UNIVERSITY OF TWENTE.

Speeding up the Simulation of Coupling Losses in ITER size CICC joints by using a Multi-Level Fast Multipole Method and GPU technology

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Energy, Materials and Systems

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Outline

- Background of the JackPot-AC model
- Calculation of the mutual coupling between two strands
- The Multi-Level Fast Multipole Method
- The Graphics Processing Unit (GPU)
- Solving the system
- Verification of the coupling loss model
- Conclusions

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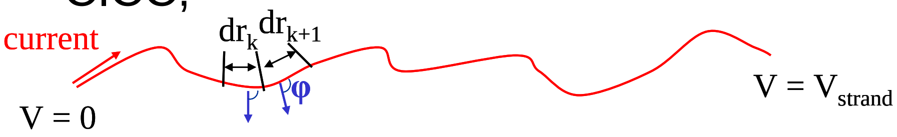
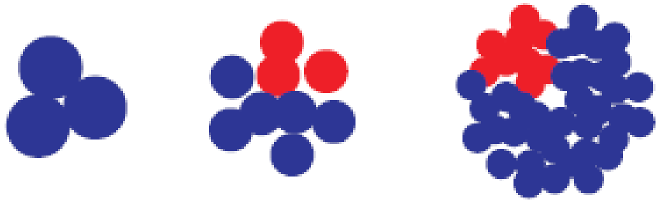
Overview of JackPot-AC network model

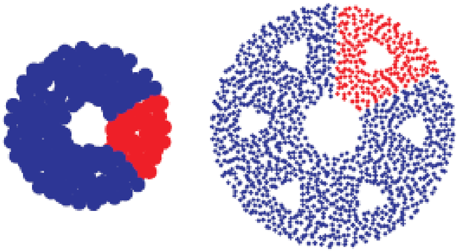
- Cable model that accurately describes *all* strand trajectories in CICC;

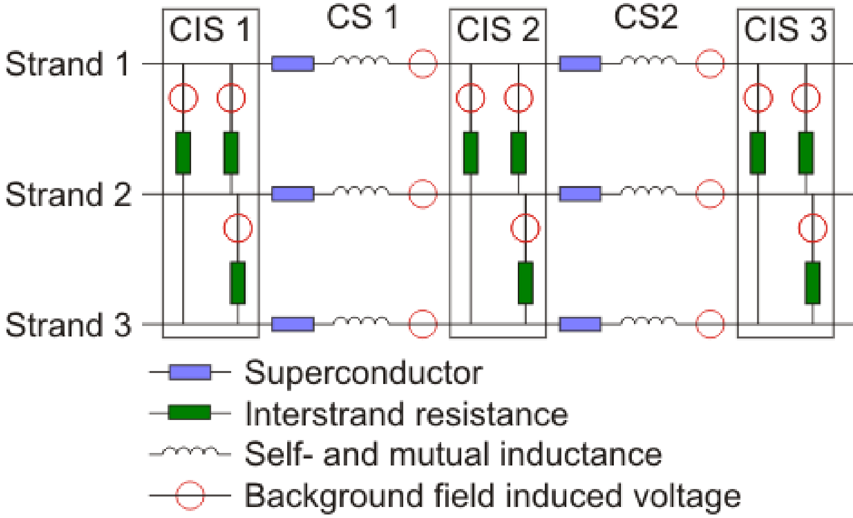


- Simulated strand trajectories are used to:
  - Calculate interstrand contact resistance distribution;
  - Strand-to-joint's copper sleeve contact resistance distribution;
  - Mutual inductances
  - Coupling with background field



Cable cross section from JackPot simulation





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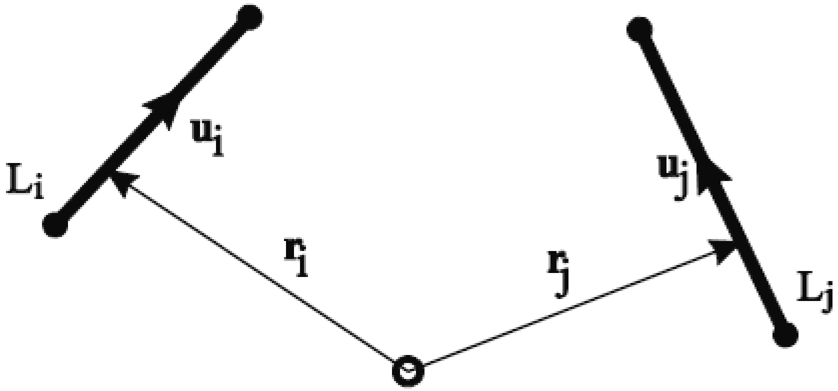
Calculation of the mutual coupling

- Strand currents are considered to be line currents flowing along their axis.
- The mutual coupling between two strand elements can then be written as

$$M_{ij} = \int_{L_i} \mathbf{u}_i \cdot \mathbf{A}_j(\mathbf{r}_j - \mathbf{r}_i)\, dL_i,$$

$$\mathbf{A}_j(\mathbf{r}) = \frac{\mu_0 \mathbf{u}_j}{4\pi} \cdot \int_{L_j} \frac{1}{\mathbf{r}}\, dL_j.$$

- When A is multiplied by the current flowing in line j, it gives the magnetic vector potential of this line at position **r**. $(\nabla \times \mathbf{A} = \mathbf{B})$



Simplification of the mutual coupling

$$M_{ij} = \int_{L_i} \mathbf{u}_i \cdot \mathbf{A}_j\left(\mathbf{r}_j - \mathbf{r}_i\right) dL_i, \qquad \mathbf{A}_j(\mathbf{r}) = \frac{\mu_0 \mathbf{u}_j}{4\pi} \cdot \int_{L_j} \frac{1}{\mathbf{r}} dL_j.$$

- Solving the double integral for the mutual coupling requires considerable effort: simplification is desirable;
- When the change of A_j along $\mathbf{L_i}$ is small, the first equation can be simplified to:

$$M_{ij} = \mathbf{u}_i \cdot \mathbf{A}_j\left(\mathbf{r}_j - \mathbf{r}_{i,c}\right) \cdot L_i,$$

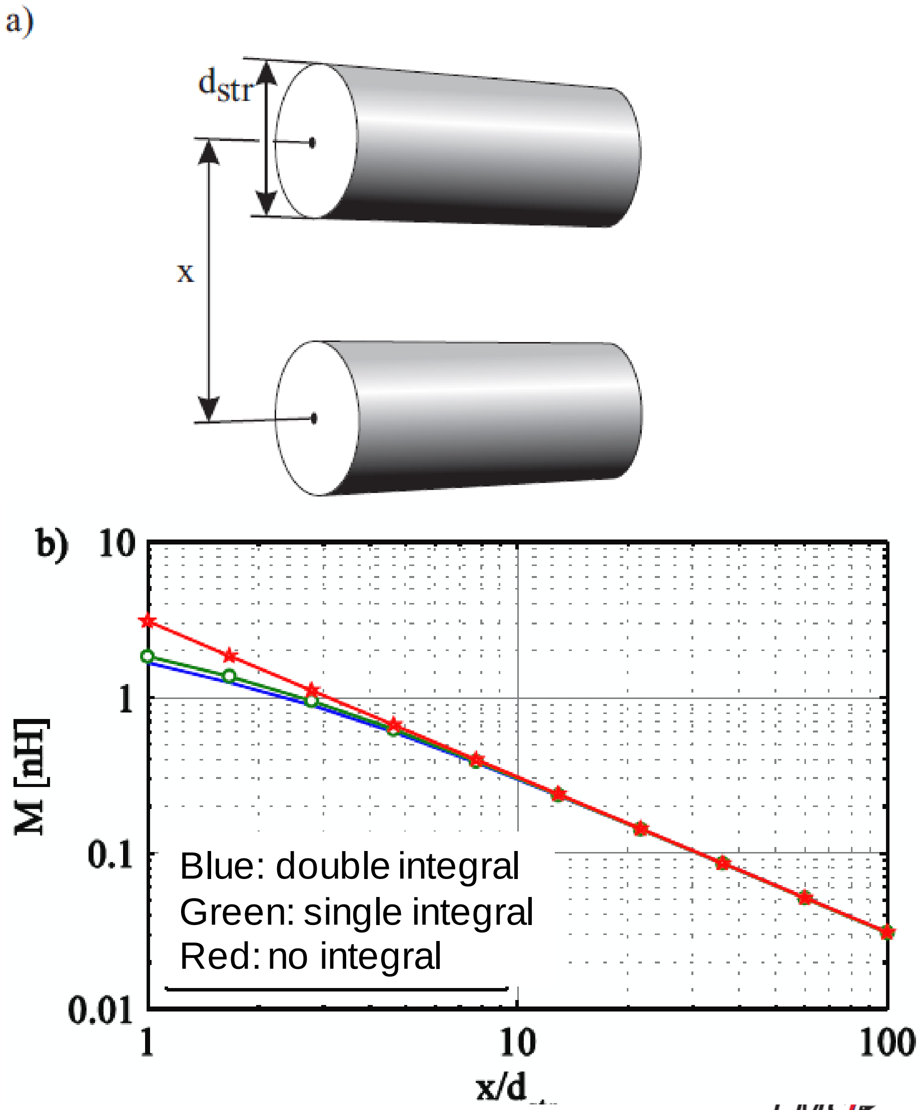
- where $r_{i,c}$ is the centre location of line i.
- If, in addition, A_j does not change much along $\mathbf{r}_j$ either, another simplification is allowed:

$$\mathbf{A}_j\left(\mathbf{r}_{j,c} - \mathbf{r}_{i,c}\right) = \frac{\mu_0}{4\pi} \frac{\mathbf{u}_j}{\mathbf{r}_{j,c} - \mathbf{r}_{i,c}} L_j,$$

- The latter simplification takes away all integrals.

Effect of approximations on the mutual inductance

- The mutual inductance is calculated between two strand sections located in parallel to each other;
- The error due to the approximation reduces rapidly with the distance between the strands;
- Both simplifications are used in JackPot-AC;
  - Single integrals for



Limitation due to the calculation of mutual inductances

- Problem: calculation effort is still $O(N^2)$ dependence. For a simulated cable with 1000 strands in 100 cross sections, this means 10^{10} interactions.
- This makes direct solution of the system undesirable because of
  - Storage of the system matrix (computer memory required);
  - Time required to solve (which has also an $O(N^2)$ dependence);
- The Multi-Level Fast Multipole Method (MLFMM) has the potential to relieve this computation[1].

[1] L.F. Greengard, "The rapid evaluation of potential fields in particle systems", Cambridge, MA, USA, 1988

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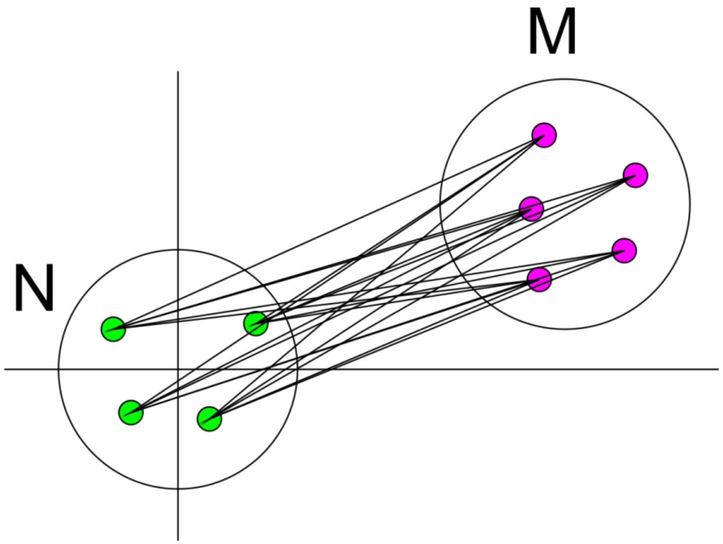
The interaction between two groups of sources

- Consider the line current elements as point sources for simplicity
- Calculate the potential of the N particles at the M locations
- The total number of direct interactions is MxN
- The objective is to facilitate this calculation with mathematical tricks

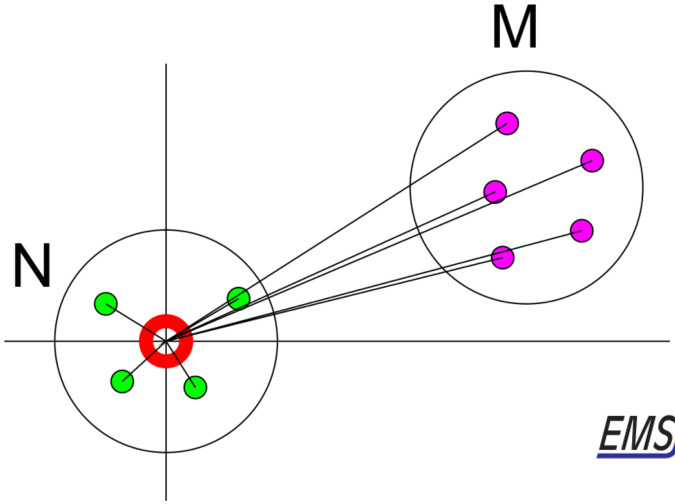


Multipole expansion of the group of sources

- The potential at one target point due to the N sources can be written as a series expansion

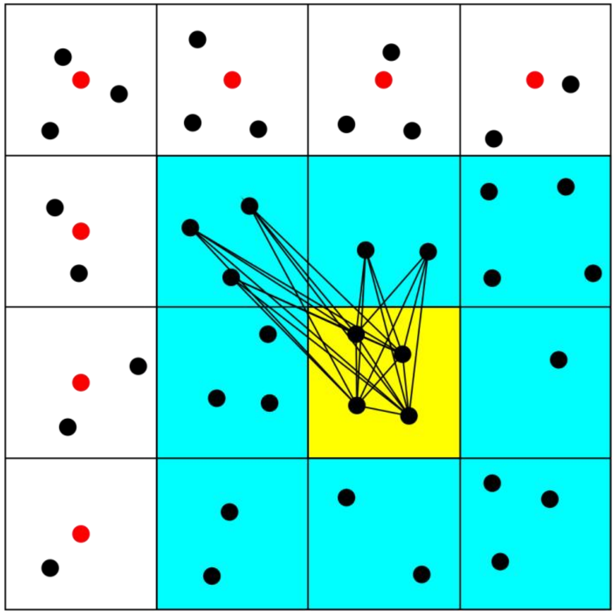
$$\phi = \sum_{k=1}^{N} \frac{q_k}{\left| r_{s,k} - r_t \right|} = \sum_{n=0}^{p} \sum_{m=-n}^{n} \frac{M_n^m}{r_t^{n+1}} Y_n^m, \qquad M_n^m = \sum_{k=1}^{N} q_k r_{s,k}^n Y_n^{-m}$$

- M_n^m (the red circle) is called a Multipole, which can be used to calculate the effect of all N sources on any of the M targets;
- Under given constraints about the distance between the sources and targets, the maximum error is a known function of p.
- The computational effort for to compute the potential in all other sources in this configuration is now O(p^2*(M + N))



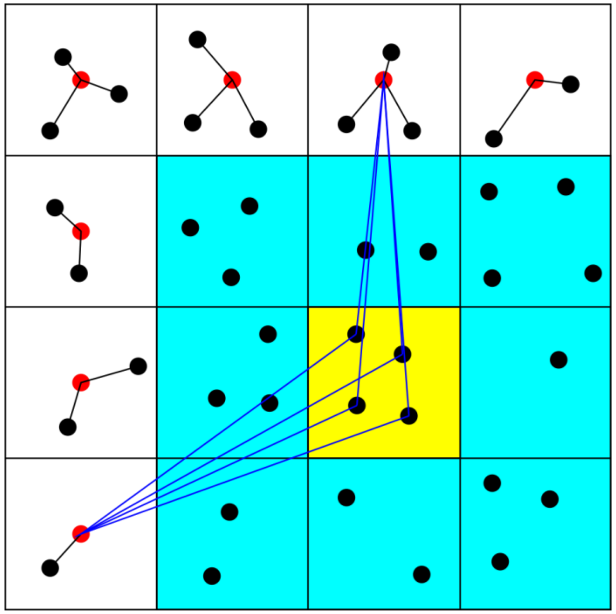
Distributing the sources among boxes

- First step in the implementation of the FMM is to distribute the sources among a grid of uniformly shaped boxes;
- The illustration shows the interactions between the particles in the yellow box and all other boxes
- Near field interactions are calculated directly between particles inside this box (yellow), and all neighbouring boxes (blue)



2D illustration: Multipole expansions

- Far field interactions can then be calculated with multipoles around the centre of each box (red circles);
- The potential at the targets can then be calculated by expanding the multipoles to the target locations;
- Expanding the multipoles on each individual target particle is still computational intense…

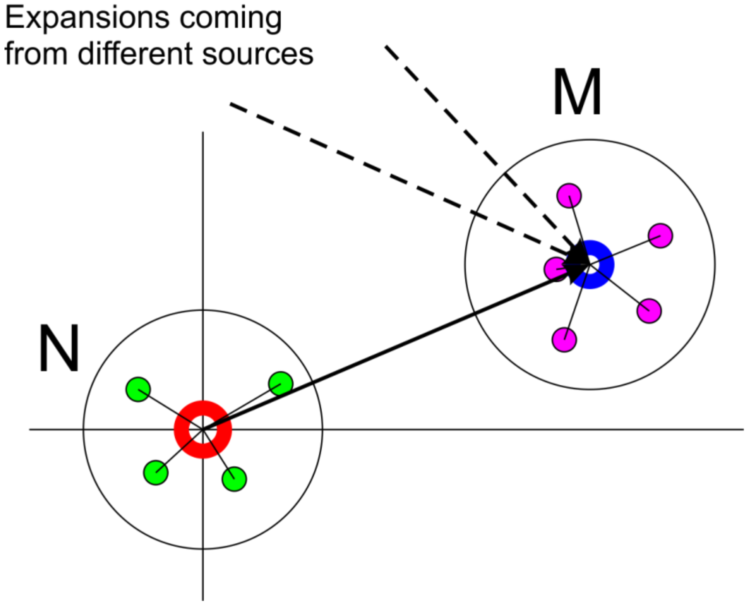


Conversion of Multipole expansions to Local expansions

- A multipole expansion can be converted into a local expansion by

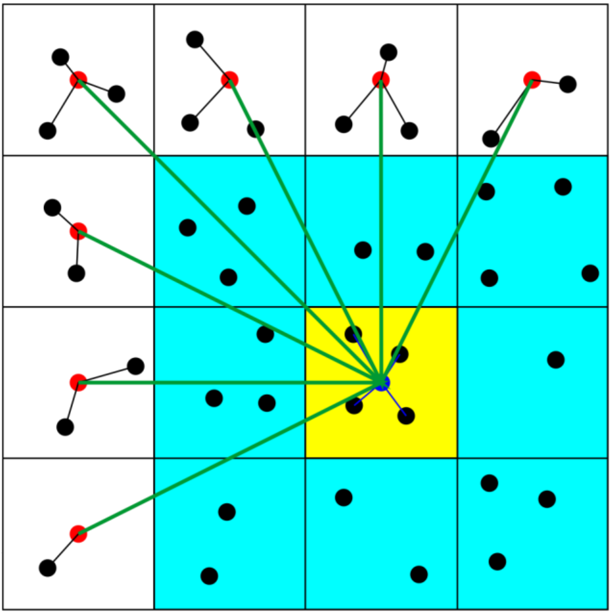
$$\phi = \sum_{j=0}^{p} \sum_{k=-j}^{j} L_j^k \cdot Y_j^k r_t^{\,j},$$

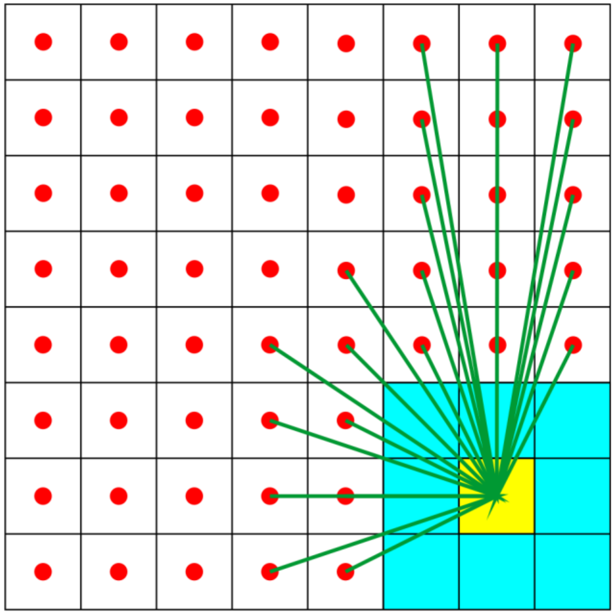
- Where $L_j^k = f\left(M_n^m, r_{mp}\right)$, $r_{mp} =$ location of the multipole expansion
- Since expansions can be added, this treatment eventually reduces the number of required operations



2D illustration: Multipole-to-Local (M2L) expansions

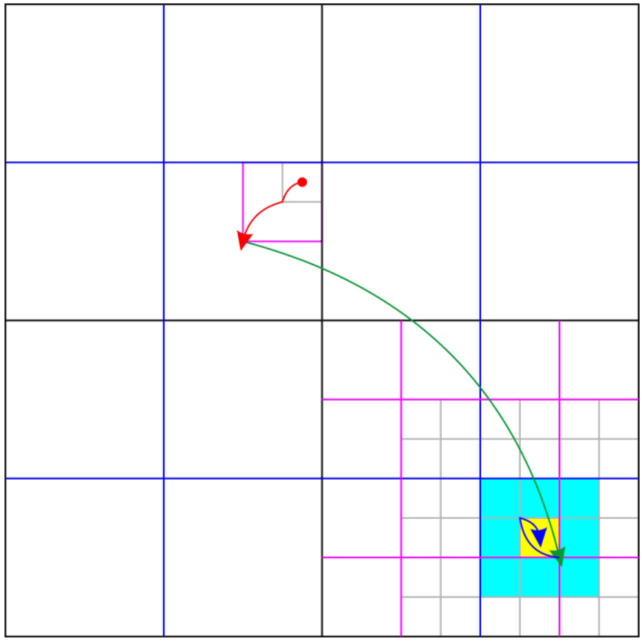
- The local expansions are now calculated at the centre of the yellow box, from where the potential at target locations can be calculated;
- However, for larger systems that need more boxes, the number of M2L computations can still be high (only a few are illustrated in the right figure);
- This is where the “multi-level” concept is introduced…





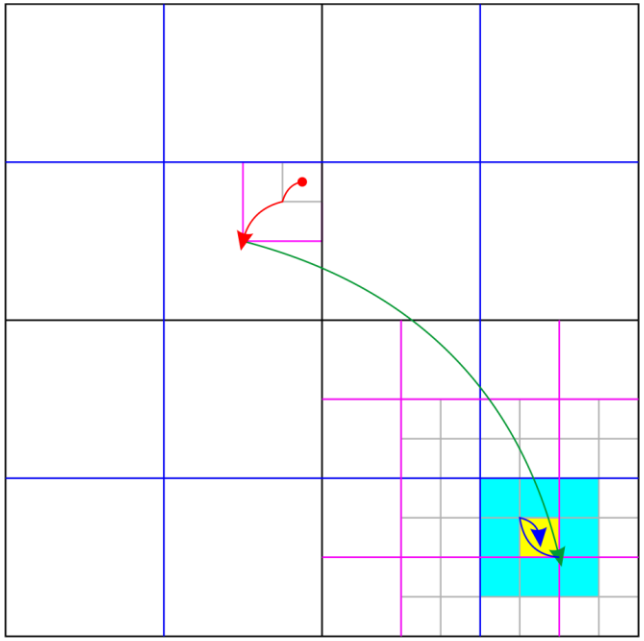
2D illustration: Multi-Level Fast Multipole Method

- The fast multipole methods also provides ways to translate multipole expansions and local expansions to different coordinates;
- Since these expansions can be added when calculated on the same coordinates, expansions from different locations can be grouped into bigger boxes;
- The further away from the target locations, the bigger the area that can be grouped without loss of accuracy;



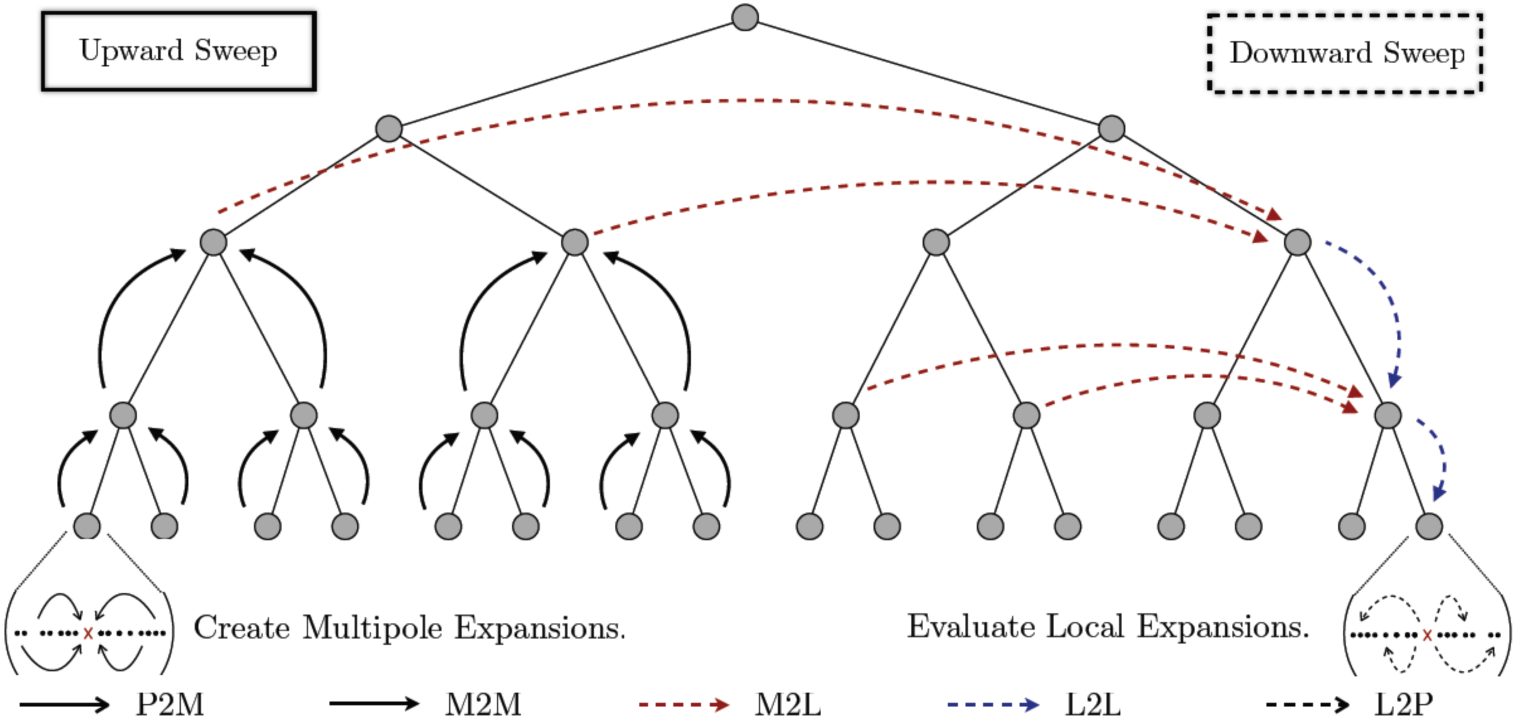
2D illustration: Multi-Level Fast Multipole Method

- This illustration shows two translations of multipole and local expansions (red and blue lines, respectively) and one M2L conversion (green line)
- There are three levels on which the multipole method is carried out.
- If N_L is the number of levels, there are always 4^{N_L} boxes in a 2D simulation, and 8^{N_L} in a 3D simulation.



Hierarchical tree concept

- This chart illustrates the MLFMM concept with a hierarchical tree:
  - P2M: Particle to Multipole expansion
  - M2M: Multipole to Multipole translation
  - M2L: Multipole to Local expansion conversion
  - L2L: Local to Local translation
  - L2P: Local to Particle conversion



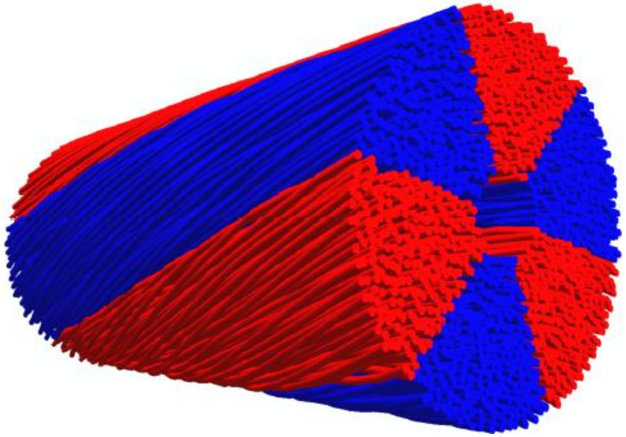
Courtesy: L.A. Barba, Boston University

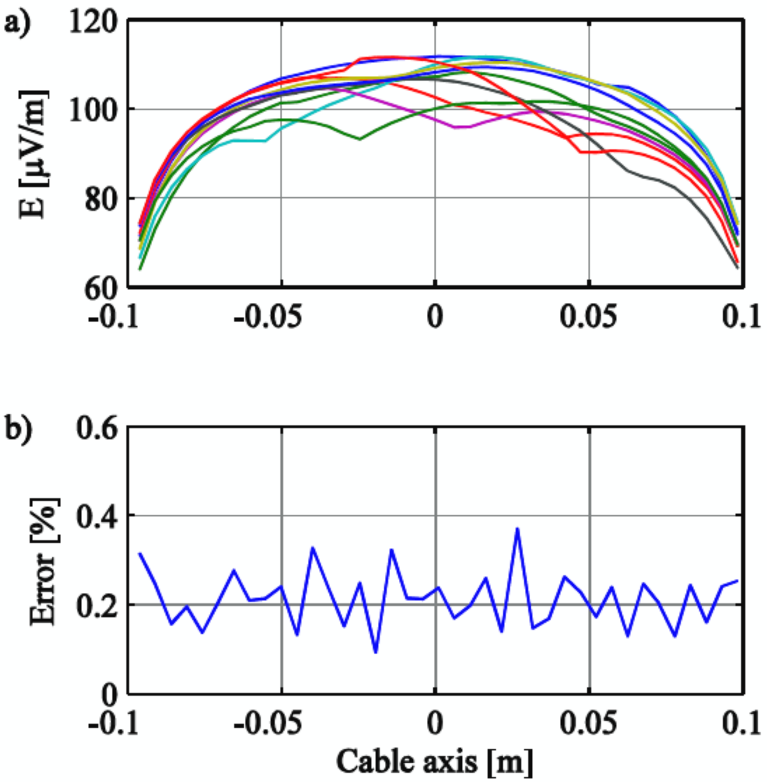
Considerations for the implementation

- The MLFMM is developed for calculating the scalar field of moving particles, from fluid dynamics to stellar systems;
- Its algorithm is suitable for parallel computation;
- Our system has to calculate the vector field of fixed objects (strand segments);
- Advantage: The distribution of objects has to be done only once, prior to a simulation;
- Drawback: The evaluation of the multipole method has to be done three times, one for each dimension.

Verification of the MLFMM model

- Verification of the MLFMM is done by comparing the results with direct calculations;
- This example shows the simulation of a 20 cm long ITER PF1 cable, which has 1440 superconducting strands;
- The induced electric field due to a 1 A/s current ramp in 10 strands is shown;
- The error is less than 0.5 % everywhere.





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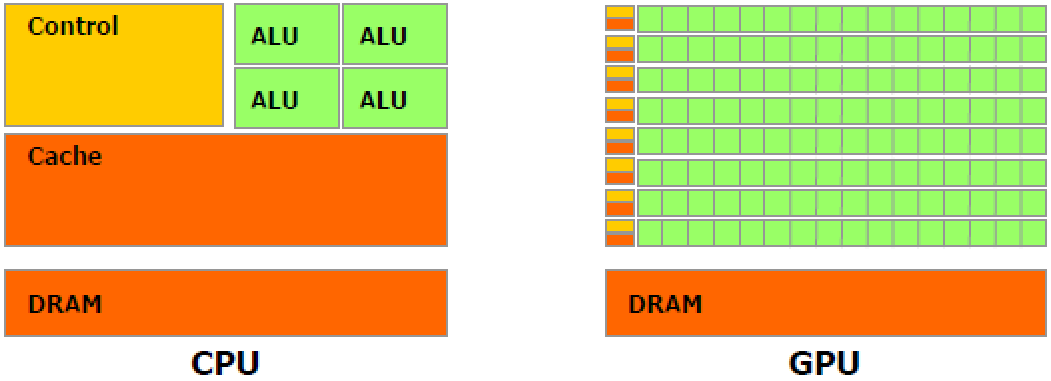
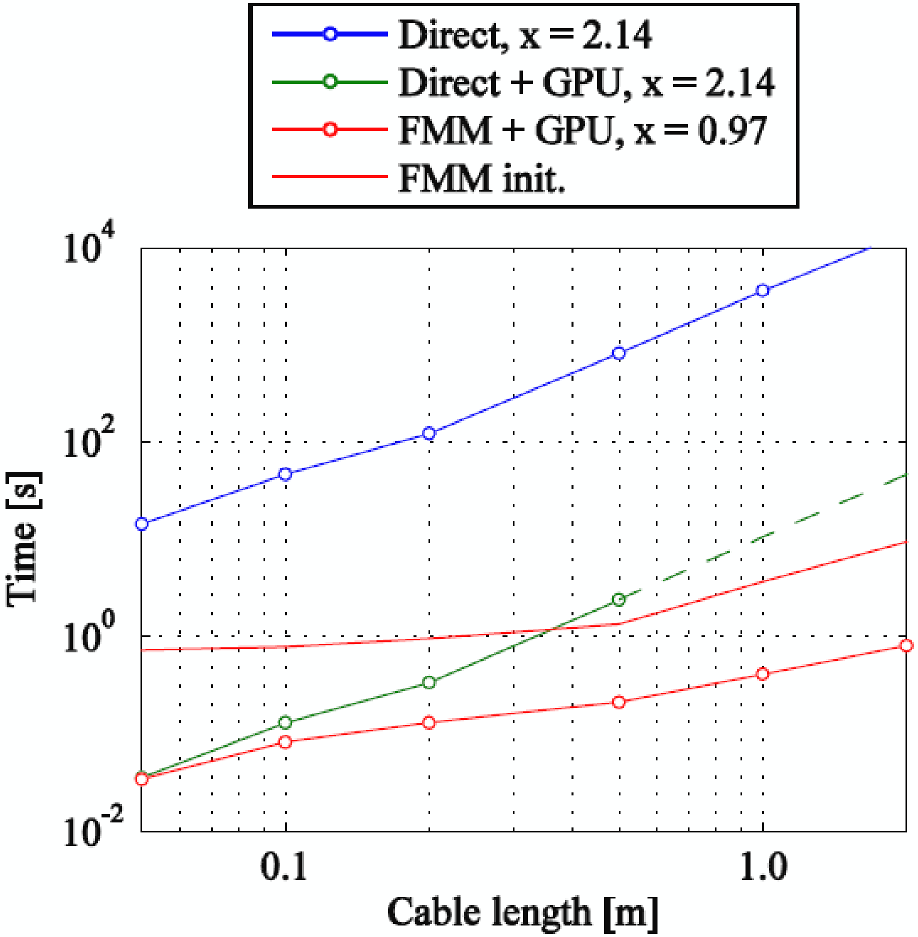
Implementation of the method on a GPU

- The mutually induced voltages can be calculated in parallel, but a CPU usually has only up to eight cores;
- In the recent years, Graphics Processing Units (GPUs) have been developed, which consist of many hundreds of cores;
- Their main advantage is that they can carry out many computations in parallel on low-cost hardware;
- For this reason, they are now also widely used for scientific calculations;
- In JackPot-AC, both the MLFMM and the direct computation for the mutual coupling are implemented for use on a GPU.



Speedup acquired with the MLFMM method

- The time is measured for the calculation of the coupling in an ITER PF1 cable with different length (and thus number of variables);
- Implementation of the direct method on the GPU lead to a more than 100 times faster solution;
- The overhead for the MLFMM (FMM init.) is required only once for every simulation;
- The x in the legend refers to the relationship:
  - time = (length)^x;
- A fast, and O(N) solution is obtained with the MLFMM on the GPU.



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Considerations for solving the system

- The system can generally be described as

$$\mathbf{M}\frac{dx}{dt} = \mathbf{A}x + S,$$

- Which, after discretisation becomes

$$(\mathbf{M} - \Delta t \cdot \mathbf{A})x_n = \mathbf{M}x_{n-1} + \Delta t \cdot S_n.$$

- This makes it impossible to solve the system directly, since the MLFMM produces matrix-vector products (M*x_n) instead of a matrix;
- As such, the system is solved iteratively, which takes away the need to construct the system matrix explicitly;

Iterative solution of the system matrix

- A strongly simplified explanation of the iterative solution of the system is as follows:
- Start with an initial guess $x_{n,0}$ for x_n, and repeat the following until x_n is accurate enough (or the residual vector r_n is small enough):

$$r_k = \left(\mathbf{M} - \Delta t \cdot \mathbf{A}\right) x_{n,k} - \mathbf{M} x_{n-1} + \Delta t \cdot S_n,$$

$$p_k = \mathbf{B}^{-1} r_k,$$

$$x_{n,k+1} = x_{n,k} + p_k,$$

- This requires a preconditioner matrix **B**, which resembles, but is not equal to the system matrix: $\mathbf{B} \approx \left(\mathbf{M} - \Delta t \cdot \mathbf{A}\right)$;
- For JackPot-AC, incomplete LU factorisation is used, where matrix **M** contains the self-inductances and the mutual inductances of only the closest strands;
- At present, calculating the preconditioner matrix is the bottleneck in the computation time for JackPot-AC.

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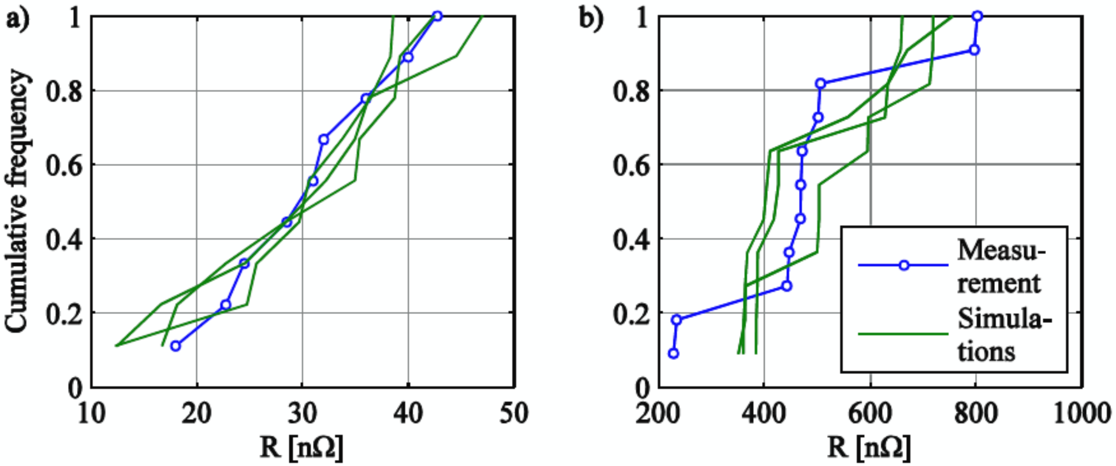
Results: obtaining the interstrand contact resistivity

- A contact resistivity parameter is required for the calculation of the interstrand resistances;
- Interstrand resistances are measured on CICCs in the Twente press by connecting sets of two strands to a power source;
- By simulating this experiment, the parameter is found by adjusting it until the simulation results match the measurements;
- For cables with sub-cable wraps, two parameters are used to account for the higher contact resistivity between strands from different sub-cables…



Intra-petal R

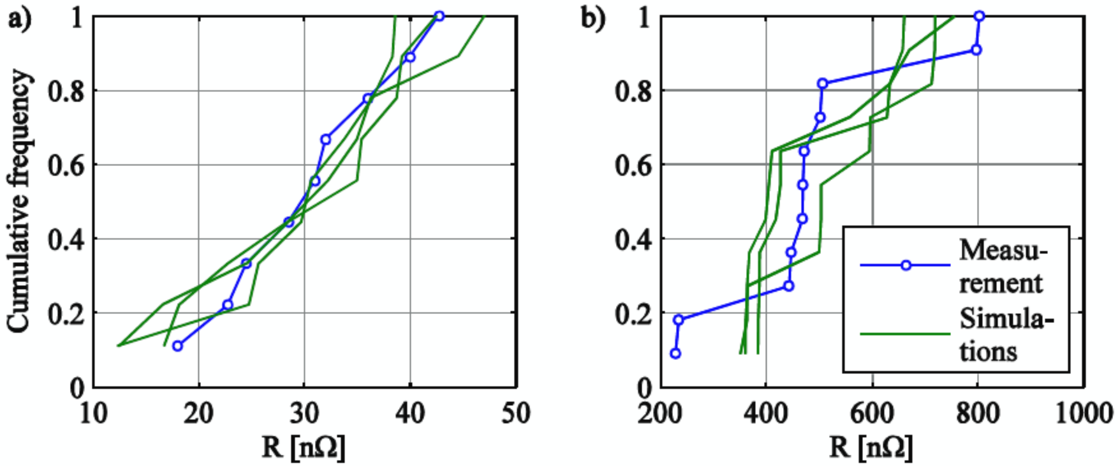
Inter-petal R

Results: obtaining the interstrand contact resistivity

- The TFJA-5 was used to find the parameters below;
  - Intra-petal R: resistance between strands from the same petal;
  - Inter-petal R: resistance between strands from different petals;
- Since the offset phase of the sub-cables are unknown, different simulations are carried out with different sets of random phase angles (three in the simulation below)

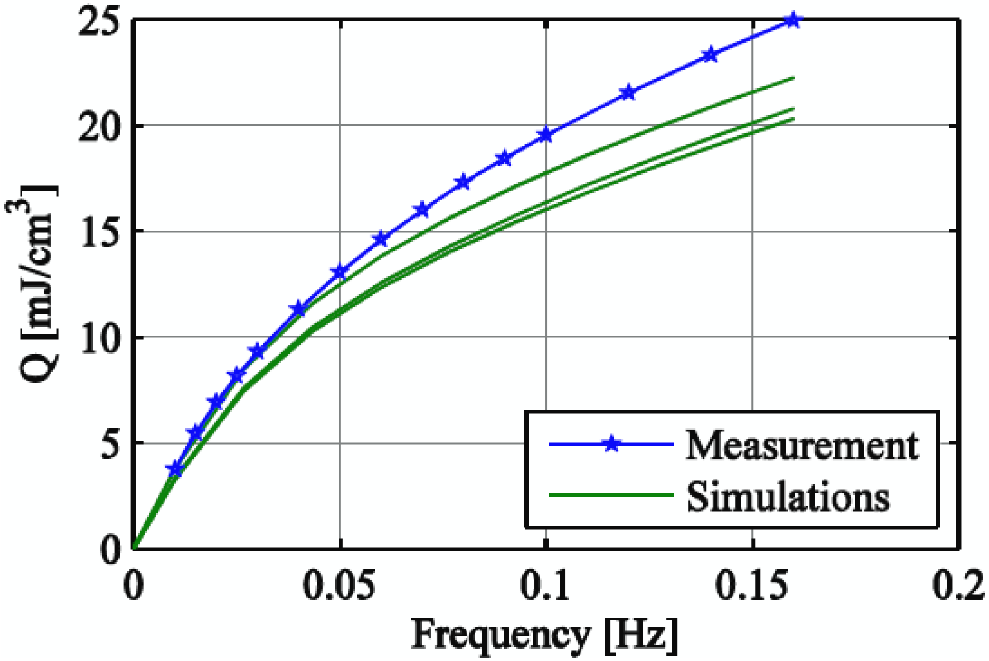
Intra-petal R

Inter-petal R



Results: Frequency response

- The power dissipation in a CICC is measured in a uniform, harmonic background field at different frequencies;
- This is also simulated, with the contact resistivity parameters obtained before;
- This leads to good agreement with the measurements.



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Conclusions

- We have implemented the MLFMM on a GPU for calculating the mutual inductance between N strand segments;
- This has lead to a considerable speedup in the calculations, and it has been demonstrated that the time dependence approaches O(N) for the O(N^2) mutual couplings;
- A small number of expansions (p) is needed to satisfy the accuracy demands of the cable model;
- The model has been validated with one measured ITER TF conductor sample;
- It has been demonstrated that the model can analyse the coupling loss of an ITER size conductor in any type of background field within a reasonable simulation time.