Investigation of Suitability of the Method of Volume Averaging for the Study of Superconducting Magnet Thermo-Hydraulics

Hervé Allain	CERN
Rob van Weelderen	CERN
Bertrand Baudouy	CEA-Saclay
Michel Quintard	IMFT-CNRS
Marc Prat	IMFT-CNRS
Cyprien Soulaine	IMFT

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Outline

- Introduction
- The method of volume averaging
- Method of volume averaging and superconducting magnets' thermal hydraulics
- Method of volume averaging and superfluid helium
- Example of magnet cooled by superfluid helium
- Conclusion

Introduction

•Extensive R&D in the development of next generation (very-) high field magnets based on Nb_3Sn and/or HTS superconductors, necessary for the LHC collimation upgrades and HL-LHC (High Luminosity LHC)

•Magnets are destined to be installed in a ***thermally very challenging environment***

•Magnets function either in sub-cooled-pressurized superfluid helium, supercritical helium or in saturated normal helium

•**Difficult to study this mixed solid-liquid environment numerically** in detail for computational reasons alone

•**Need of a mathematical description, suitable for numerical modelling**, which preserves as much as possible the geometrical information and the physics

•In porous media field, same problem: use of ***upscaling methods*** to model the physical behavior of porous media

•***Idea:*** *considering the interior of a superconducting magnet as a porous media and applying the method of volume averaging in order to get macro-scale equations that model the behavior of the magnet, suitable for numerical simulation*

The method of volume averaging (1/2)

- **Principle**

  - Technique that can be used to derive continuum equations for ***multiphase systems***

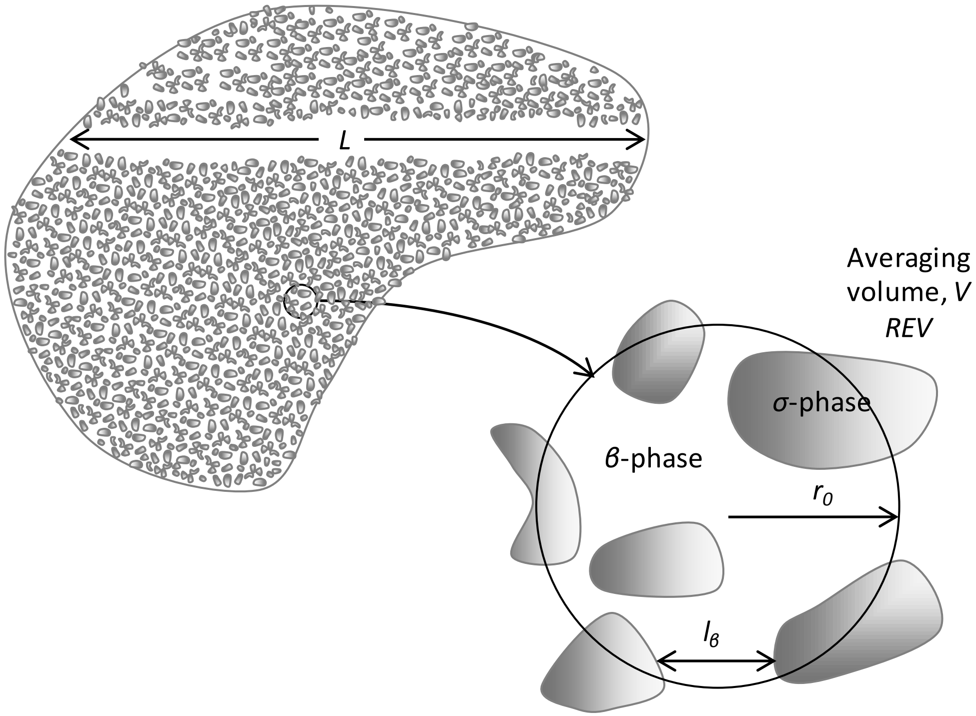
  - ***Equations*** which are valid within a particular phase can be ***spatially smoothed*** to produce equations that are valid everywhere

  - For example, in the process of transient heat transfer in porous media, we are interested in the temperature of the fluid and of the solid at a given location of space and time in the porous medium

    - Direct analysis of this problem, in terms of equations valid at the pore scale, is not possible in general because of the complex structure of a porous medium

Equations, boundary conditions and initial conditions at the pore scale are used to derive local volume averaged equations that have validity everywhere

The method of volume averaging (2/2)



Intrinsic average $\langle c_\beta \rangle^\beta = \frac{1}{V_\beta}\int_{V_\beta} c_\beta \, dV$

Superficial average $\langle c_\beta \rangle = \frac{1}{V}\int_{V_\beta} c_\beta \, dV$

Porosity $\varepsilon_\beta = \frac{V_\beta}{V}$

Length-scale constraint $l_\beta \ll r_0 \ll L$

Pore scale problem:
System of equations
Boundary conditions
Initial conditions

→ Spatial smoothing

→ Variables Decomposition
$c_\beta = \langle c_\beta \rangle^\beta + \tilde{c}_\beta$

→ Averaged equations:
Intrinsic averaged variables
Spatial deviation variables

→ Pore-scale deviations equations:
closure problem
(periodicity needed)

→ Deviations as functions of averaged variables and effective parameters

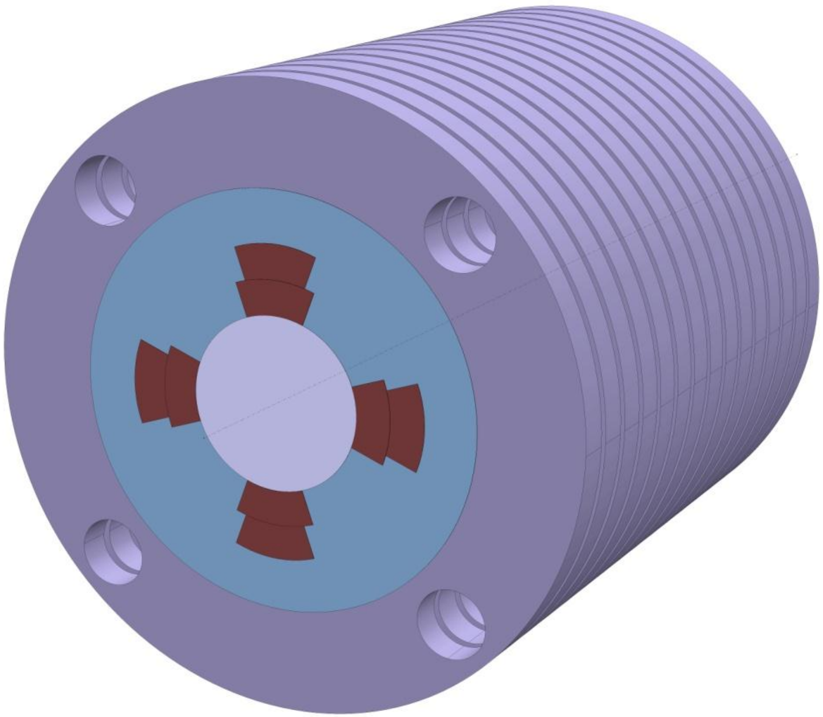
→ **Macroscopic equations**

Method and superconducting magnets (1/2)

- σ-phase: coils, collars and yoke

- β-phase: liquid helium filling all the voids space

The method of volume averaging can be applied to any system of equation and this system depends on the thermodynamics state of helium:
- pressurized superfluid helium
- supercritical helium
- saturated normal helium



Drawing by Jeremy Mouleyre

Method and superconducting magnets (2/2)

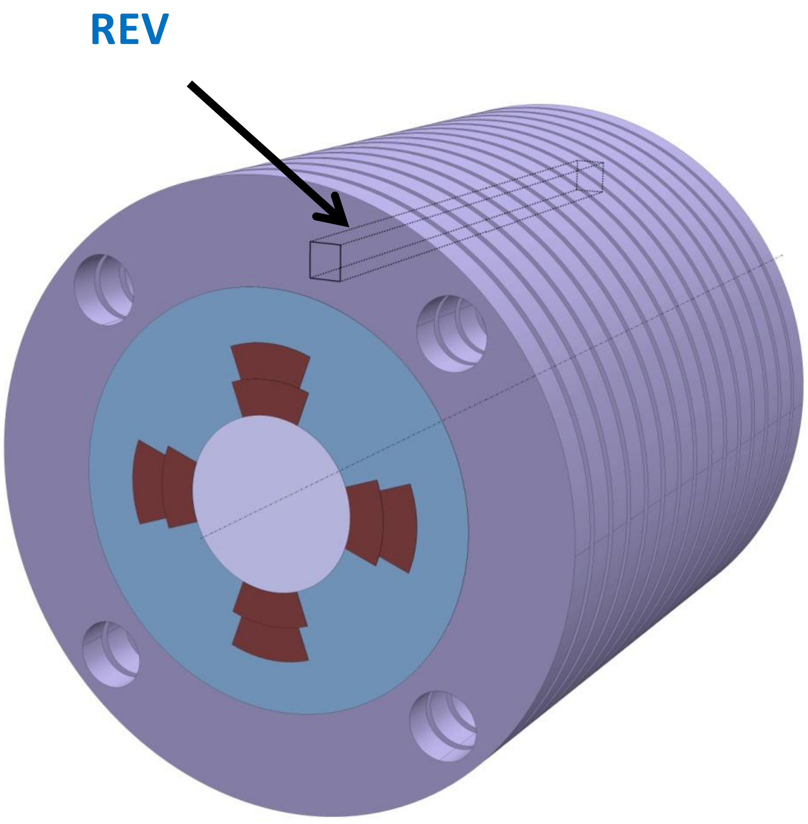
Unit cell identifications

Length-scale constraint $l_\beta \ll r_0 \ll L$

Magnet is typically several meter long: **L ≈ 10 m**
Space between laminations: **l_β ≈ 0.1 mm**

Laminations defines periodicity
Unit cell length: lamination thickness and spacing (few millimeters)
REV ≈ 10 unit cells at least
Taking **r_0 ≈ 0.1 m** (more than 10 UC)

The length-scale constraint is satisfied



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Previous work on the method of volume averaging and He II

The method of volume averaging has been applied to the two-fluids model of superfluid helium

- Assumptions: Landau regime
  steady state regime
  no heat transfer between solid and liquid

Macroscopic equations obtained:

•Continuity equation $\nabla.\left(\rho_n(T^\beta)\mathbf{v}_n+\rho_s(T^\beta)\mathbf{v}_s\right)=0$

•Momentum equation for the superfluid component $\mathbf{0}=\frac{\rho_s(T^\beta)s(T^\beta)}{\tau}\nabla T^\beta-\frac{\rho_s(T^\beta)}{\rho(T^\beta)}\frac{1}{\tau}\nabla P^\beta+\rho_s(T^\beta)$

•Momentum equation for the normal component $\mathbf{0}=-\frac{\rho_s(T^\beta)s(T^\beta)}{\tau}\nabla T^\beta-\left(1-\frac{\rho_s(T^\beta)}{\rho(T^\beta)\tau}\right)\nabla P^\beta-\frac{\eta(T^\beta)}{K}\mathbf{v}_n+\rho_n(T^\beta)\mathbf{g}$

•Entropy equation $\nabla.\left(\varepsilon_\beta\rho(T^\beta)s(T^\beta)\mathbf{v}_n\right)=0$

Effective parameters: *K* -> permeability, τ -> tortuosity

Method of volume averaging, He II and magnets

(differences with previous work)

Superconducting magnets: heat mainly produced in the coils, collars and yoke
heat driven to a heat sink via the superfluid

➢ *Heat transfer between the solid and the liquid*

At the solid surface $$\mathbf{q} = \rho s T \mathbf{v}_n$$

Instead of classical no-slip boundary condition for $\mathbf{v}_n$, generation of momentum of $\mathbf{v}_n$

And Kapitza resistance?

And extension from the laminar to the Gorter-Mellink (turbulent) flow regime

→ New closure problem to solve (much more complicated)
Need of a superfluid helium two-fluid model numerical code

→ Case currently under development!

Preliminary heat transfer macro-scale model (1/2)

Heat diffusion model of He II

$$\rho C_p \frac{\partial (T)}{\partial t} + \rho C_p \mathbf{v}.\nabla(T) = \nabla.\left(\frac{1}{f(T)}\nabla T\right)^{1/3} + q_{vol}$$

- Need to solve the two-fluid model to have the velocity for the convective term
- Let assume that the Péclet number is small and consider only the diffusive term
- Linearization of the diffusive term

$$\rho C_p \frac{\partial (T)}{\partial t} = \nabla.\left[\left(\frac{1}{f(T).|\nabla T|^2}\right)^{1/3} \nabla T\right] + q_{vol}$$

Effective thermal conductivity:

$$k_{eff} = \left(\frac{1}{f(T).|\nabla T|^2}\right)^{1/3}$$

➡ Classical heat diffusion equation (beside k_{eff} non linearity)

Preliminary heat transfer macro-scale model (2/2)

Simplified heat transfer problem at the pore scale

In the liquid $$\left(\rho c_p\right)_\beta \frac{\partial T_\beta}{\partial t} = \nabla \cdot \left(k_\beta \nabla T_\beta\right)$$

In the solid $$\left(\rho c_p\right)_\sigma \frac{\partial T_\beta}{\partial t} = \nabla \cdot \left(k_\sigma \nabla T_\sigma\right) + \Phi_\sigma$$

Boundary conditions $$T_\beta = T_\sigma$$ at $A_{\beta\sigma}$
$$\mathbf{n}_{\beta\sigma} \cdot k_\beta \nabla T_\beta = \mathbf{n}_{\beta\sigma} \cdot k_\sigma \nabla T_\sigma + \Omega$$

Ω and Φ_σ are heterogeneous and homogeneous source terms respectively

➡ *Method of volume averaging*: 2 possibilities

- Local thermal equilibrium, one-equation model $\langle T_\beta \rangle = \langle T_\sigma \rangle = \langle T \rangle$
- Non-local thermal equilibrium, two-equation model

If local thermal equilibrium: $$\langle \rho \rangle C_p \frac{\partial \langle T \rangle}{\partial t} = \nabla \cdot \left(\mathbf{K}^* \cdot \nabla \langle T \rangle\right) + a_V \langle \Omega \rangle_{\beta\sigma} + \varepsilon_\sigma \langle \Phi_\sigma \rangle^\sigma$$

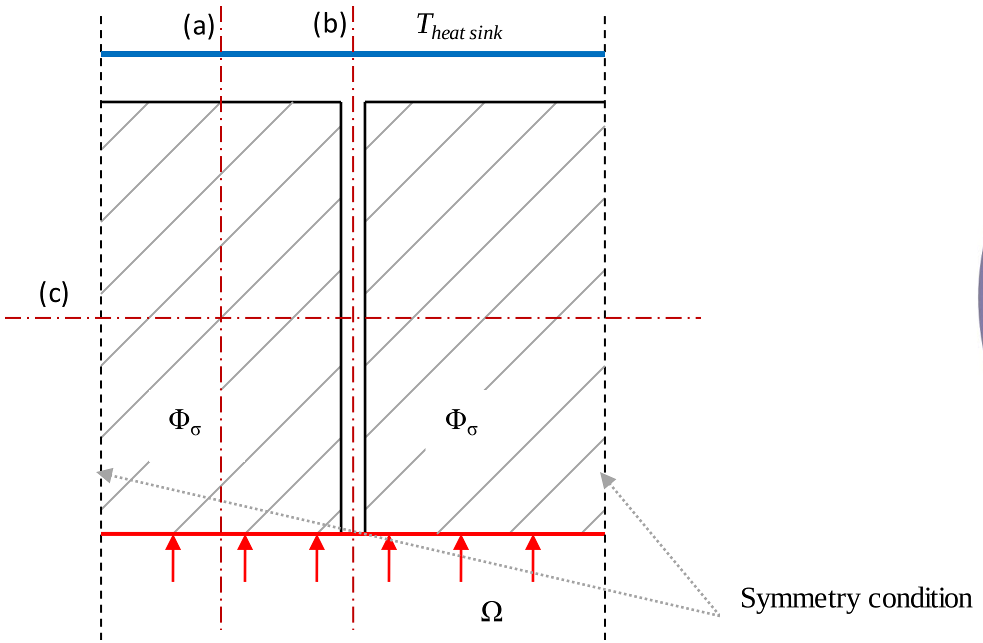
K* effective thermal diffusion tensor
a_V is the specific area
$\langle \Omega \rangle_{\beta\sigma}$ area averaged value of the heterogeneous thermal source
Heat capacity per unit volume $\langle \rho \rangle C_p = \varepsilon_\beta \left(\rho c_p\right)_\beta + \varepsilon_\sigma \left(\rho c_p\right)_\sigma$

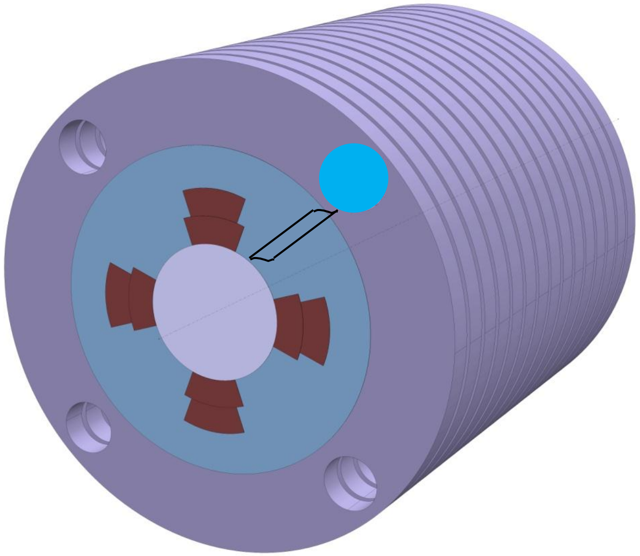
Validity of local thermal equilibrium? (1/3) Evaluation of $\langle T_\beta \rangle = \langle T_\sigma \rangle = \langle T \rangle$

•**Numerical code** implemented with *Comsol Multiphysics* in **steady-state**

•Equations considered: *classical heat diffusion equation* for the **solid** and the **liquid** with $k_{eff} = \left(\frac{1}{f(T).|\nabla T|^2}\right)^{1/3}$ for the superfluid

•Boundary conditions: - continuity of temperature and heat flux
- Kapitza resistance $q = h_K \Delta T_s$



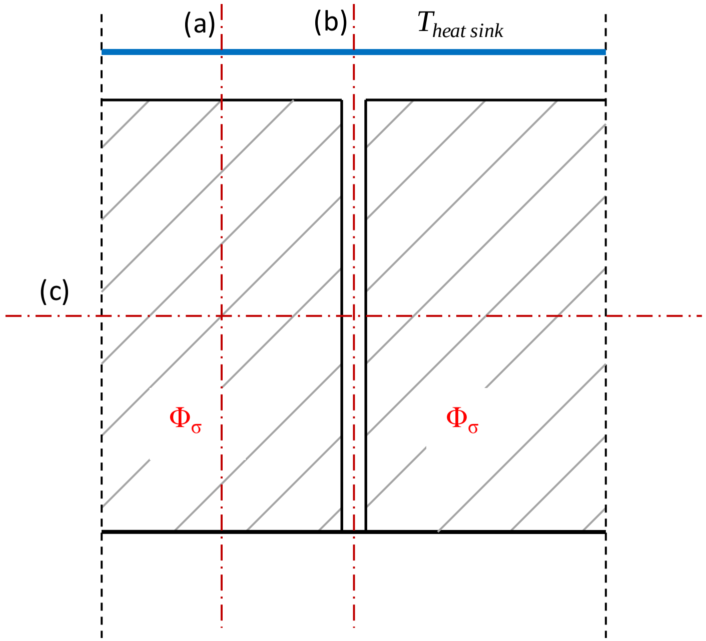


Validity of local thermal equilibrium? (2/3)

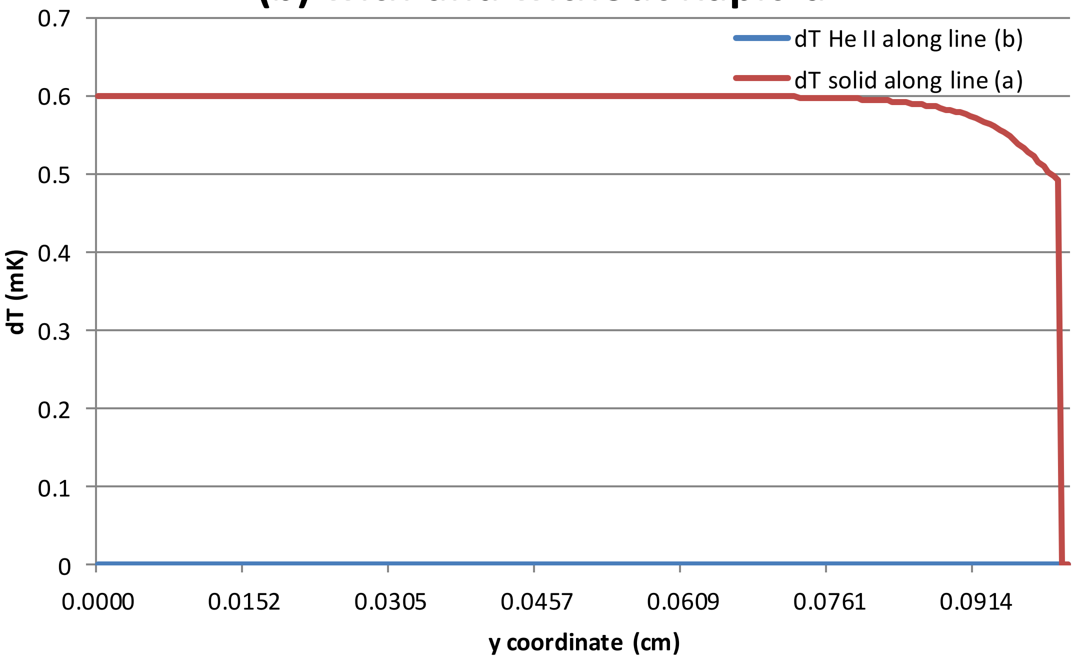
Evaluation de $\langle T_\beta \rangle = \langle T_\sigma \rangle = \langle T \rangle$

Parameters	Value	Unit
k_{solid} (thermal conductivity of the solid)	109	W/m.K
h_k	5000	$W/m^2.K$
Ω	1000	W/m^2
Φ_σ	2000	W/m^3
e	$1.5\ 10^{-3}$	m

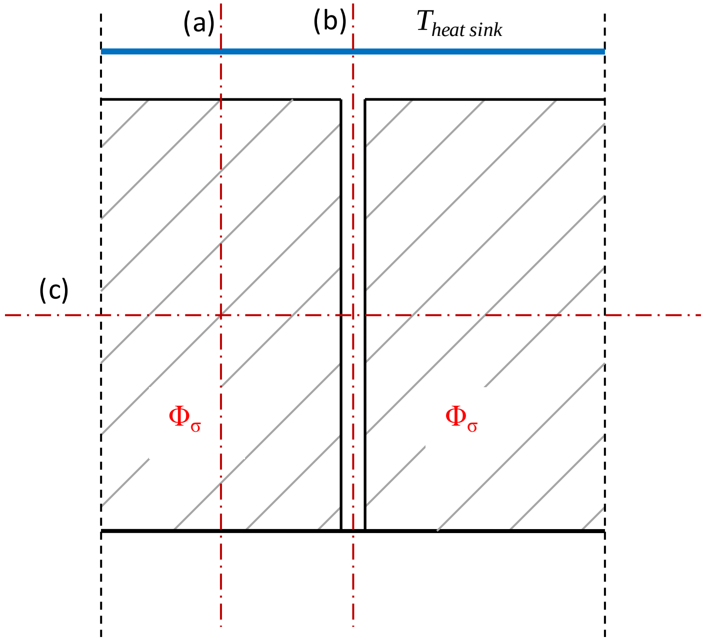
Order of magnitude of heat source terms **(Ω, Φ_σ)** based on calculus done for *LHC upgrade phase I*

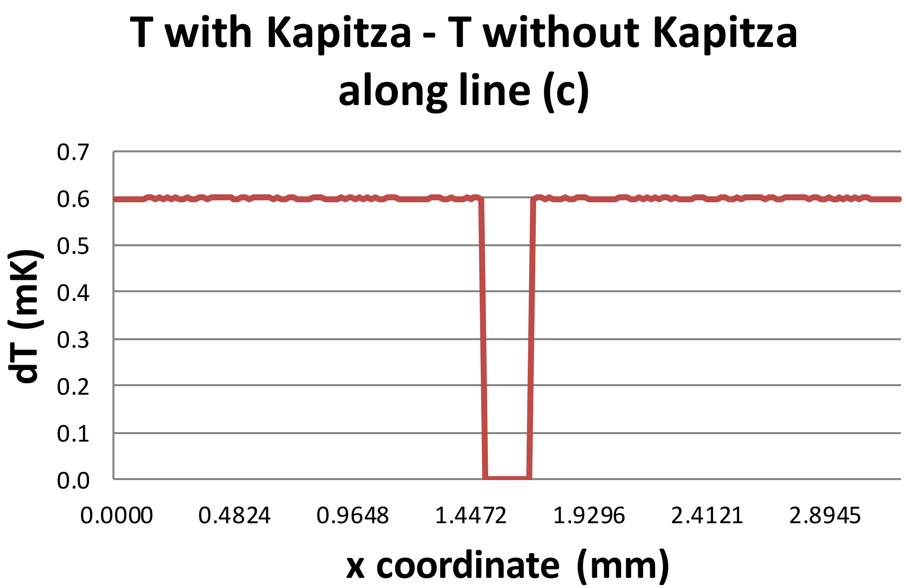


Difference in temperature along line (a) and (b) with and without Kapitza



Validity of local thermal equilibrium (3/3)
Evaluation de $\langle T_\beta \rangle = \langle T_\sigma \rangle = \langle T \rangle$





Conclusions: - *temperature* in the *solid* and in the *liquid* can be considered **equal** within **9 %**
- Kapitza resistance does not influence path taken by the heat
- more than **99.5 %** of the heat is driven to the heat exchanger by the superfluid
- same has been done for surface heat source, same results

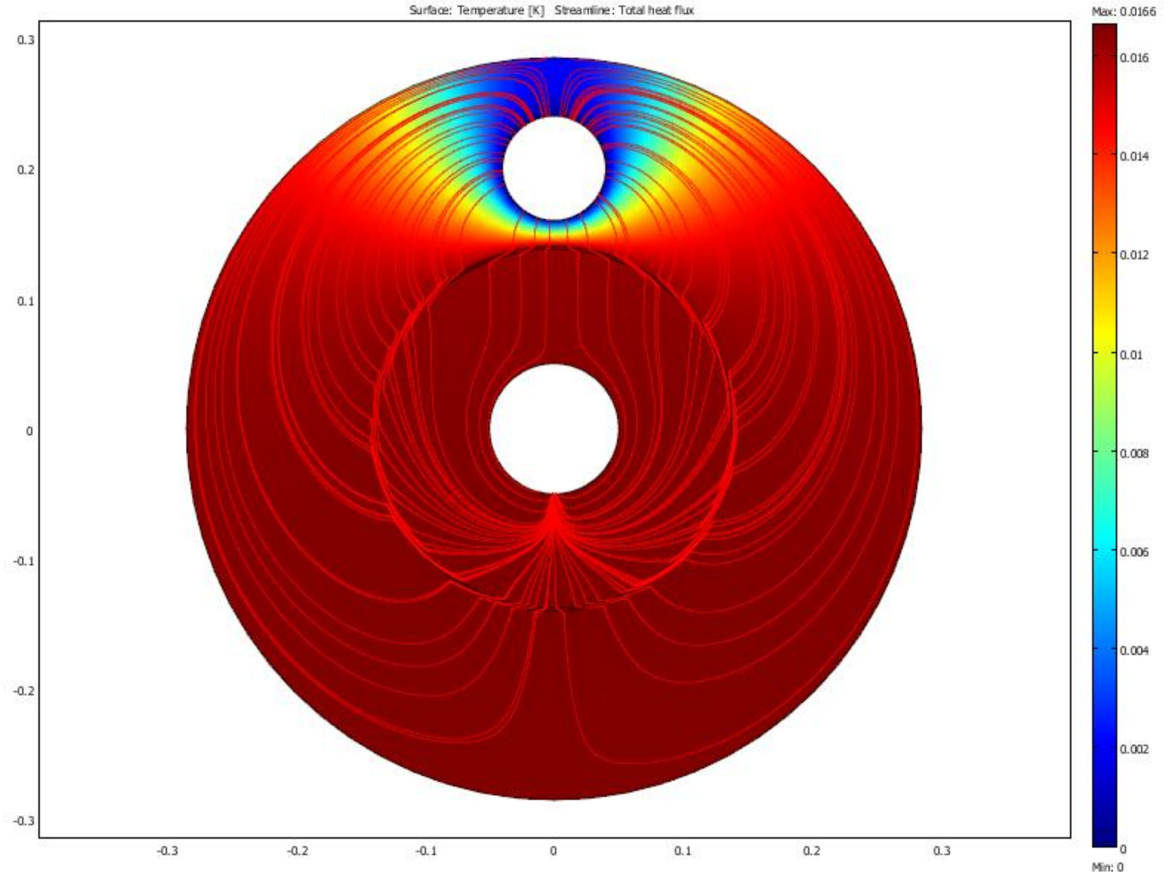
→ ***One equation model for local thermal equilibrium valid***

Application to MQXC (NbTi option) magnet design (1/2)

Assumptions: - *homogeneous heat source* along the axis in the collars so 2 D view sufficient, value based on calculus done for LHC upgrade phase I at peak power deposition
- ***thermal diffusion tensor*** reduced to a diagonal matrix with $\varepsilon_\beta \times k_{eff}$ for coefficient
- ε_β = 6 % for the collars, 2 % for the yoke (two porous media)

One heat sink at 1.9 K

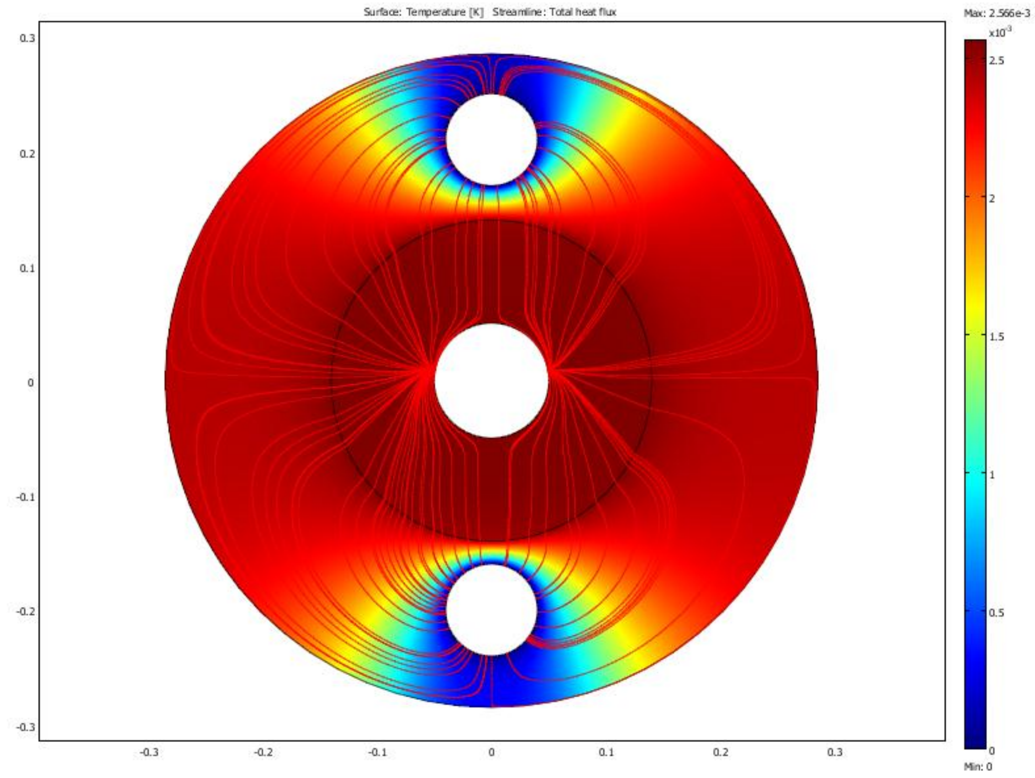
Maximum **ΔT ≈ 17 mK**



Application to MQXC (NbTi option) magnet design (2/2)

Two heat sinks at 1.9 K

Maximum **ΔT ≈ 2.5 mK**



Considering only the collar and yoke, neglecting details of the coil, the **method of volume averaging** already allows us to appreciate the influence on temperature difference regarding the use of **one or two** heat sinks

Application to MQXC (NbTi option) magnet design (3/3)

•Extension of the previous model

- **include the coil and beam pipe in the geometry of the model**

- extend the model to *transient regime* (local thermal equilibrium valid?)

- Treat the *coil* as a third *porous medium*

Conclusion

- The **method of volume averaging** has been investigated to study **superconducting magnets' thermo-hydraulics** considering the interior of a magnet as a **porous media**
- Three porous media can be pointed out: coils, collars and yoke
- Method can be apply to different type of cooling (pressurized He II, supercritical helium,…)
- The different *phases* and *length-scales* necessary to apply the method have been identified
- Method applied to the heat diffusion model of superfluid helium
- In steady state ***local thermal equilibrium*** assumption ***valid***: one equation model can be used
- Application to the design of MQXC magnet (NbTi option), only two porous media considered (collars and yoke)
- One order of magnitude wined on the maximum temperature difference with two-heat sinks

- **Need to extend this method for He II two-fluids model for magnet application (heat transfer between solid and liquid, Gorter-Mellink regime and transient regime)**

Thank you for attention