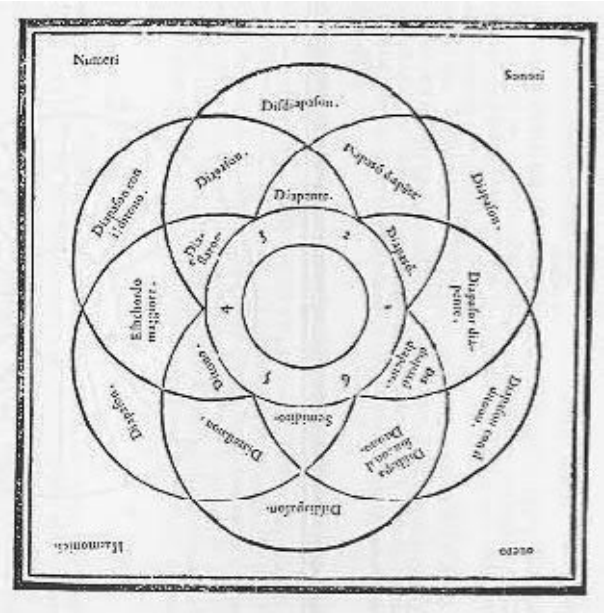
Redefined the Pythagorean problem of consonance, establishing that the harmonic ratios contained in the *senario,* consisting of the integers 1 to 6, were consonant.

Ptolemy had already admitted the 5:4 ratio as consonant.

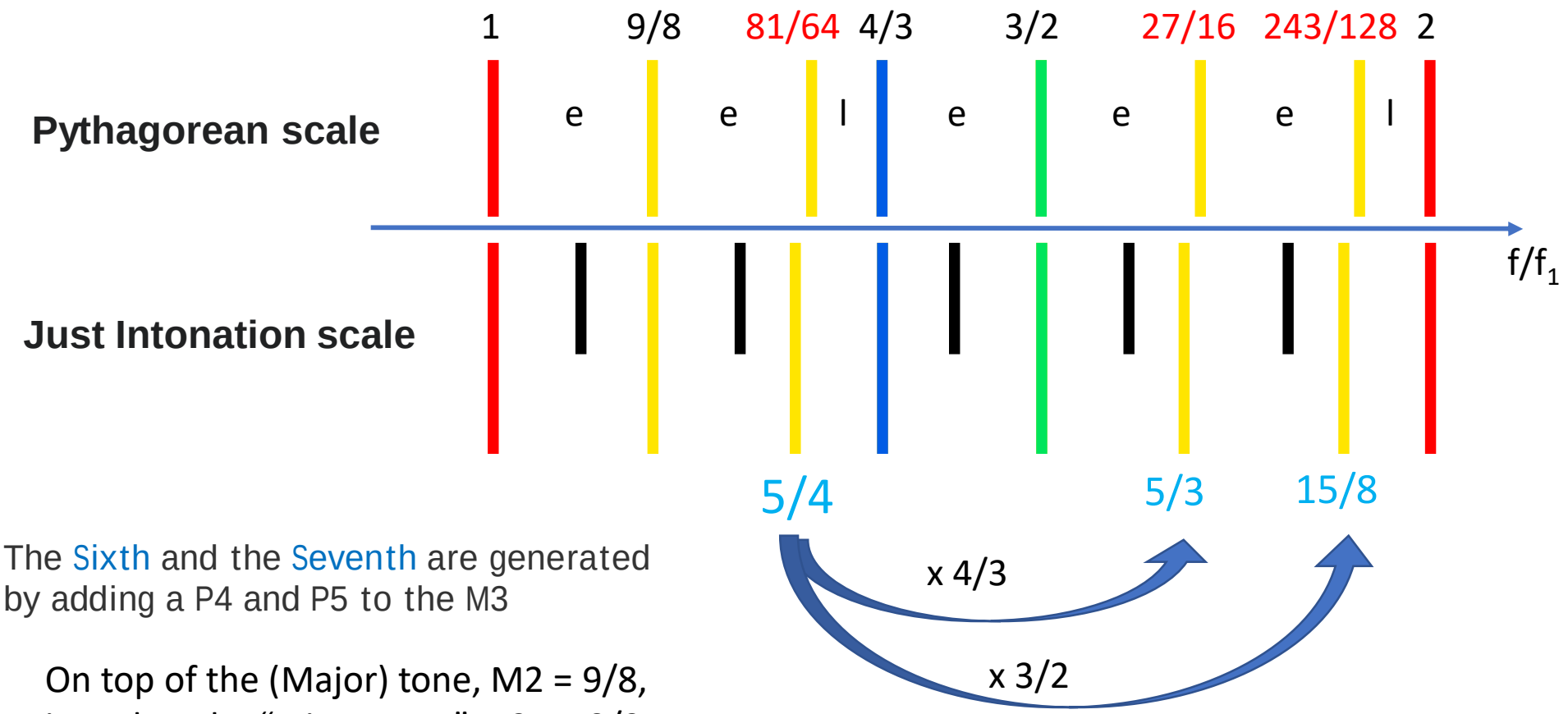
→ The ratios 5:3 (M6), 5:4 (M3), 6:5 (m3), 8:5 (m6) are all promoted to valid (imperfect) consonances.

This scale is considered natural or just. It aims at removing all possible beatings between harmonics.

Gioseffo Zarlino, Chioggia (Venice), 1517-1590



Building the Just Intonation Scale



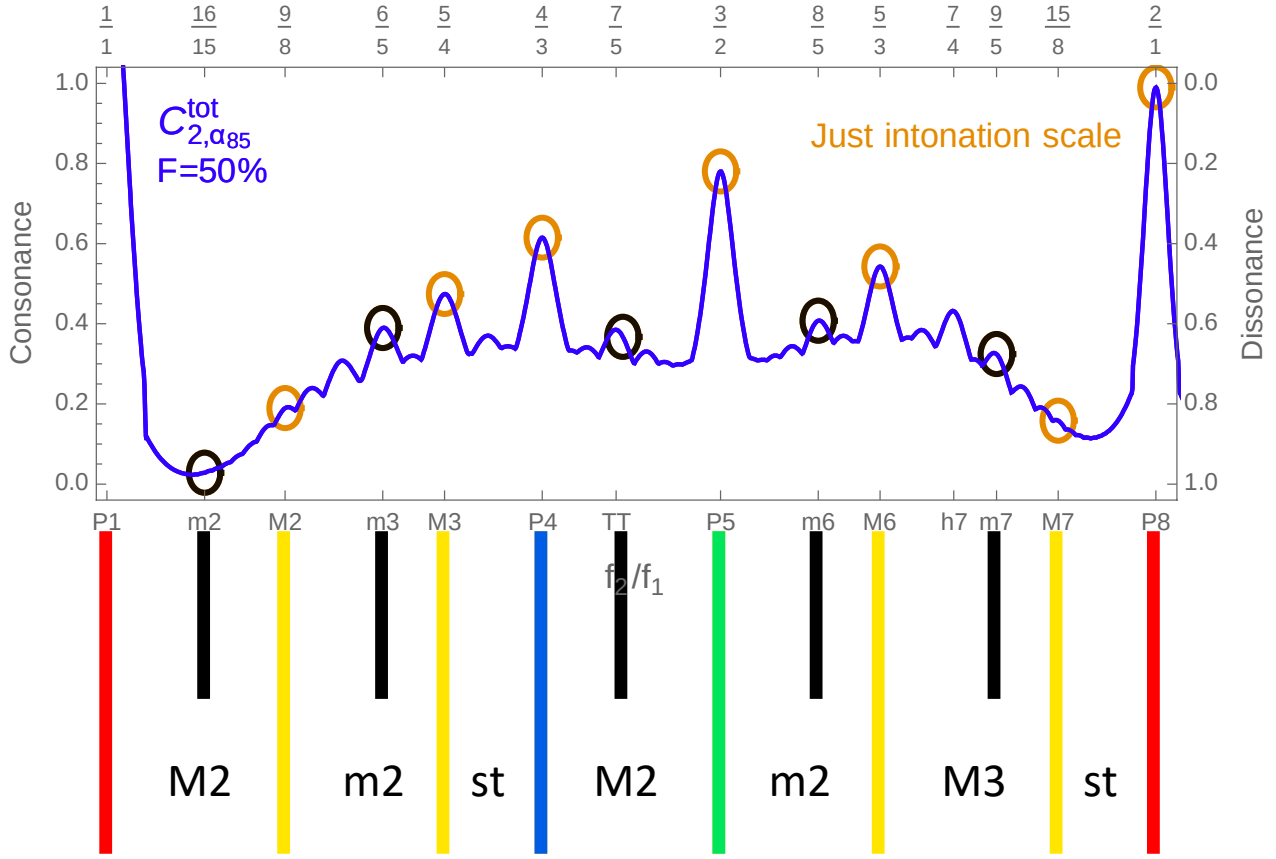
The Sixth and the Seventh are generated by adding a P4 and P5 to the M3

On top of the (Major) tone, M2 = 9/8, introduced a “minor tone” m2 = 10/9, with “semitone” st = 16/15

How does it play with respect to the consonance of its intervals?

Complete Just Intonation Scale

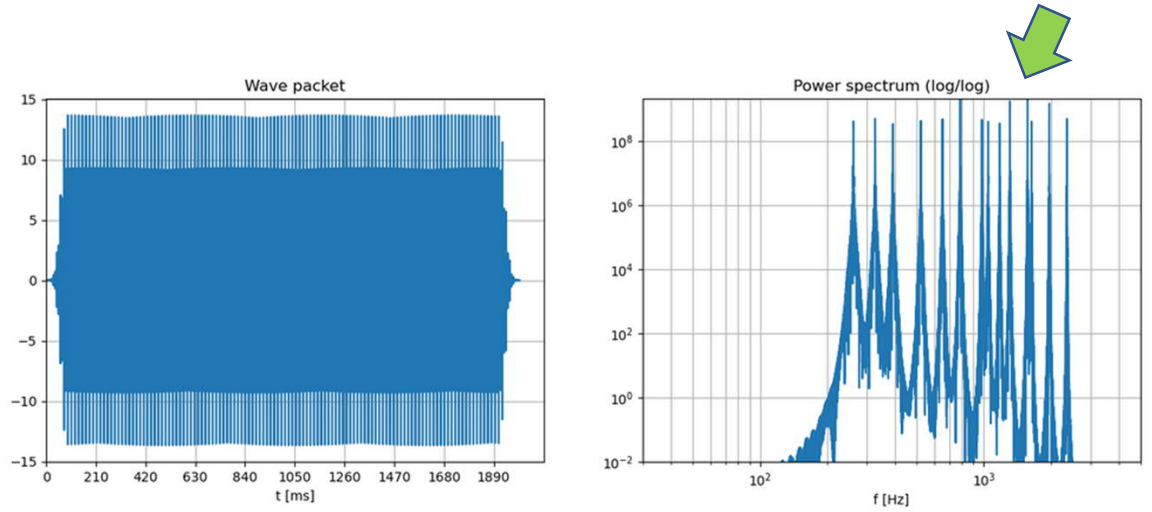
- Advantages
  - All intervals are pure => highest possible consonance
  - Naturally adopted by *a cappella* choirs
- Disadvantages
  - Increased complexity
    - Major tone (tM) and minor tone (tm)
  - Hard to introduce modulations
    - Wolf Fifths and “Wolf Thirds”
    - Key changes during a composition were not common practice…



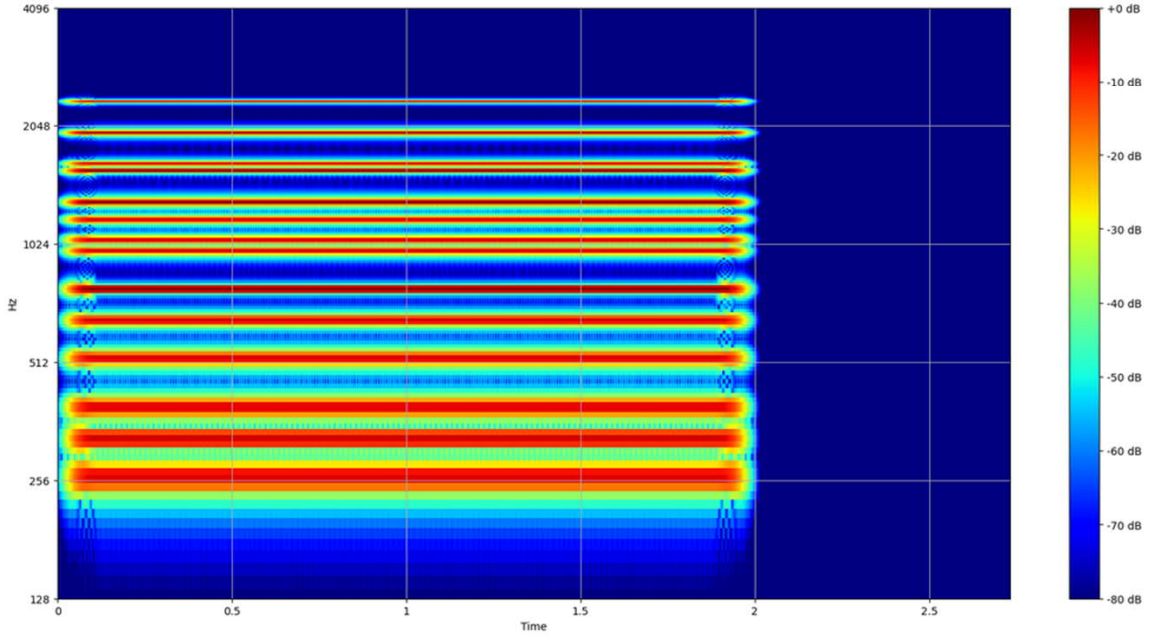
Just Intonation Chords

C Major: the purest possible

- Multiple harmonics are overlapping, increasing the perception of stability
- Strong “Missing Fundamental”
- No beatings between any of the partials

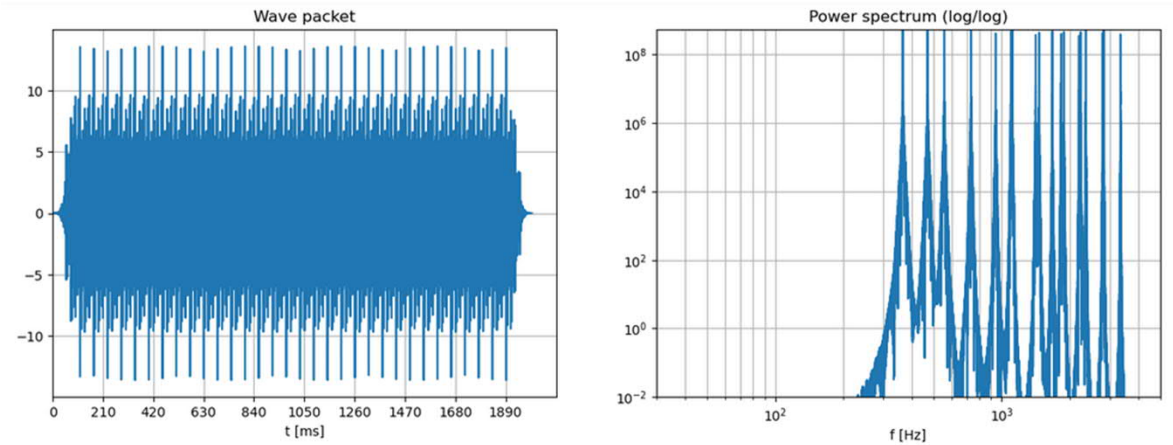


C minor is also very stable

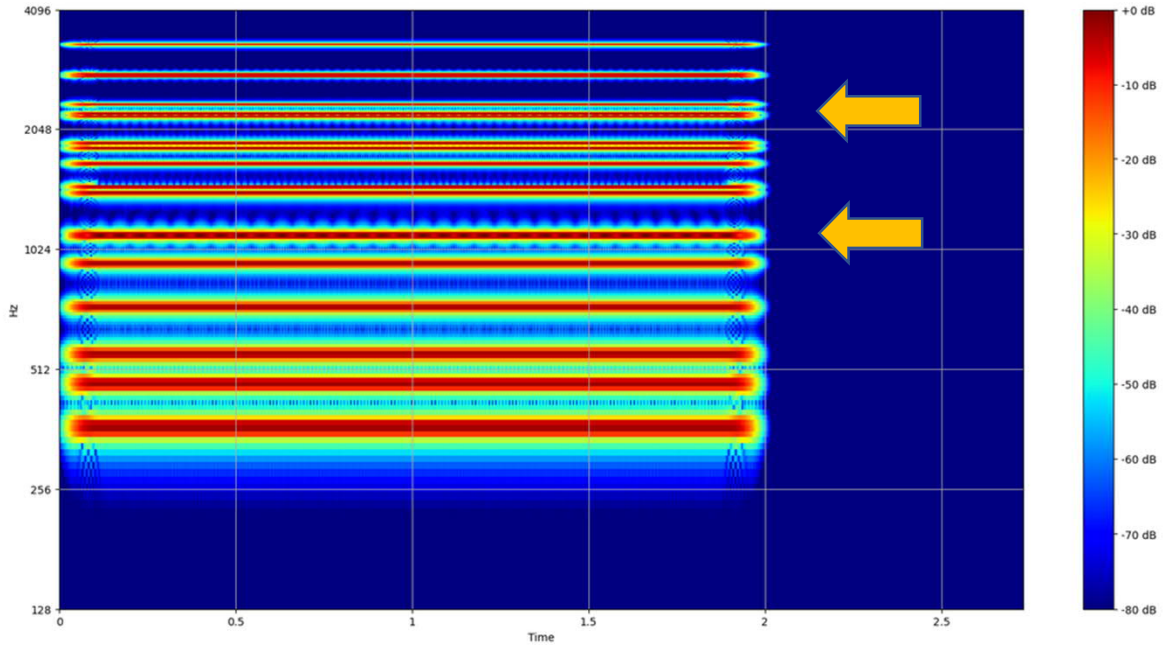


Just Intonation Chords

F# Major is quite unbearable!

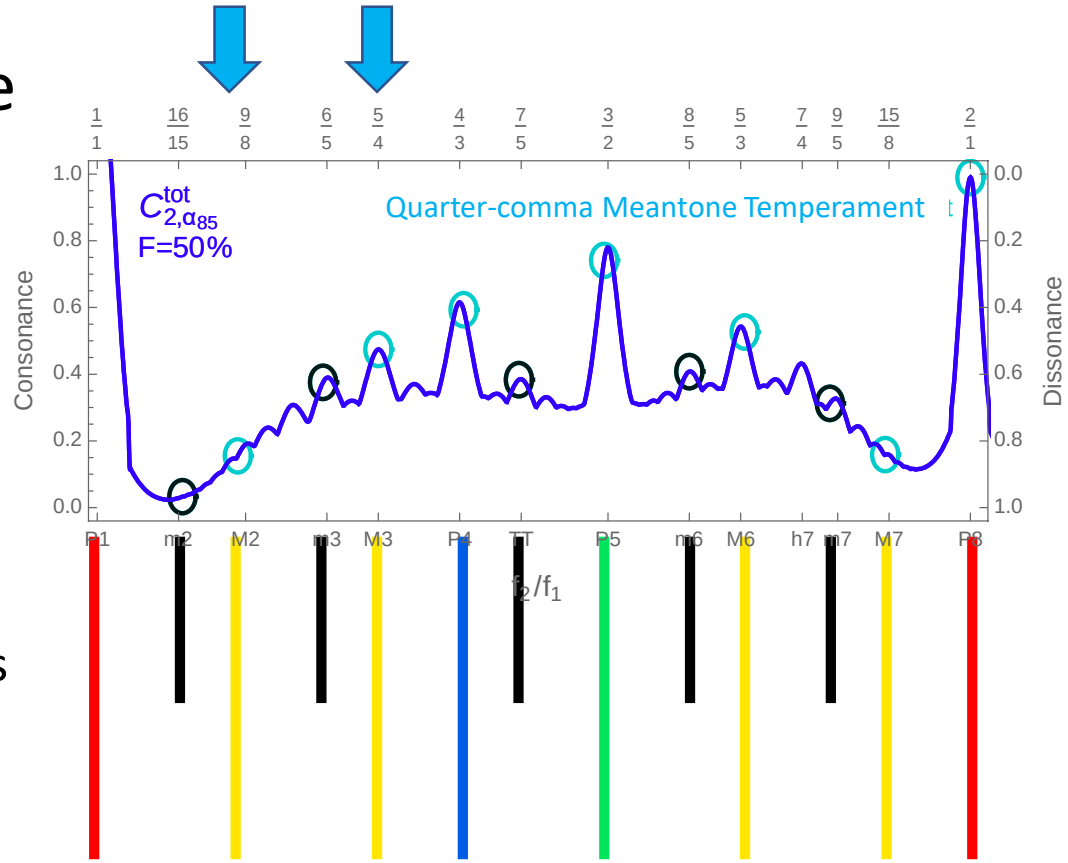


=> Unpractical for fixed-tuning instruments
(Clavichords, Organs, etc.)



Meantone Temperaments

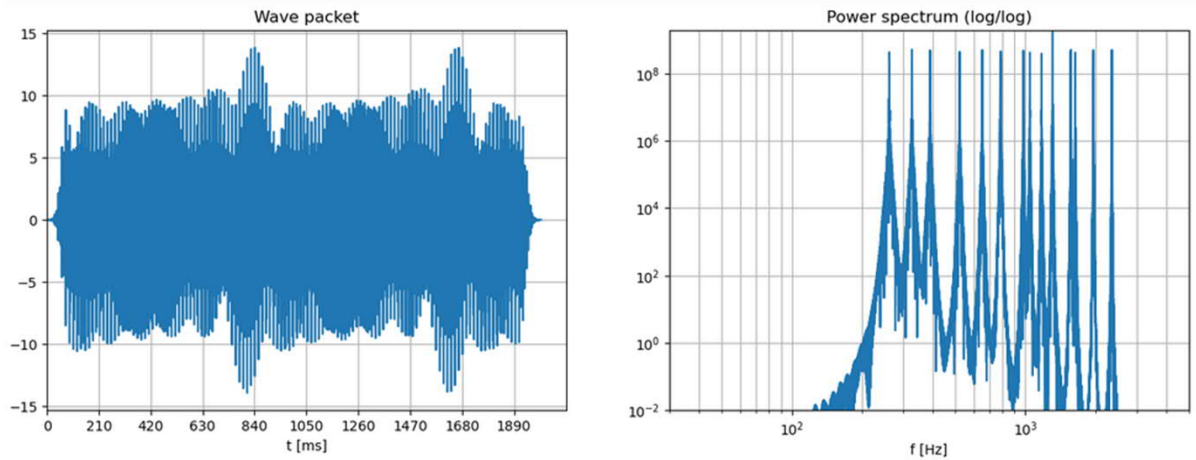
- Principle: keep as many pure Major Thirds as possible, and tune the Fifths accordingly. 4 Fifths = $(3:2)^4$ = 2 Octaves + 1 Major Third + S.C.
  - S.C. = *Syntonic Comma* = 81:80
- Most common: Quarter-Comma Meantone
  - "Mean" Tone: mean of M3 = $\sqrt{5/4}$
  - Why Quarter-Comma? Because it can be obtained by flattening the Fifths by ¼ of a S.C.
- Advantages
  - Tolerable beatings of the Fifths in most tonalities
  - Suitable for Orgues and fixed-tuning instruments
  - Procedures exist to tune instruments based on beatings
- Disadvantages
  - Wolf Fifth still present => "remote" tonalities must be avoided

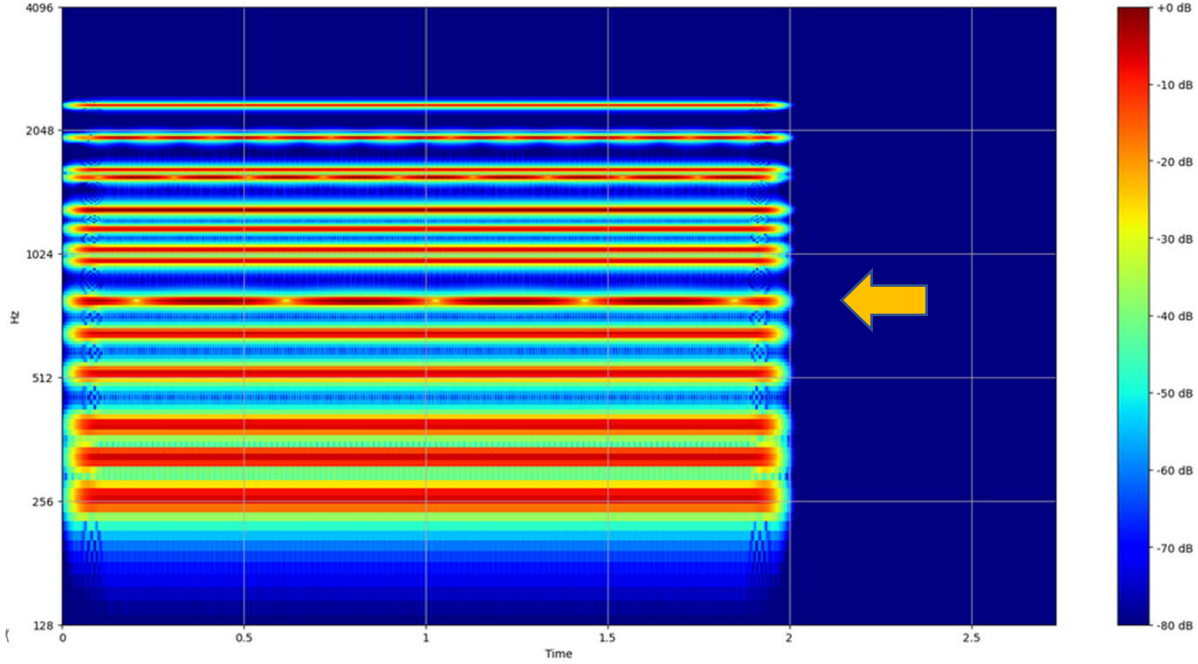


Q. Comma Meantone Temperament Chords

C maj: the tempered Fifth introduces some beatings, but they are “tolerable”:
within the DL of a P5!

Organs may have a “Tremolo” effect on top



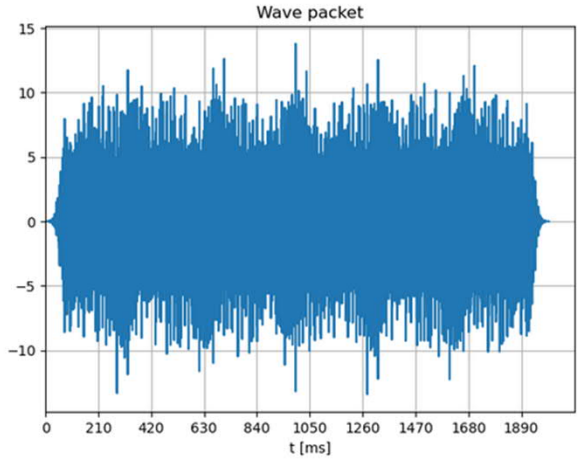


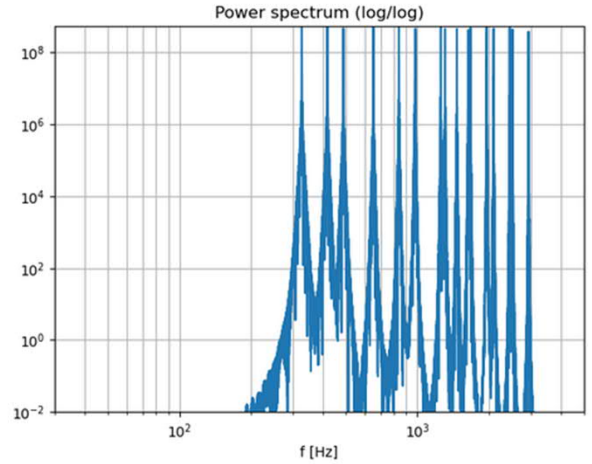
Q. Comma Meantone Temperament Chords

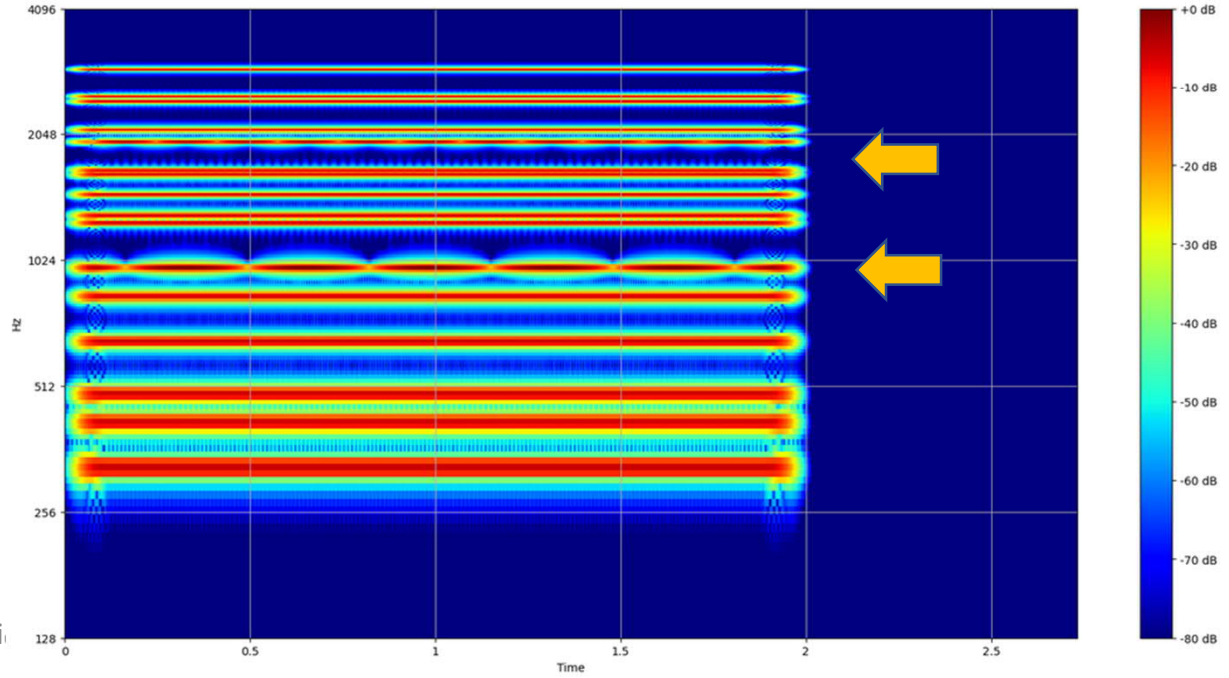
E maj: not as good

Music composed for organs in 1500-1600 deliberately avoided using remote tonalities with many sharps or many flats.

The Wolf Fifth was moved far away, never to be used!

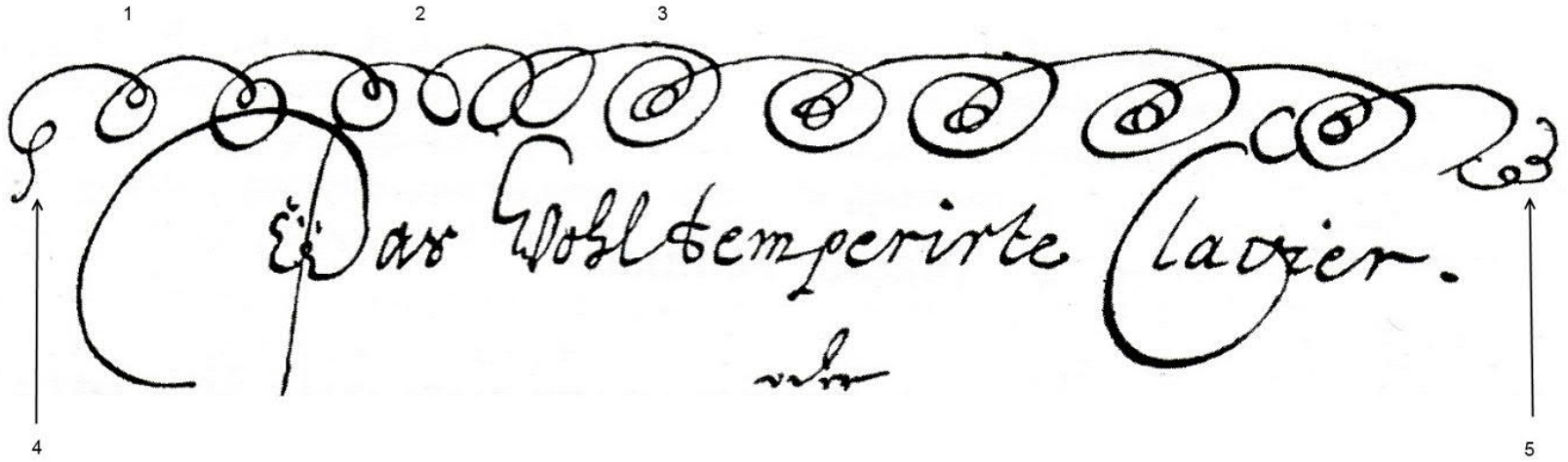




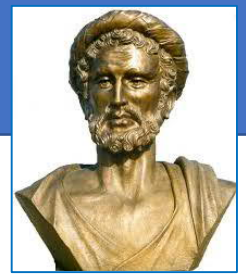


Other Unequal Temperaments

- Can we do better? Yes!
  - Several proposals in the XVIII and XIX centuries faced the challenge of including some pure Fifths and some pure Major Thirds, but “closing the loop” without a Wolf Fifth
- An important example (A. Werckmeister, 1691): an unequal “well temperament” with a mix of pure Fifths and meantone Fifths
  - It is generally accepted that J. S. Bach intended to suggest one such temperament in his *Das Wohltemperierte Clavier* masterpiece

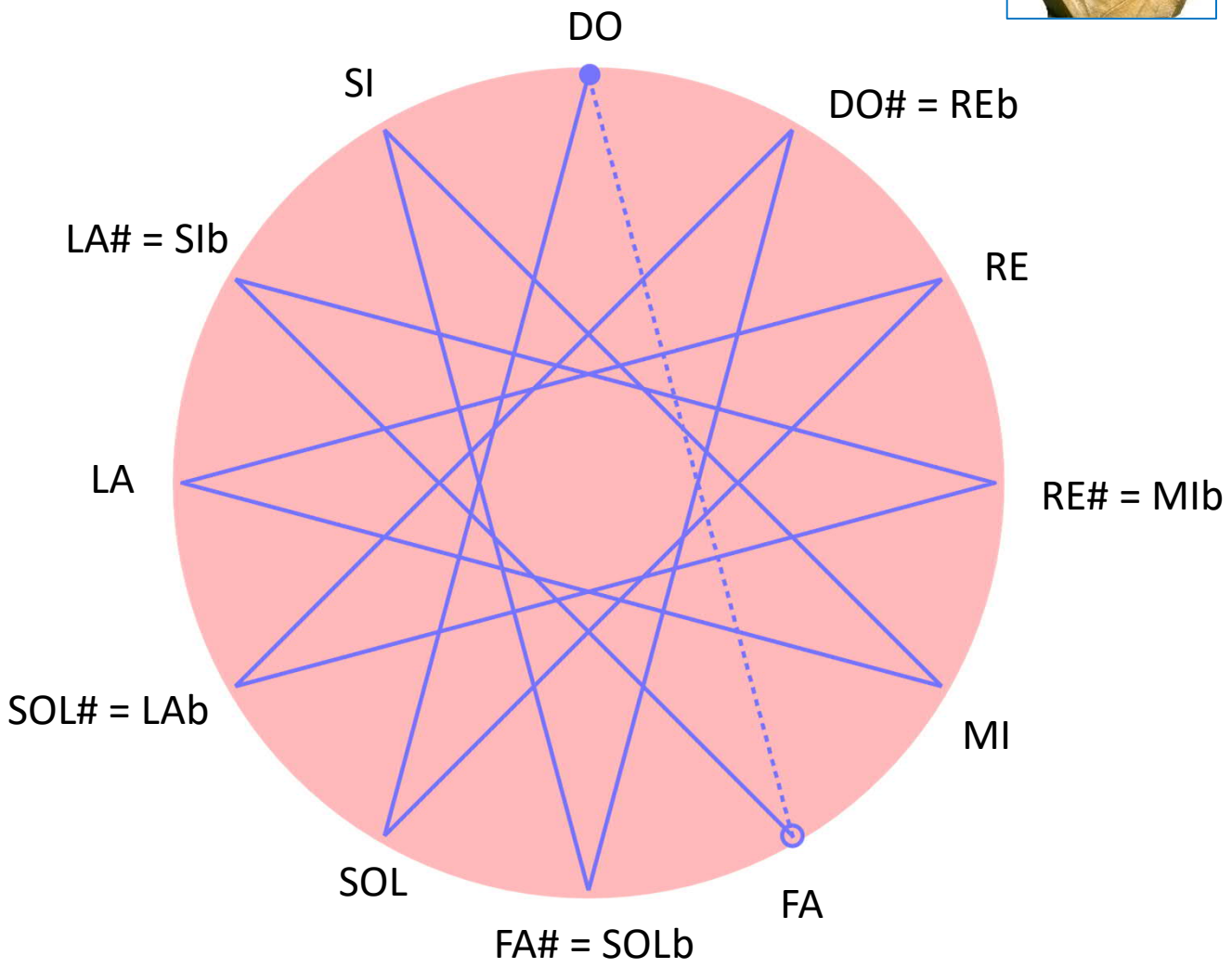


12-TONE EQUALLY TEMPERED SCALE

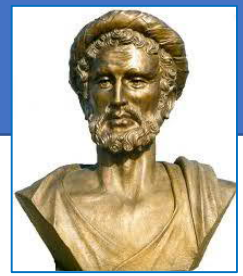


- Back to Archyta (Taranto)!
- Reconsidered in the XVI century for lutes (V. Galilei), keyboard instruments hard to tune until late XIX century
  - Irrational ratios imply difficulties in defining a tuning procedure!
- The principle though is very simple: temper all Fifths by 1/12 of the Pythagorean Comma
  - *Enharmonic* tones
  - All semitones are equal, such that:

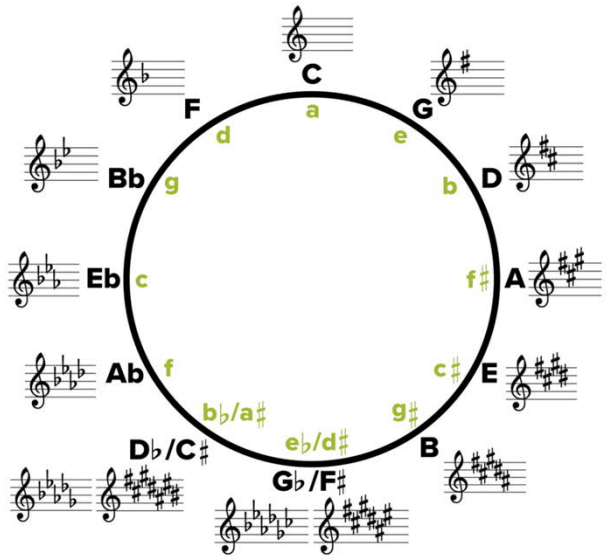
$$S^{12} = 2$$

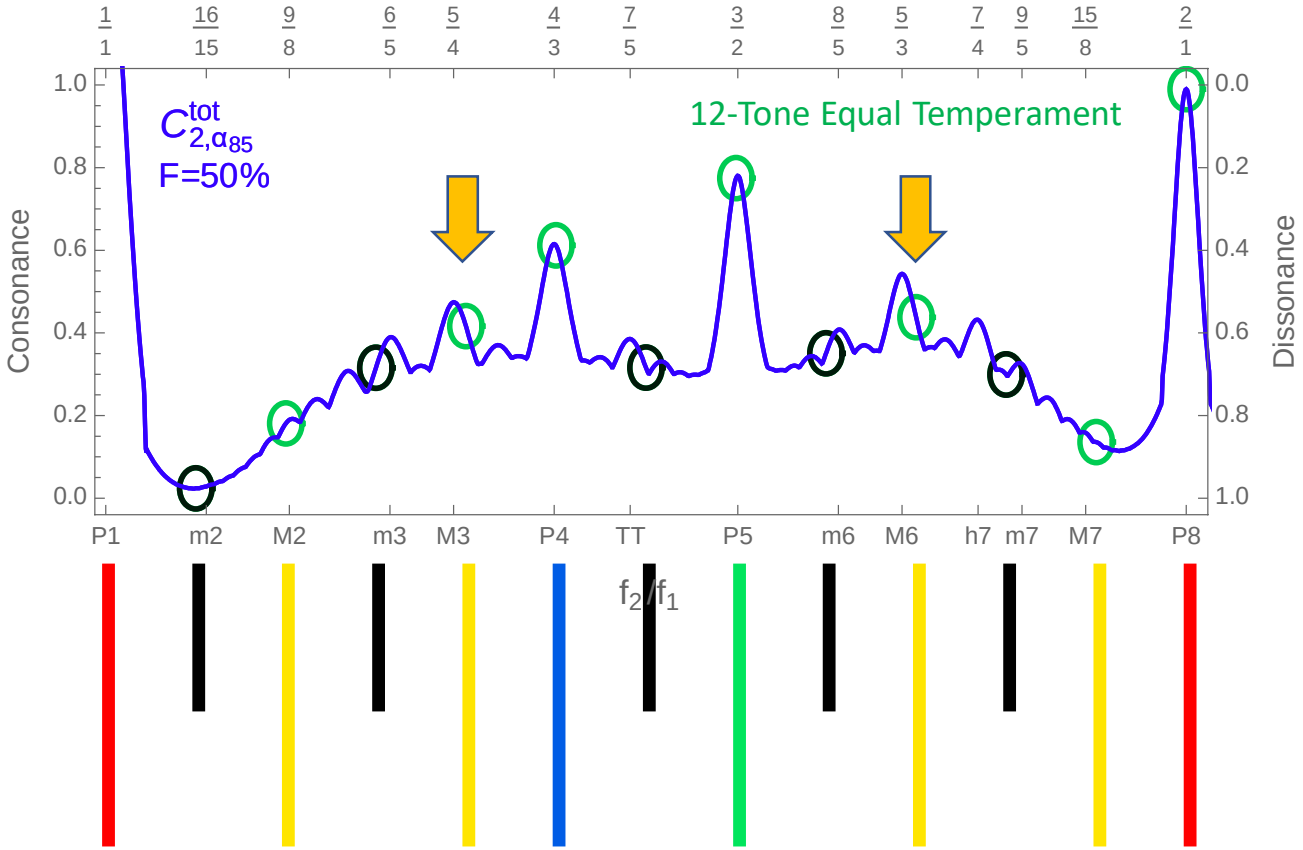


12-TONE EQUALLY TEMPERED SCALE



- Disadvantages
  - Major Thirds and Sixths are not great
  - Minor Thirds are similarly mistuned
- Advantages
  - Excellent approximation of P5
  - Modulations (key changes) allowed for **all tonalities**



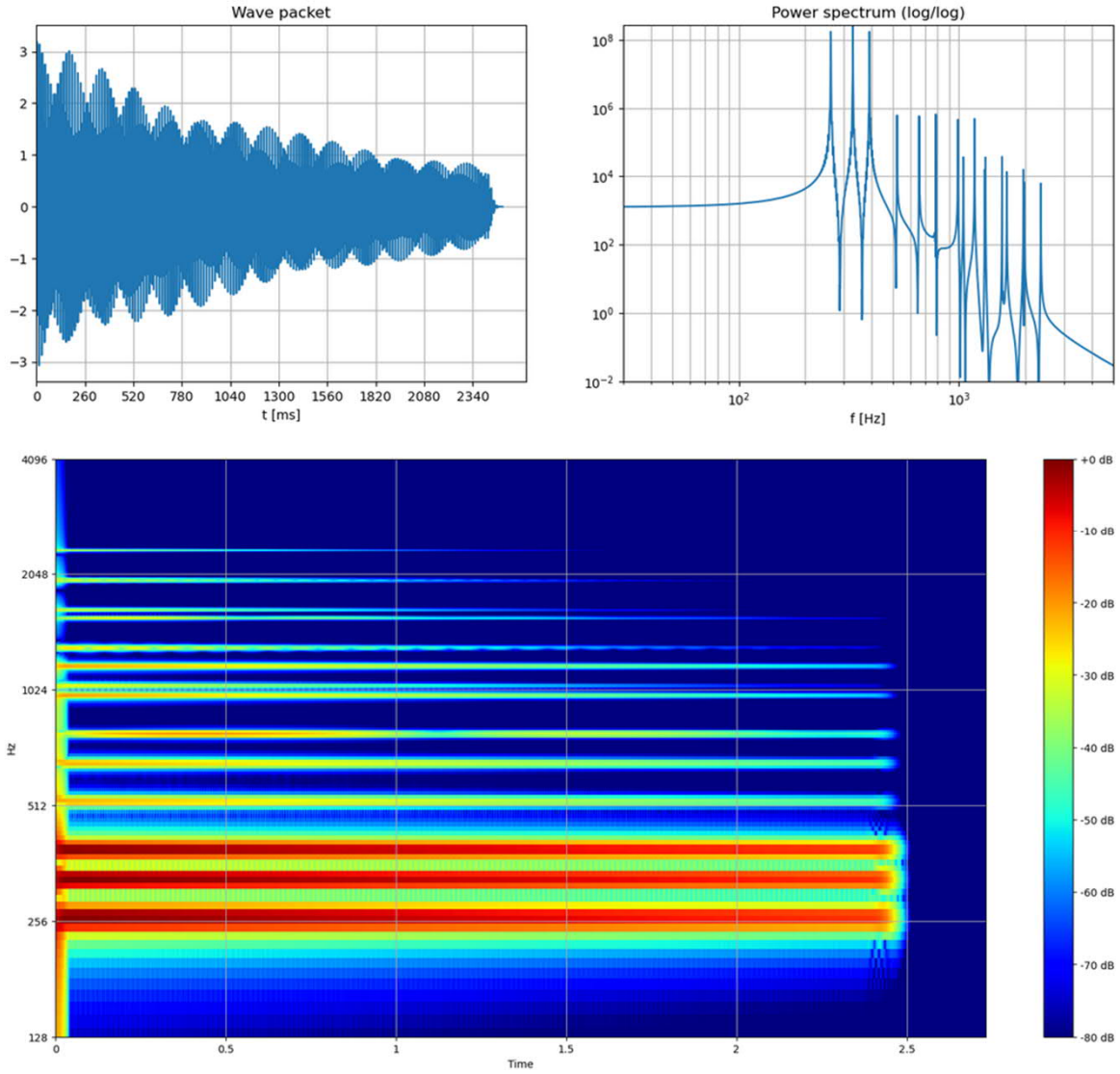


12-TET Chords (Plucked string)

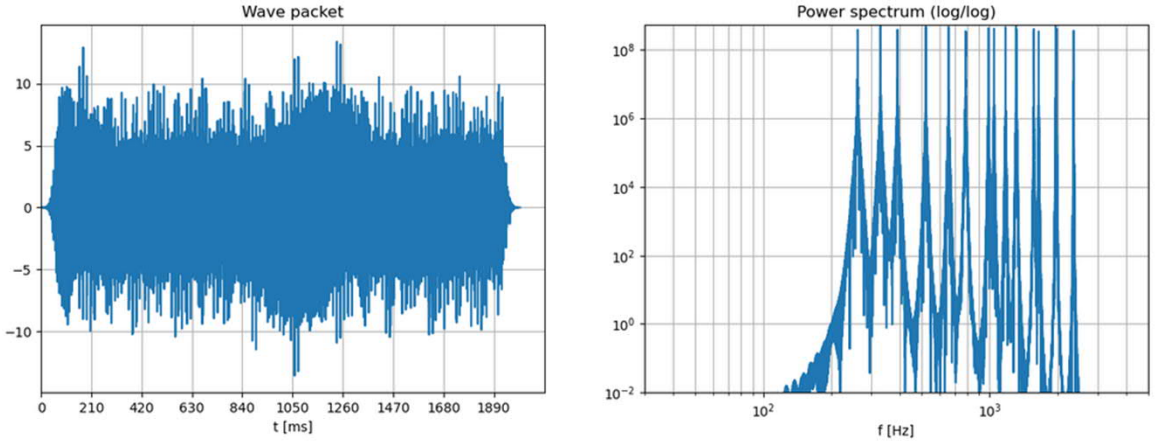
C maj: beatings barely heard as higher harmonics are quickly suppressed

Excellent compromise for all non-sustained instruments!

A Piano tuner knows how to further masks such beatings by mistuning the three strings that make each tone

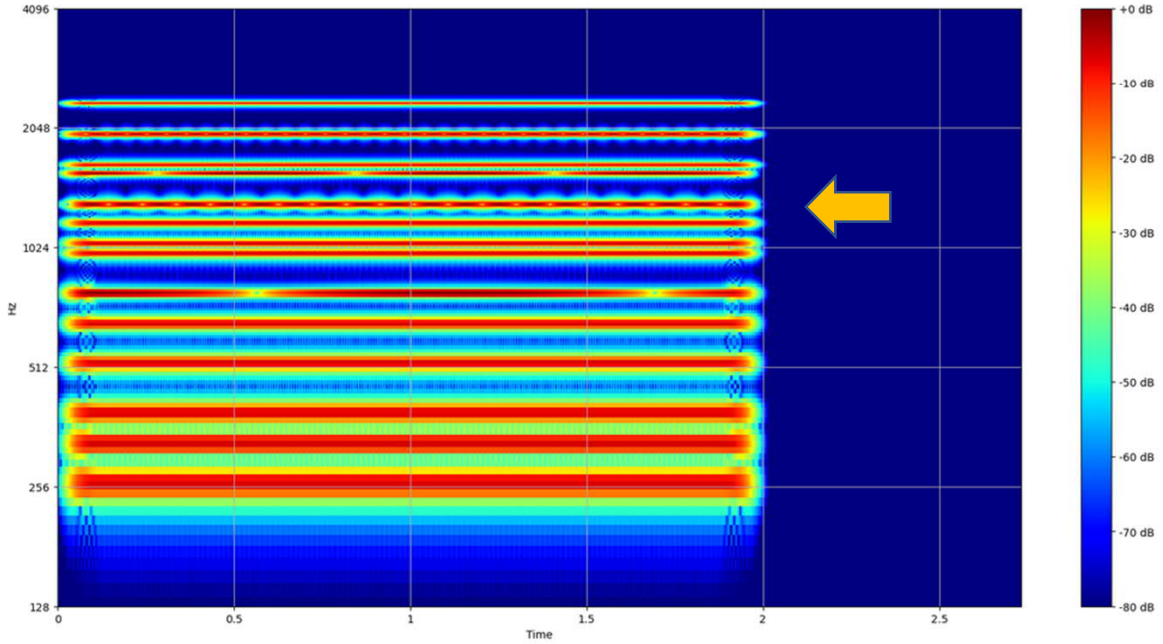


12-TET Chords (Pipe)



C maj: a pipe-like timbre makes those beatings unsatisfactory, in comparison to other tunings

Unequal temperaments typically remain preferred for organs, to limit such beatings

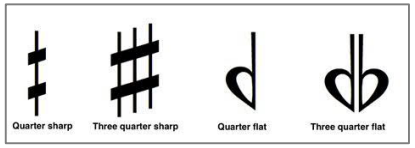


ALTERNATE TUNINGS AND MICROTONES

From https://www.youtube.com/watch?v=dFb1fECwk2o

Microtonal Tunings

- Alternative to “fix” the out-of-tune 12-TET yet using an **equal** tuning: **go beyond 12 notes!**
- Several attempts have been made over history by several cultures
  - Middle-East and Asian tunings regularly make use of microtones
- Musical notation extended to represent more sharps and flats



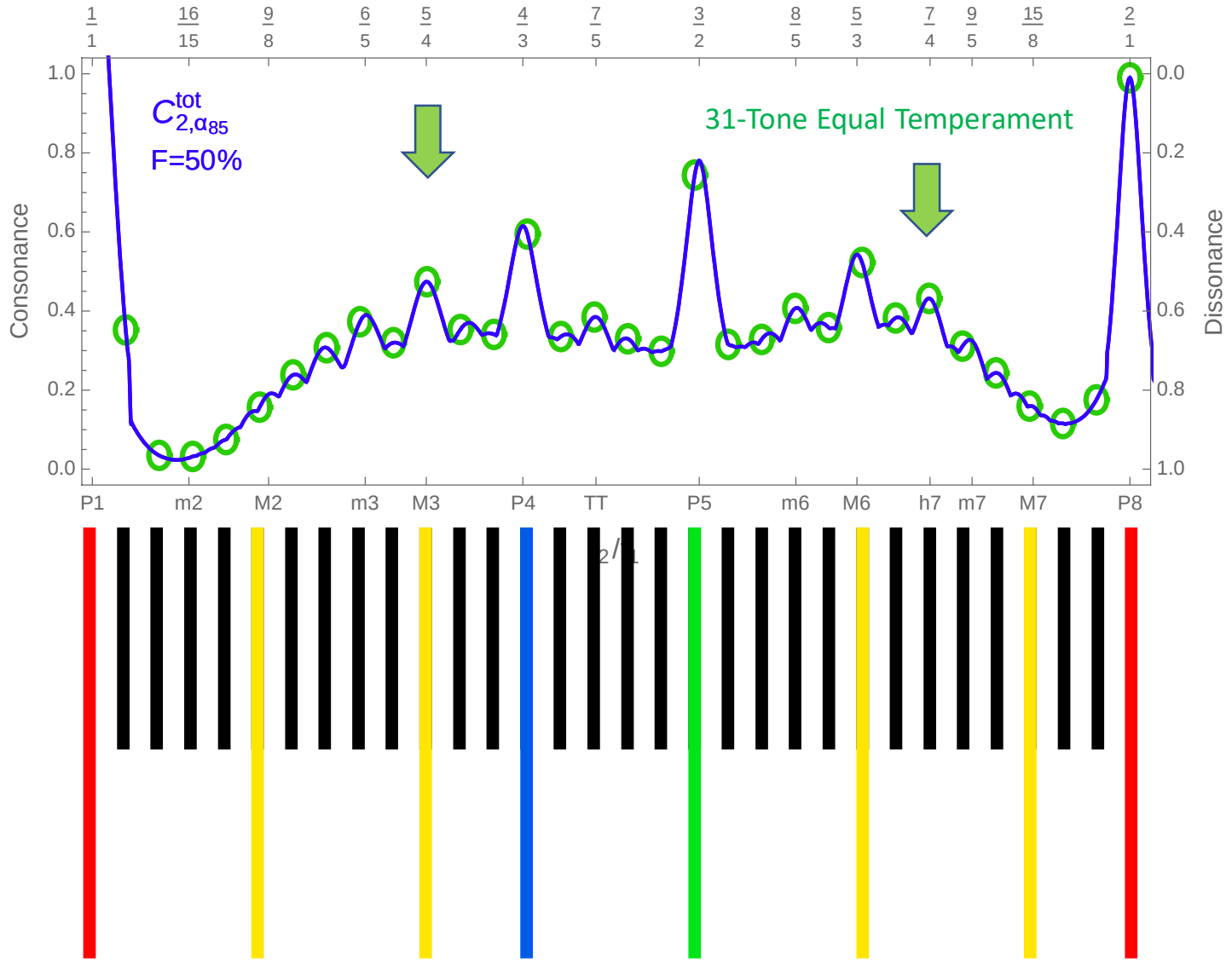
- But: increased complexity both in crafting instruments and in musical execution
  - Acceptable for keyboards, guitars, less so for strings
  - Much harder for woodwinds
  - Recent revamp of microtonal music thanks to electronic synthesizers

Most significant microtonal tunings

- 19-TET (F. de Salinas, 1513; G. Zarlino, 1517; …)
  - Good approximation of all known consonant intervals
- 31-TET (N. Vicentino as Ext. Meantone, 1511; M. Mersenne, 1588; …)
  - Excellent approximation of M3, M6, and … H7 (more later)
- 24-TET (L. Euler, 1707; H. von Helmholtz, 1821)
  - Easier to play compared to the above, but no significant improvements vs 12-TET
- Higher “order” tunings, up to 205-TET and beyond: Polychromatic tunings

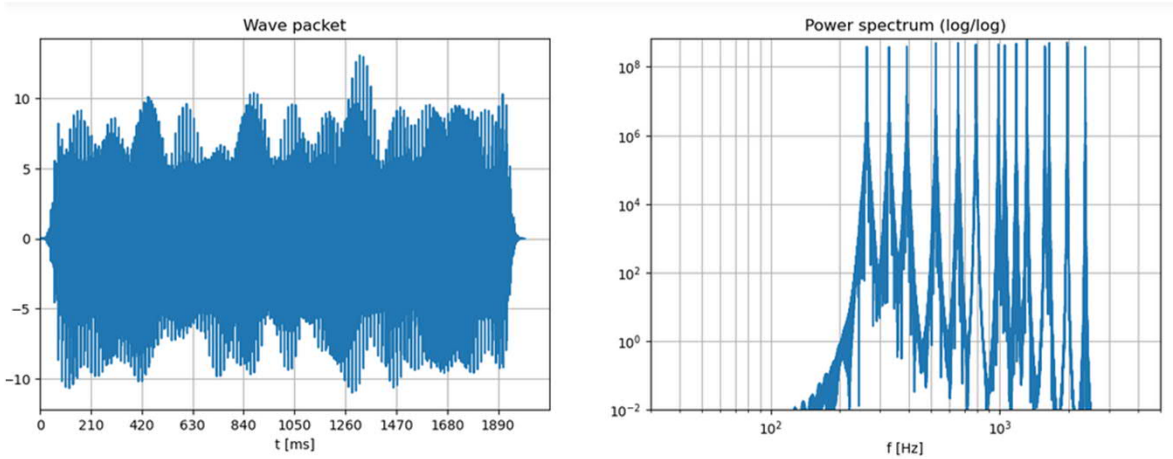
31-Tones Equal Temperament

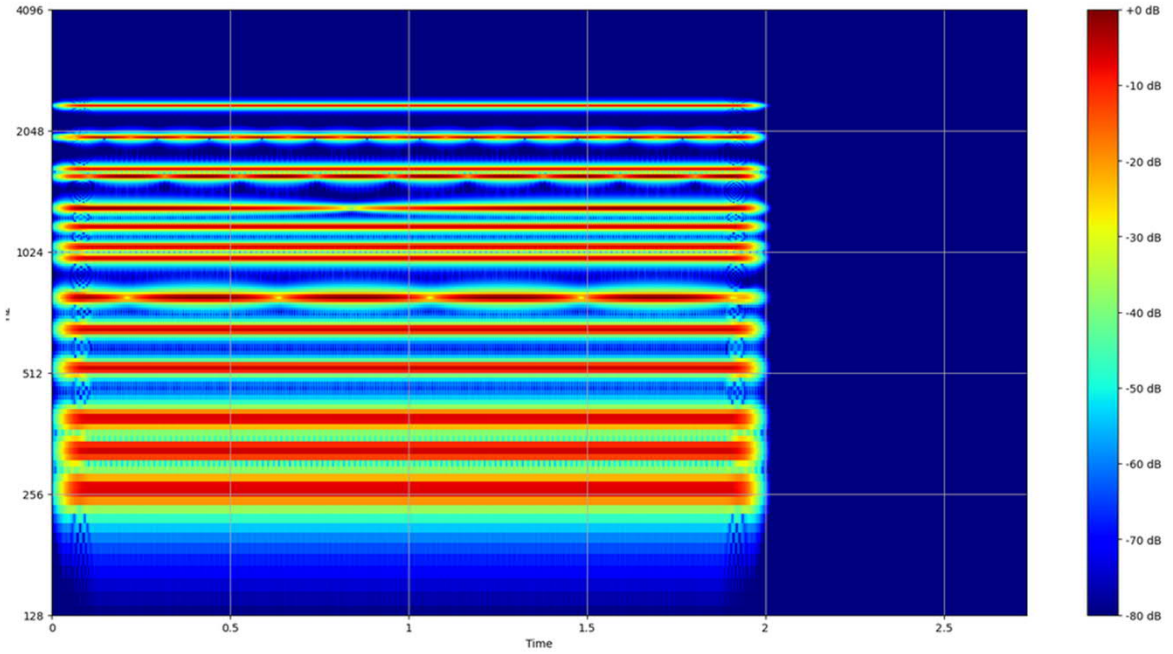
- It was realized that 31-TET is close to Quarter-Comma Meantone
  - Excellent M3
  - Very good P5
- It also matches well some 7-limit ratios, most notably 7:4 (the Harmonic Seventh)



31-TET Chords

C maj: significantly similar to the Quarter-Comma Meantone temperament

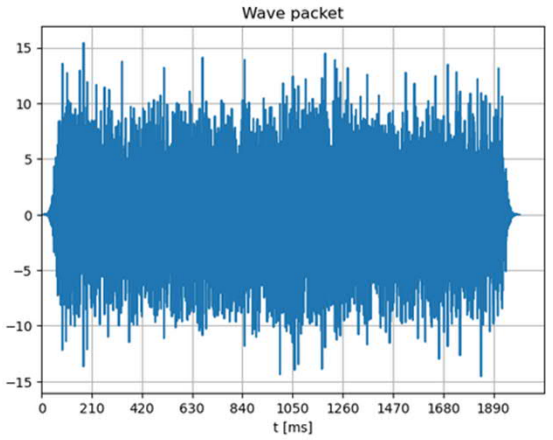


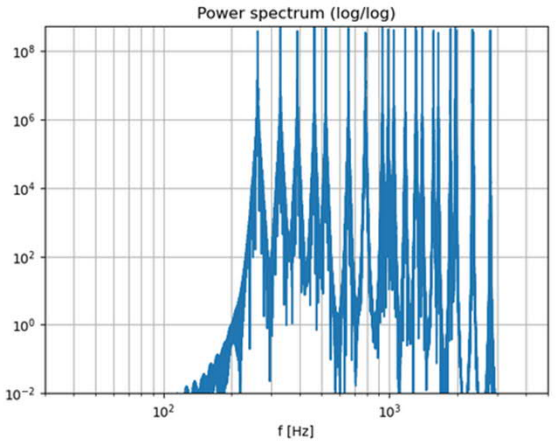


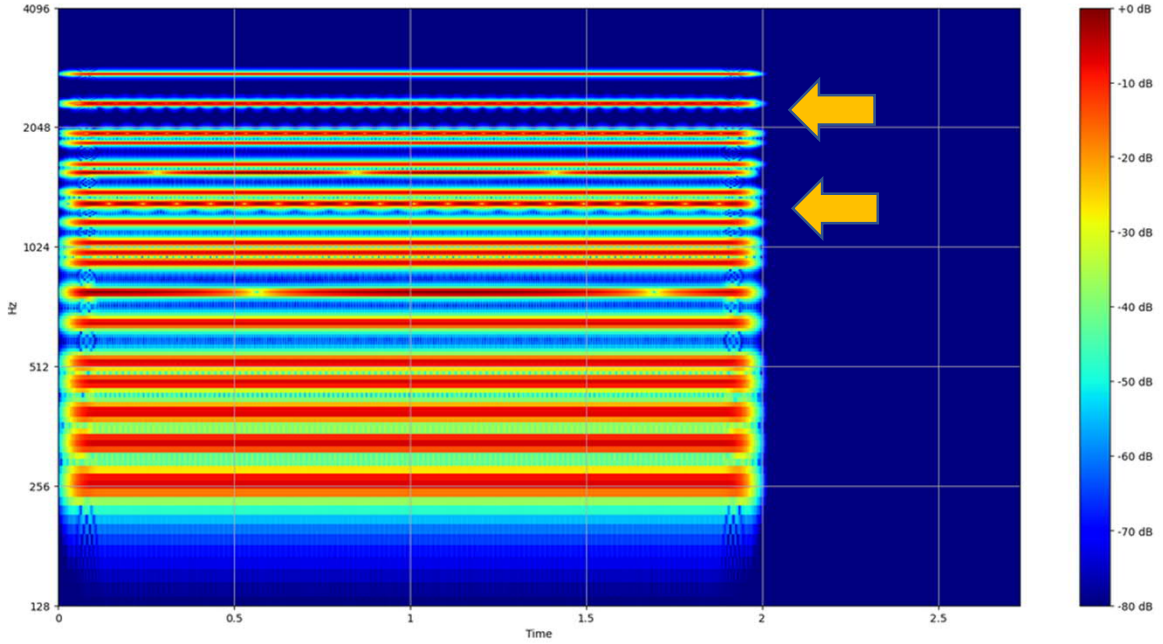
The 7th Chord

A standard C7 chord in 12-TET features beatings

It is a well-known chord to create tension



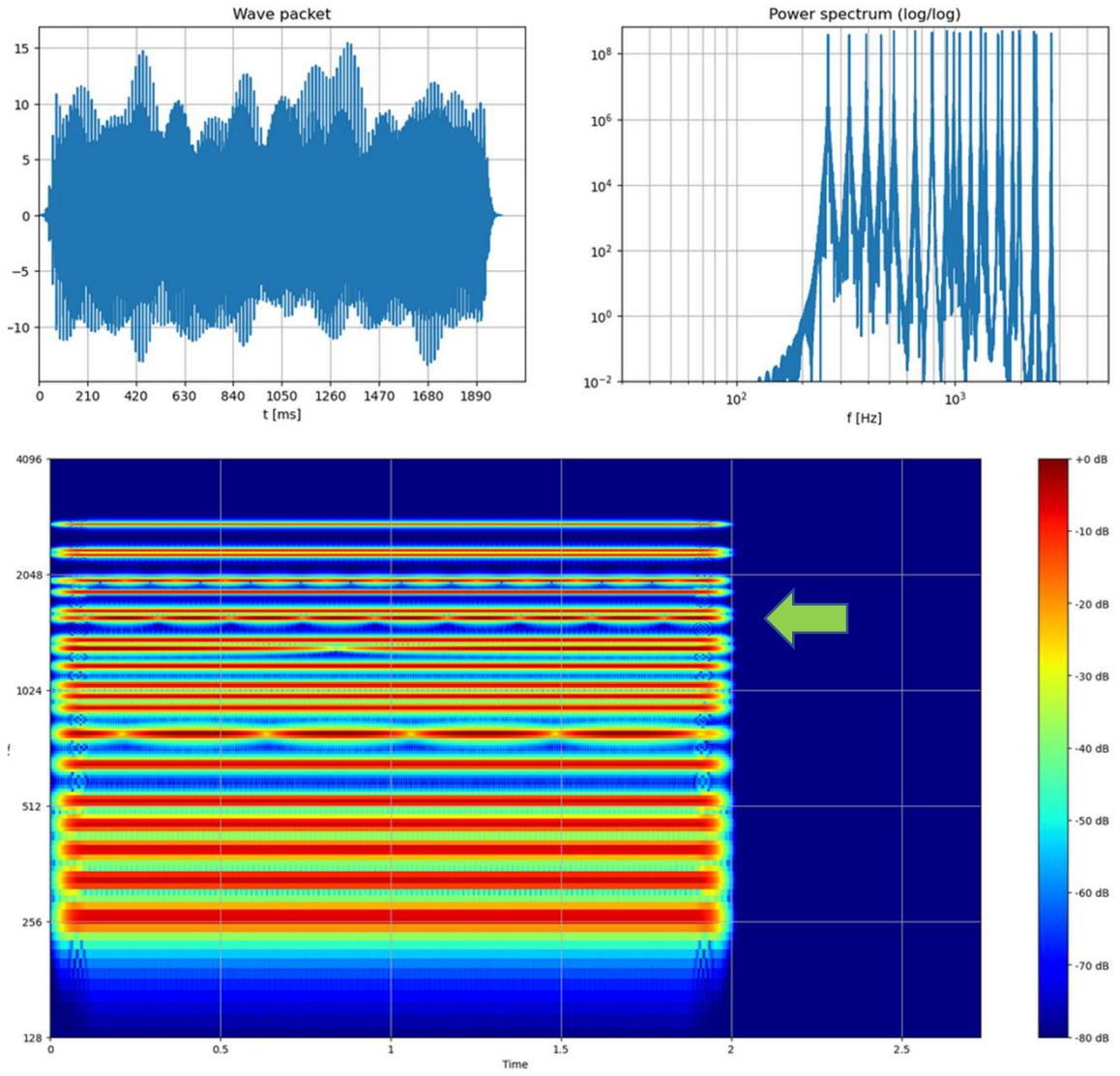




The 7th Chord

C7(h7): the 7th degree in 31-TET can be played with a “B superflat”, which approximates the 7:4 ratio better than 12-TET, producing less beatings

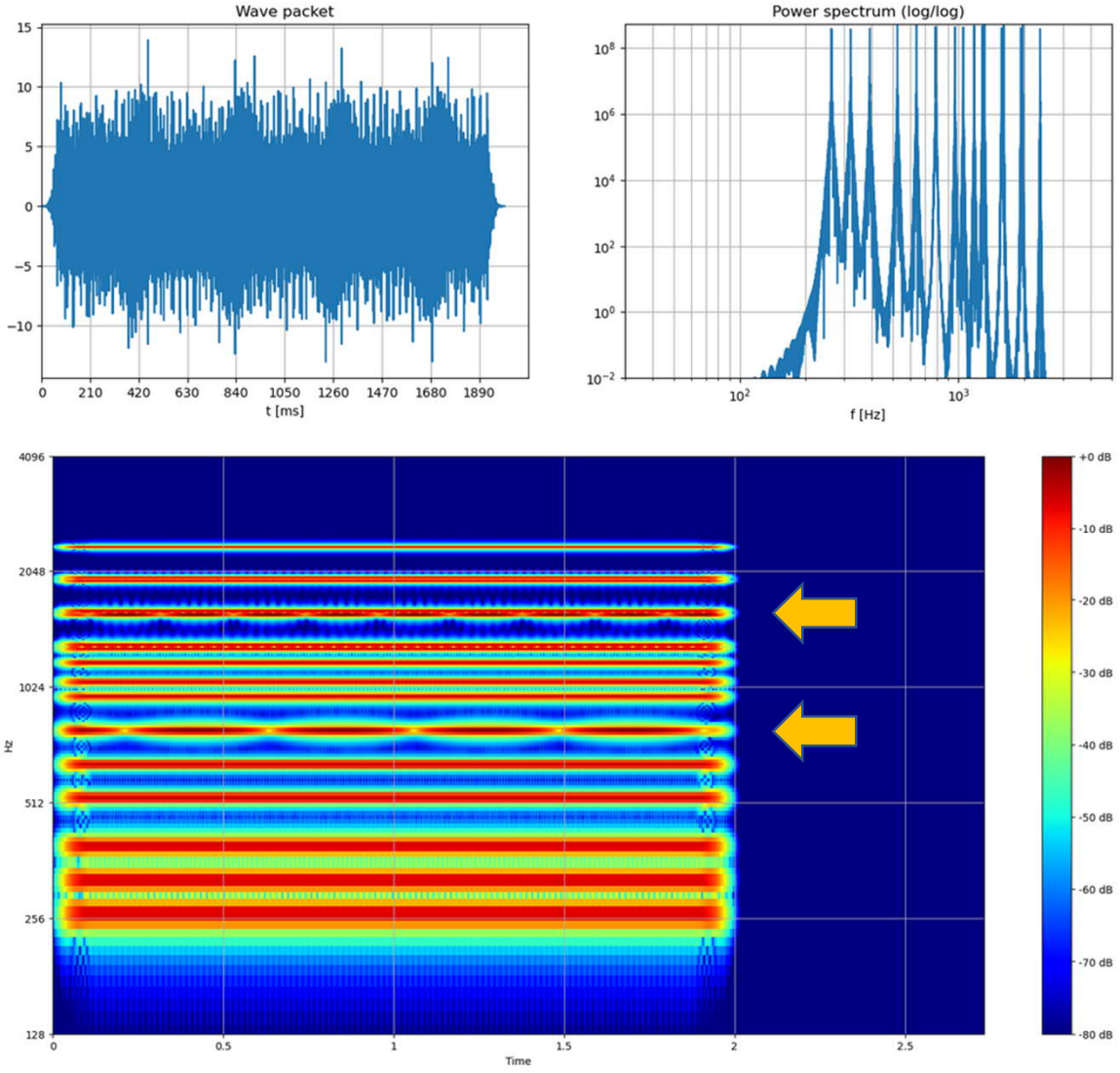
As a result, there is less “tension” as the chord is more stable.



The Neutral 3rd

C neut, with a “neutral” Third (close to 11:9)

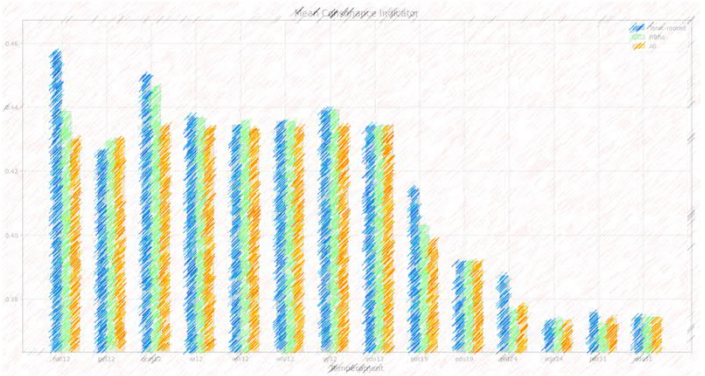
More dissonant than a pure minor Third (6:5), significant beatings reinforce the perception of out-of-tune



Recap

- Unequal Temperaments are especially suited for Organs, even nowadays
- Microtonal Temperaments allow for some exploratory music with uncommon frequency ratios
  - BUT: need to be careful with so many additional dissonant intervals
- 12-TET remains an excellent compromise for a large variety of musical instruments and styles

- *Work in progress:* we are trying to define a *Temperament Consonance Indicator*, based on our consonance model, which would allow us to quantitatively compare the different temperaments.



To conclude, a Gran Finale

Most consonant intervals following the analysis so far:

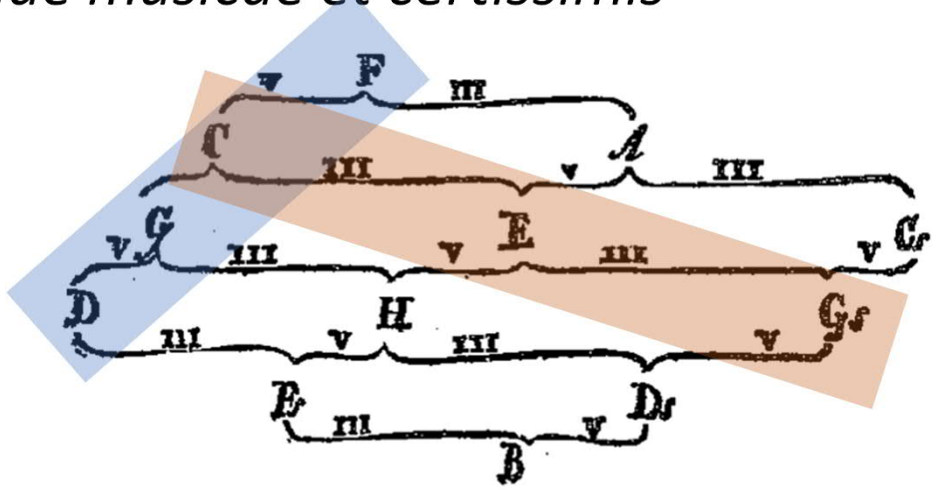
- 3/2 = P5 (Perfect 5th)
- 4/3 = P4 (Perfect 4th)
- 5/4 = M3 (Major 3rd)
- 8/5 = m6 (minor 6th)
- 6/5 = m3 (minor 3rd)
- 5/3 = M6 (Major 6th)

What about representing such a space of intervals?

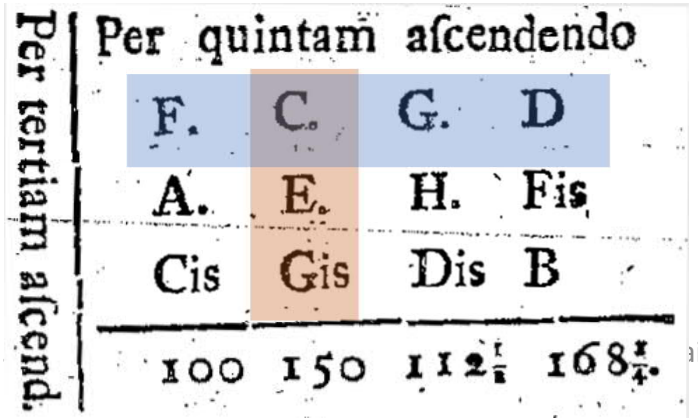
- On top of the well-known circle of fifths, and focusing on 12-Tone

L. EULER AND THE TONNETZ

- In German, “Network of Tones”
- First appearance in 1739
  Tentamen novae theoriae musicae et certissimis harmoniae principiis
  - Aperiodic!



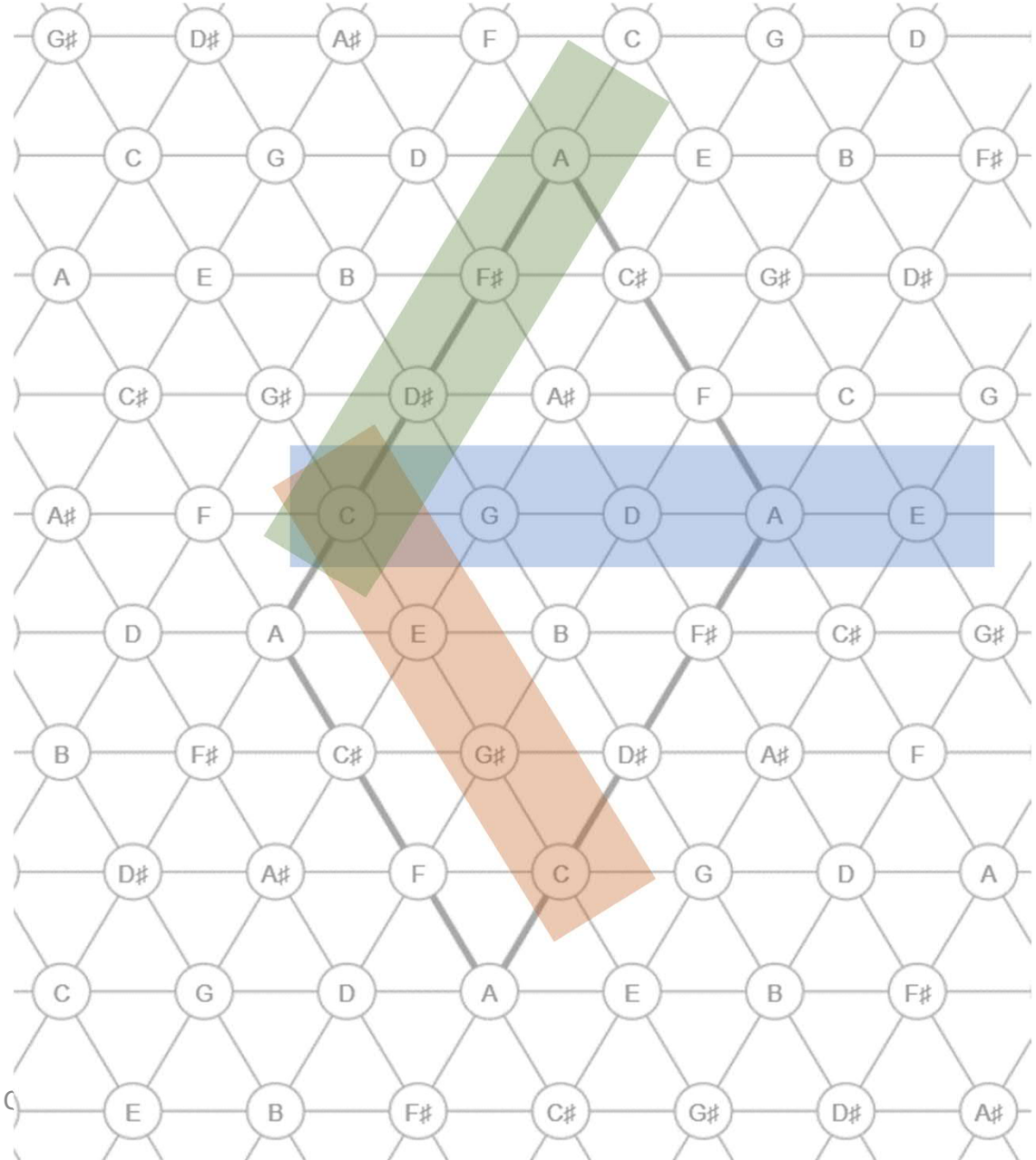
- Revised in 1774
  Speculum musicum



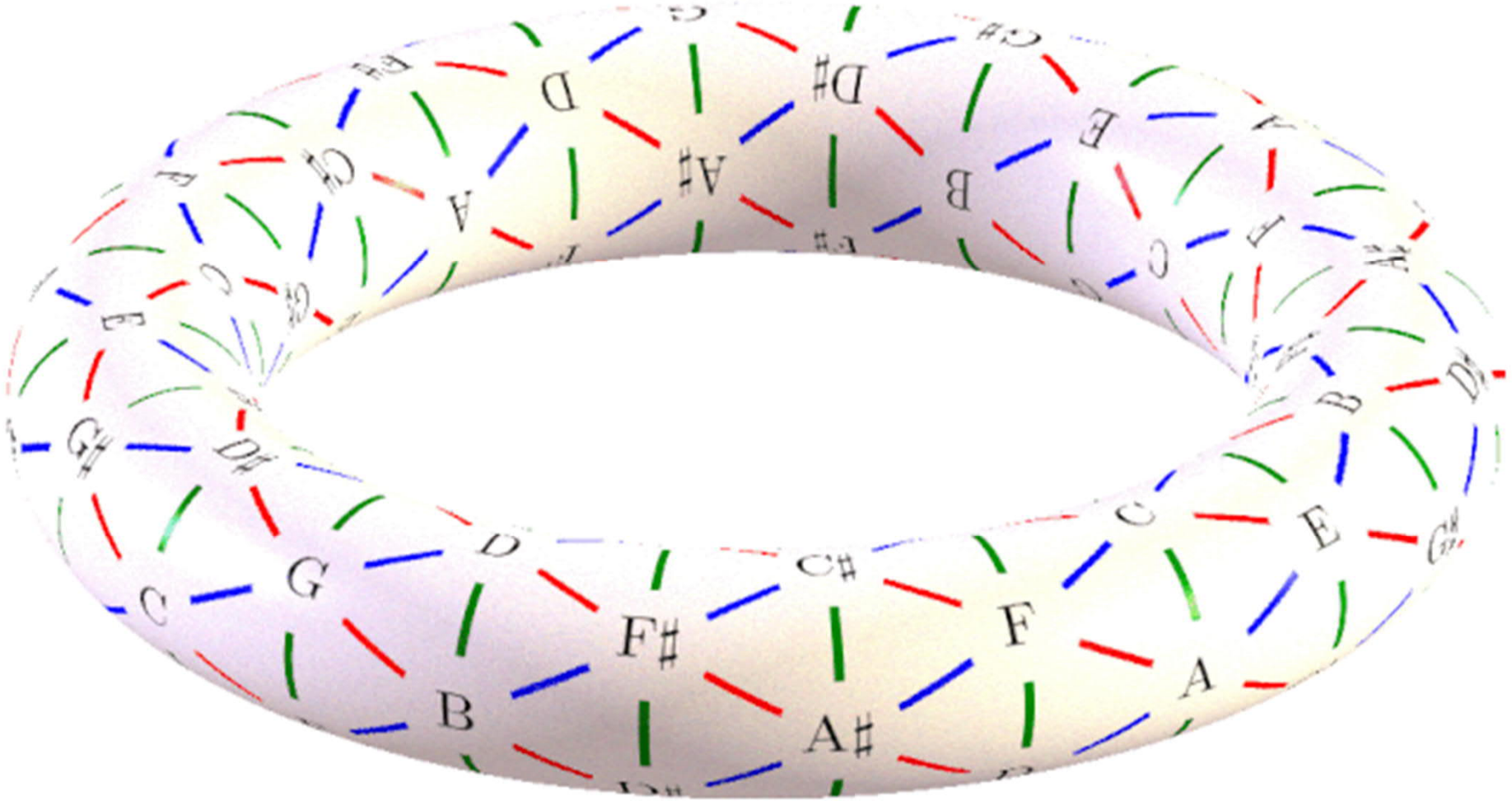
- Rediscovered in 1880 by H. Riemann (Musicologist) and A. von Oettingen (Physicist)

A Modern View of the Tonnetz

- From the interactive representation available at: https://cifkao.github.io/tonnetz-viz
- Relevant “axes”
  - Fifths
  - Major Thirds
  - Minor Thirds
- Features
  - Chords built on subsequent fifths are close
    - Typical *cadences* clearly visible
  - Chords belonging to different *tonalities* may be “graphically close”, though non “musically close”



Representation on the Torus

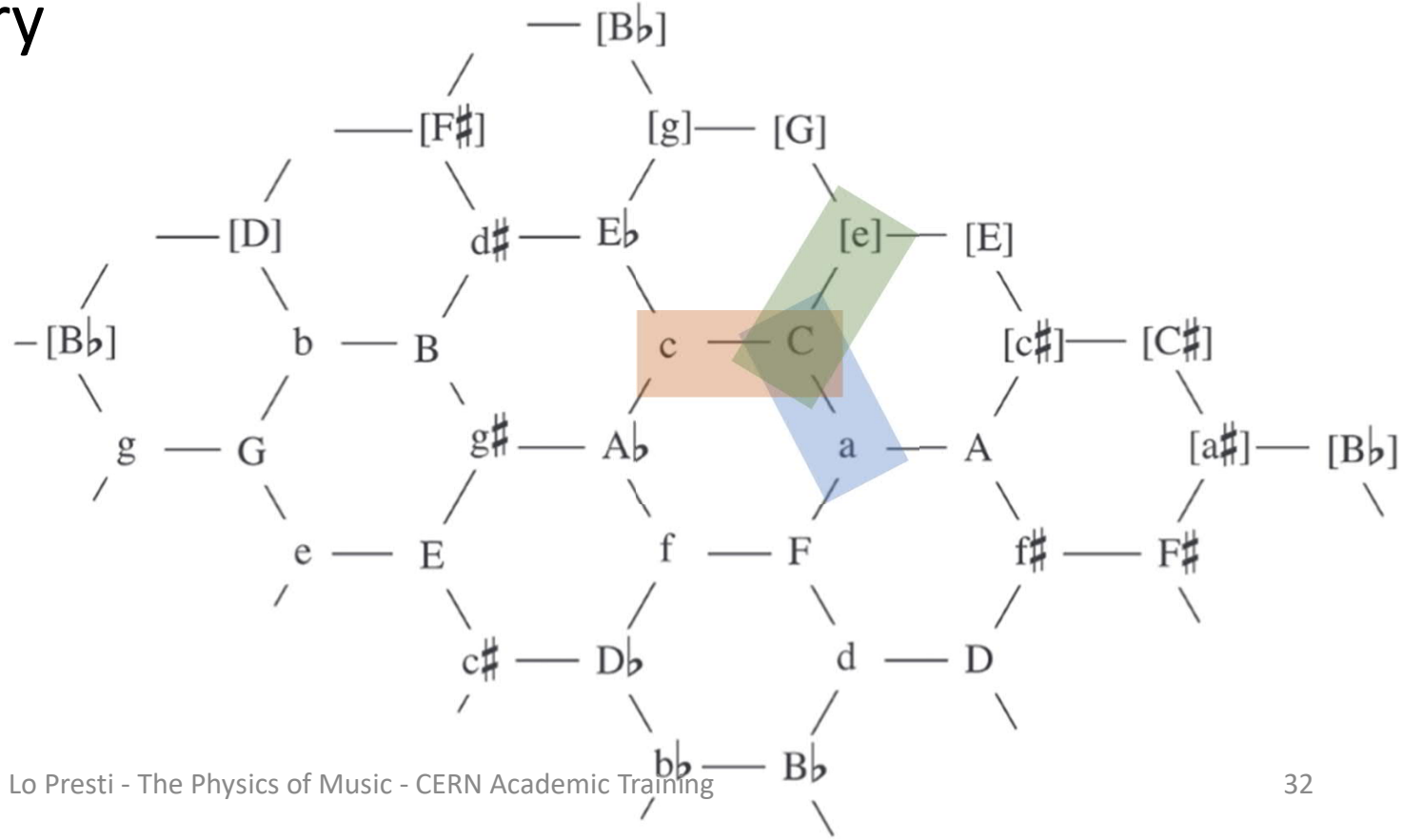


The Dual Tonnetz

- A Dual representation can be defined by replacing each triangle with a node, which represents a **Major** or **minor** triad
- The Dual Tonnetz includes 24 nodes and repeats itself with a similar toroidal symmetry

Relevant transformations ("Dual axes")

- Relative
- Parallel
- Leading-Tone

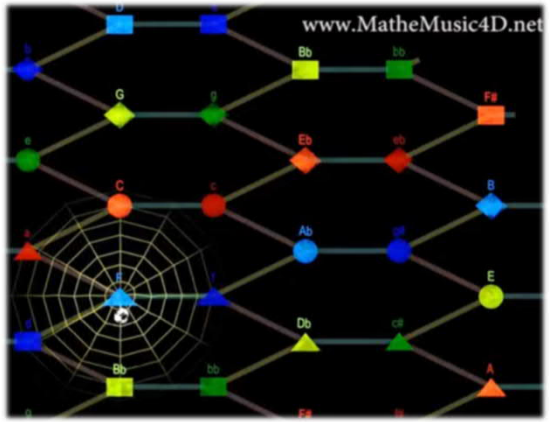


Examples

- Chopin, *Preludio in Mi*, Op 28 n.9
  https://www.youtube.com/watch?v=hjTaut-h-Tk
  - Musical analysis over the Tonnetz

- Rossini, *Guglielmo Tell, Finale*
  https://www.youtube.com/watch?v=TvrjRbiIArs
  - Rossini goes on a line in the Dual Tonnetz, for about a half circle of fifths

- M. Andreatta, Strasbourg Univ., *La Sera* (2014)
  https://www.youtube.com/watch?v=1TlwFLYCYhY
  - Tonnetz extensions to 3D and 4D representations



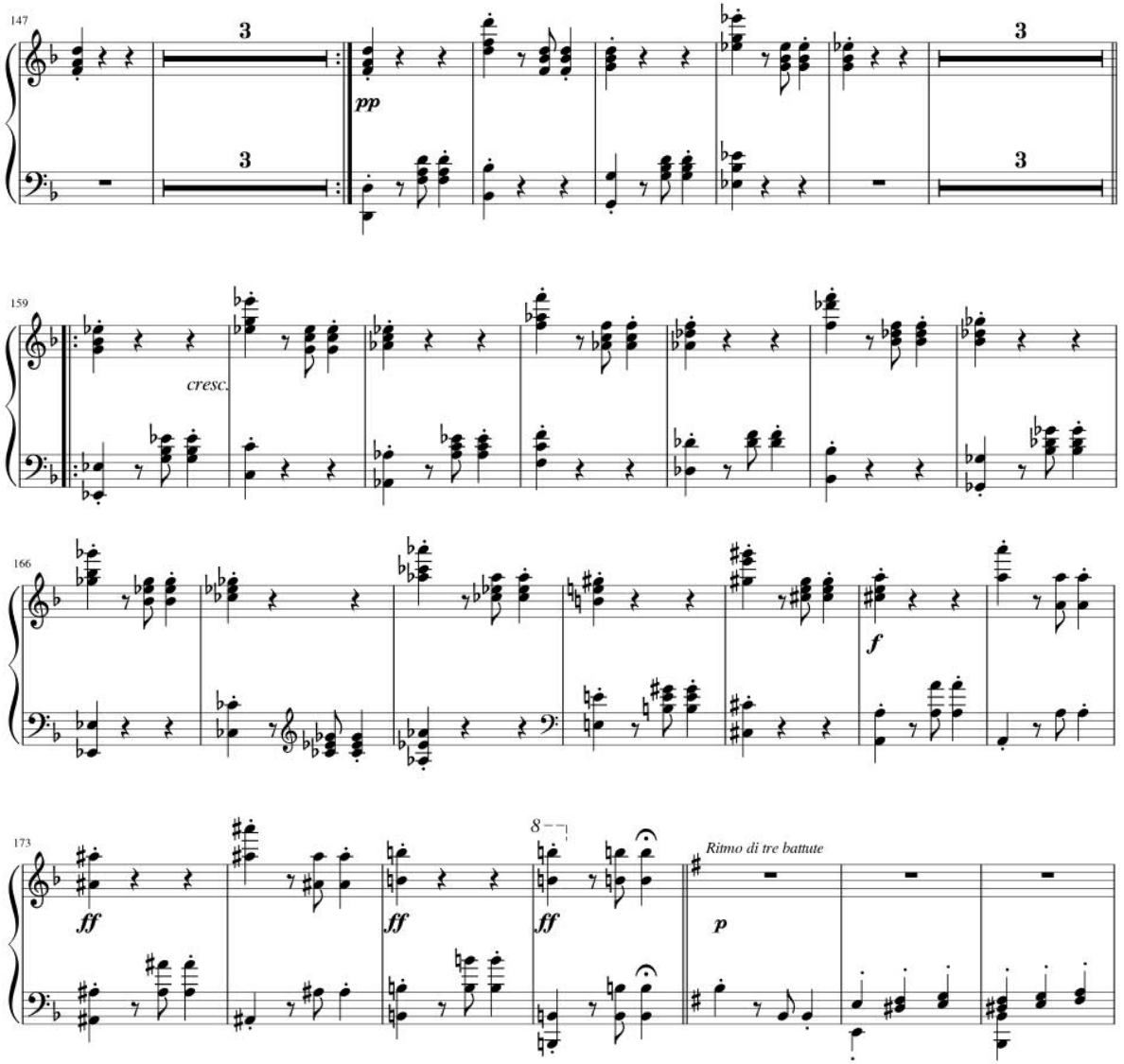
Examples

Ludwig van Beethoven, *9th Symphony,* II Mvt.

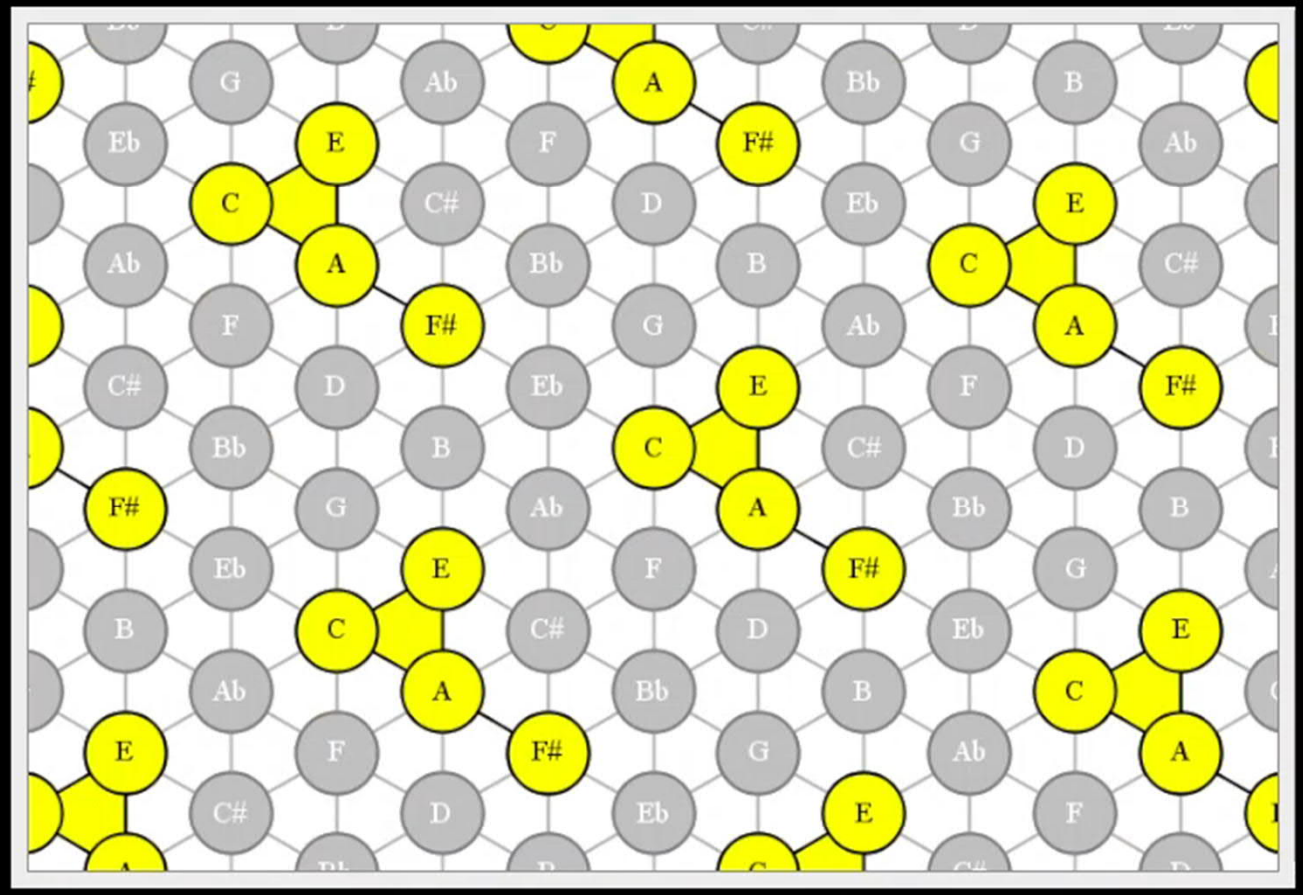
Recording: Barenboim/West-Eastern Divan Orchestra
https://www.youtube.com/watch?v=Tn4lk8fRskA

Tonnetz Software: Hexacord (Bigo, Andreatta et al.),
https://louisbigo.com/hexachord

Self-made synchronization



Beethoven and the Tonnetz



CONCLUSIONS

- This is a niche, yet active research field!
- We hope we have raised some awareness about the Physics underlying musical harmony
  - In particular where the perception of Consonance and Dissonance is involved
- We can only be happy if now you have more questions than when we started!

Thank you and happy jamming!