Implementation of the superfluid helium phase transition using 3D FE modeling: simulations of hot spot burnout, recovery and phase front movement

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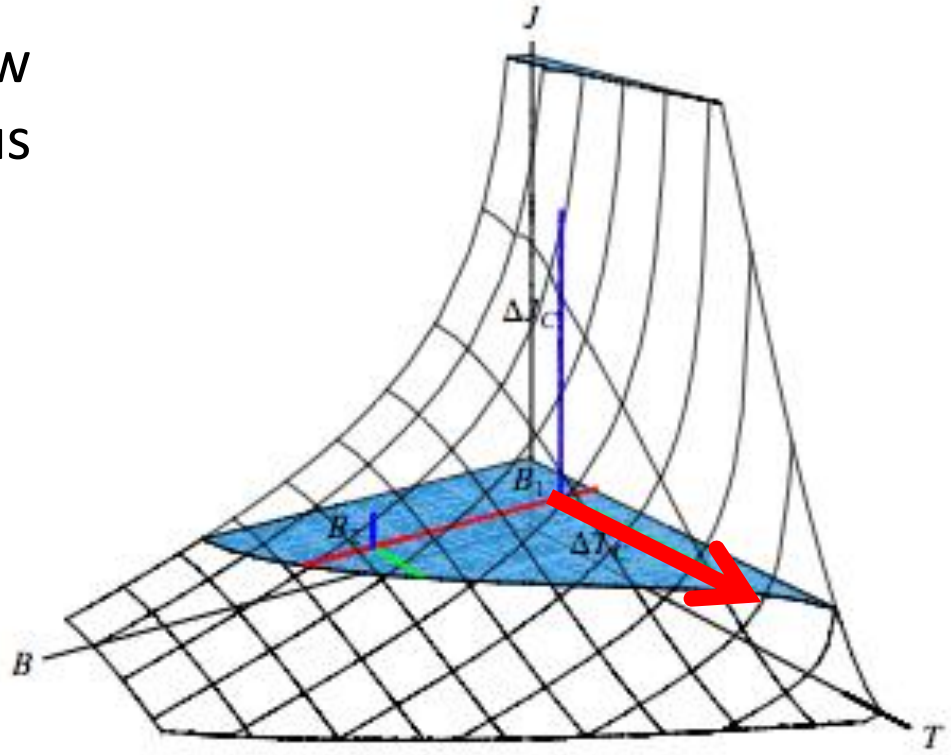
Contents

Introduction

The operating point of superconducting magnets needs to lie underneath the critical surface.

Lower operating temperatures allow for higher current densities and thus stronger magnetic fields.

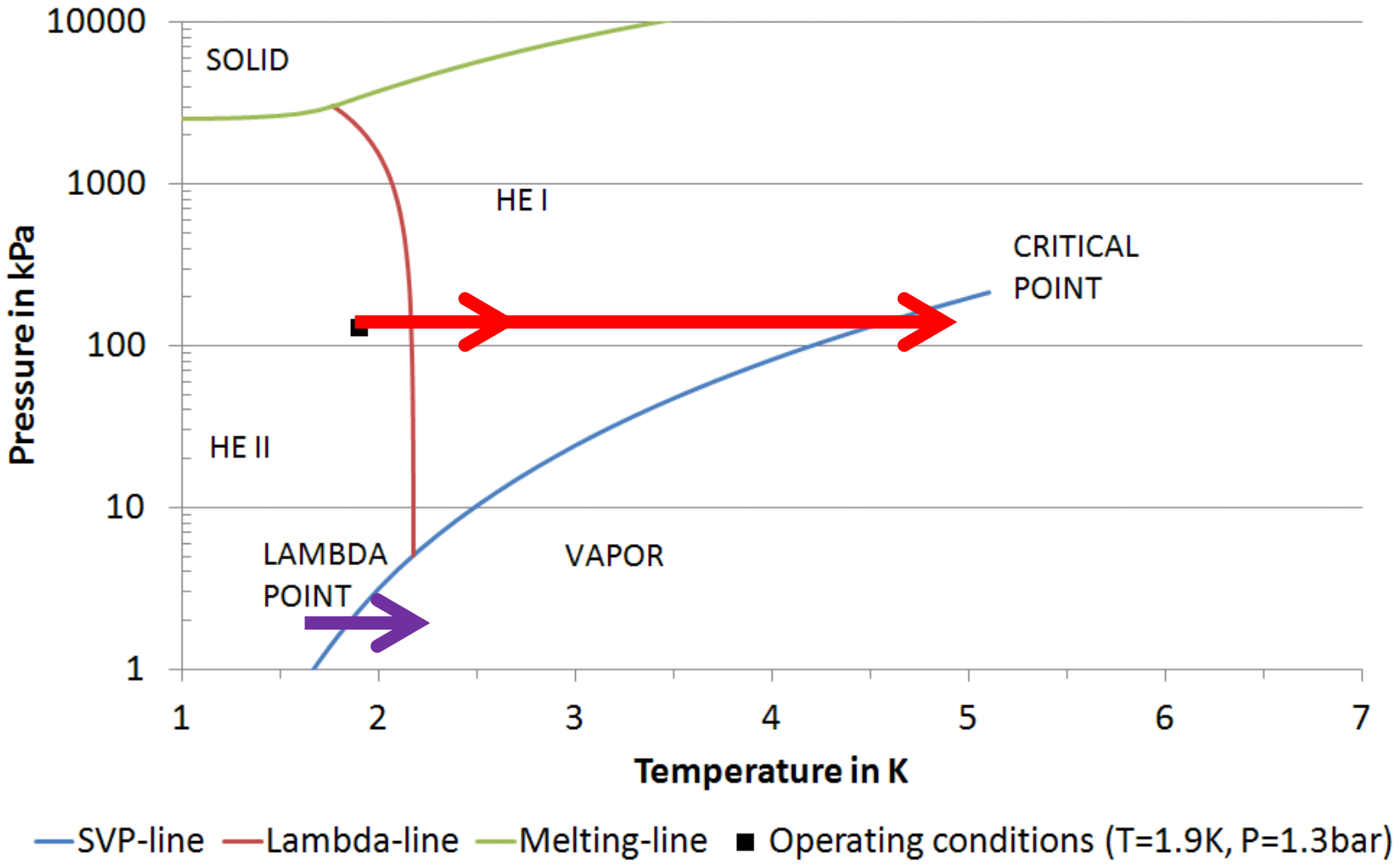
In the case of Nb-Ti technology, magnets are typically cooled with helium. The operating temperature of the inner triplet quadrupole magnets of the LHC is 1.9 K in a static superfluid helium bath.



Critical surface of Nb-Ti

COMSOL Multiphysics is used to implement helium bahavior

Helium phase diagram 1



Helium phase diagram 2

Small differences in published data occur. **HEPAK** gives:
Lambda point pressure = 5.041 kPa
Lambda point temperature = 2.1768 K
Critical point pressure = 227.46 kPa
Critical point temperature = 5.1953 K

Therefore two typical situations can be distinguished:
Saturated condition below the lambda point (both temperature and pressure).
Subcooled condition at temperatures below the lambda temperature and pressures above the lambda point pressure.

Inner triplet quadrupole magnets operate in the subcooled condition.

Critical Heat Flux

Critical Heat Flux (CHF) is defined as the maximum applied heat flux at a solid-helium interface such that the helium at the interface just stays superfluid and does not undergo a lambda-transition.

It must be stretched that heat transport in He II places no fundamental limit to the maximum steady state heat transfer. The allowable temperature difference is the limiting factor and depends strongly on the applied geometry.

Two important questions arise for engineering purposes:

What does the temperature distribution in the helium (and the total structure) looks like and how can you stay below the CHF?

How does the heat transfer behave when the critical heat flux is exceeded and is recovery possible?

q<CHF using $f(T)^{-1}$

For q<CHF temperature distributions in one-dimensional (1D) channels can be calculated easily assuming that the heat transfer can be described using the Gorter-Mellink equation for turbulent heat transport in Tisza's two-fluid model:

$$\frac{\mathrm{d}T}{\mathrm{d}x} = -f(T)q^m$$

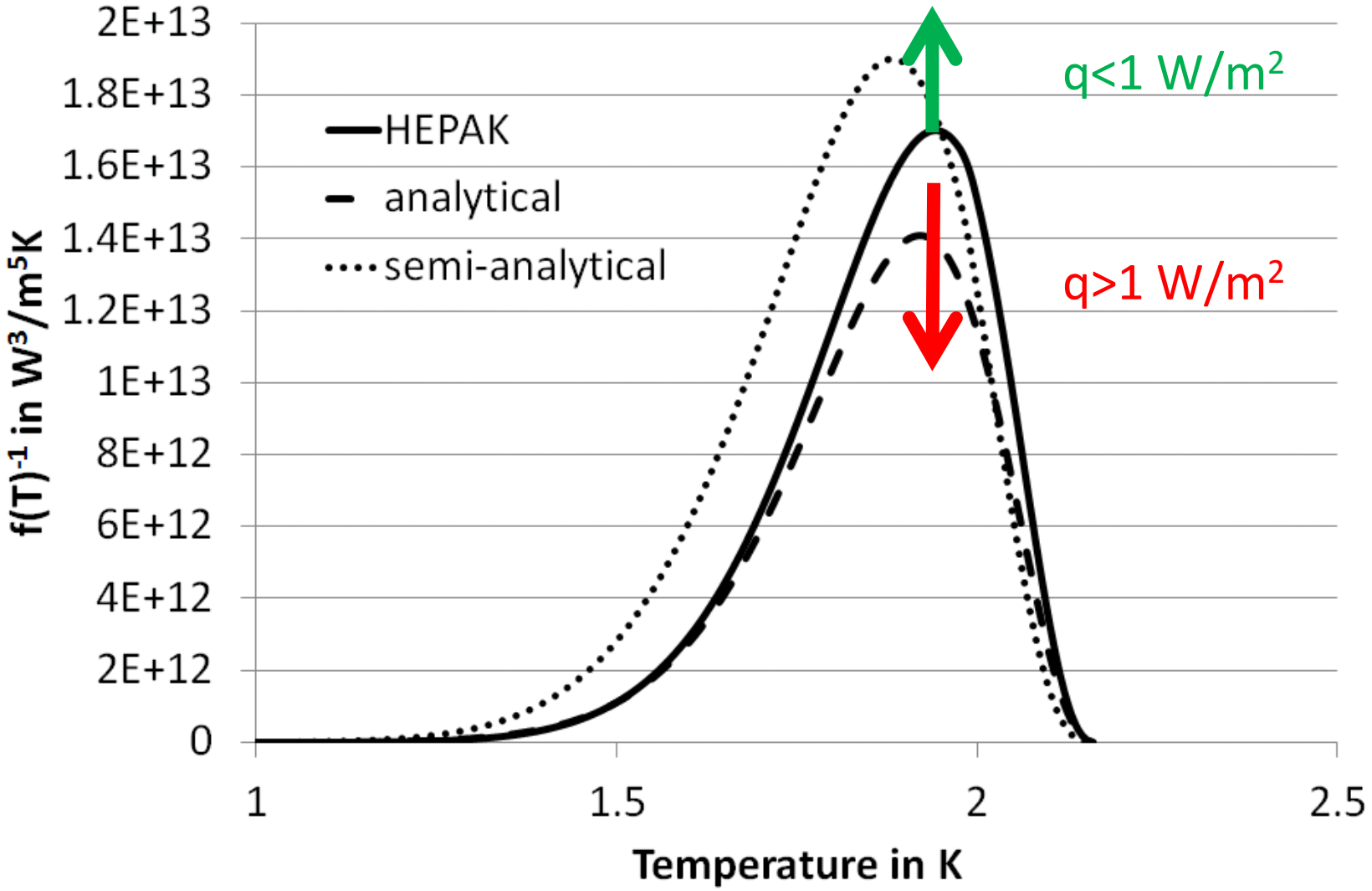
Assuming the theoretical value of m=3 this results in:

$$q^3 = -\frac{1}{f(T)}\frac{\mathrm{d}T}{\mathrm{d}x}$$

Rewriting this into a Fourier conduction like equation implies the use of an effective thermal conductivity:

$$q = -k_{eff}\nabla T \qquad q = -\frac{1}{q^2 f(T)}\frac{\mathrm{d}T}{\mathrm{d}x} \qquad q = -\left(\left(\frac{1}{|f(T)|}\right)^{\frac{1}{3}}\left(\frac{1}{|\nabla T|}\right)^{\frac{2}{3}}\right)\nabla T$$

$f(T)^{-1}$ graph


2E+13
1.8E+13
1.6E+13
1.4E+13
1.2E+13
1E+13
8E+12
6E+12
4E+12
2E+12
0
f(T)-1 in W3/m5K
HEPAK
analytical
semi-analytical
q<1 W/m2
q>1 W/m2
1
1.5
2
2.5
Temperature in K

$f(T)^{-1}$ function

$$f^{-1}(T,p) = g(T_\lambda)\left(t^{5.7}\left(1 - t^{5.7}\right)\right)^3$$

$$g(T_\lambda) = \frac{\rho^2 s_\lambda^4 T_\lambda^3}{A_\lambda} \qquad t = \frac{T}{T_\lambda}$$

$$s_\lambda = 1559\ \frac{\mathrm{J}}{\mathrm{kg\ K}} \qquad A_\lambda = 1450\ \frac{\mathrm{ms}}{\mathrm{kg}}$$

q<CHF using 'Claudet'

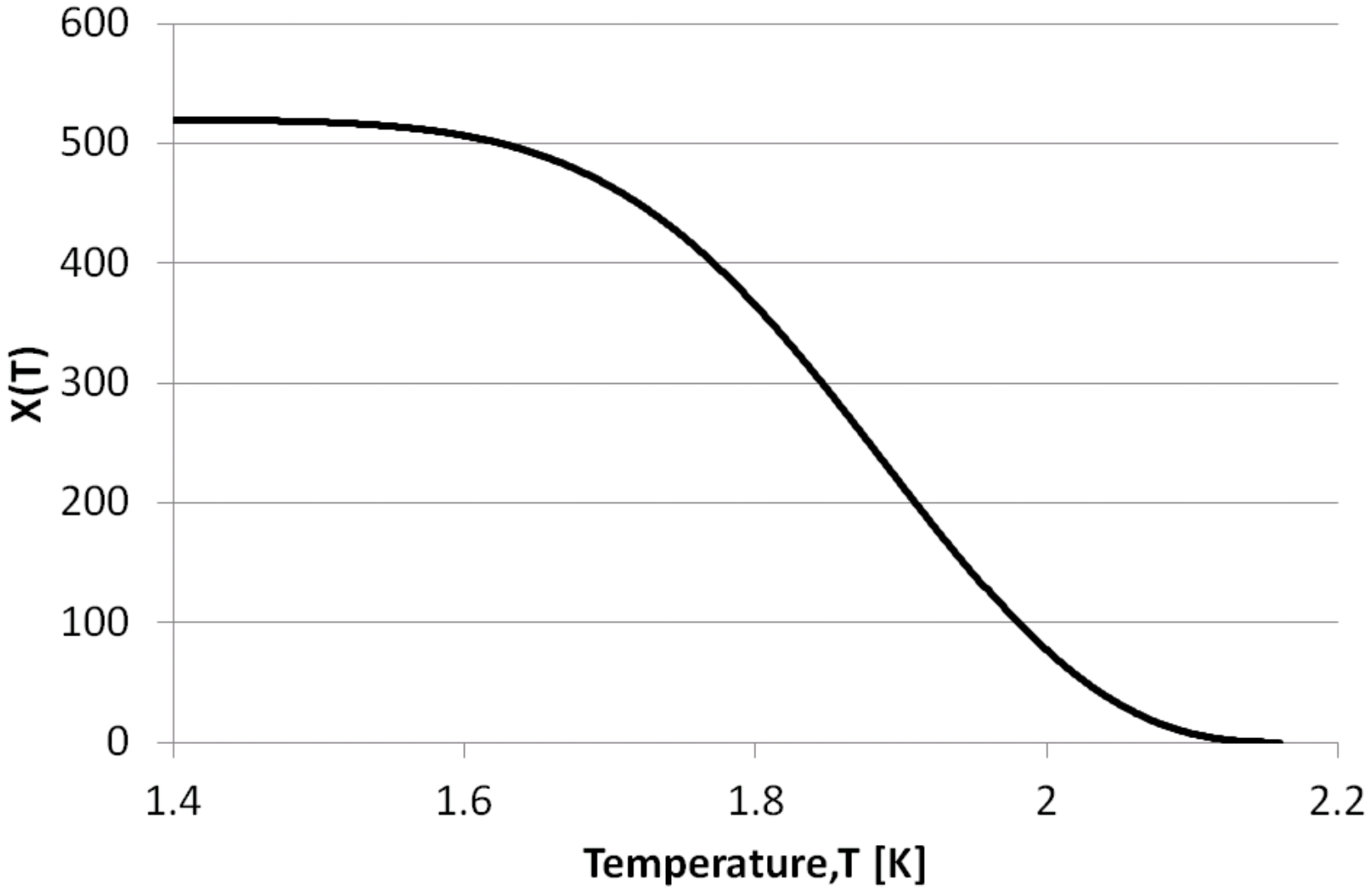
For 1D channels a engineering relation was obtained by Claudet and co-workers. Based on measurements a relation was found between:

- the warm end of a heated channel,
- the heat flux through the channel,
- the channel's length,
- the open end bath temperature.

In principle this is nothing else then integrating the before mentioned $f^{-1}(T)$ function over the correct interval.

Advantage of the $f^{-1}(T)$ function is that it can be applied in FEM applications as a material property and therefore in 2D and 3D as well.
Advantage of the 'Claudet' approach is that it can be used easily in spreadsheets or spice models, but only at atmospheric pressure and 1D.

‘Claudet’ graph



‘Claudet’ function

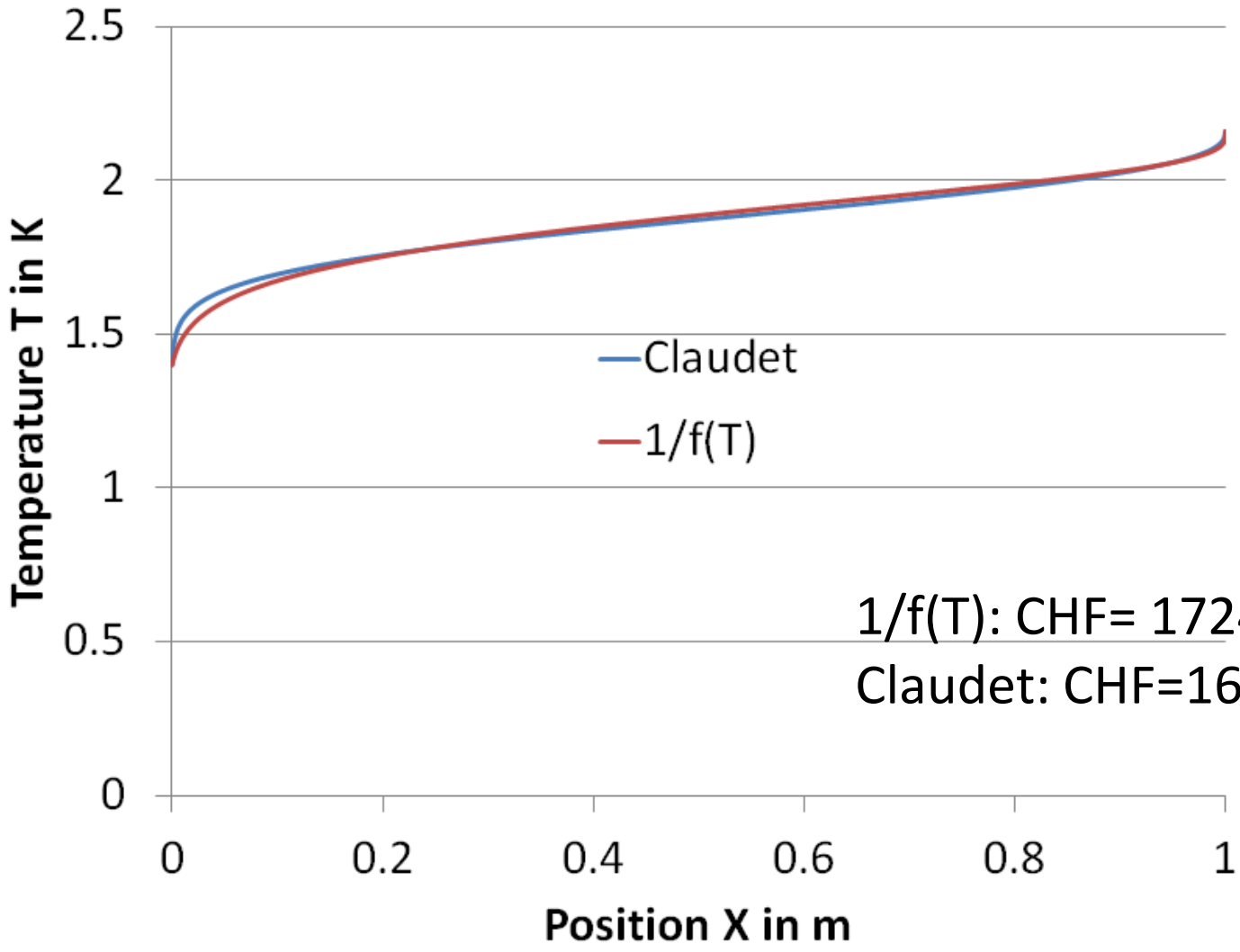
$$X(T) = 520\left(1 - e^{(3(2.16-T))^{2.5}}\right)$$

$$q = \left(\frac{X(T_c) - X(T_w)}{l}\right)^{0.29}$$

For comparison recall:

$$q^3 = -\frac{1}{f(T)}\frac{\mathrm{d}T}{\mathrm{d}x} \longrightarrow q = -\left(\frac{1}{f(T)}\right)^{\frac{1}{3}}\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)^{\frac{1}{3}}$$

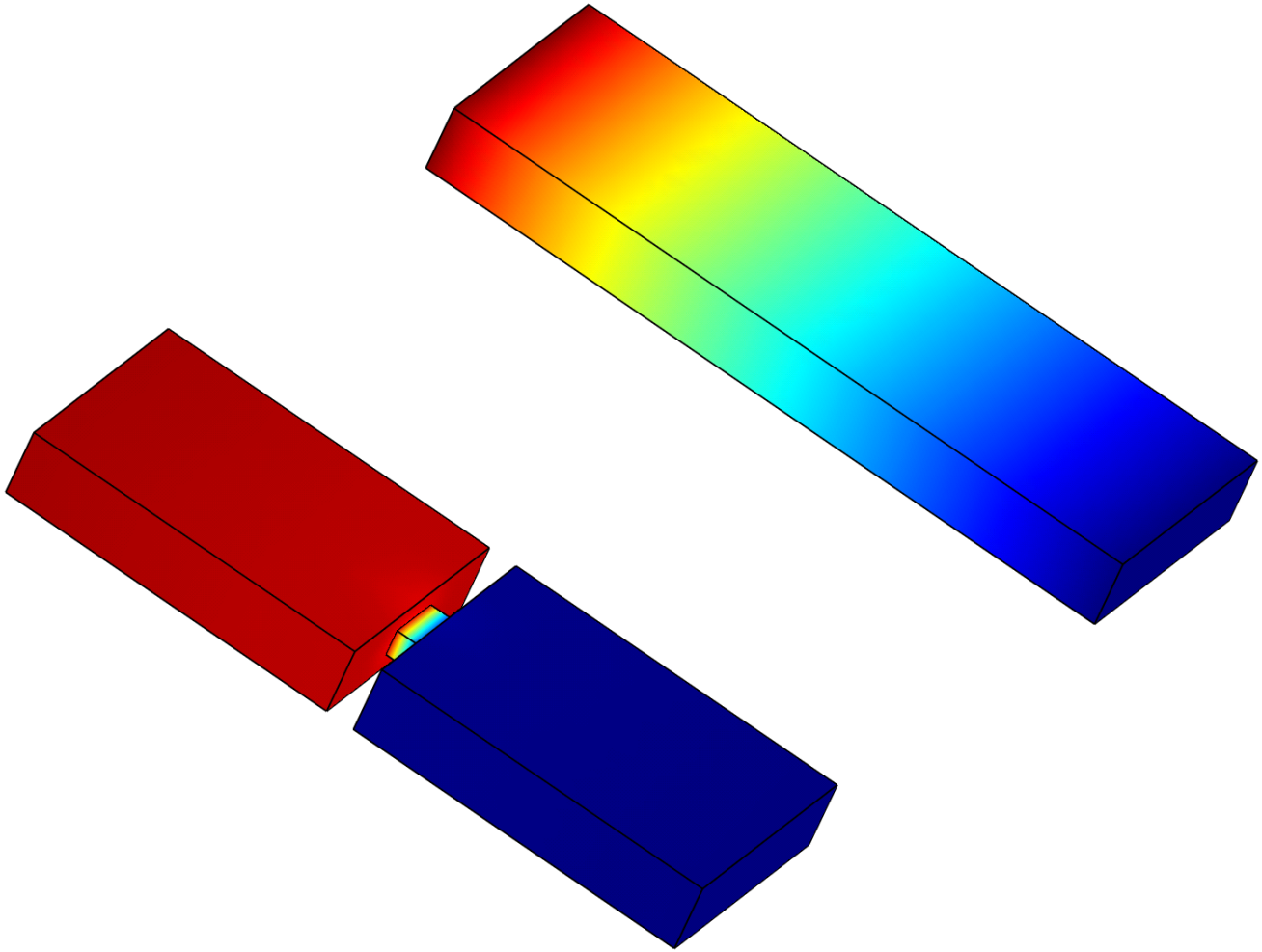
Temperature distributions $<T_\lambda$



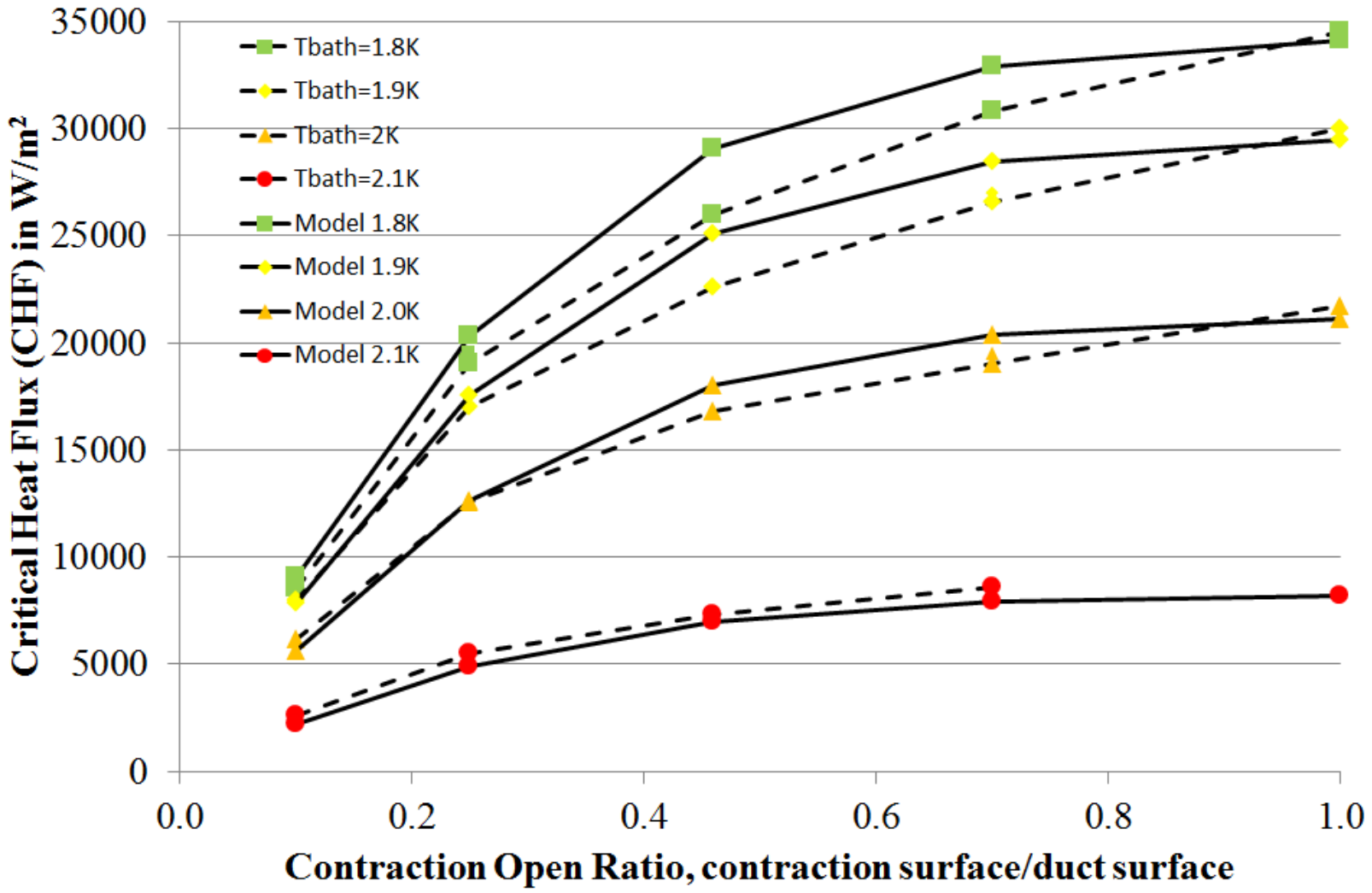
1/f(T): CHF= 17240 W/m²

Claudet: CHF=16128 W/m²

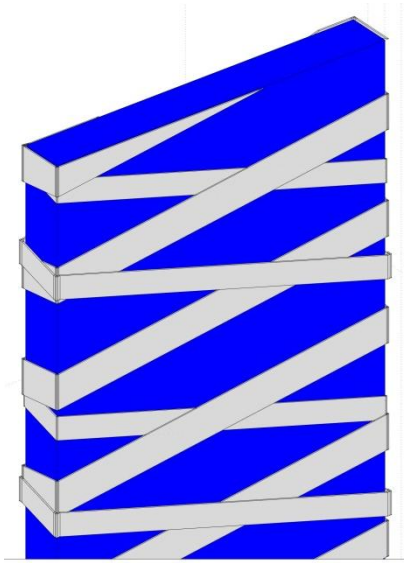
q=CHF

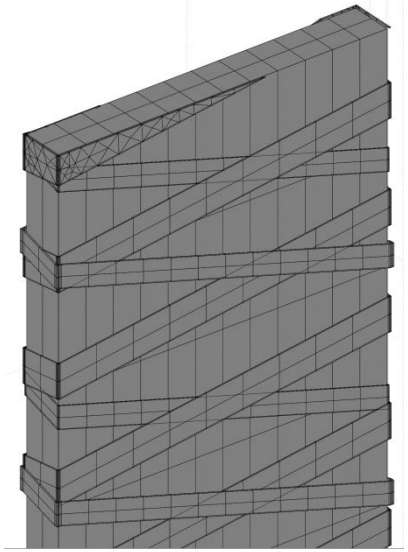


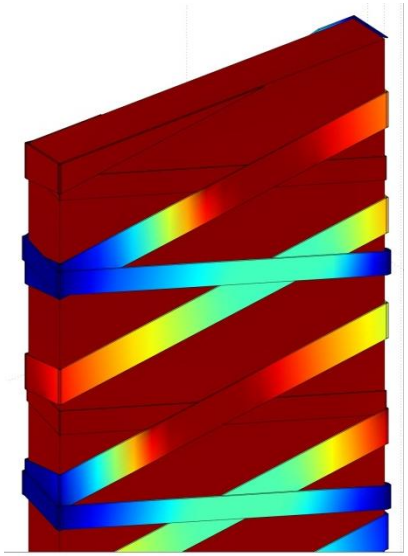
q=CHF


Tbath=1.8K
Tbath=1.9K
Tbath=2K
Tbath=2.1K
Model 1.8K
Model 1.9K
Model 2.0K
Model 2.1K
Critical Heat Flux (CHF) in W/m²
35000
30000
25000
20000
15000
10000
5000
0
0.0
0.2
0.4
0.6
0.8
1.0
Contraction Open Ratio, contraction surface/duct surface

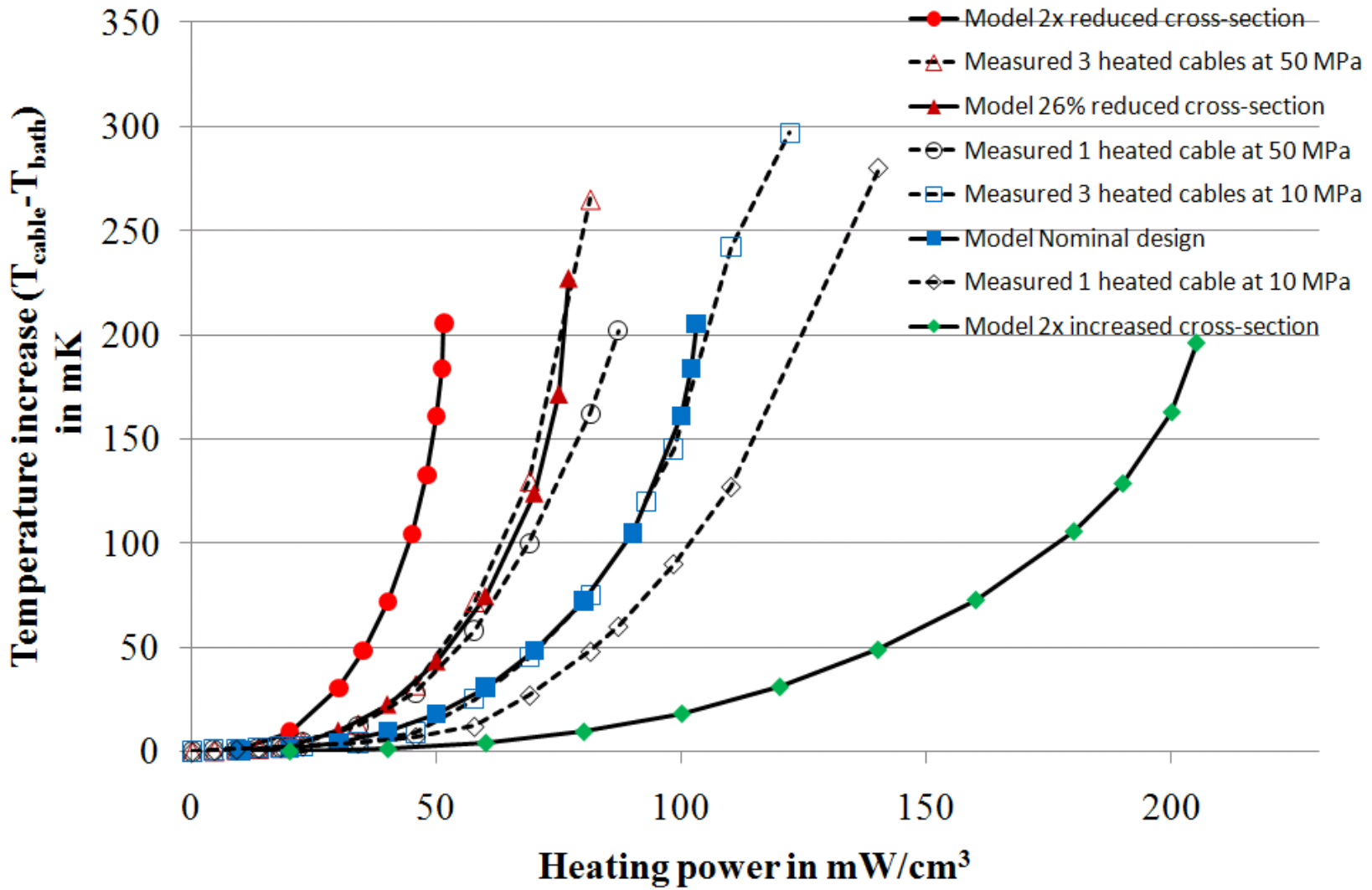
Cable insulation



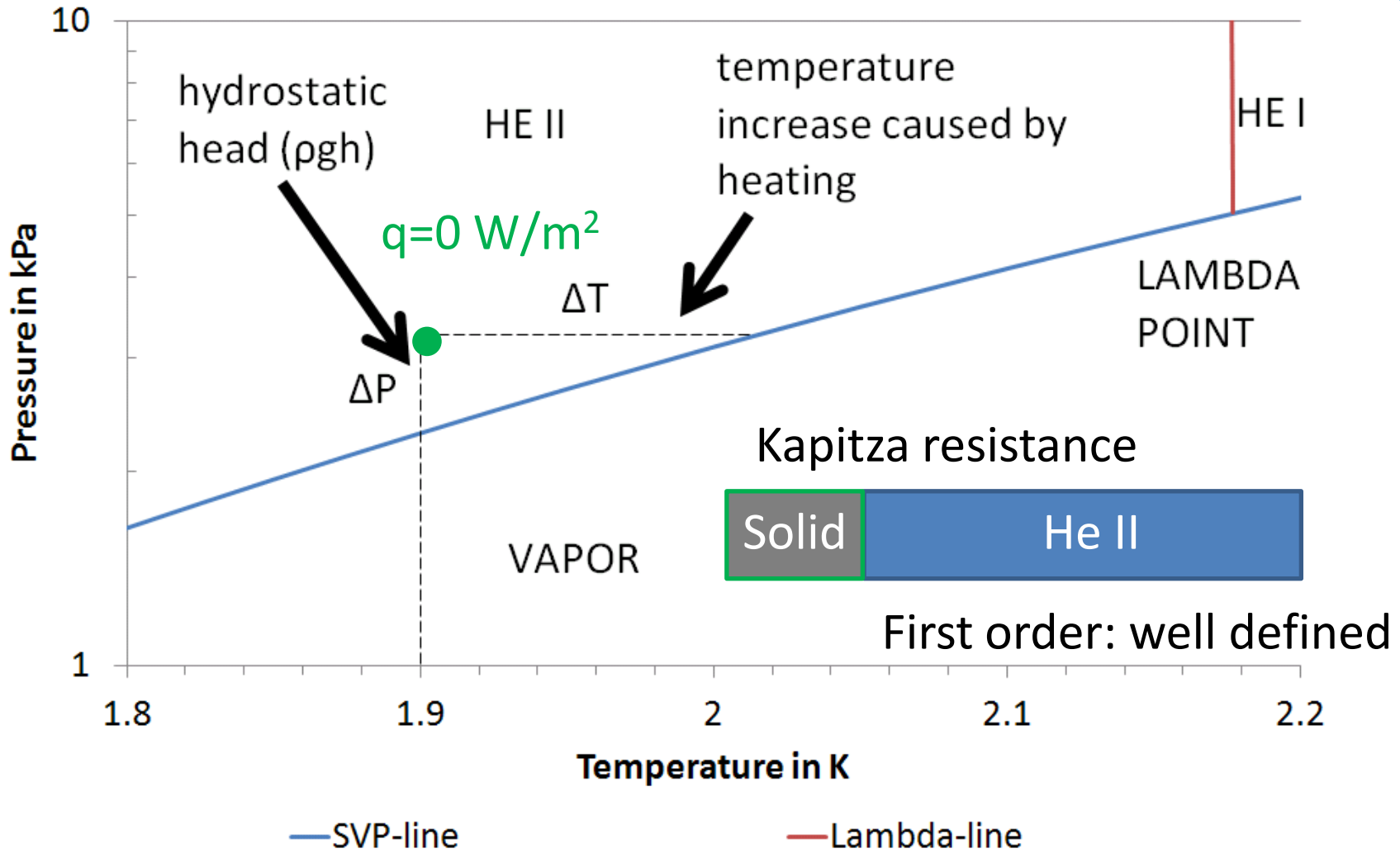




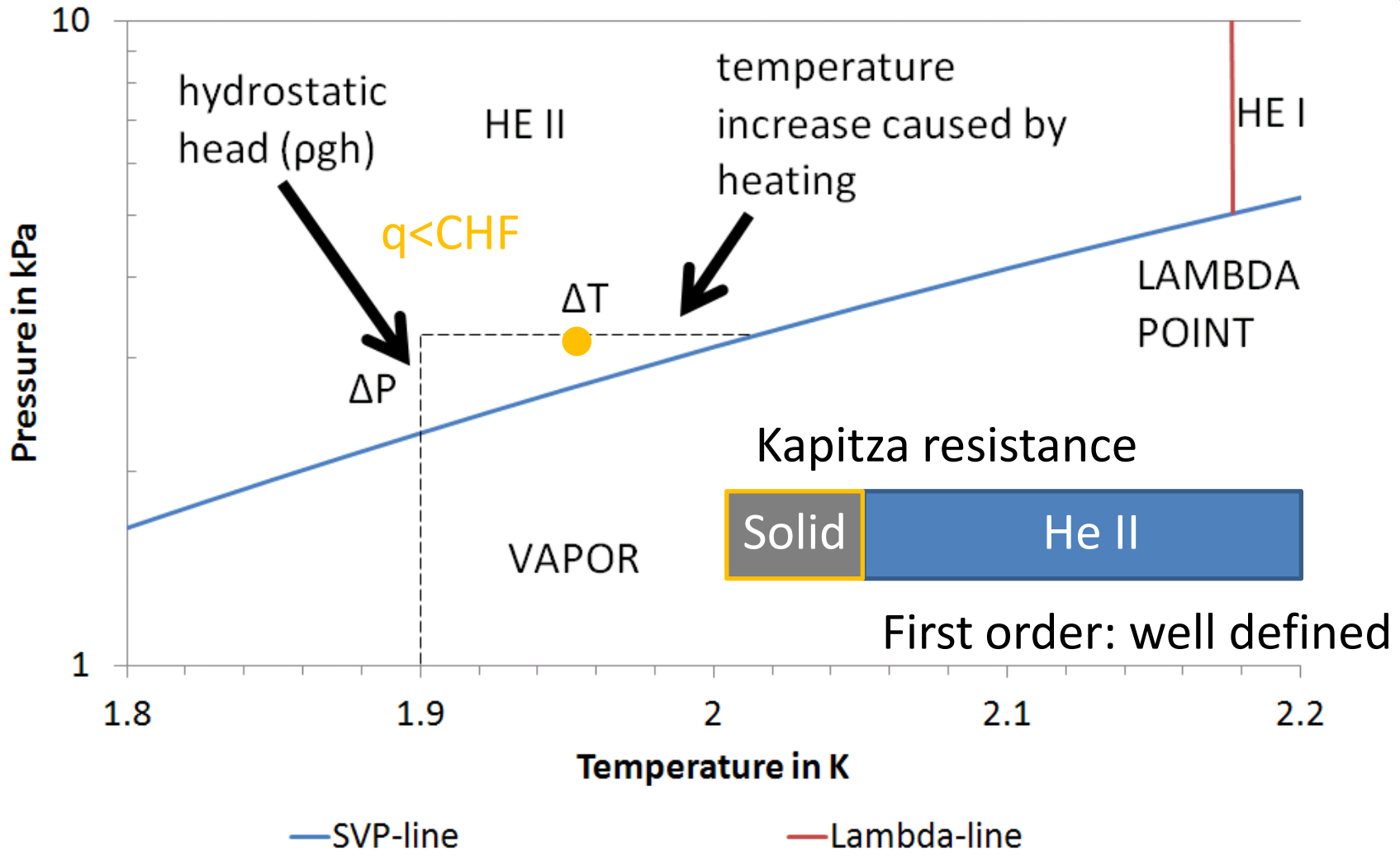
Cable insulation



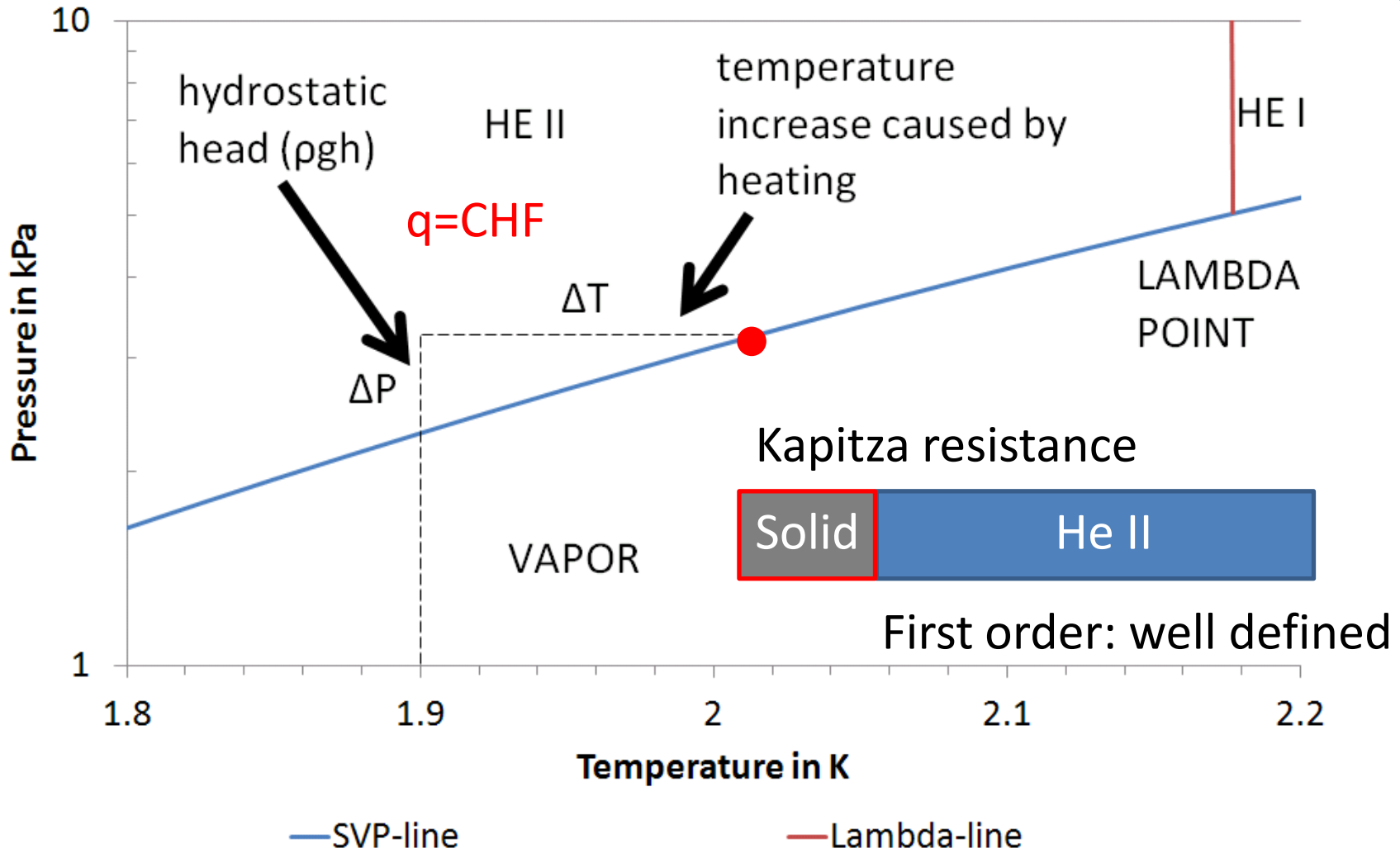
Phase change: saturated condition



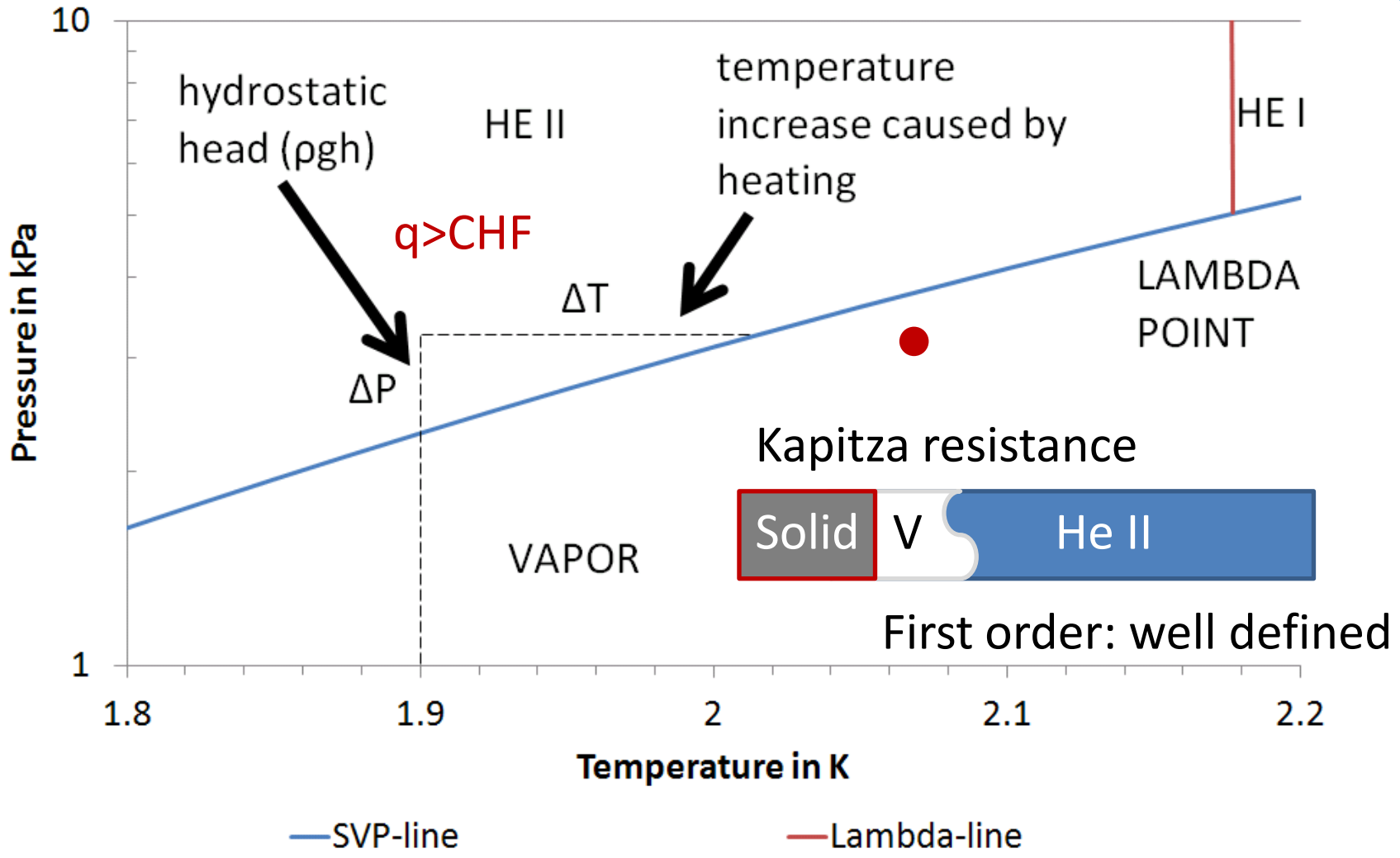
Phase change: saturated condition



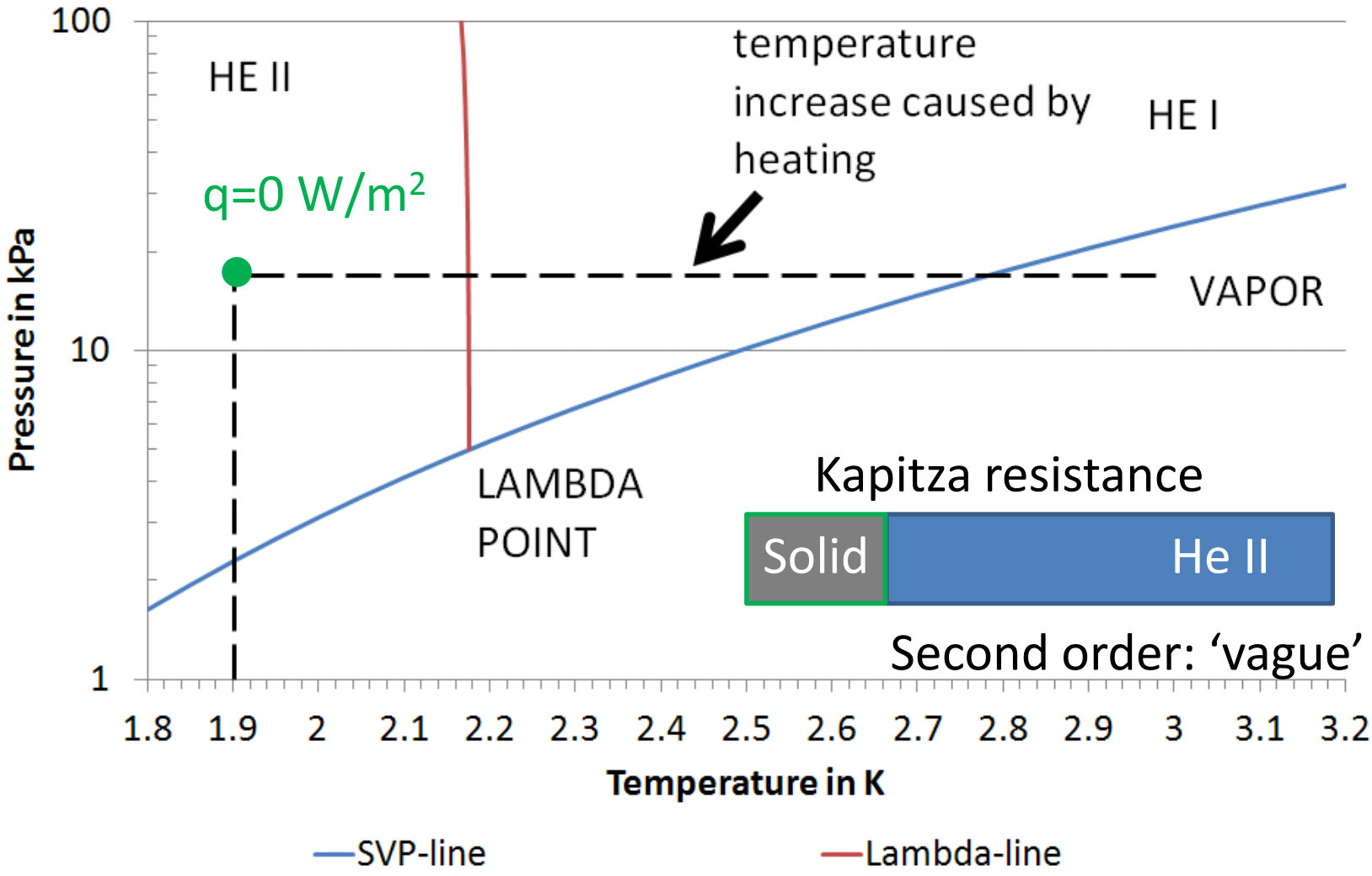
Phase change: saturated condition



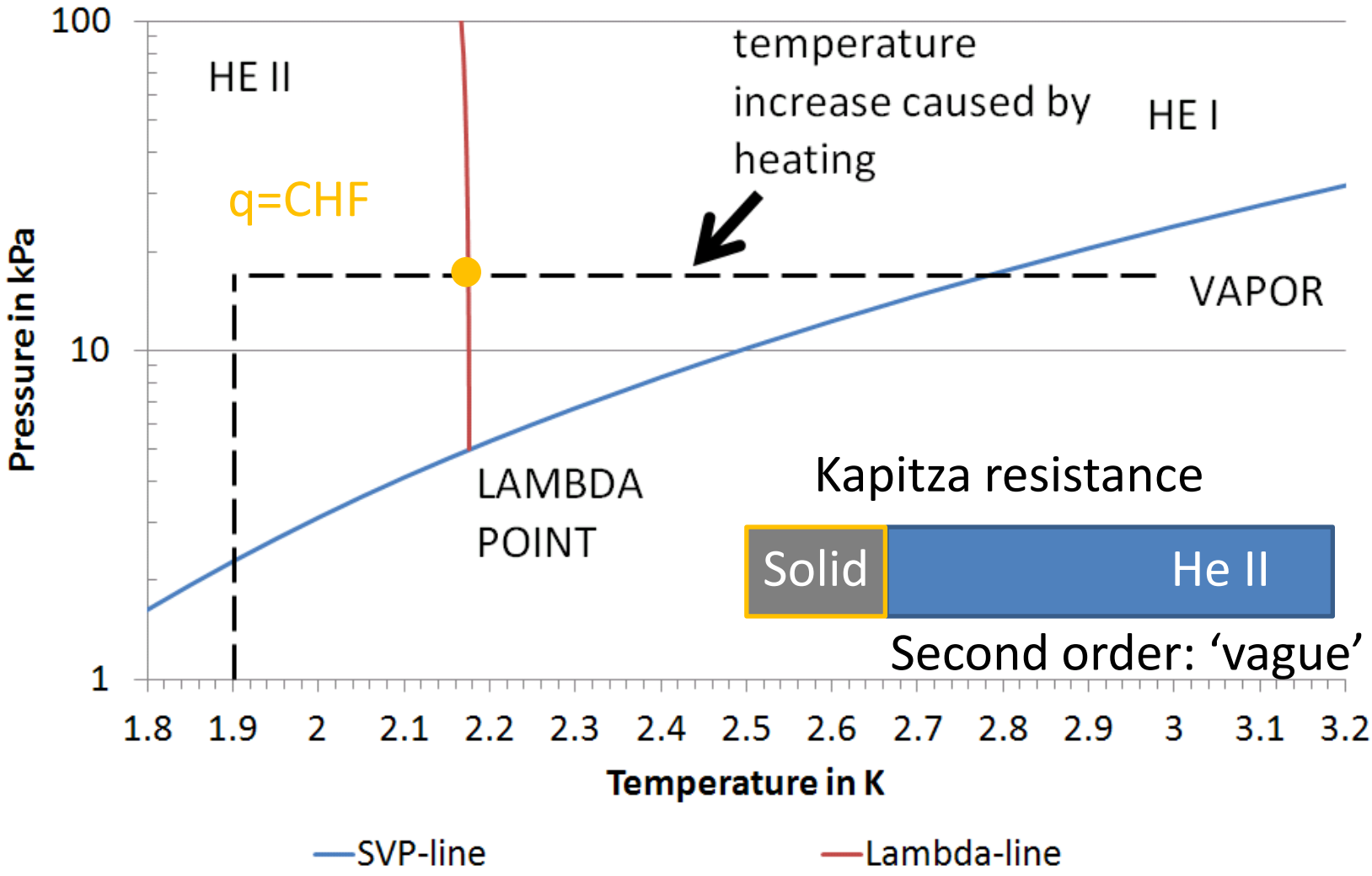
Phase change: saturated condition



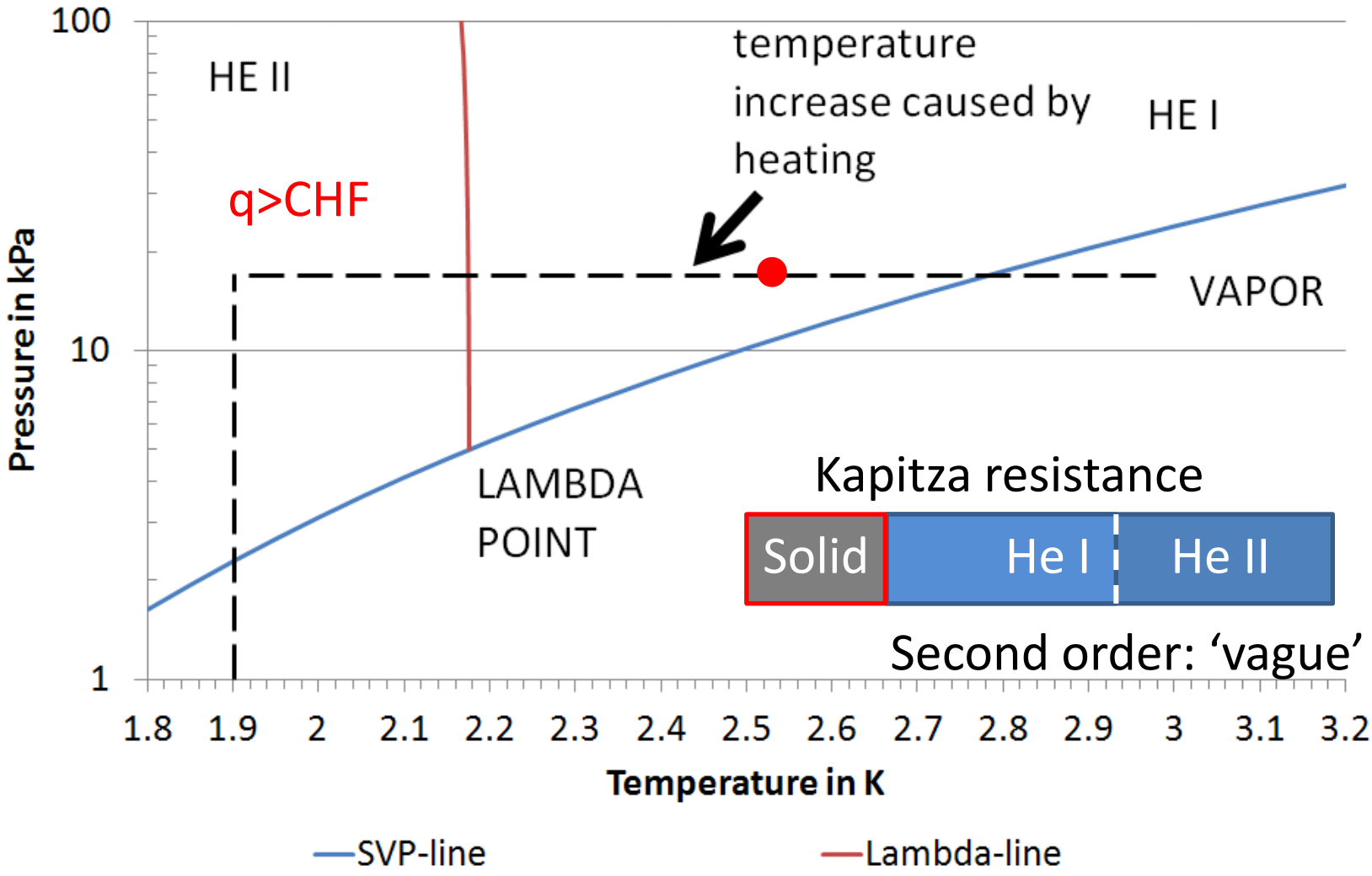
Phase change: subcooled condition



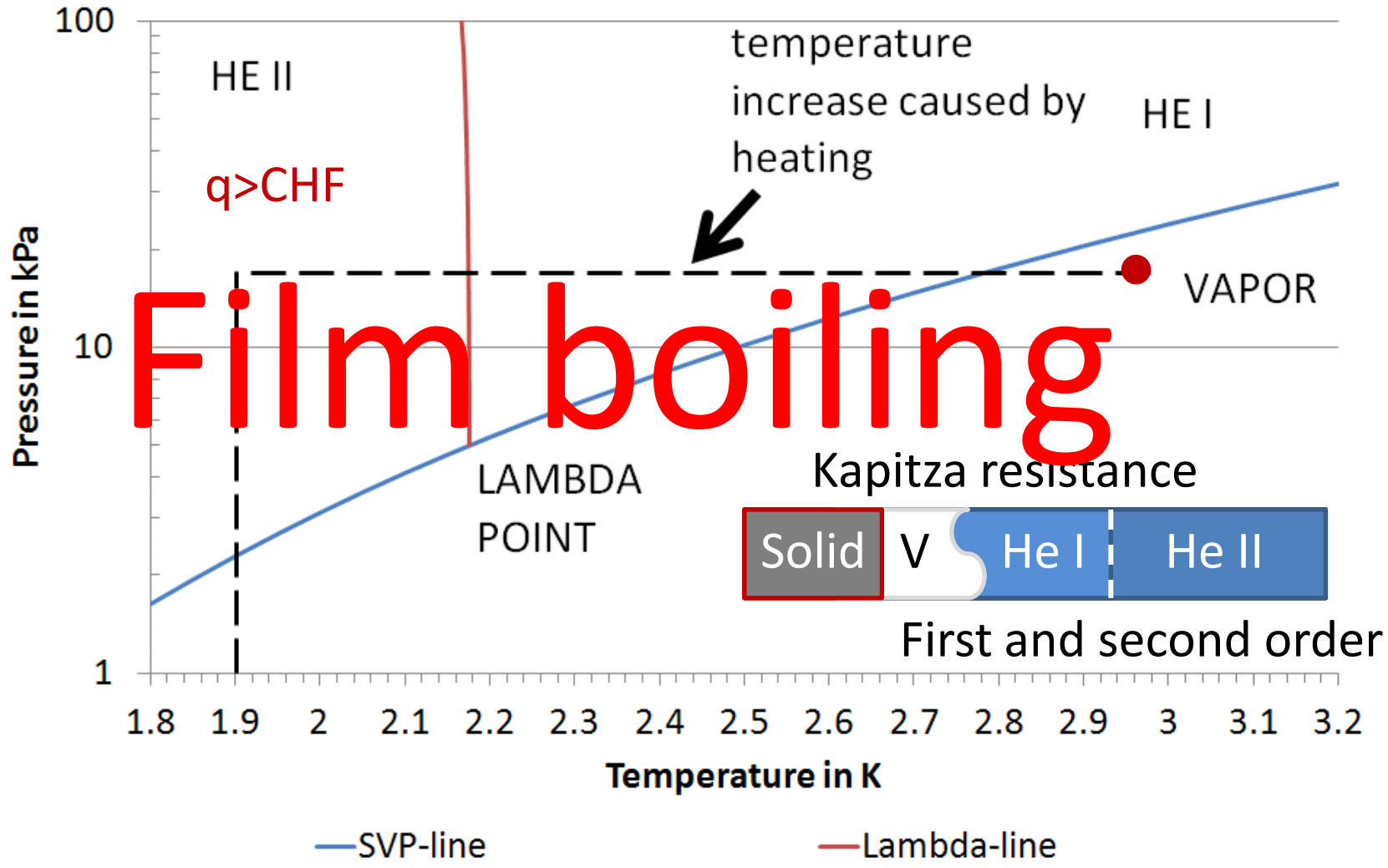
Phase change: subcooled condition



Phase change: subcooled condition



Phase change: subcooled condition



Heat equation

$$\rho_{\mathrm{D}}(T)c_{\mathrm{p}}(T,B)\left(\frac{\mathrm{d}T}{\mathrm{d}t}+(\boldsymbol{u}\cdot\nabla)T\right)+\nabla\cdot(-k(T,B)\nabla T)=q$$

ρ_D is mass density in kgm^{-3}
c_p is the volumetric specific heat in $J\ kg^{-1}\ K^{-1}$
k is the thermal conductivity in $Wm^{-1}K^{-1}$
$\boldsymbol{u}$ is the velocity vector in ms^{-1}
q is the volumetric heat source in Wm^{-3}
T is the temperature in K
t is the time in s
B is the magnetic field in T

Transient, non-constant properties <u>with</u> fluid flow

Heat equation

$$\rho_D(T)c_p(T,B)\frac{\partial T}{\partial t} + \nabla \cdot (-k(T,B)\nabla T) = q$$

ρ_D is mass density in kgm^{-3}
c_p is the volumetric specific heat in $J\ kg^{-1}\ K^{-1}$
k is the thermal conductivity in $Wm^{-1}K^{-1}$
$\boldsymbol{u}$ is the velocity vector in ms^{-1}
q is the volumetric heat source in Wm^{-3}
T is the temperature in K
t is the time in s
B is the magnetic field in T

Transient, non-constant properties without flow

Heat equation

$$\alpha = \frac{k}{\rho_D c_p}$$

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} - \nabla^2 T = \frac{q}{k}$$

ρ_D is mass density in kgm^{-3}
c_p is the volumetric specific heat in $J\ kg^{-1}\ K^{-1}$
k is the thermal conductivity in $Wm^{-1}K^{-1}$
$\boldsymbol{u}$ is the velocity vector in ms^{-1}
q is the volumetric heat source in Wm^{-3}
T is the temperature in K
t is the time in s
B is the magnetic field in T

Transient, rewritten

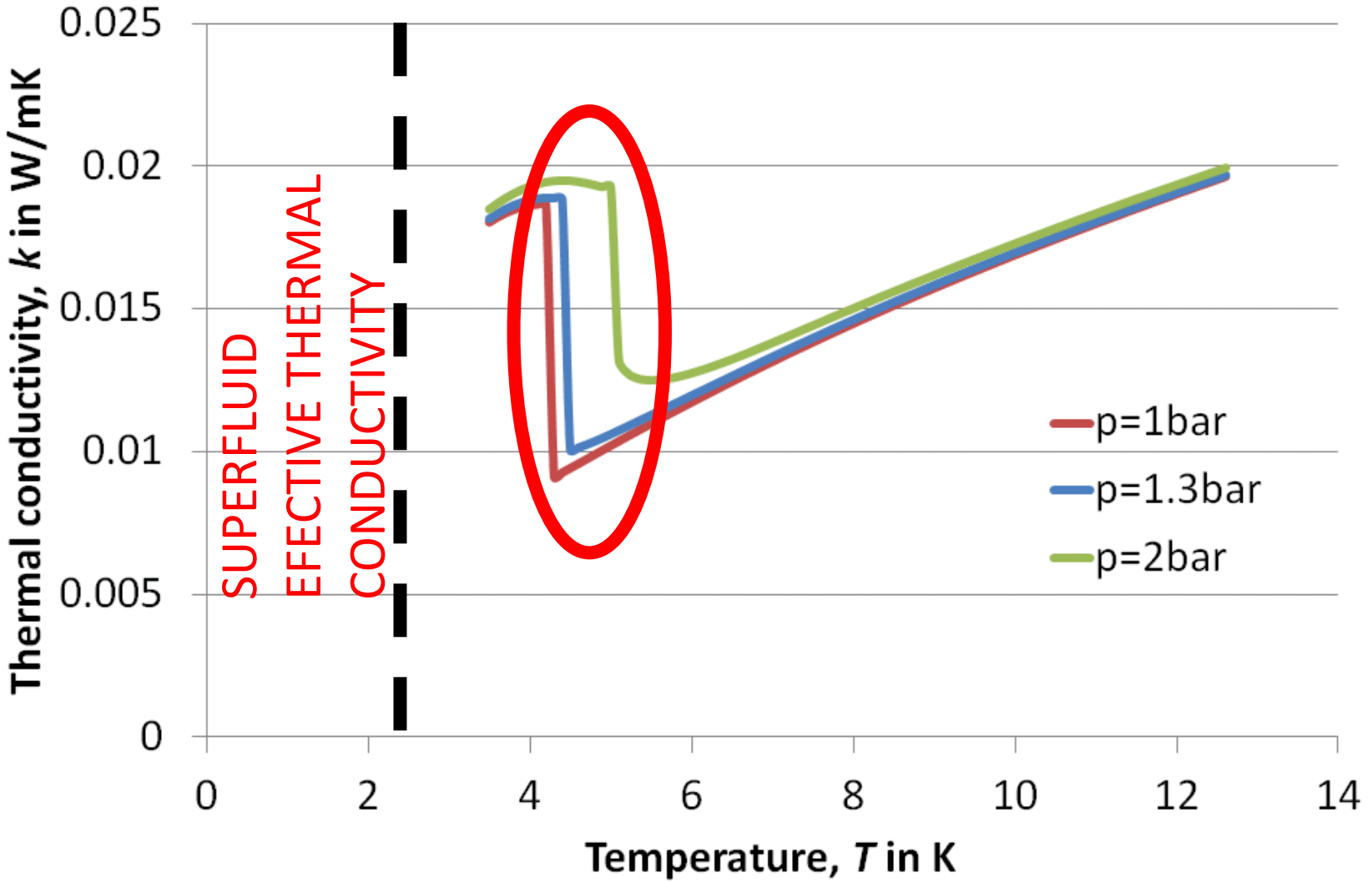
Heat equation

$$-\nabla^2 T = \frac{q}{k}$$

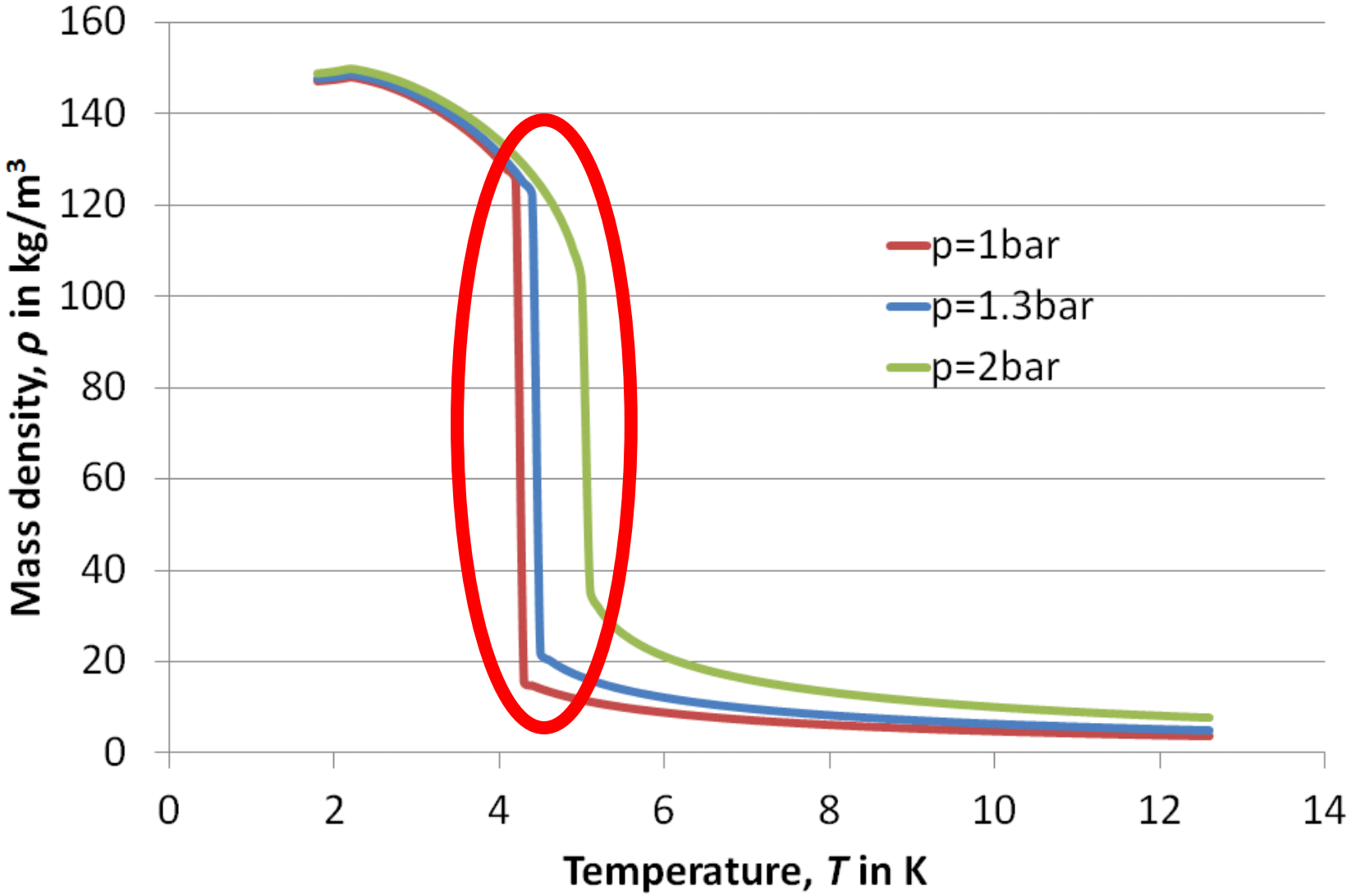
ρ_D is mass density in kgm^{-3}
c_p is the volumetric specific heat in $J\ kg^{-1}\ K^{-1}$
k is the thermal conductivity in $Wm^{-1}K^{-1}$
$\boldsymbol{u}$ is the velocity vector in ms^{-1}
q is the volumetric heat source in Wm^{-3}
T is the temperature in K
t is the time in s
B is the magnetic field in T

Steady state

Thermal conductivity



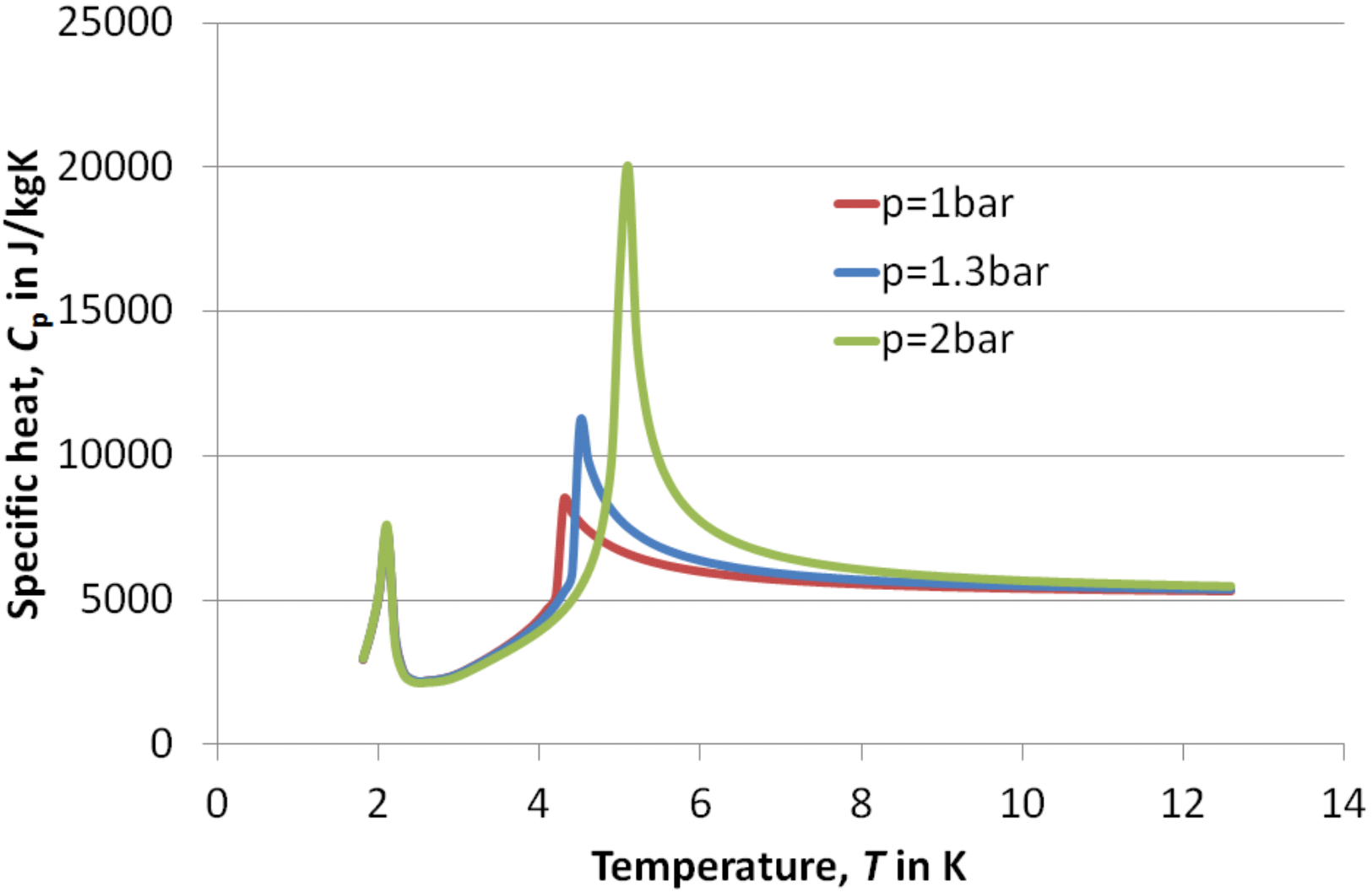
Mass density



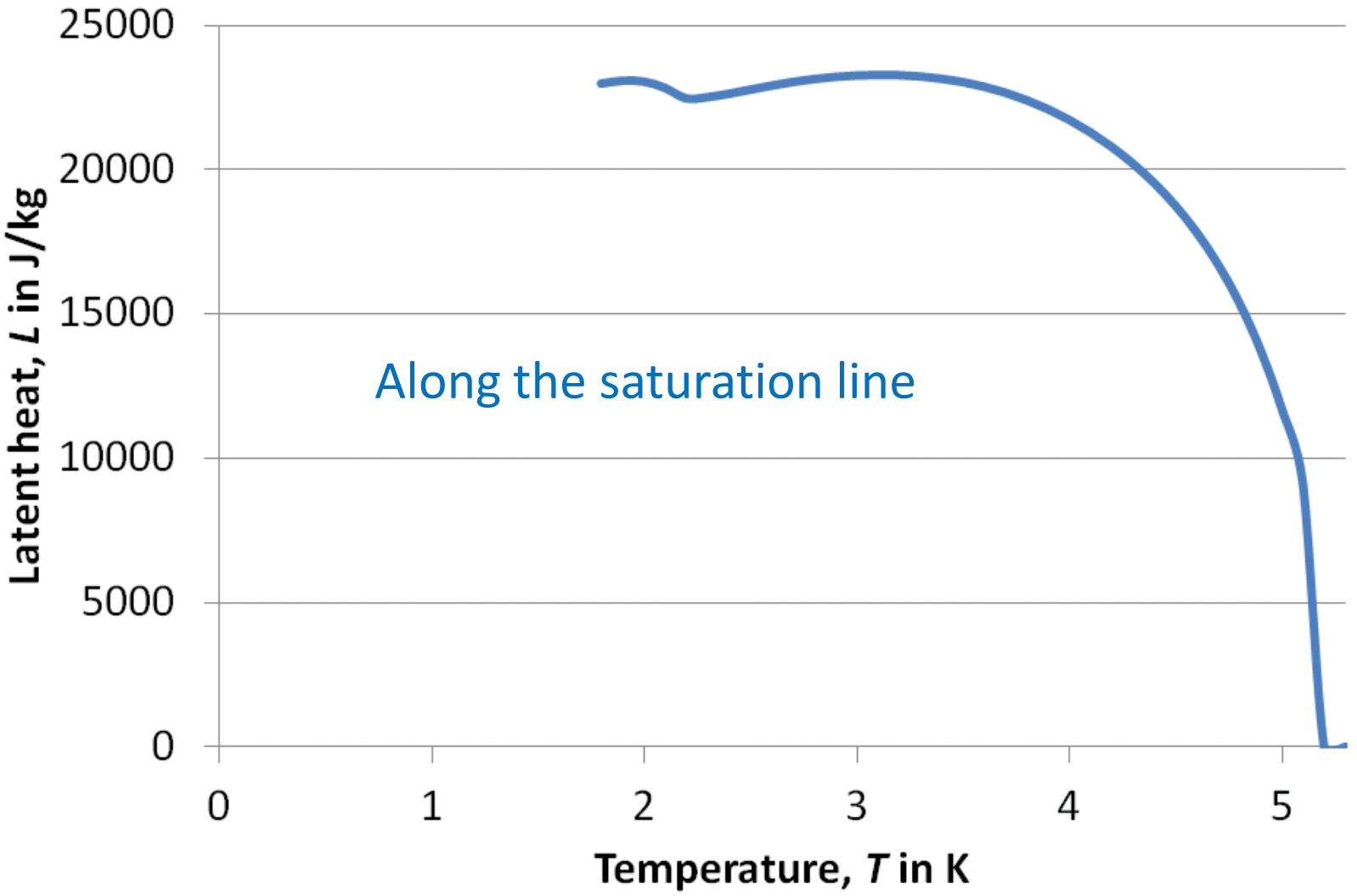
Specific - and latent heat

For first order phase transitions a latent heat is recognized, which can be modeled numerically as an (additional) peak in the specific heat at the transition temperature.
The lambda transition is called like that since the specific heat as a function of temperature has the shape of the Greek letter lambda.
Although a peak occurs, the phase transition is of second order.

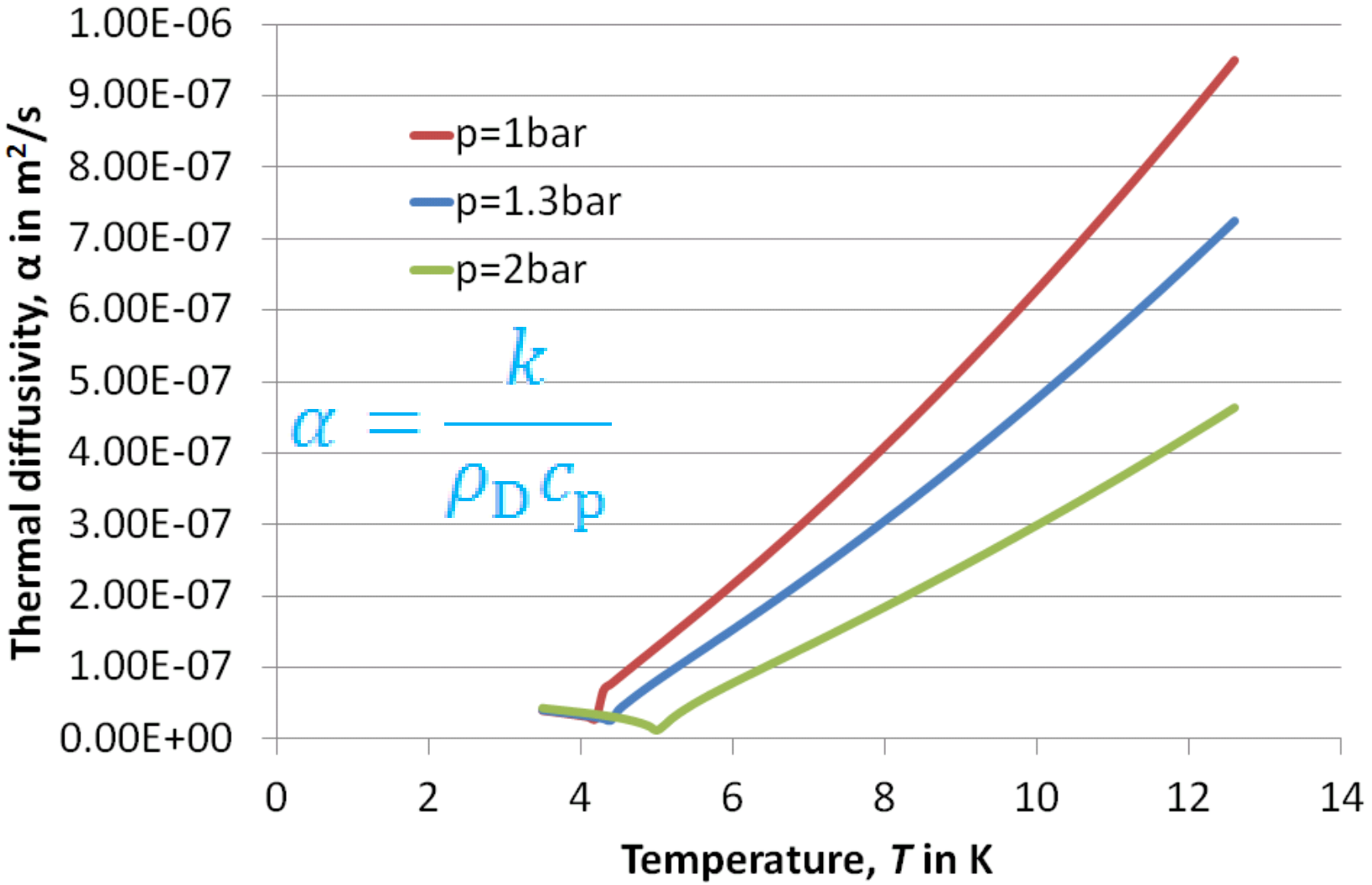
Specific - and latent heat



Specific - and latent heat

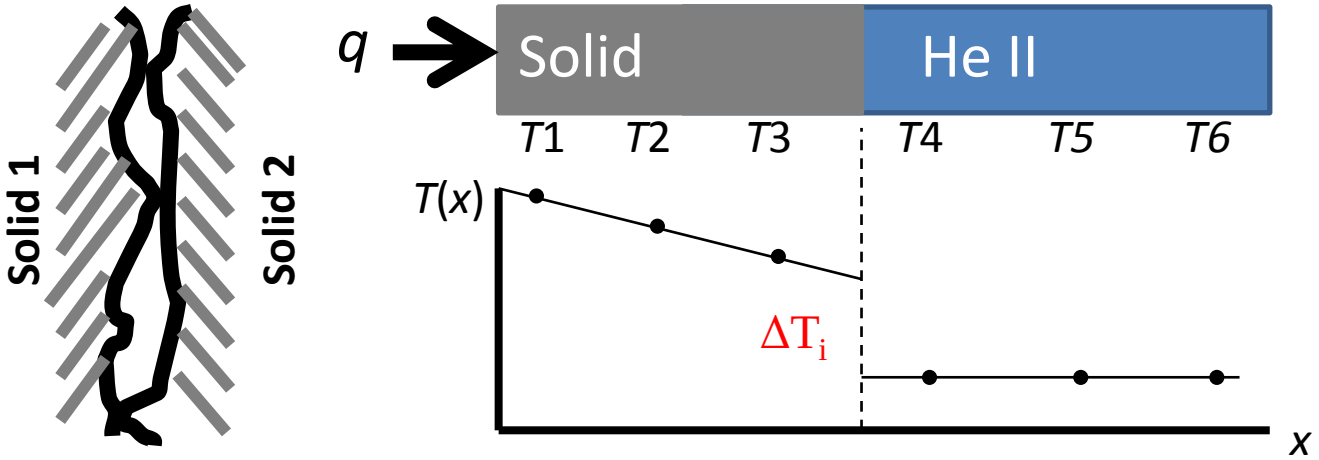


Thermal diffusivity



Kapitza conductance 1

On interfaces between solids, a thermal and electrical resistance is found: the contact resistance. This phenomenon can be explained by imperfections in the contact surface.



$$h_K = \frac{q}{\Delta T_i}$$

Prediction by acoustic mismatch (AM or Khalatnikov) theory underestimates and by diffuse mismatch (DM or phonon radiation limit) theory overestimates this so-called Kapitza conductance. This discontinuity occurs between any two dissimilar materials where electronic transport does not play a role.

Kapitza conductance 2

The Kapitza conductance is strongly dependent on temperature, by theory a cubic relation is found for small heat fluxes ($\Delta T \ll T$) :

$$h_K = cT_b^3$$

For large heat fluxes ($\Delta T \approx T$), a general form fitting experimental data gives:

$$h_K = a(T_s^n - T_b^n)$$

Implementation of these relations is not straightforward in COMSOL:

-Dependent variable is continuous if a composite geometry is used

-Discontinuity of the dependent variable is possible only in an assembly

-In the assembly “pairs” of boundaries have to be defined

-A standard relation is implemented in the “thermal module”:

$$-\boldsymbol{n} \cdot \boldsymbol{q} = \frac{k}{d}(T_1 - T_2)$$

Kapitza conductance 3

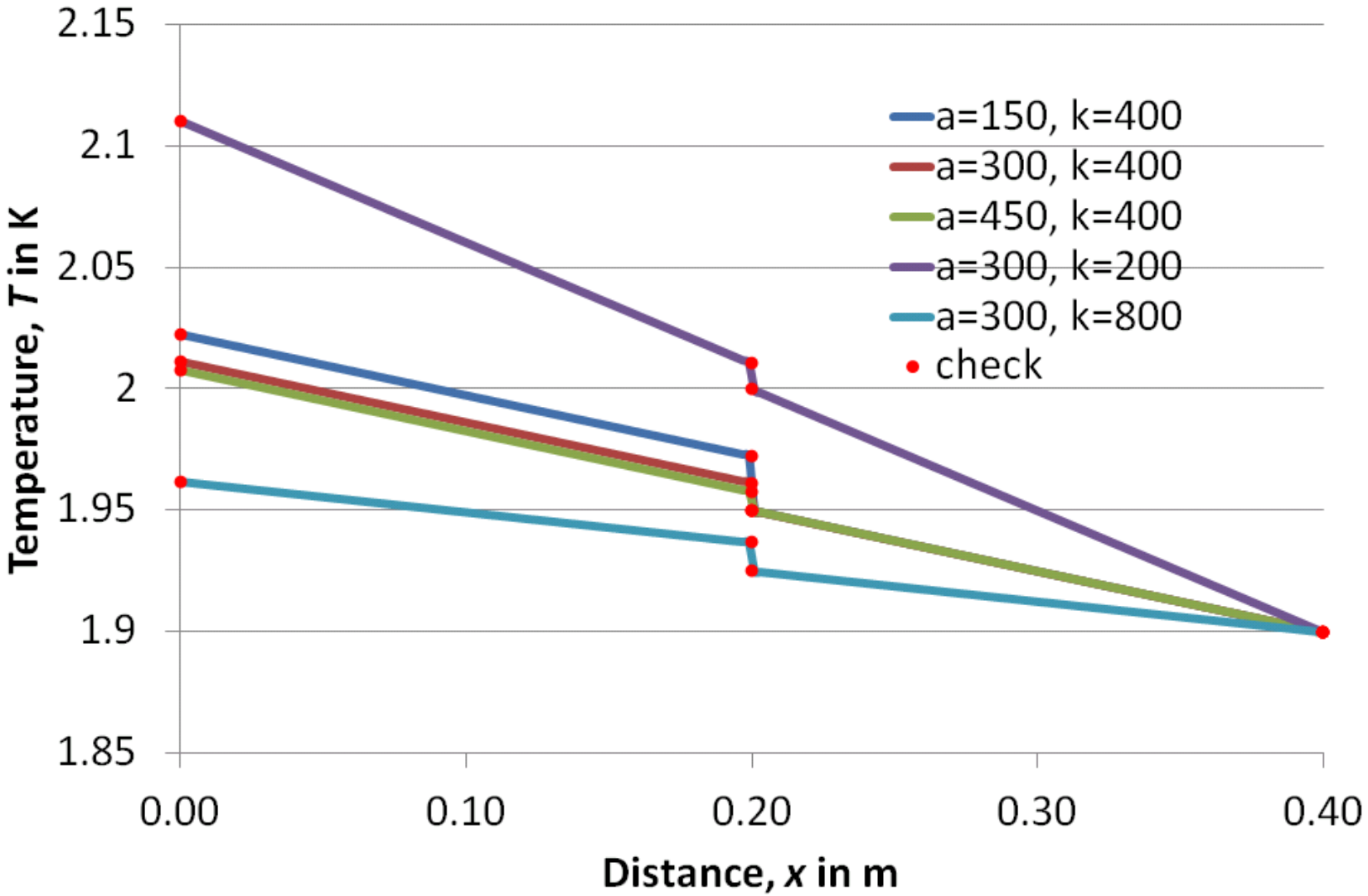
Using the theoretical value of n=4 the following is valid:

$$a\left(T_s^4 - T_b^4\right) = a\left(T_s^2 + T_b^2\right)\left(T_s^2 - T_b^2\right)$$

$$= \underbrace{a\left(T_s^2 + T_b^2\right)(T_1 + T_2)}_{h_K = \frac{k_L}{d_L}}(T_1 - T_2)$$

In COMSOL this can be implemented by using so called integration-coupling-variables to obtain the correct values for the temperature on the boundaries.

Kapitza conductance 3



Phase front tracking

- As noticed before, when the CHF is exceeded a phase change occurs. This leads to film boiling in the subcooled situation. If a steady state heat flux is applied temperatures rise strongly and burn-out results.
- Level-set method: A contour line of the globally defined level set function, ϕ, describes the interface between phases. The interface can move within any velocity field. In COMSOL, ϕ is a smooth heaviside function. A fixed mesh is used.
- Phase field: additional to the level-set method, the governing Cahn-Hilliard equation (4^{th} order PDE) diminishes the total energy of the system. A fixed mesh is used.
- ALE: Arbitrary Lagrangian Eulerian method describes a moving mesh where dependent variables (T) represent the mesh movement.
- Non-linear material properties: the heat equation is solved with material coefficients which incorporate the phase change

ALE

- In the Lagrangian method, the mesh movement follows the movement of the physical material. This can be used when displacements are small.
- In the Eulerian method, the mesh is fixed and cannot account for moving boundaries but can be used to describe more complicated material motion.
- In the Arbitrary Lagrangian Eulerian method, allows moving boundaries, but the mesh does not need to follow the material.

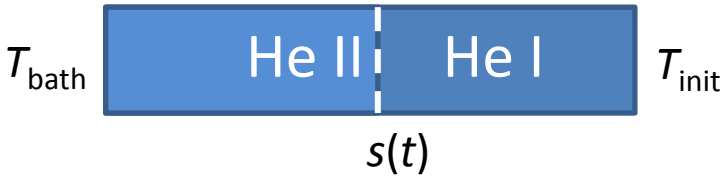
The ALE method makes use of a reference frame, where the mesh is fixed and a spatial frame, where the mesh is moving. The movement of the boundaries is described by functions of the dependent variable. The ALE method asks for input for the position or velocity of the phase boundary. This can be dependent on the transient solution.

Stefan problems

At phase boundaries, the heat equation is not valid. An additional condition is needed to obtain closure of the Partial Differential Equation: the Stefan condition. It expresses the local velocity of a moving boundary. The physical constraint in heat transfer problems is the conservation of energy. The local velocity depends on the discontinuity of the heat flux over the boundary (for first order transitions).

Typically in absence of flow, in both the He II and the He I domain, the heat equation has to be solved. In a semi-infinite geometry, the following boundary conditions occur: adiabatic at the end, bath temperature at the other end and for the interface the Stefan condition is in place. The heat flux is balanced by the product of latent heat, density and interface velocity.

Subcooling of He channels 1



$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \qquad 0 \le x \le s(t) \qquad t > 0$$ Governing equations

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \qquad s(t) \le x \le x_{inf} \qquad t > 0$$

$$T = T_b \qquad x = 0 \qquad t > 0$$ Boundary conditions

$$T = adiab. \qquad x = x_{inf} \qquad t > 0$$

$$T = T_{init} \qquad x > 0 \qquad t = 0$$ Initial conditions

$$s(0) = 0$$

Two additional conditions are needed on s, one to give the second boundary conditions for the heat equations, and one to determine the position of the interface itself.

Subcooling of He channels 2

$$T_1 = T_2 = T_\lambda$$ Transition temperature

$$k_2 \frac{\partial T_2}{\partial x} - k_1 \frac{\partial T_1}{\partial x} = L\rho \frac{ds}{dt}$$ Stefan condition

$$x = s(t), \quad t > 0$$

Since the lambda transition is of second order and latent heat does not play a role, the heat flux across the boundary is continuous. It is therefore also not possible to obtain a velocity of the phase front this way.
Assuming that the position of the lambda front is known beforehand, the corresponding heat fluxes and velocity of the phase boundary can be easily calculated.

Subcooling of He channels 2

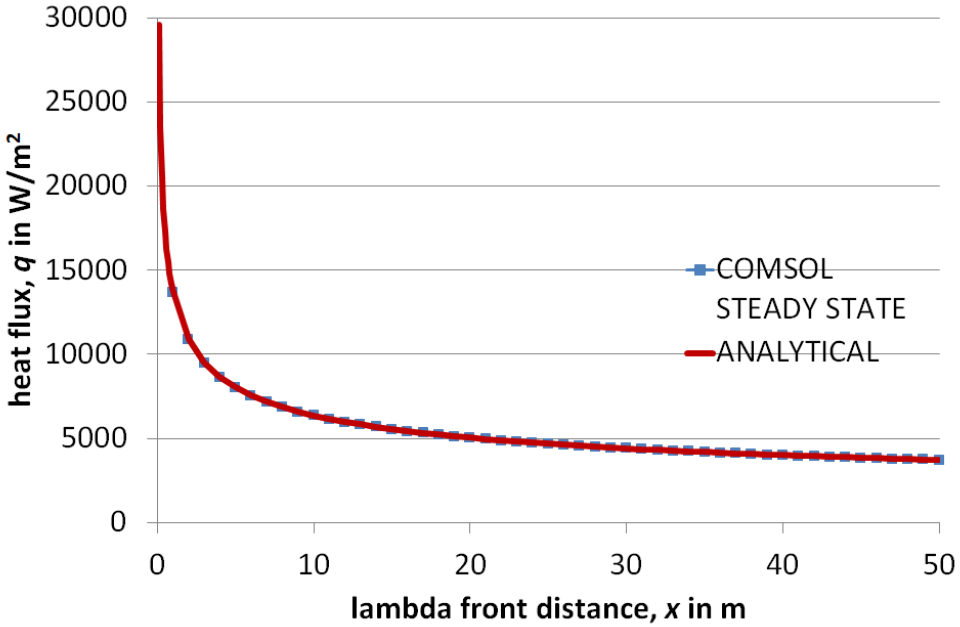
$$T_1 = T_2 = T_\lambda$$

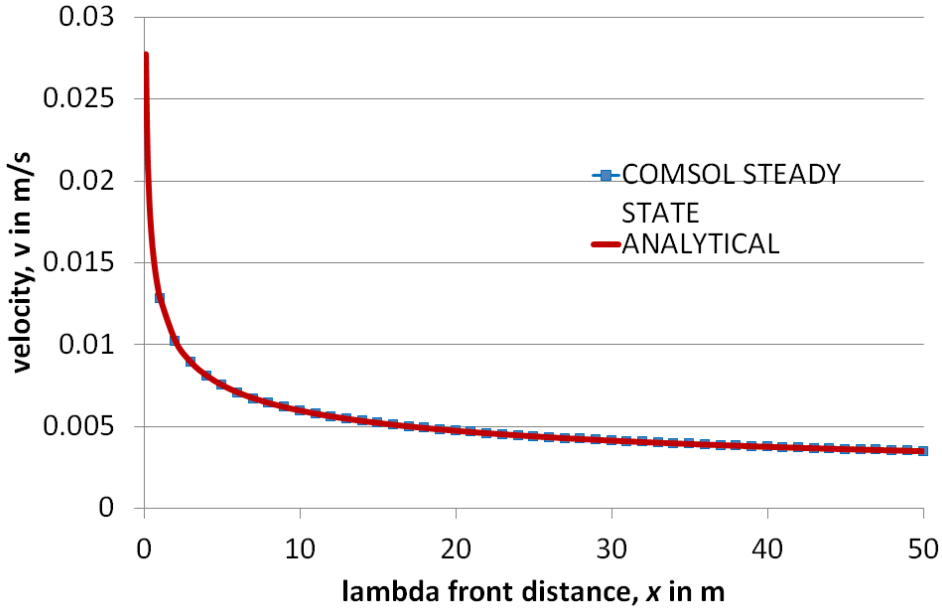
Transition temperature

$$k_2 \frac{\partial T_2}{\partial x} - k_1 \frac{\partial T_1}{\partial x} = L\rho \frac{ds}{dt}$$

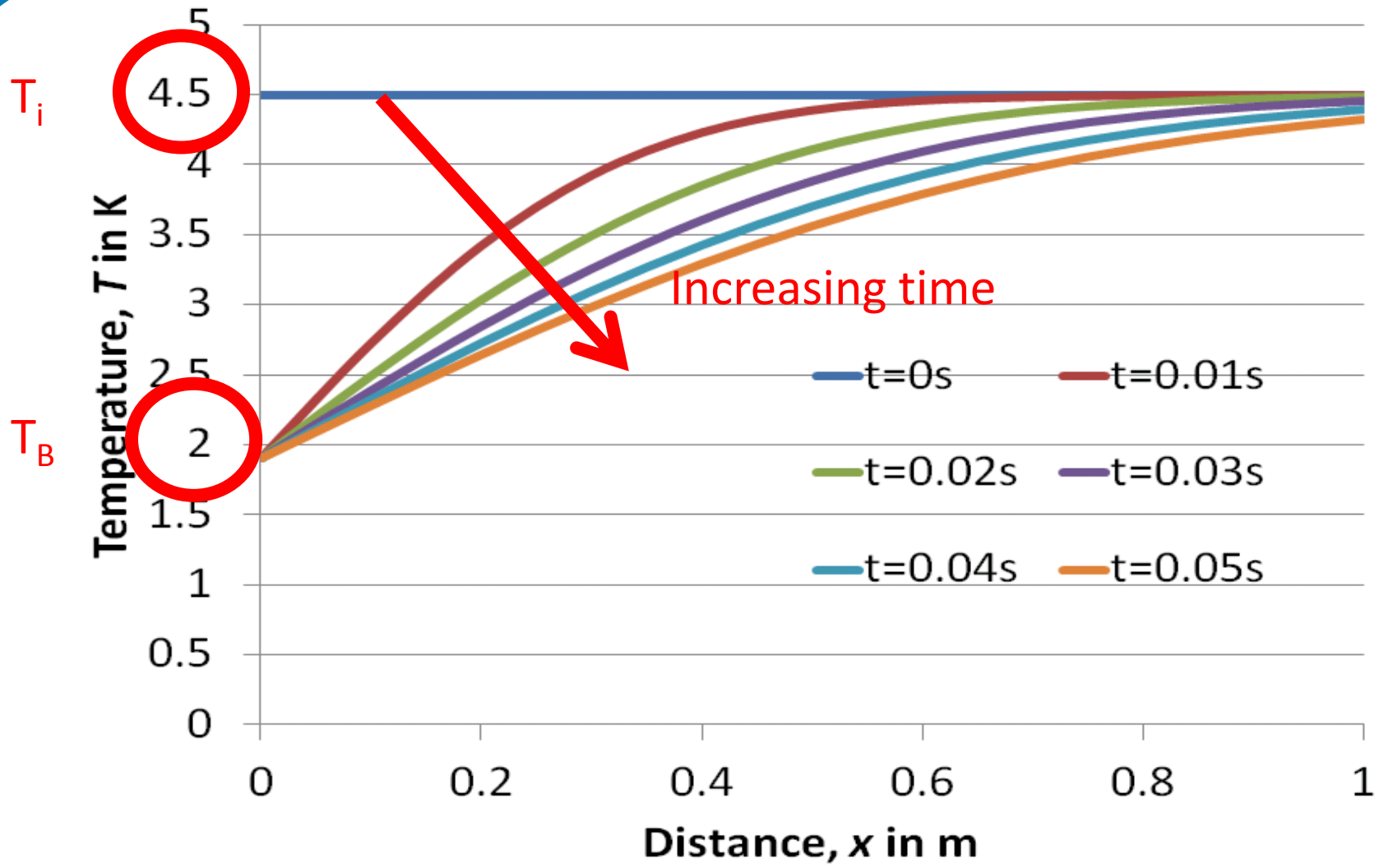
Stefan condition

$$x = s(t)$$
$$t > 0$$

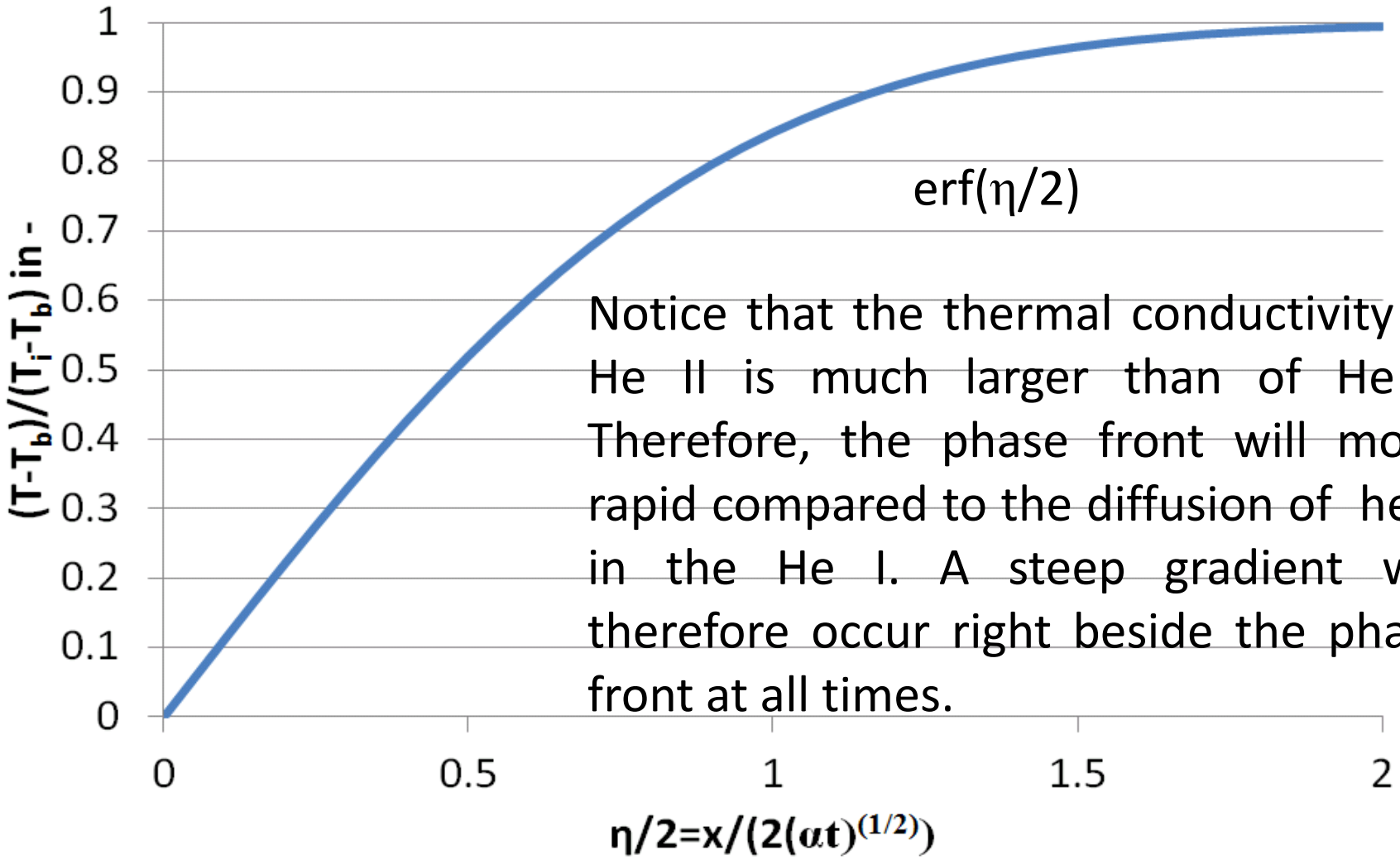




Similarity solutions



Similarity solutions



Notice that the thermal conductivity of He II is much larger than of He I. Therefore, the phase front will move rapid compared to the diffusion of heat in the He I. A steep gradient will therefore occur right beside the phase front at all times.

Burnout and recovery 1

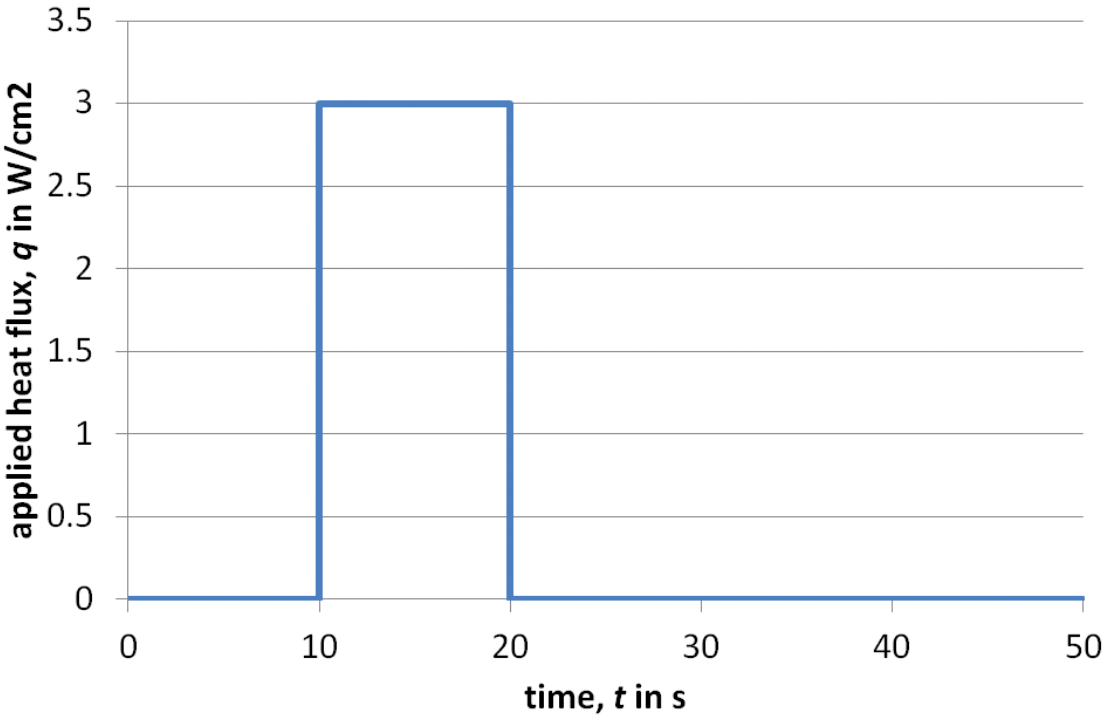
$$q = -\left(\left(\frac{1}{|f(T)|}\right)^{\frac{1}{3}}\left(\frac{1}{|\nabla T|}\right)^{\frac{2}{3}}\right)\nabla T$$

Gives good results for steady state conditions, however for transient calculations, this approach is not suitable since the time step taken by the solver oscillates and the problem does not converge.

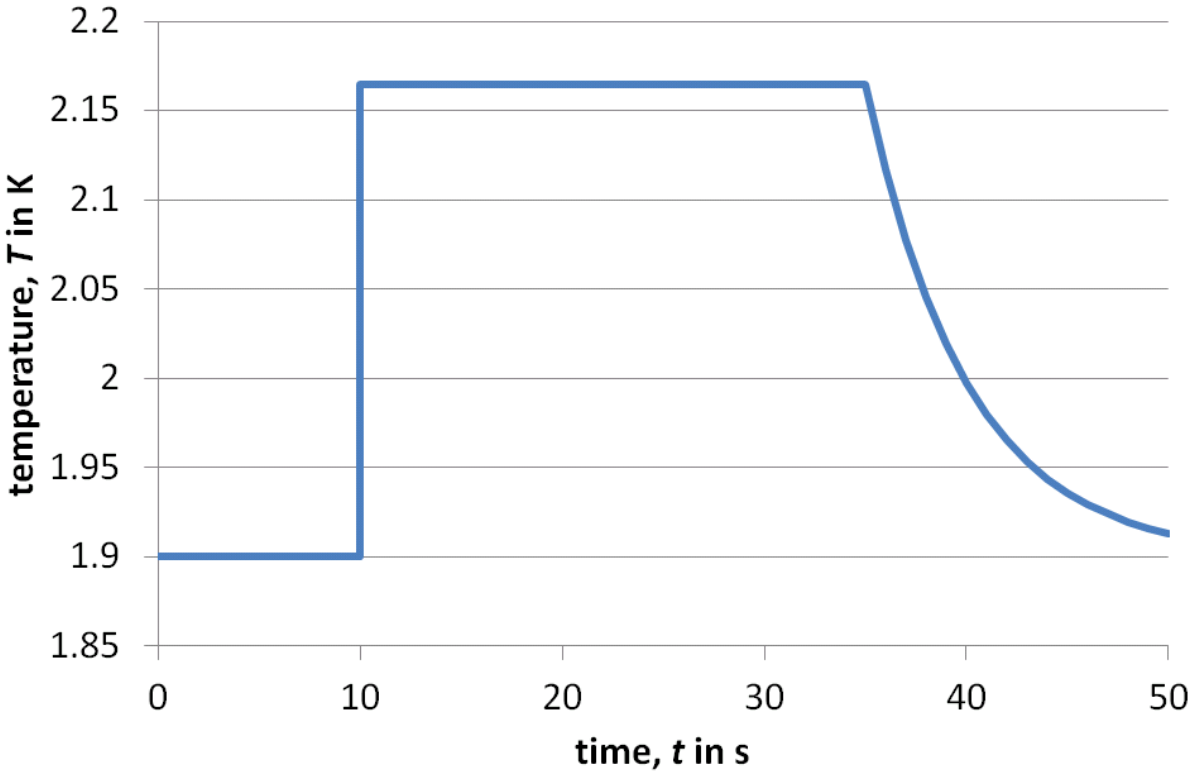
$$q = -\frac{1}{q^2 f(T)}\frac{\mathrm{d}T}{\mathrm{d}x}$$

Gives a circular dependency. However, in 1D steady state problems, the applied surface heat flux can be used to solve this problem. For the transient case, part of the applied heat is absorbed by the heat capacity. Since diffusion is fast and the vapor layer thin, a heat transfer coefficient can be applied on the interface and the hot end of the helium equals T_λ.

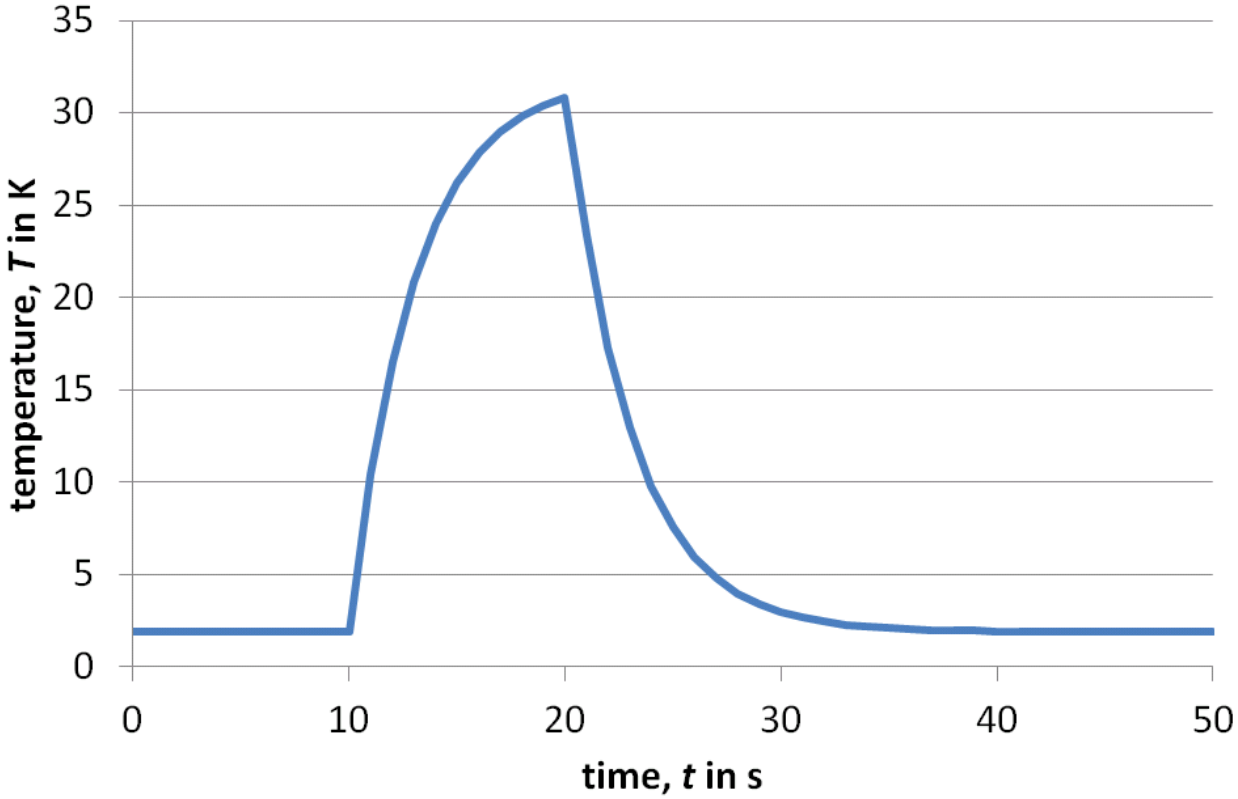
Burnout and recovery 2



Burnout and recovery 2



Burnout and recovery 2



Burnout and recovery 2

The channel length is known as well as the end temperatures and thus the heat flux is known too. The difference between the heat flux extracted through the channel and the applied heat flux heats the solid. The total temperature rise depends on the specific heat and size.

Conclusion

- Temperature distributions below T_λ can be simulated quite easily in 1D geometries, but also complex 3D networks of micro-channels filled with superfluid helium can be modeled below T_λ with a good correlation to experimental results.
- 3D Critical Heat Fluxes are obtained with good correlation to experiments.
- Non-linear Kapitza conductance can be correctly modeled by making use of boundary integration variables.
- Stefan problems can be modeled by strongly non-linear material properties in a straightforward way, but convergence difficulties arise.
-Stefan problems in 1-dimensional problems can be correctly modeled using the Arbitrary Lagrangian Eulerian method.
- Transient burnout and recovery are very difficult to model, since convergence problems arise.

Outlook

Experiments on cable stacks to increase the understanding of the cable insulation show that a transition occurs in the heat transfer when the cable reaches the lambda temperature. Probably not all the superfluid helium in the micro-channels is converted into normal liquid helium, but only a thin layer at the surface of the cable. The transition of the heat flowing through the helium towards the cable insulation is not modeled correctly yet. The correct modeling of this transition is planned for the near future.

Thank you for your attention,
please ask your questions!

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