Numerical study of the thermal behavior of an Nb_3Sn high field magnet in He II

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CHATS on Applied Superconductivity (CHATS-AS), October 12-14 2011, CERN

Outline

- ☐ Motivation
- ☐ Two – fluid model
- ☐ Simplified model of He II
- ☐ Validation of simplified model

Steady state modeling

- ☐ Modeling of thermal – flow process during AC losses in Nb_3Sn magnet – steady state
  - ○ Description of Fresca II magnet;
  - ○ 3D computational region, assumption and boundary conditions;
  - ○ Mesh;
  - ○ Numerical results.

Unsteady state modeling

- ☐ Modeling of thermal process during quench heating – unsteady state model
  - ○ Geometry and mesh;
  - ○ Numerical results.

- ☐ Conclusions

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Motivation

- ☐ Within the framework of the European project EuCARD, a Nb_3Sn high field accelerator magnet is under design to serve as a test bed for future high field magnets and to upgrade the vertical CERN cable test facility, *Fresca 2*.

- ☐ Calculation of the maximum temperature rise in the magnet during AC losses.

- ☐ Calculation of magnet thermal – flow behavior during the quench detection event.

- ☐ Implementation of superfluid helium in comercial software - ANSYS CFX software.

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Two – fluid model

- □ Density of superfluid helium $\rho = \rho_n + \rho_s$ (1)

- □ Density flux $\rho\, u = \rho_n u_n + \rho_s u_s$ (2)

- □ Continuity equation $\frac{\partial \rho}{\partial \tau} + \nabla \cdot (\rho_n u_n + \rho_s u_s) = 0$ (3)

- □ Momentum equations for the total fluid

$$\frac{\partial}{\partial \tau}(\rho_n u_n + \rho_s u_s) = -\nabla \cdot (\rho_n u_n u_n + \rho_s u_s u_s) - \nabla p + \eta\left[\nabla^2 u_n + \tfrac{1}{3}\nabla(\nabla \cdot u_n)\right] + \rho g \quad (4)$$

- □ Momentum equations for the superfluid component

$$\frac{\partial u_s}{\partial \tau} = -(u_s \cdot \nabla)u_s + s\nabla T - \frac{1}{\rho}\nabla p + \frac{\rho_n}{2\rho}\nabla|u_n - u_s|^2 + A\rho_n|u_n - u_s|^2(u_n - u_s) + g \quad (5)$$

- □ Entropy equation

$$\frac{\partial}{\partial \tau}(\rho s) = -\nabla \cdot (\rho s u_n) + \frac{A\rho_n\rho_s|u_n - u_s|^4}{T} \quad (6)$$

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Simplified model of He II (Kitamura et al.)

□ The momentum equations for the superfluid component is simplified to the form

$$s\nabla T = -A\rho_n|u_n - u_s|^2(u_n - u_s)$$

(the thermomechanical effect term and the Gorter-Mellink mutual friction term are larger than the other)

Superfluid component:

$$u_s = u - \frac{\rho_n}{\rho}(u_n - u_s) = u + \left(\frac{\rho_n^3 s}{A\,\rho^3\rho_n|\nabla T|^2}\right)^{1/3}\nabla T$$

Normal component:

$$u_n = u + \frac{\rho_s}{\rho}(u_n - u_s) = u - \left(\frac{\rho_s^3 s}{A\,\rho^3\rho_n|\nabla T|^2}\right)^{1/3}\nabla T$$

Momentum equation

$$\rho\frac{\partial u}{\partial \tau} = -\rho(u\cdot\nabla)u - \nabla p - \nabla\cdot\left[\frac{\rho_n\,\rho_s}{\rho}\left(\frac{s}{A\rho_n|\nabla T|^2}\right)^{2/3}\nabla T\nabla T\right]$$
$$+\eta\left[\nabla^2 u + \frac{1}{3}\nabla(\nabla\cdot u) - \left(\frac{\rho_s^3 s}{A\rho^3\rho_n|\nabla T|^2}\right)^{1/3}\left\{\nabla^2(\nabla T) + \frac{1}{3}\nabla(\nabla\cdot\nabla)T\right\}\right] + \rho g$$

The system of equation for He II simplified model

- Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, u) = 0 \tag{1}$$

- Momentum equation:

$$\rho \frac{\partial u}{\partial \tau} = -\rho (u \cdot \nabla) u - \nabla p - \nabla \cdot \left[\frac{\rho_n\ \rho_s}{\rho} \left(\frac{s}{A \rho_n |\nabla T|^2} \right)^{2/3} \nabla T \nabla T \right] +$$

$$\eta \left[\nabla^2 u + \frac{1}{3} \nabla (\nabla \cdot u) - \left(\frac{\rho_s^3 s}{A \rho^3 \rho_n |\nabla T|^2} \right)^{1/3} \left\{ \nabla^2 (\nabla T) + \frac{1}{3} \nabla (\nabla \cdot \nabla) T \right\} \right] + \rho g \tag{2}$$

where:

$\nabla \cdot \left[\frac{\rho_n\ \rho_s}{\rho} \left(\frac{s}{A \rho_n |\nabla T|^2} \right)^{2/3} \nabla T \nabla T \right]$ - the convectional acceleration;

$\left(\frac{\rho_s^3 s}{A \rho^3 \rho_n |\nabla T|^2} \right)^{1/3} \left\{ \nabla^2 (\nabla T) + \frac{1}{3} \nabla (\nabla \cdot \nabla) T \right\}$ - the viscous effect.

- Energy equation:

$$\rho c_p \frac{\partial T}{\partial \tau} = -\rho c_p (u \cdot \nabla) T - \nabla \cdot \left\{ \left(\frac{1}{f(T) |\nabla T|^2} \right)^{1/3} \nabla T \right\} \tag{3}$$

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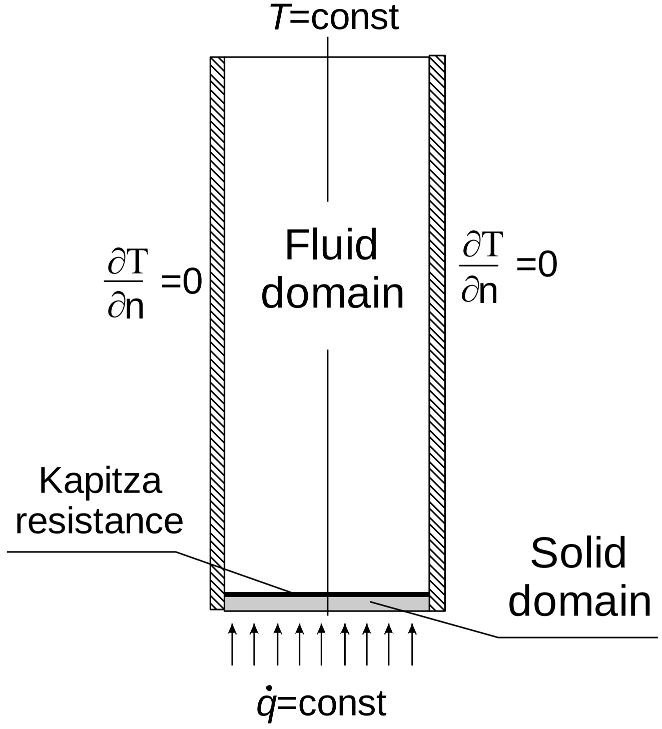
Validation of the simplified model



1. For He II (fluid domain)

- $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, u) = 0$$
- $$\rho \frac{\partial u}{\partial \tau} = -\rho(u \cdot \nabla)u - \nabla p - \nabla \cdot \left[\frac{\rho_n\, \rho_s}{\rho}\left(\frac{s}{A\rho_n|\nabla T|^2}\right)^{2/3} \nabla T \nabla T\right] + \eta\left[\nabla^2 u + \frac{1}{3}\nabla(\nabla \cdot u) - \left(\frac{\rho_s^3 s}{A\rho^3 \rho_n |\nabla T|^2}\right)^{1/3} \left\{\nabla^2(\nabla T) + \frac{1}{3}\nabla(\nabla \cdot \nabla)T\right\}\right] + \rho g$$
- $$\rho c_p \frac{\partial T}{\partial \tau} = -\rho c_p (u \cdot \nabla)T - \nabla \cdot \left\{\left(\frac{1}{f(T)|\nabla T|^2}\right)^{1/3} \nabla T\right\}$$

2. Insulation (solid domain)

- $$\rho_{solid}\ c_p(T)\frac{\partial T}{\partial \tau} = \left[\frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right)\right]$$

3. Kapitza resistance R_k as a function of temperature

Boundary conditions

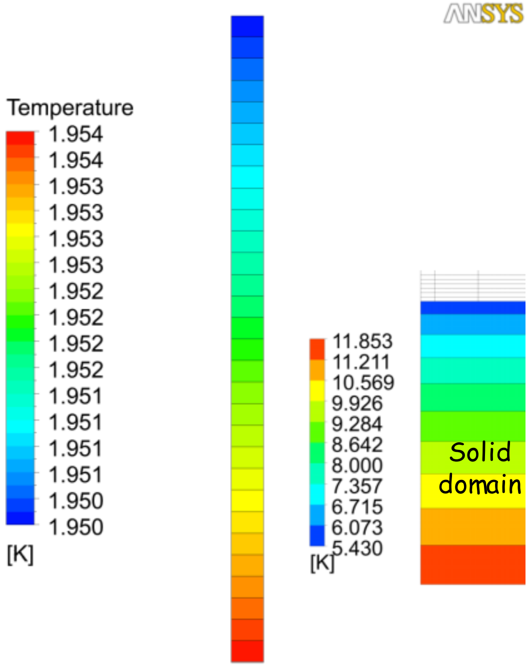
on left and right – adiabatic condition $\frac{\partial T}{\partial n} = 0$

on the top – constant temperature T_b=1.95 K

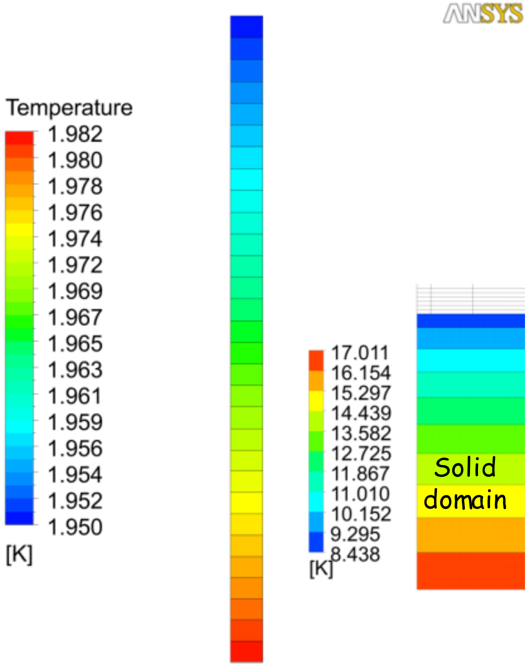
on the bottom – constant heat flux q=const

on all walls $u_\perp = 0$ and $u_\parallel = \left(\frac{\rho_s^3 s}{A\, \rho^3\, \rho_n |\nabla T|^2}\right)^{1/3} (\nabla T)_\parallel$

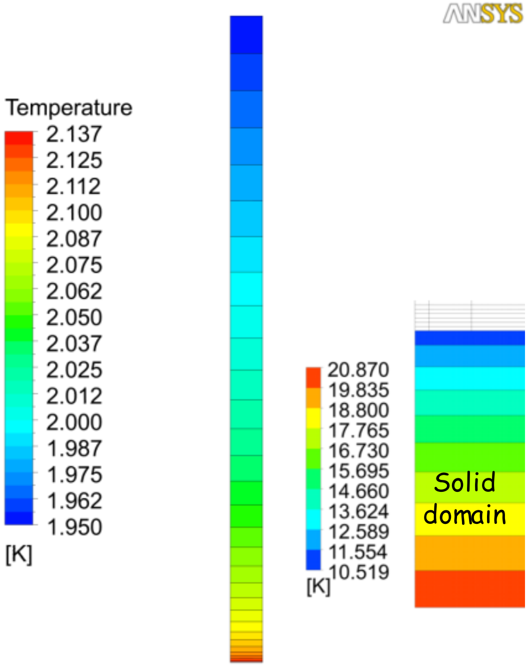
Validation of the simplified model



q = 0.70 W/cm^2



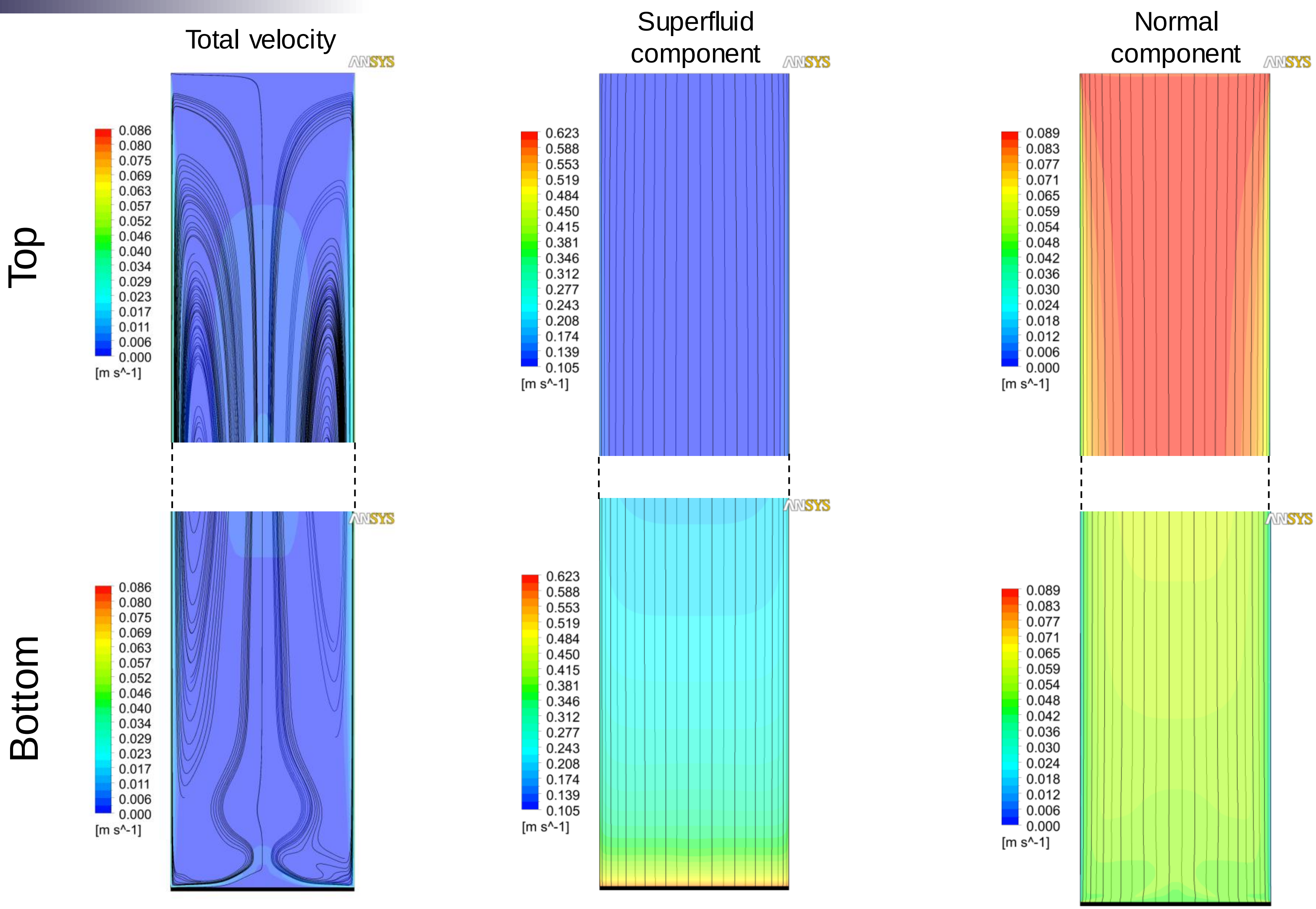
q = 1.39 W/cm^2



q = 2.09 W/cm^2
(near CHF)

Temperature distribution in He II and solid (partially) domains for the temperature of 1.95 K and 0.70, 1.39 and 2.09 W/cm2 of heat fluxes at steady state condition

Validation of the simplified model



The velocity distribution and the streamlines of total velocity, superfluid and normal components in the region near the bottom and the top of He II domain for the highest heat flux of 2.09 W/cm^2

Validation of the simplified model

The comparison between analytical and numerical maximum temperature for applied heat flux at the bottom of solid domain

Applied heat flux	Maximum temperature		*Error*
	Analytical	Numerical	
W/m²	K	K	%
20877	2,1500	2,1371	0,602
13918	1,9823	1,9823	0,002
6959	1,9540	1,9540	0,000

One dimensional turbulent heat transport equation analitycal solution

$$\dot{q} = \left(\frac{1}{f(T)|\nabla T|^2} \right)^{1/3} \nabla T$$

The comparison between applied and calculated (from difference between normal and superfluid components) heat fluxes at the bottom and top of He II domain

Applied heat flux	*(u_n – u_s)*		*$q=\rho_s sT(u_n-u_s)$*		*Error*	
	at bottom	at top	at bottom	at top	at bottom	at top
W/m²	m/s	m/s	W/m²	W/m²	%	%
20877	0,624	0,193	20826	20877	0,25	0,000
13918	0,132	0,129	13935	13918	0,12	0,000
6959	0,066	0,065	7078	7011	1,71	0,007

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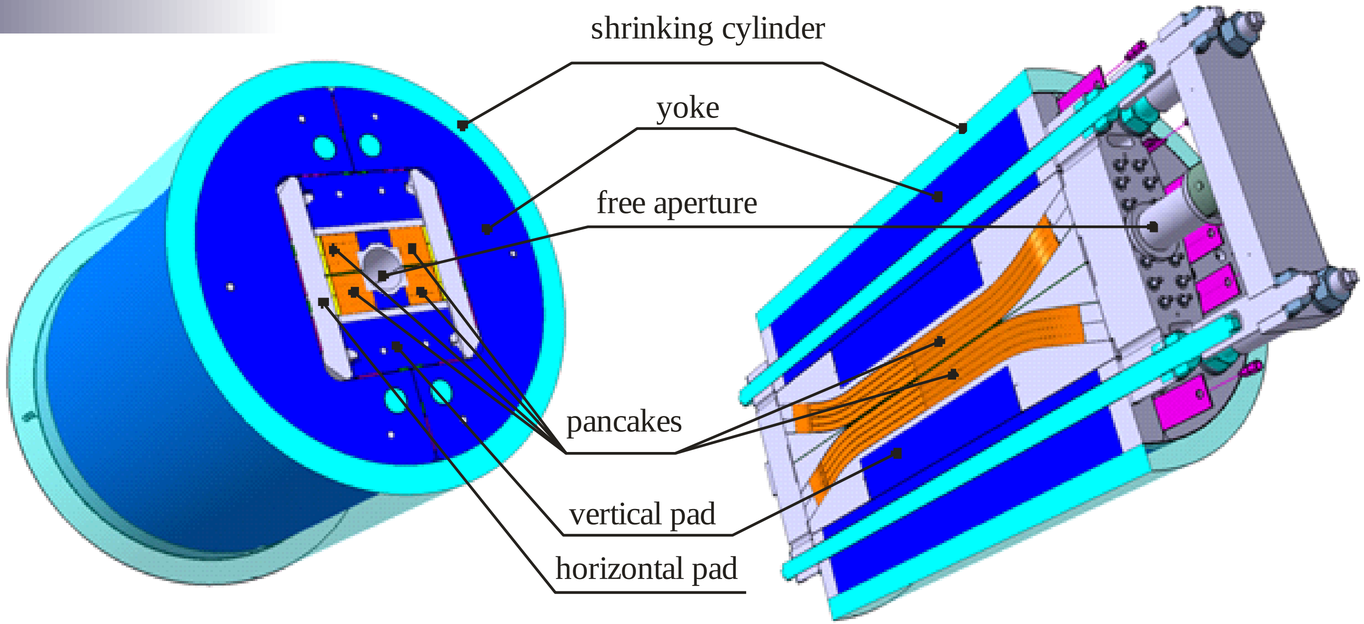
Description of *Fresca II* magnet



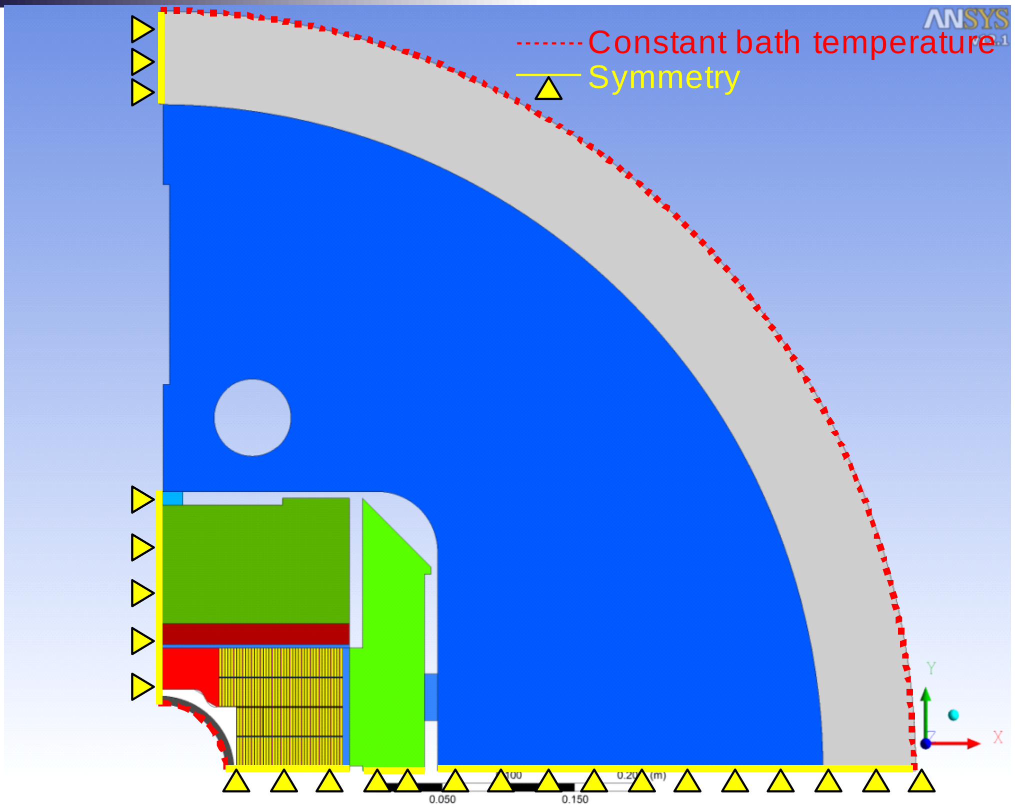
MAGNET SPECIFICATION

- type: block coil, 156 conductors in one pole;
- free aperture: 100 mm;
- total length: 1600 mm;
- outside diameter: 1030 mm;
- magnetic field: 13 T;

OPERATING PARAMETERS

- coolant: superfluid and/or saturated helium;
- temperature: 1.9 K and/or 4.2 K;
- temperature operating margin: 5.84 at 1.9 K and 3.54 K at 4.2 K

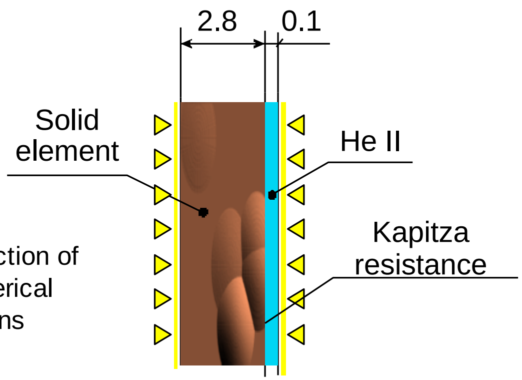
3D computational region, assumption and boundary conditions



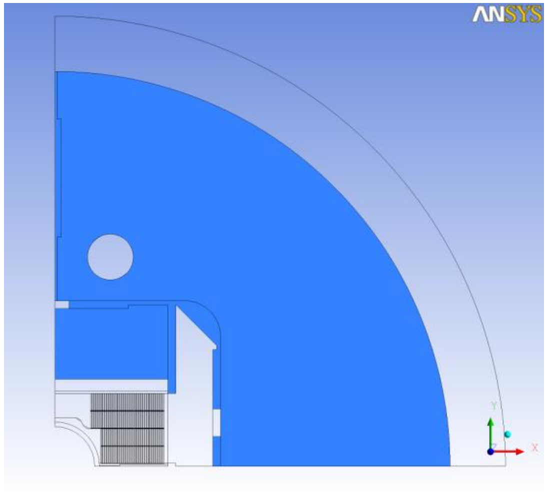
Geometry and boundary conditions applied during simulations

Assumptions

- Two types of boundary conditions:
  1. Constant bath temperature of 1.9 K on walls (red lines);
  2. Symmetry (yellow lines);
- Thermal conductivity as function of temperature;
- Perfect contact between solid elements;
- Calculations are carried out for CUDI model (AC loss due to ISCC losses, non-homogenous spreads)
- He II between yoke and pad laminations (200 μm)

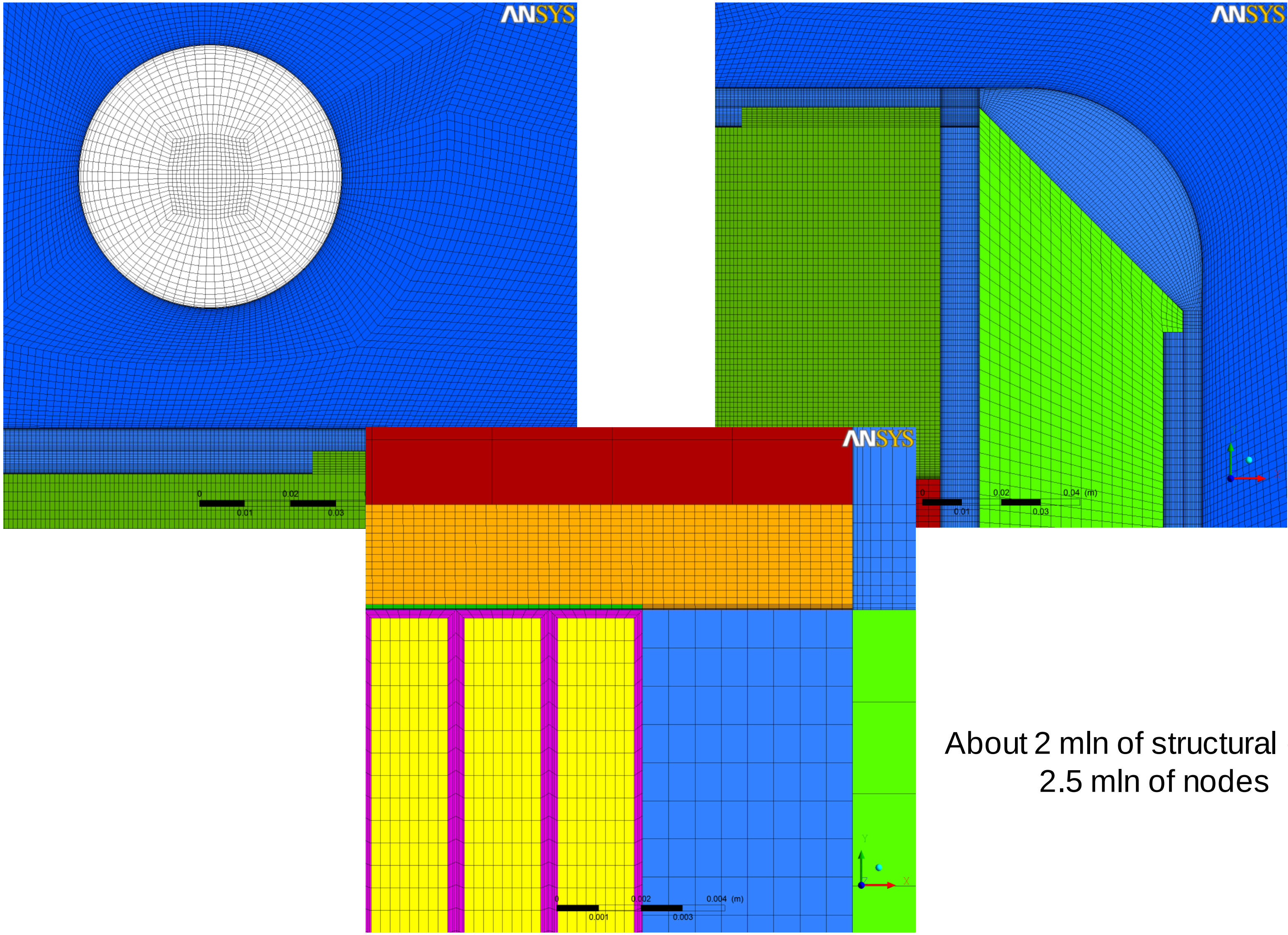


Cross –section of the numerical domains



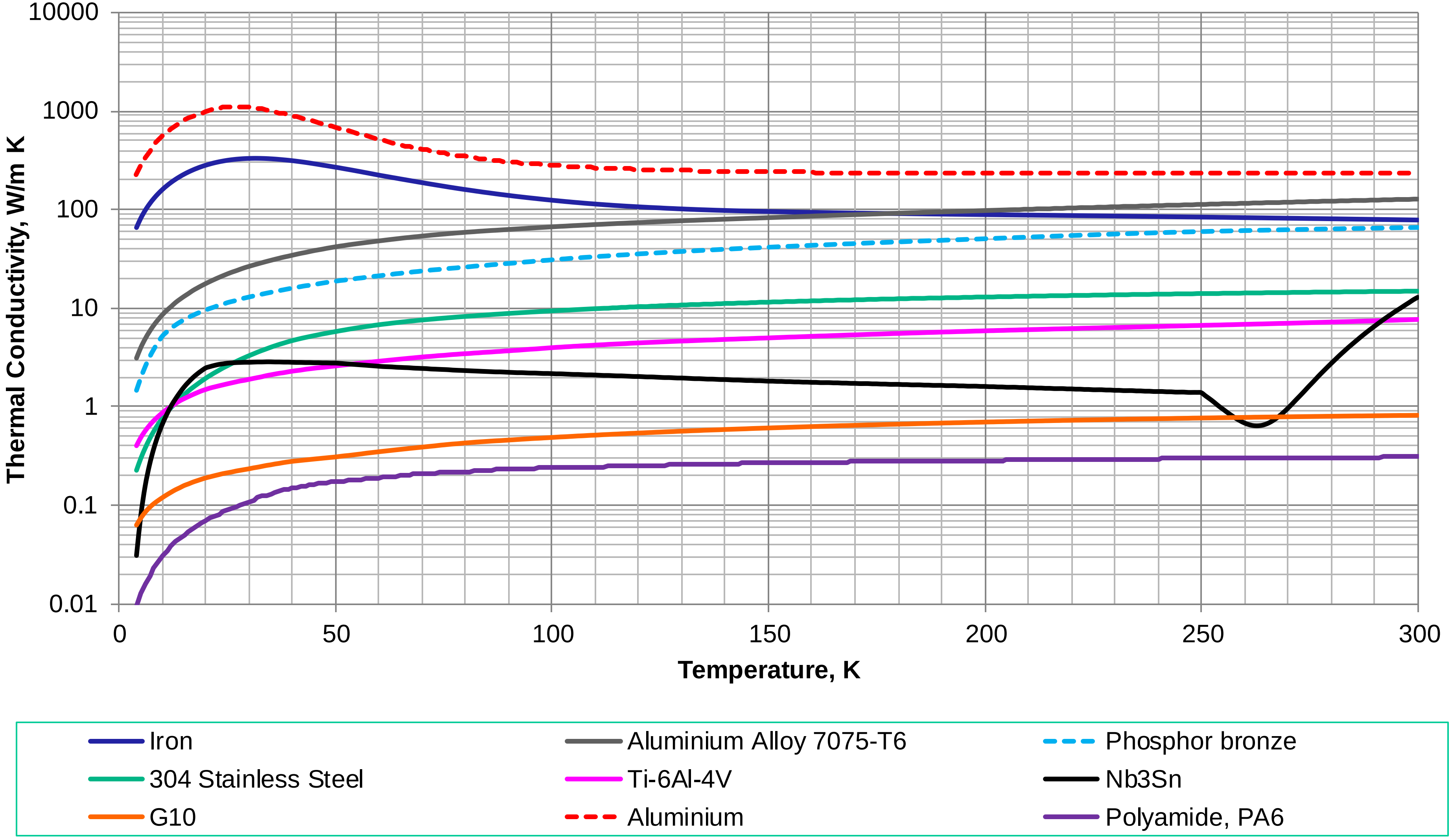
He II region

Mesh



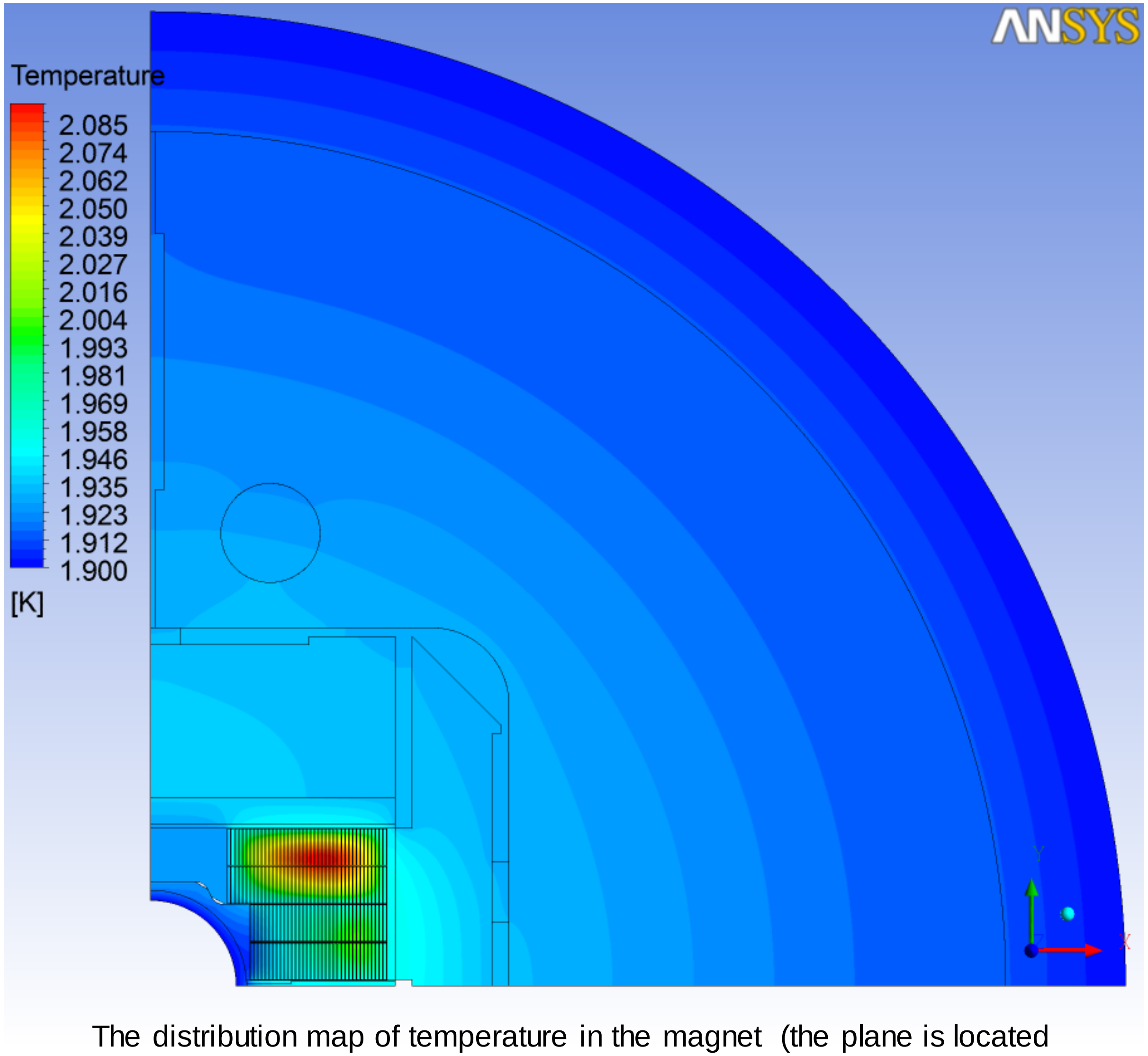
About 2 mln of structural elements
2.5 mln of nodes

Thermal conductivity of materials

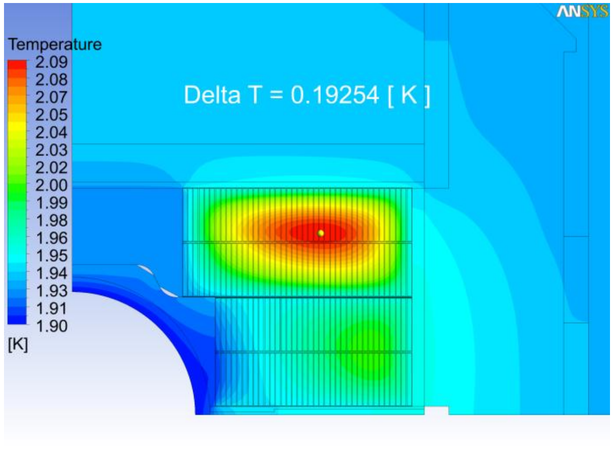


Source:
Cryocomp Software v 3.06
Metalpak Software v 1.00

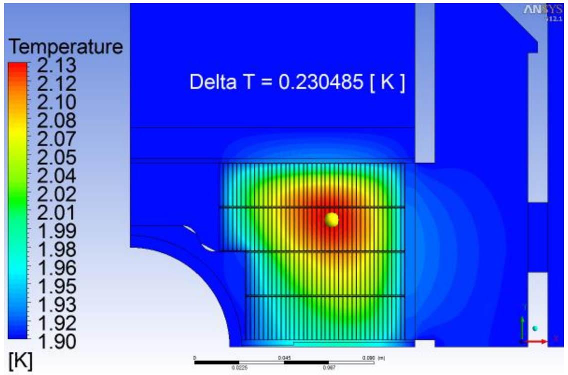
Numerical results



The distribution map of temperature in the magnet (the plane is located on symmetry of helium side)

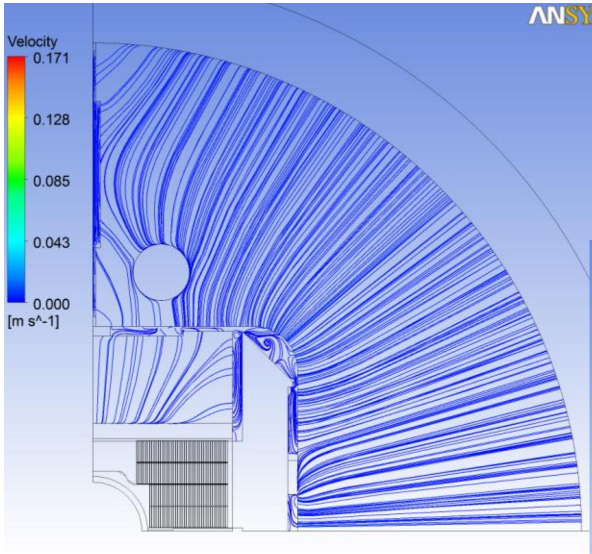


Details of the temperature map in the conductors

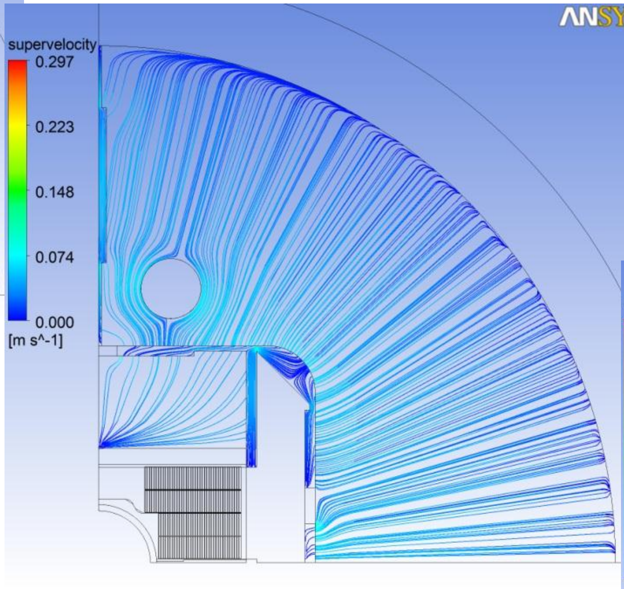


Details of the temperature map in the conductor for solid model (S. Pietrowicz, B. Baudouy, *Thermal design of an Nb$_3$Sn high field accelerator magnet, CEC Conference, 2011, Spokane, USA*)

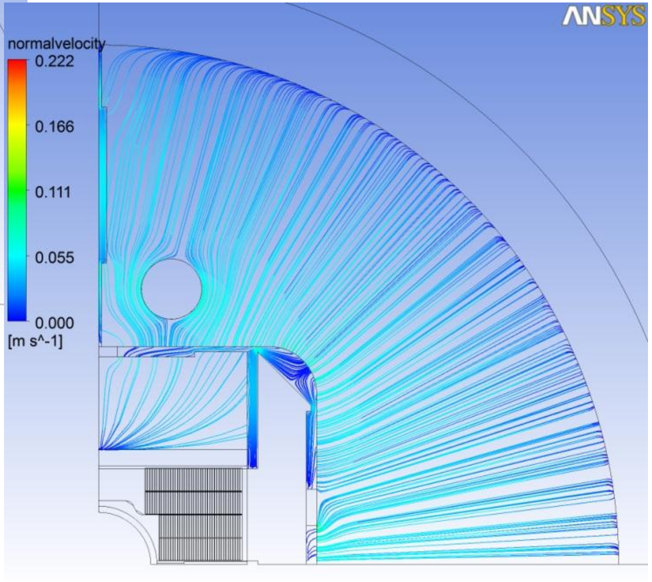
Numerical results



Total velocity



Superfluid component



Normal component

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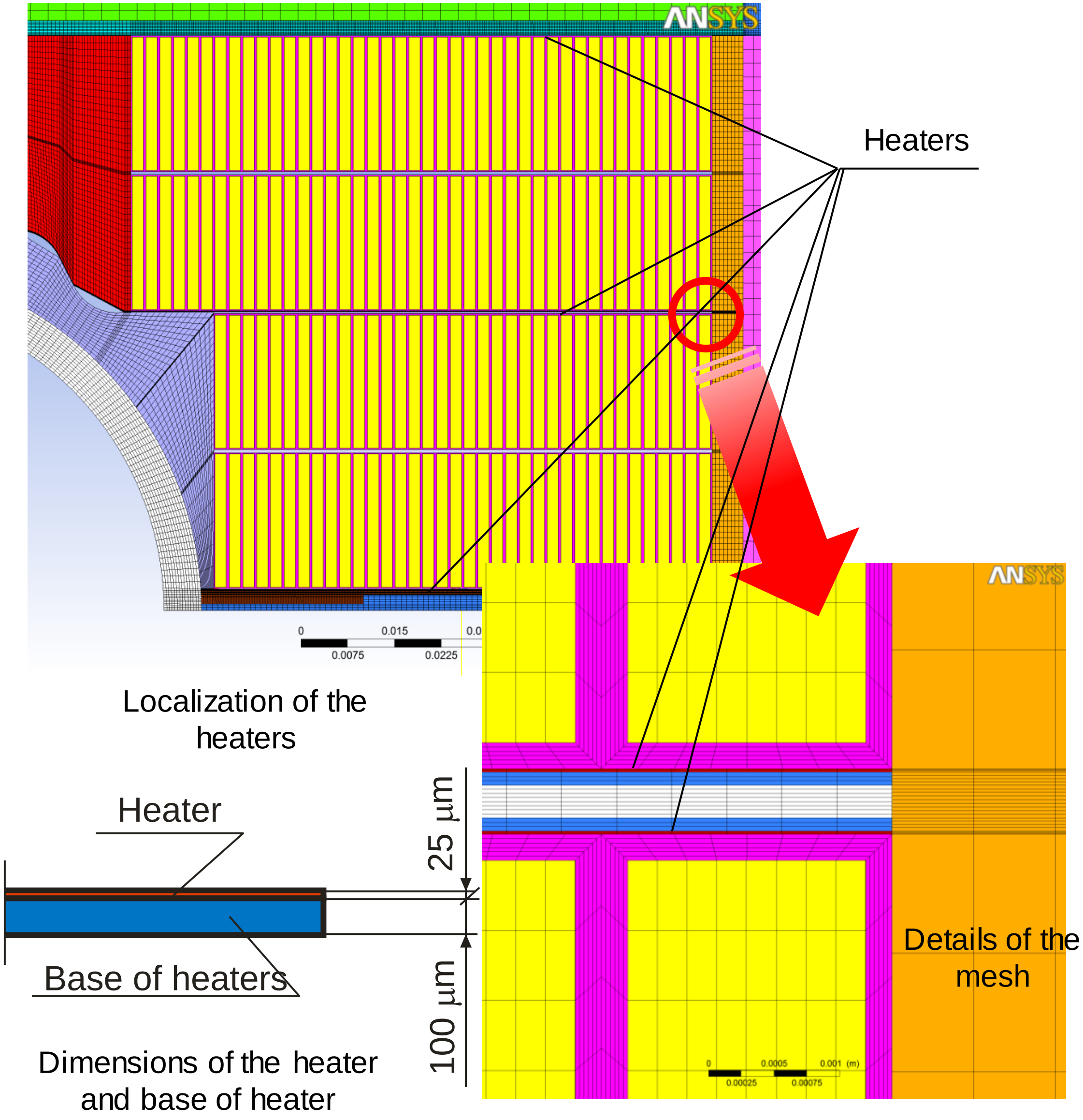
Modeling of thermal process during quench heating – unsteady state model



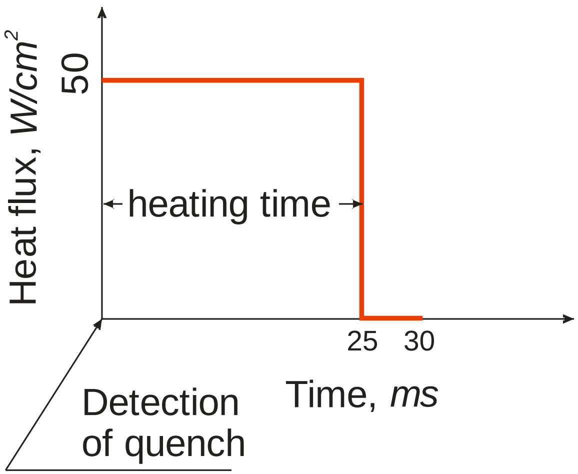
Localization of the heaters

Dimensions of the heater and base of heater

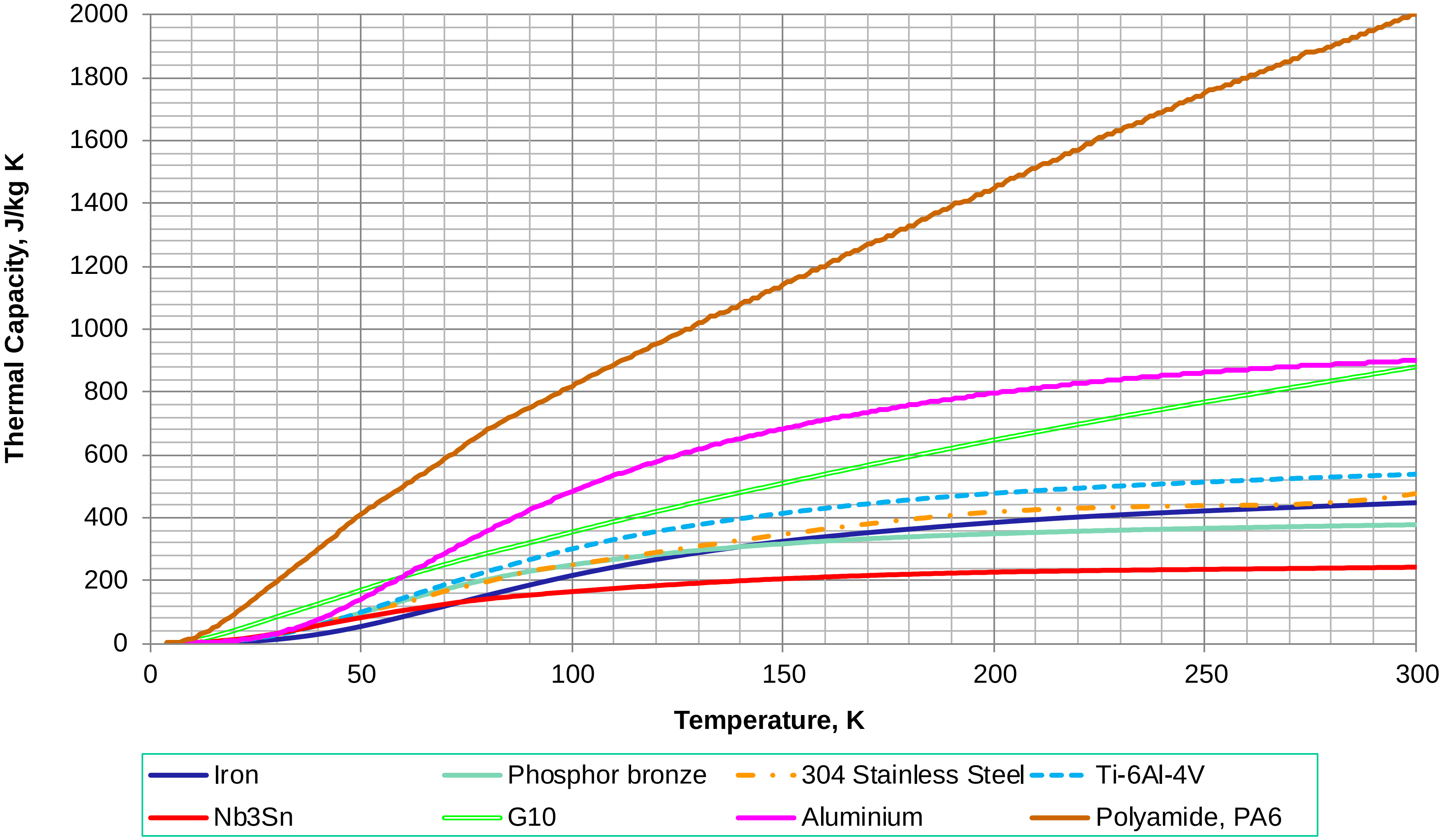
Details of the mesh

Assumptions

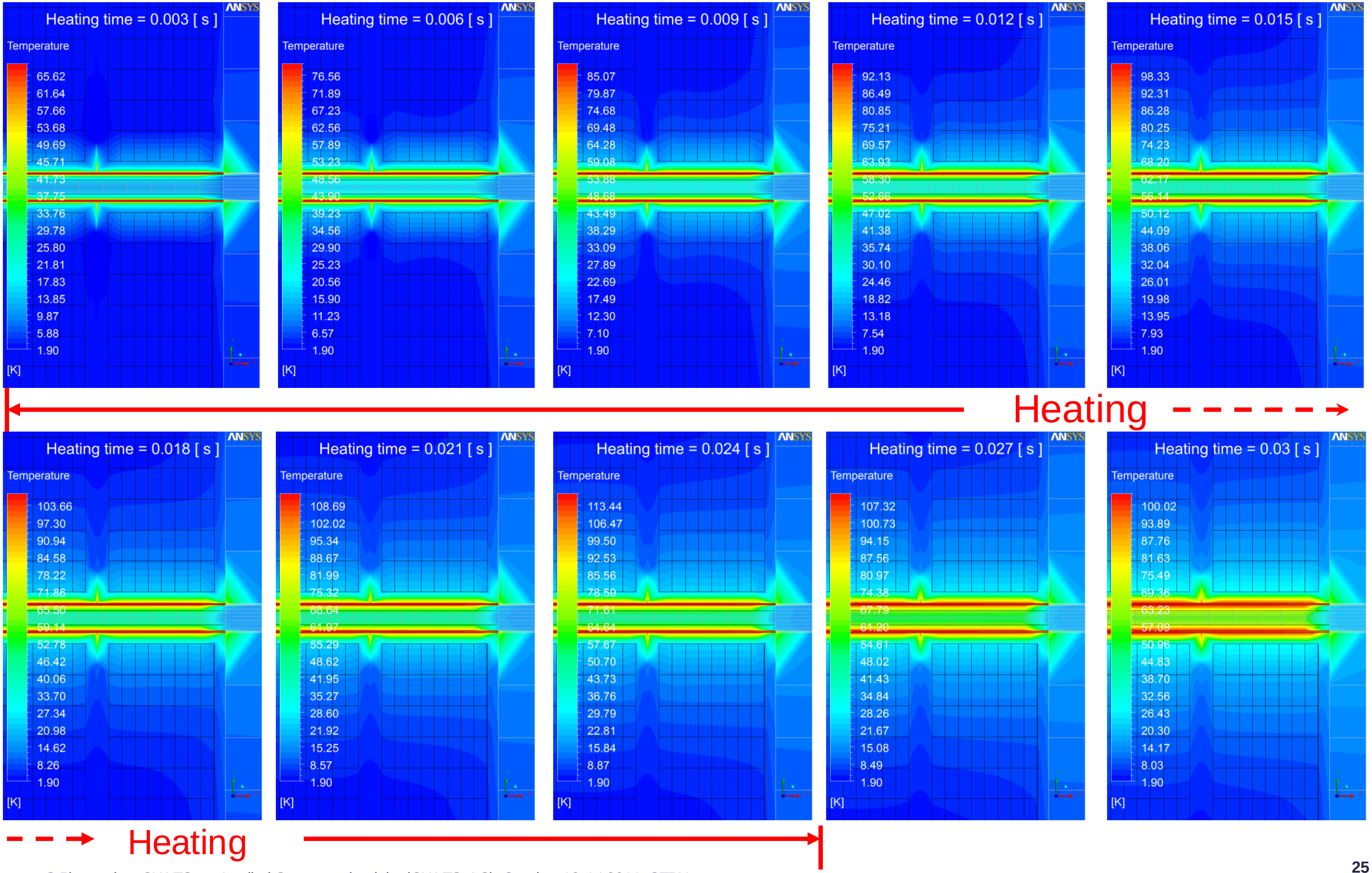
- Two types of boundary conditions:
  1. Constant temperature of the bath and Kapitza resistance on walls (red lines);
  2. Symmetry (yellow lines);
- Thermal conductivity and capacity as a function of temperature;
- Perfect contact between solid elements;
- Bath temperature 1.9 K
- Heating power of quench heaters **50 W/cm^2** (the magnet is heated 25 ms after quench detection)



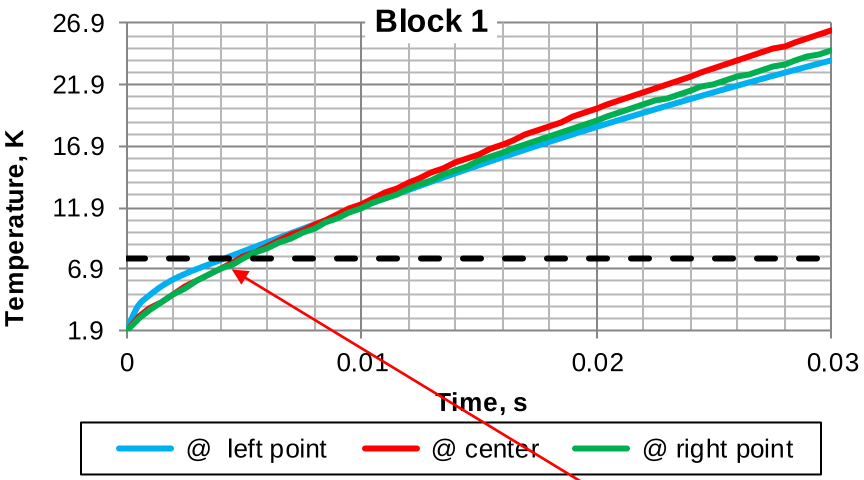
Thermal capacity of the materials

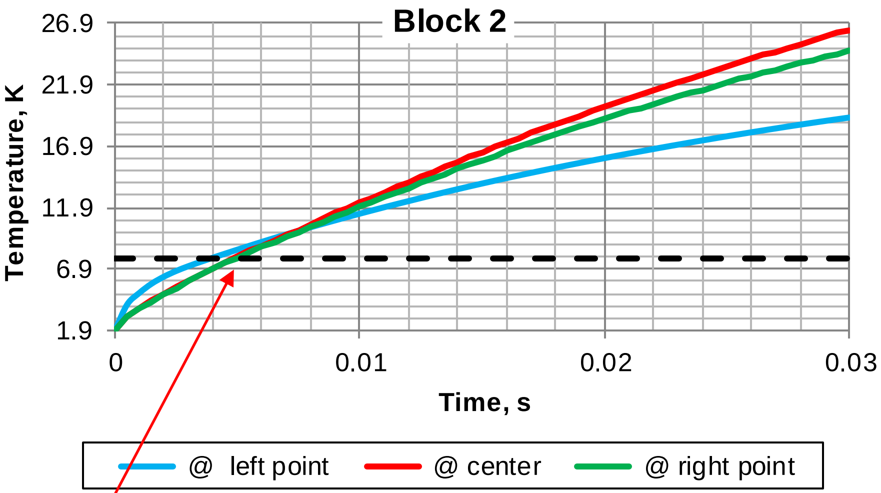


Modeling of thermal process during quench heating – unsteady state model



Modeling of thermal process during quench heating – unsteady state model

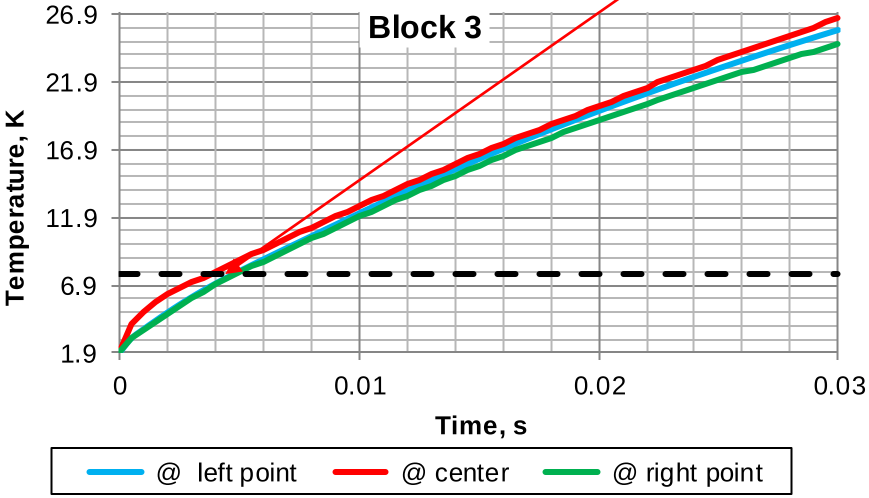


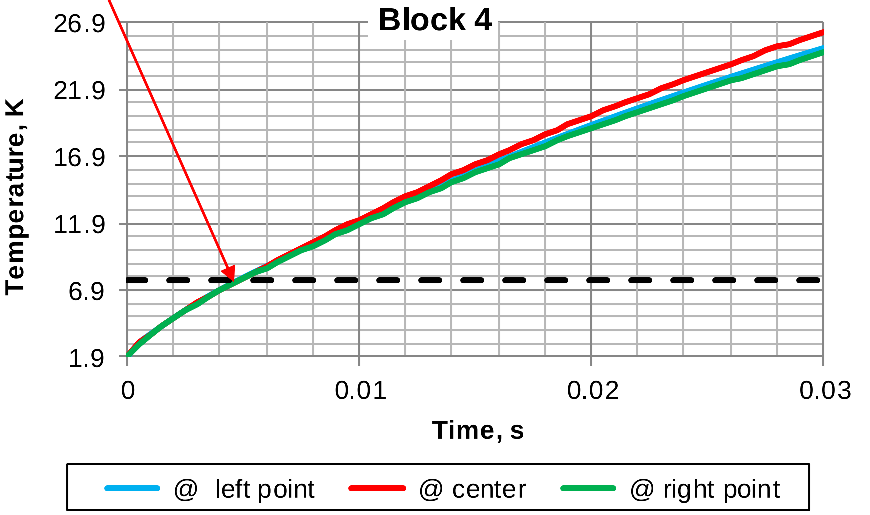


Has to be validated

4 – 5 *ms*

Temperature evolution at selected points





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Conclusions

- The simplified model for calculations of the thermal – flow phenomena was developed and applied in ANSYS CFX Software which is based on FVM (Finite Volume Method)

- The model has been validated on the simple geometry and compared with the analytical solution. The maximum error is varied between 0.6 and 1.7 %.

- The calculation at steady state conditions for the CUDI model of AC losses of Nb_3Sn magnet – Fresca II has been carried out. In comparison to (with) the solid model (without helium), adding He II to the structure decreased maximum temperature rise.

- The simulation of transient process in He II during the quench heating has been performed.