Usage of manifolds in electromagnetism and superconductor modelling

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Chats 2011
Applied Superconductivity

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- Motivation of the topic / Introduction
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- Practical examples
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- Modelling twisted conductors in 2D without loss of any information
  - Magnetostatics: defining critical current of conductor in self-field
  - Eddy current problems: hysteresis losses of a conductor
- Summary

Motivation / Introduction

- Phenomena are not coordinate dependent, but only evaluation in computations requires coordinates, or parametrizations, to present the problem.

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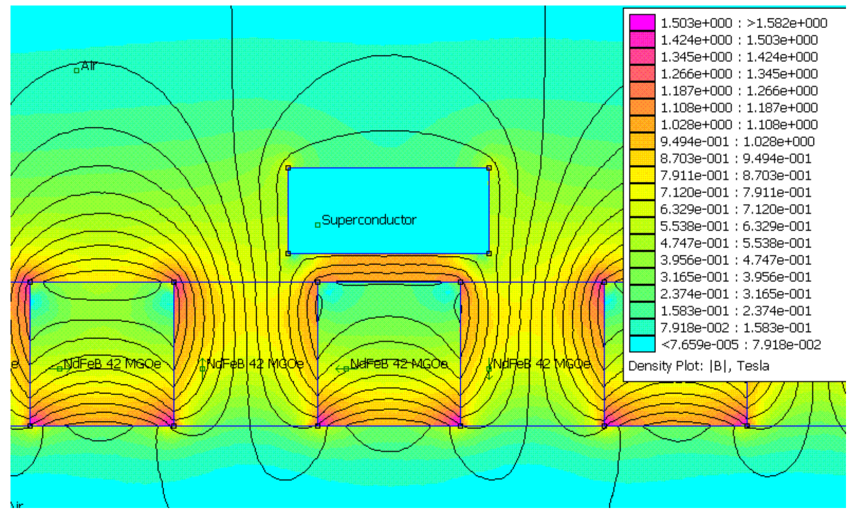

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Motivation / Introduction

- Phenomena do not depend on metric, but material characterization is done in predetermined metric, because it enables comparison and unified approach.

[Sawo]

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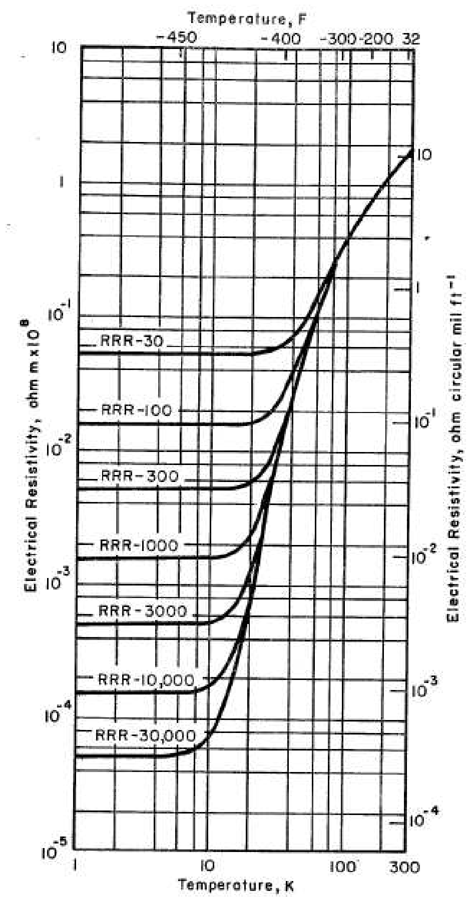

[Selected cryogenic data notebook]

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- To utilize symmetry easily or pose problems conveniently, e.g. commercial FEM programs include set of predetermined coordinate systems.
- However, these are not all that we could think to use.
- Sometimes new computations are enabled if we leave the Euclidean world behind.

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Loosely: A generalization of some coordinate system,
like cartesian or cylindrical,
is known in mathematics by name **manifold**.

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- Manifold is something that is parametrizable.
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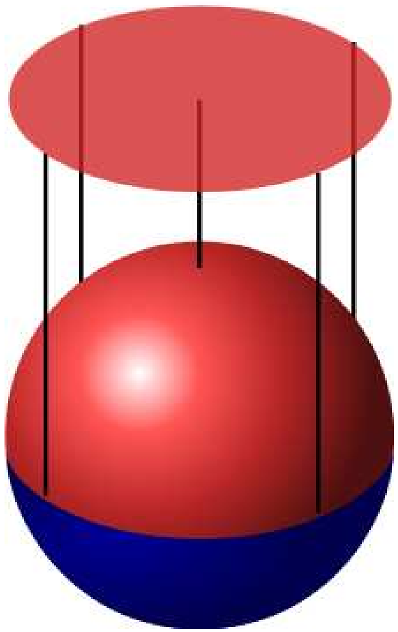

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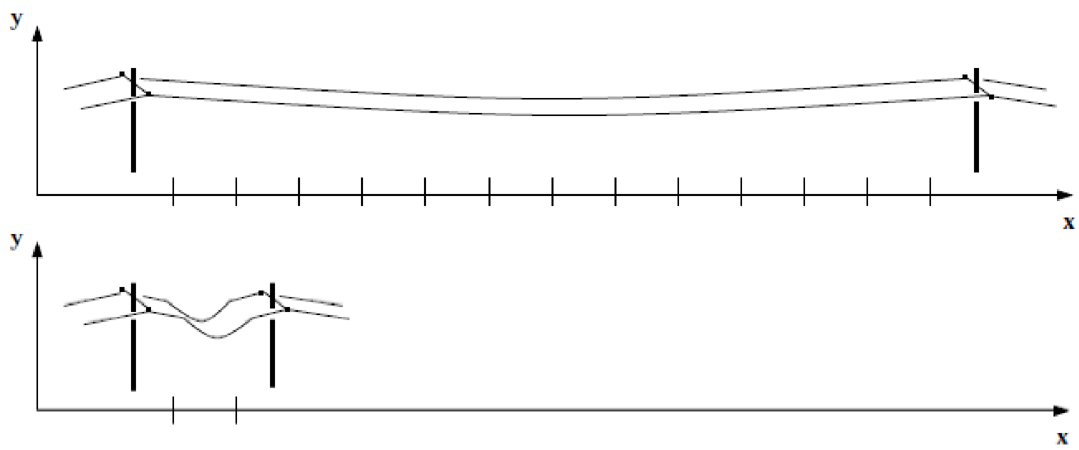

[P Raumonen]

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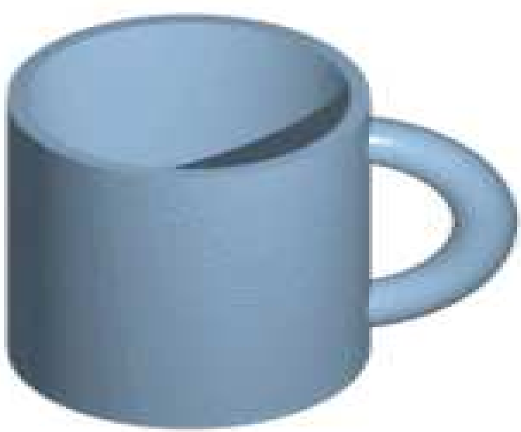

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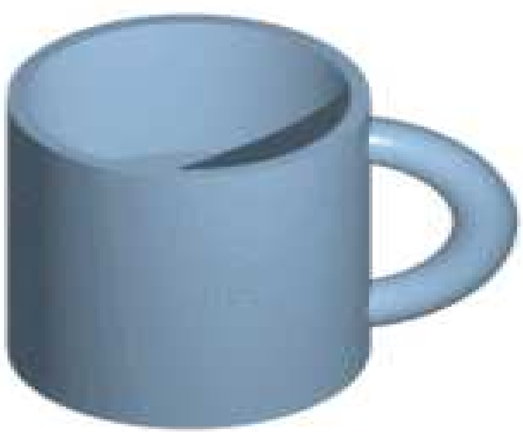

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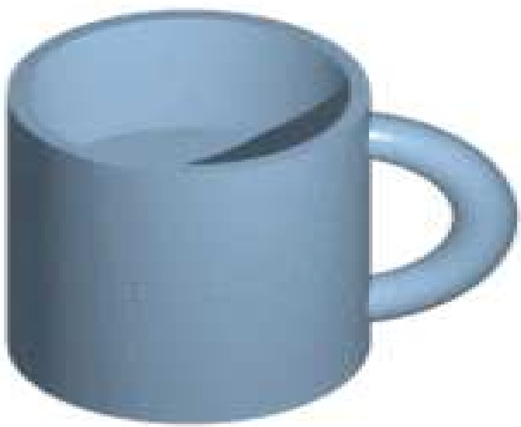

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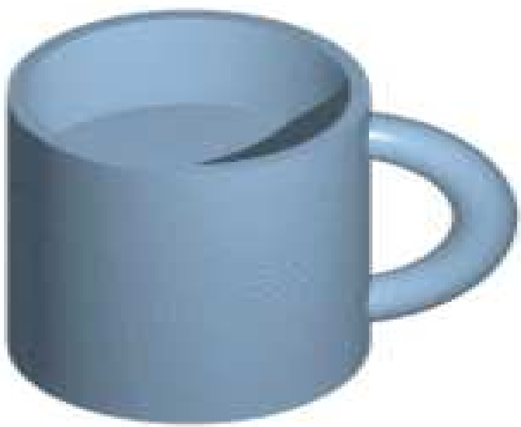

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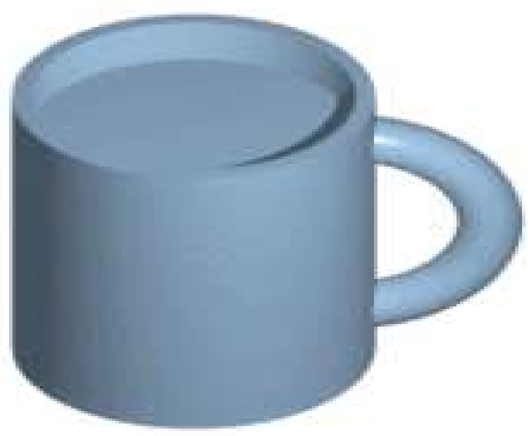

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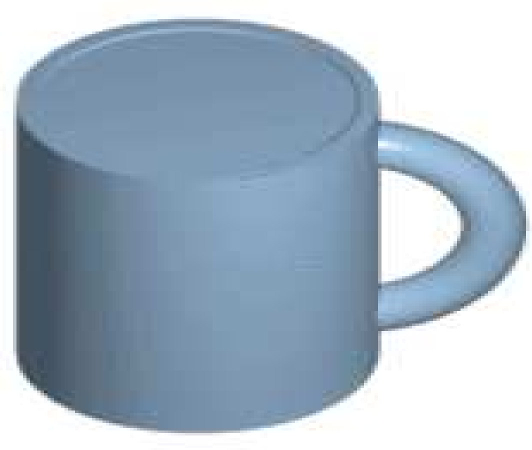

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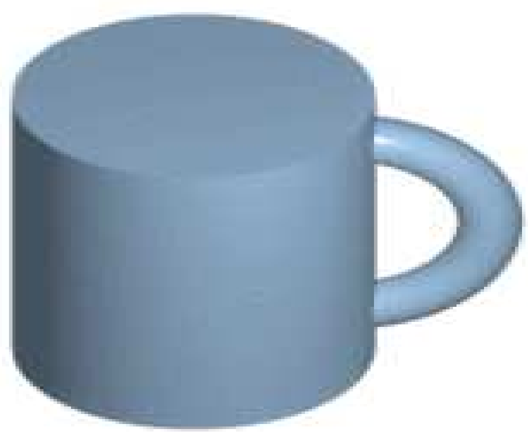

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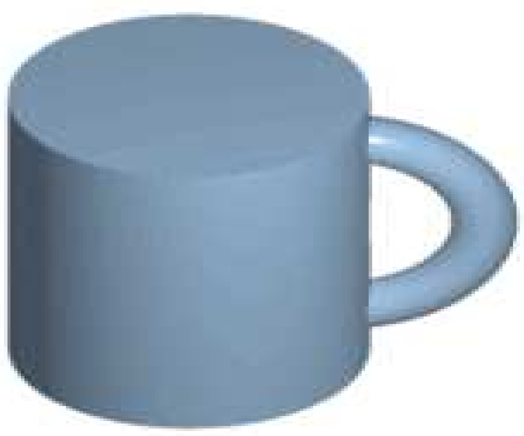

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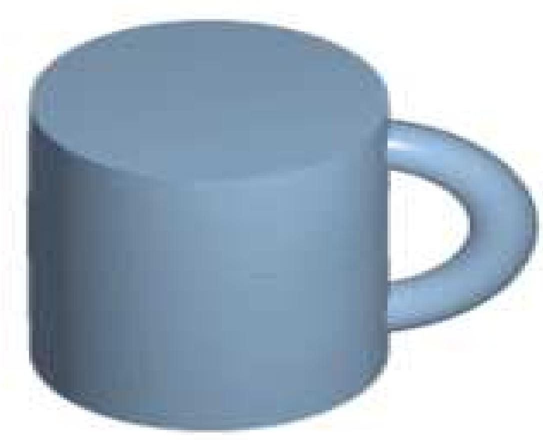

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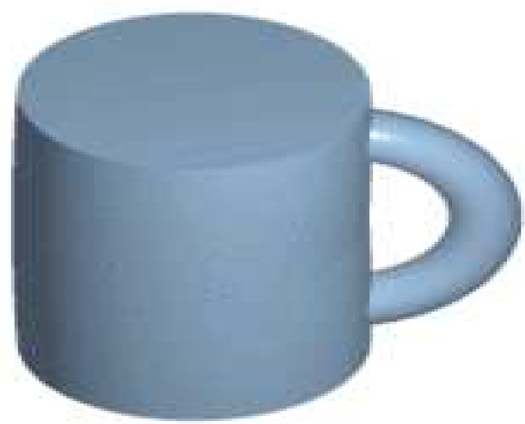

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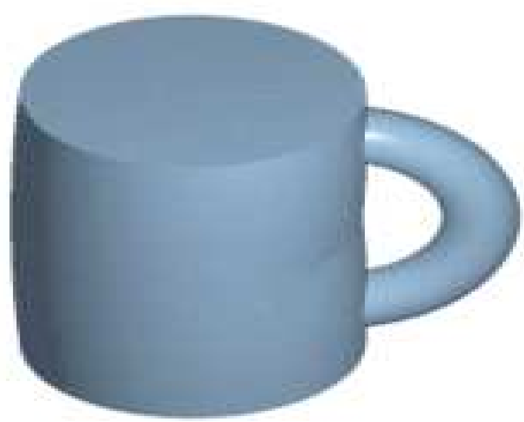

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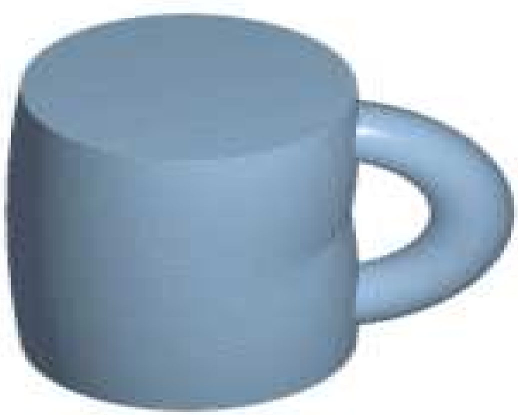

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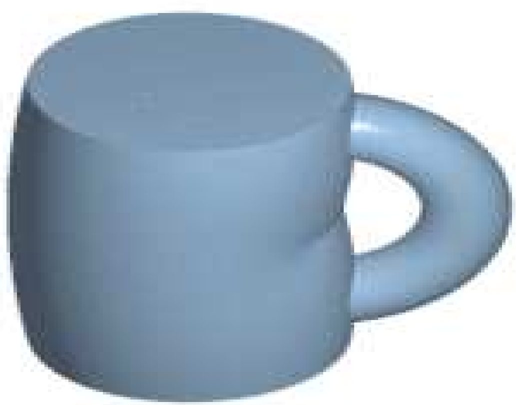

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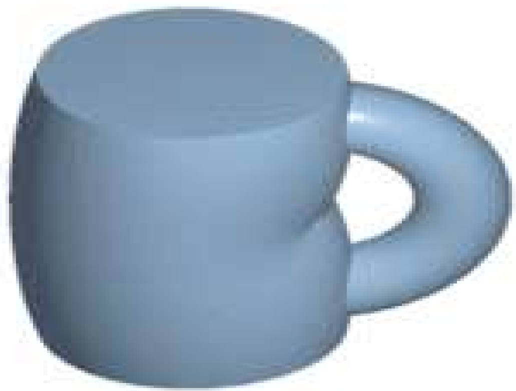

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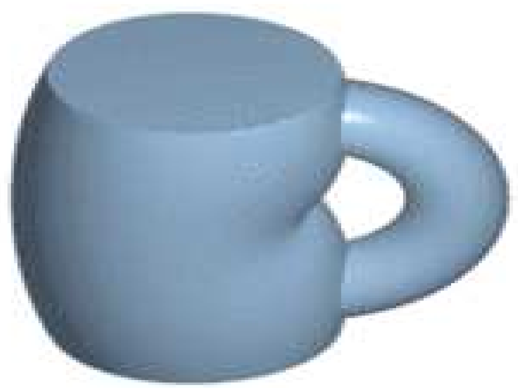

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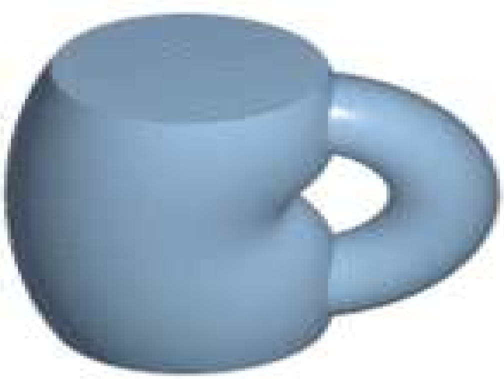

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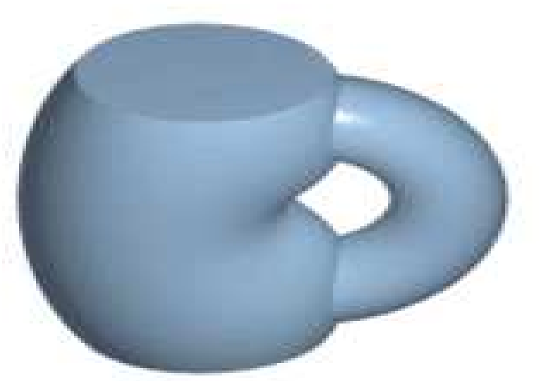

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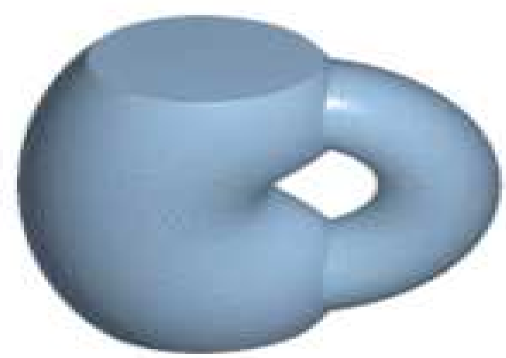

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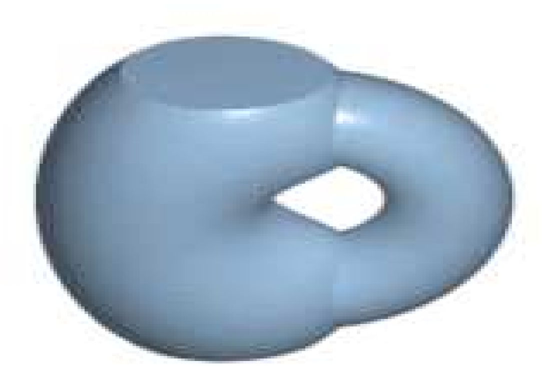

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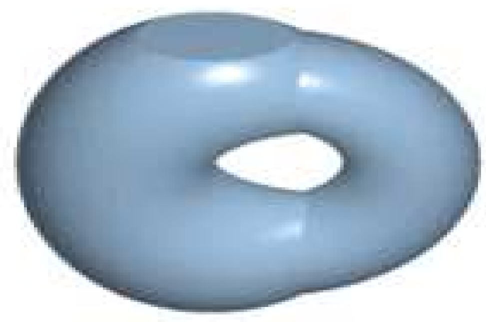

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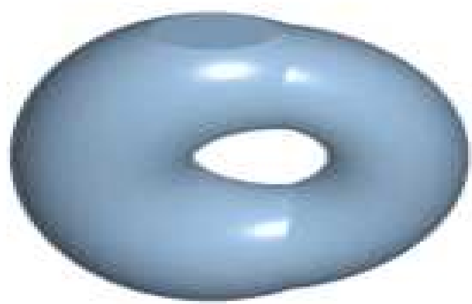

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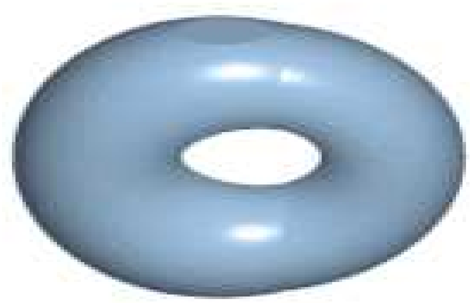

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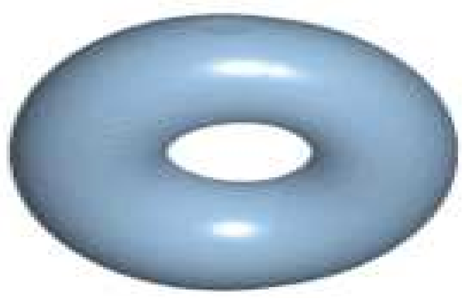

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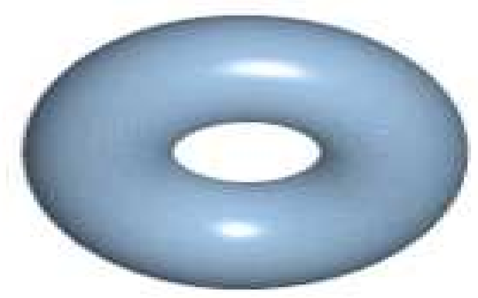

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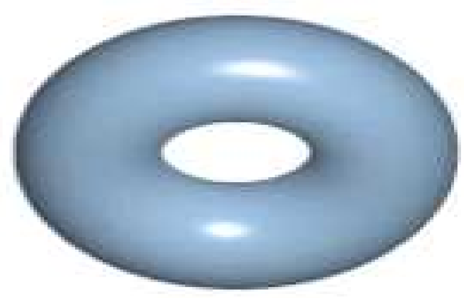

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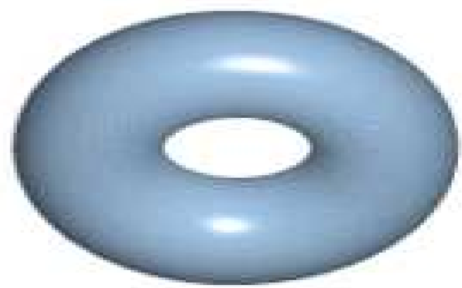

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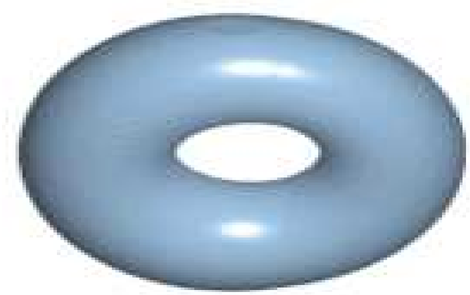

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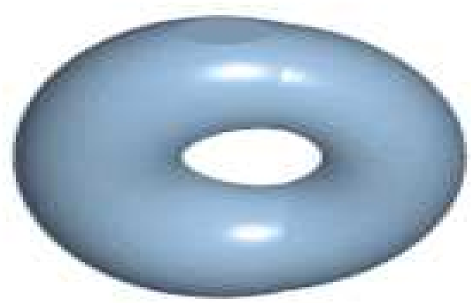

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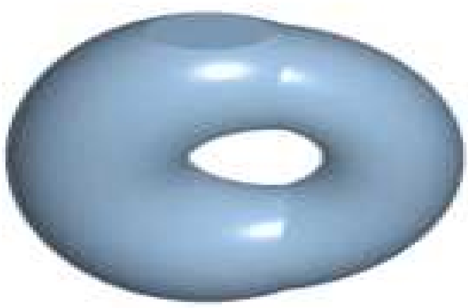

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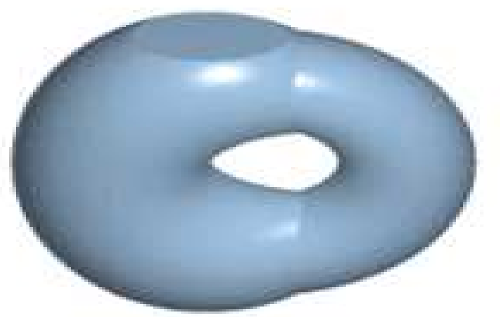

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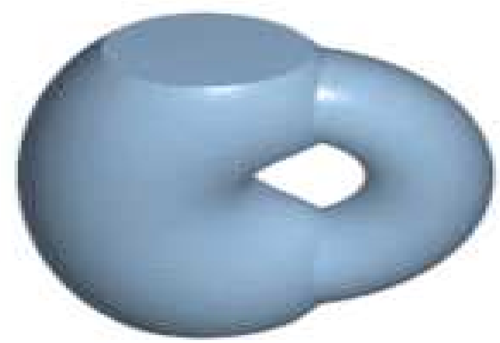

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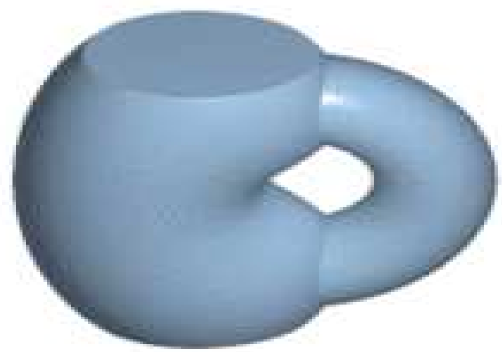

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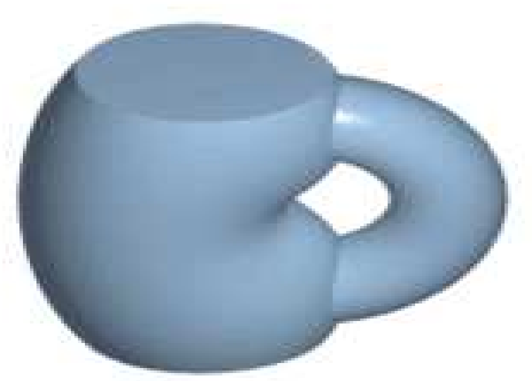

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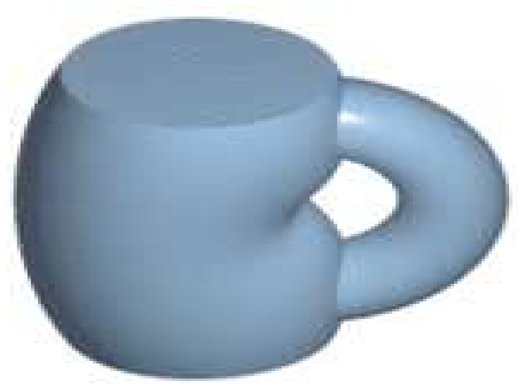

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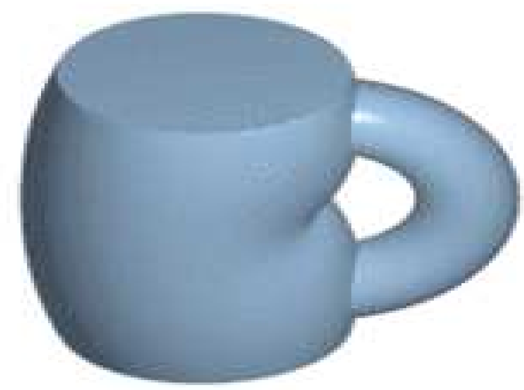

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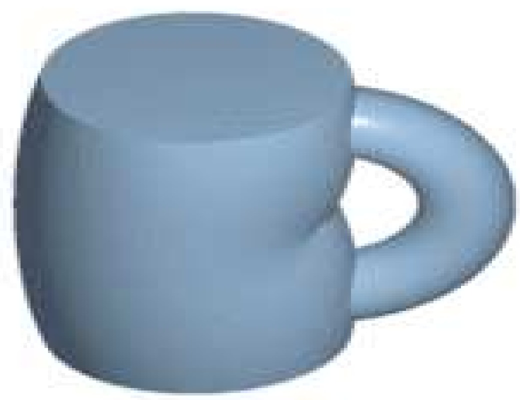

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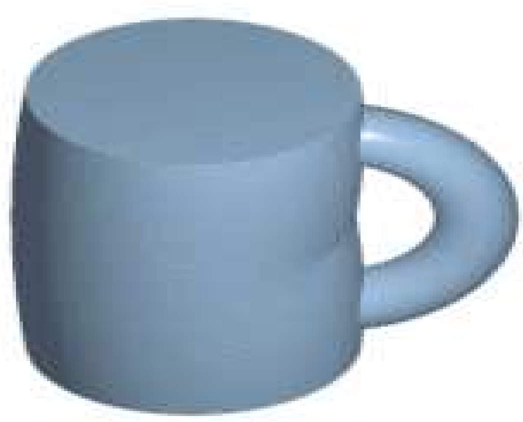

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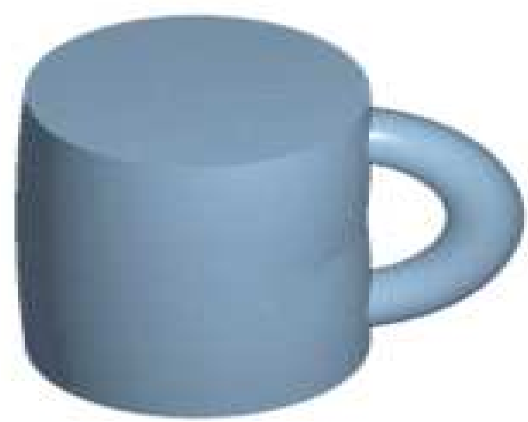

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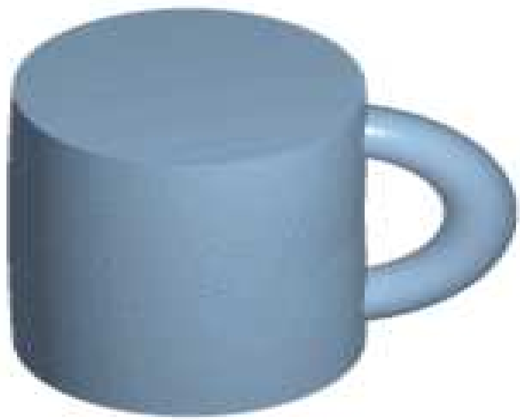

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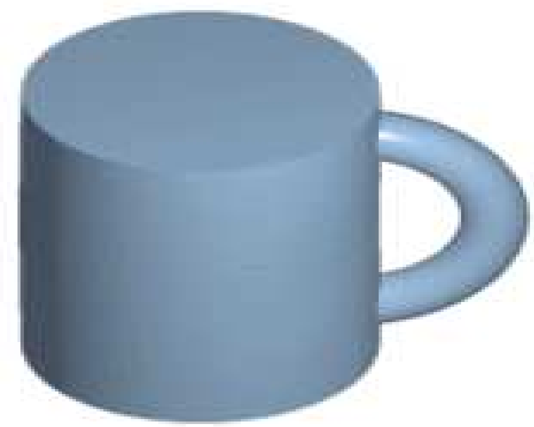

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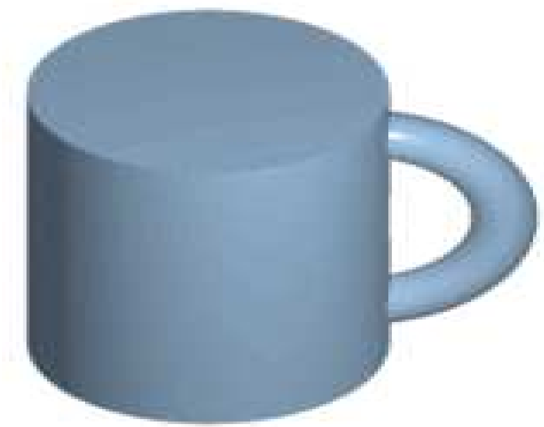

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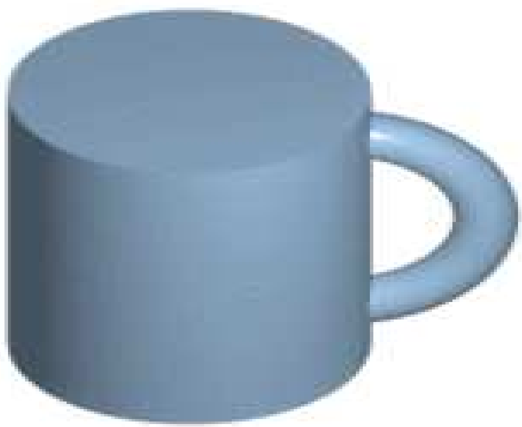

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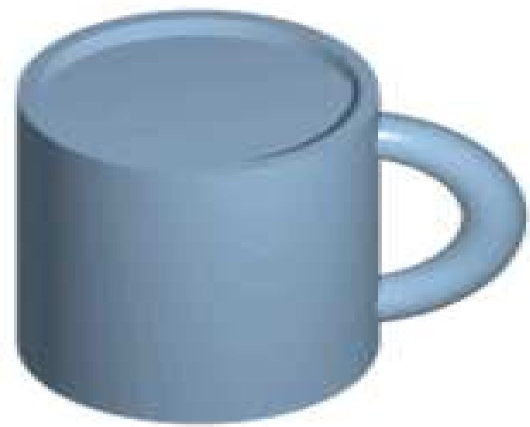

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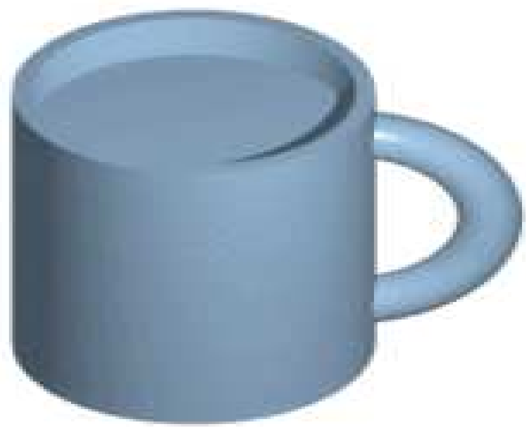

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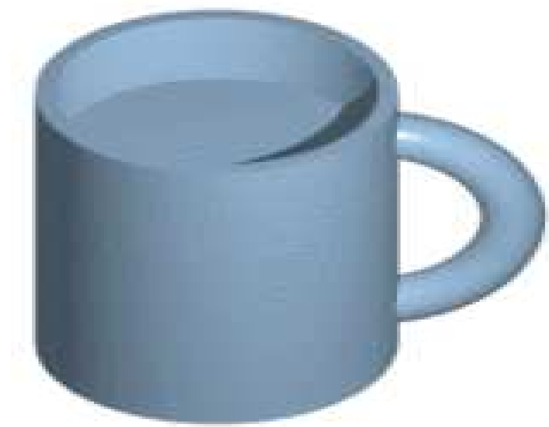

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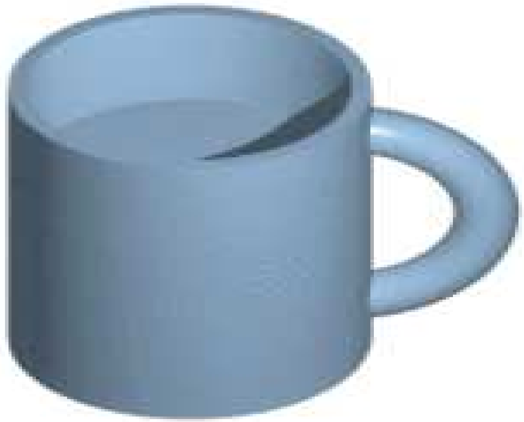

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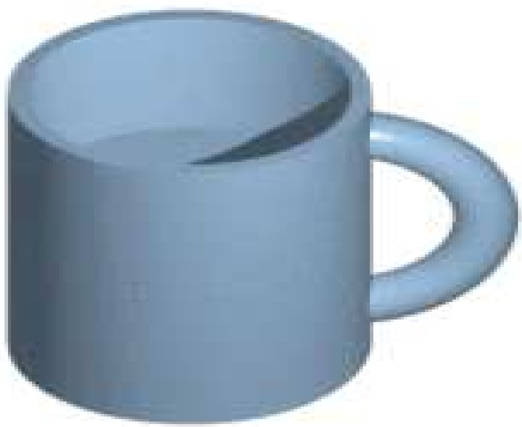

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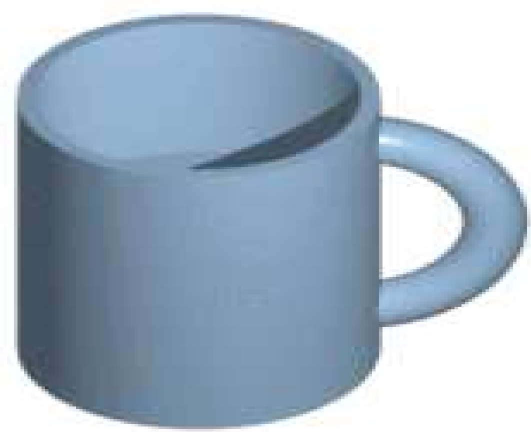

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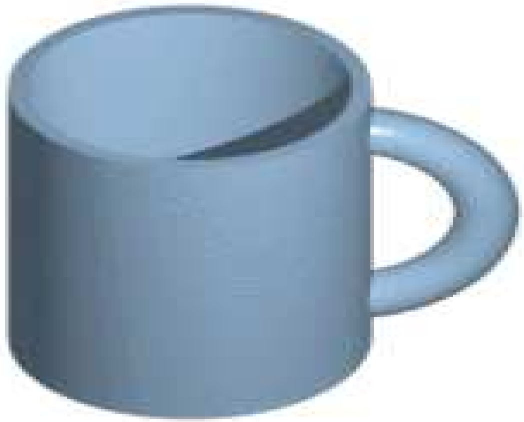

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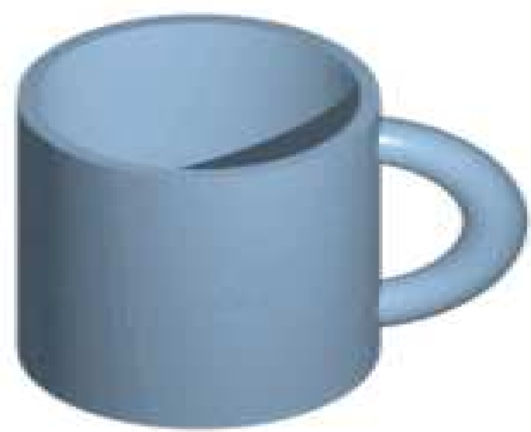

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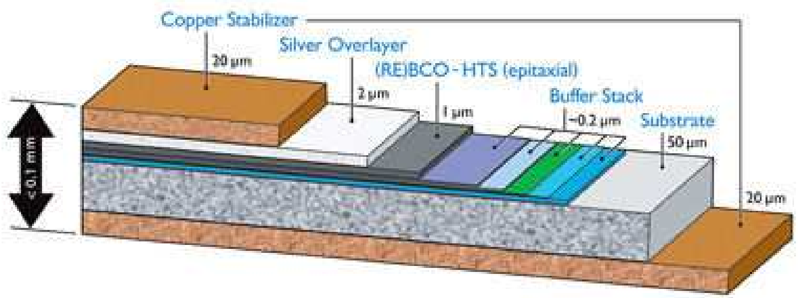

[SuperPower]

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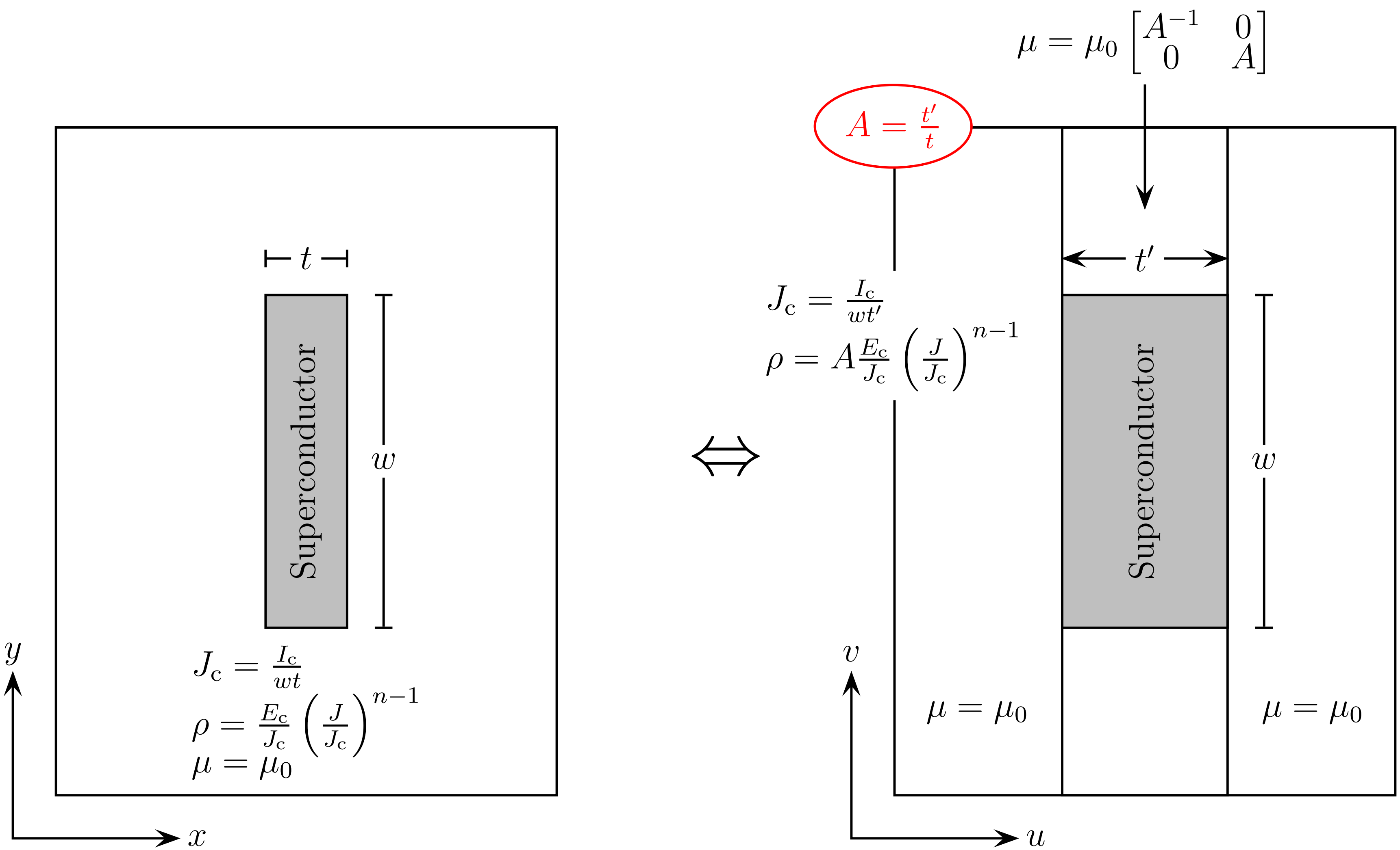

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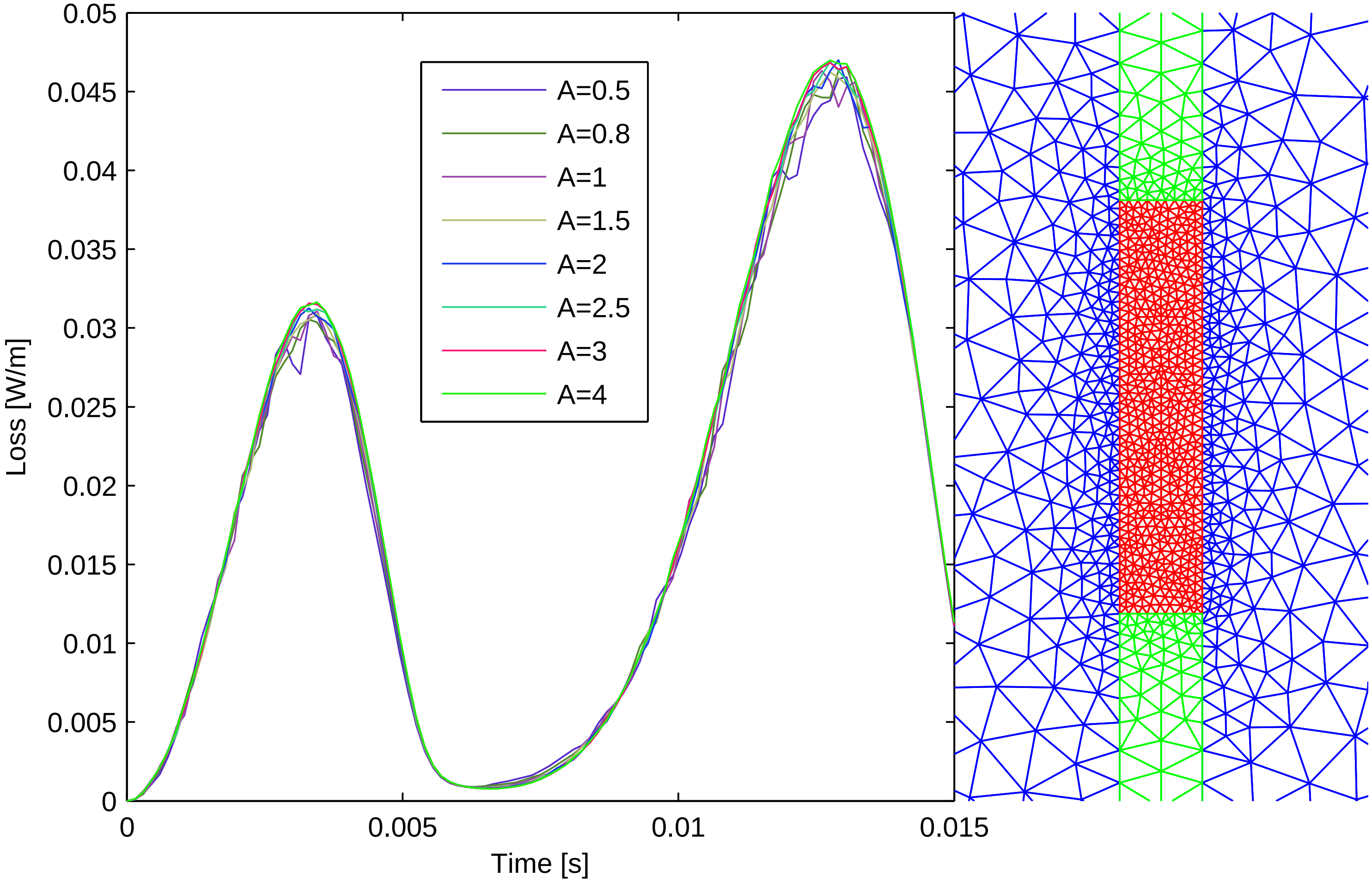

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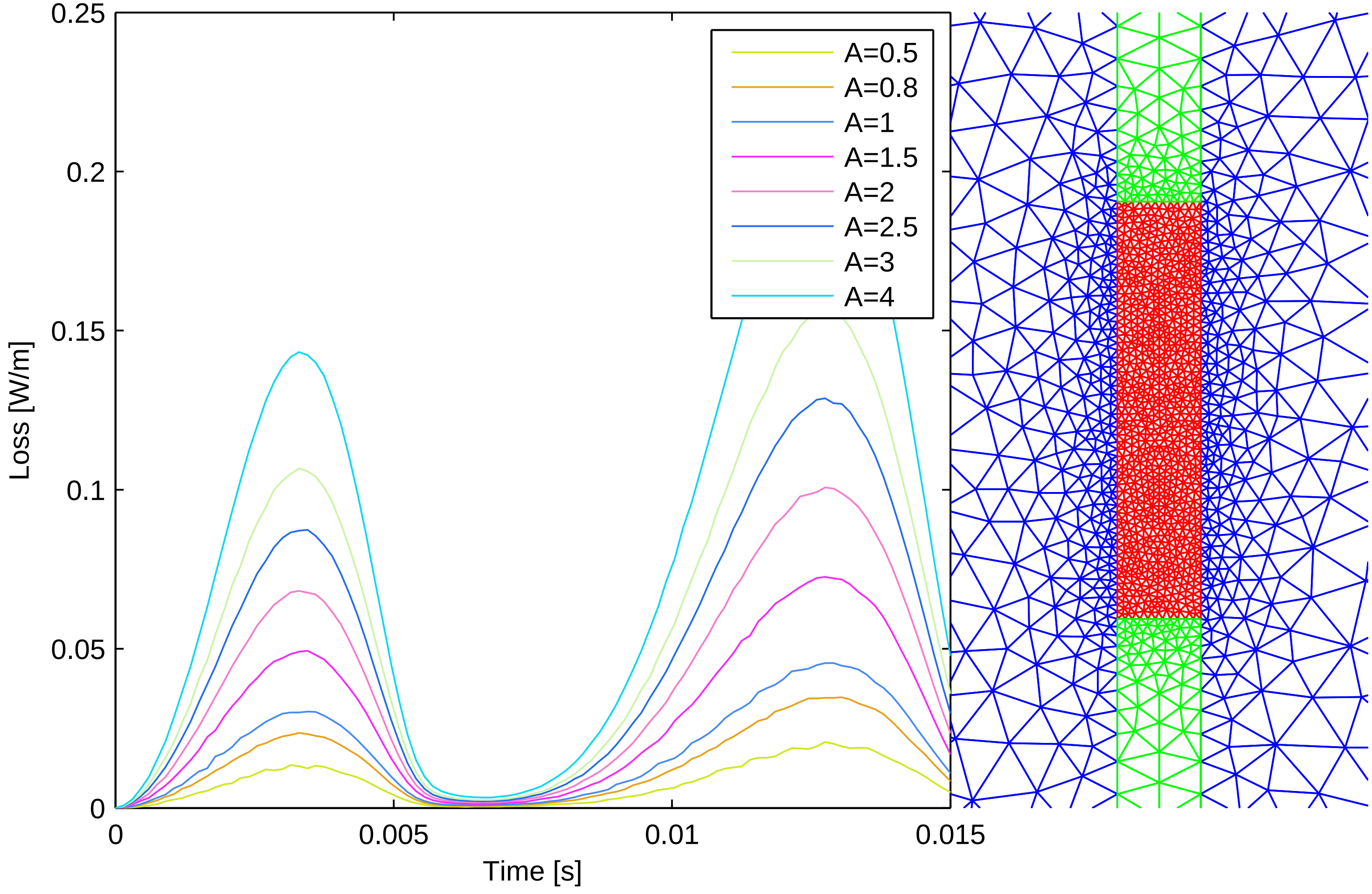
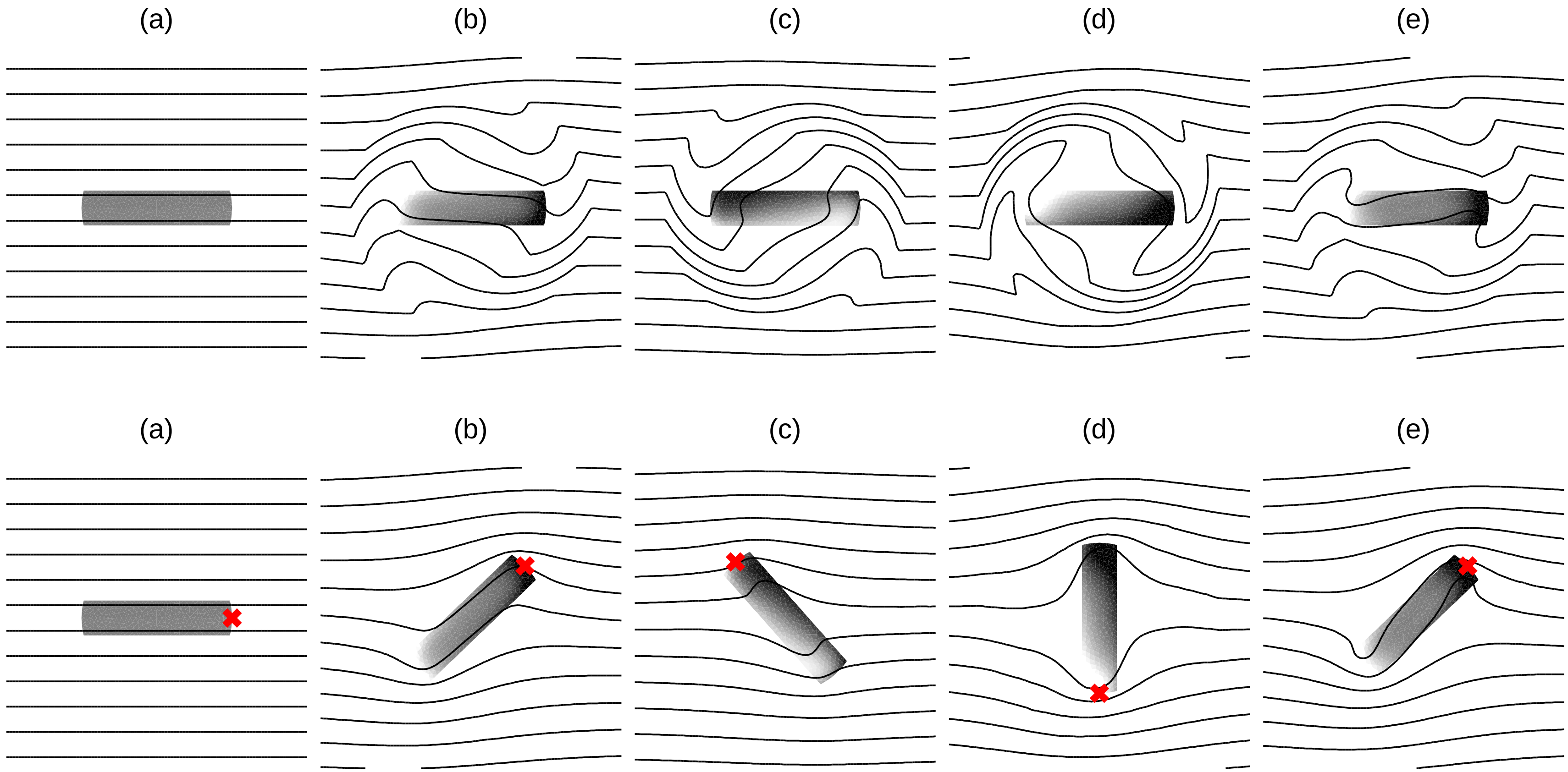


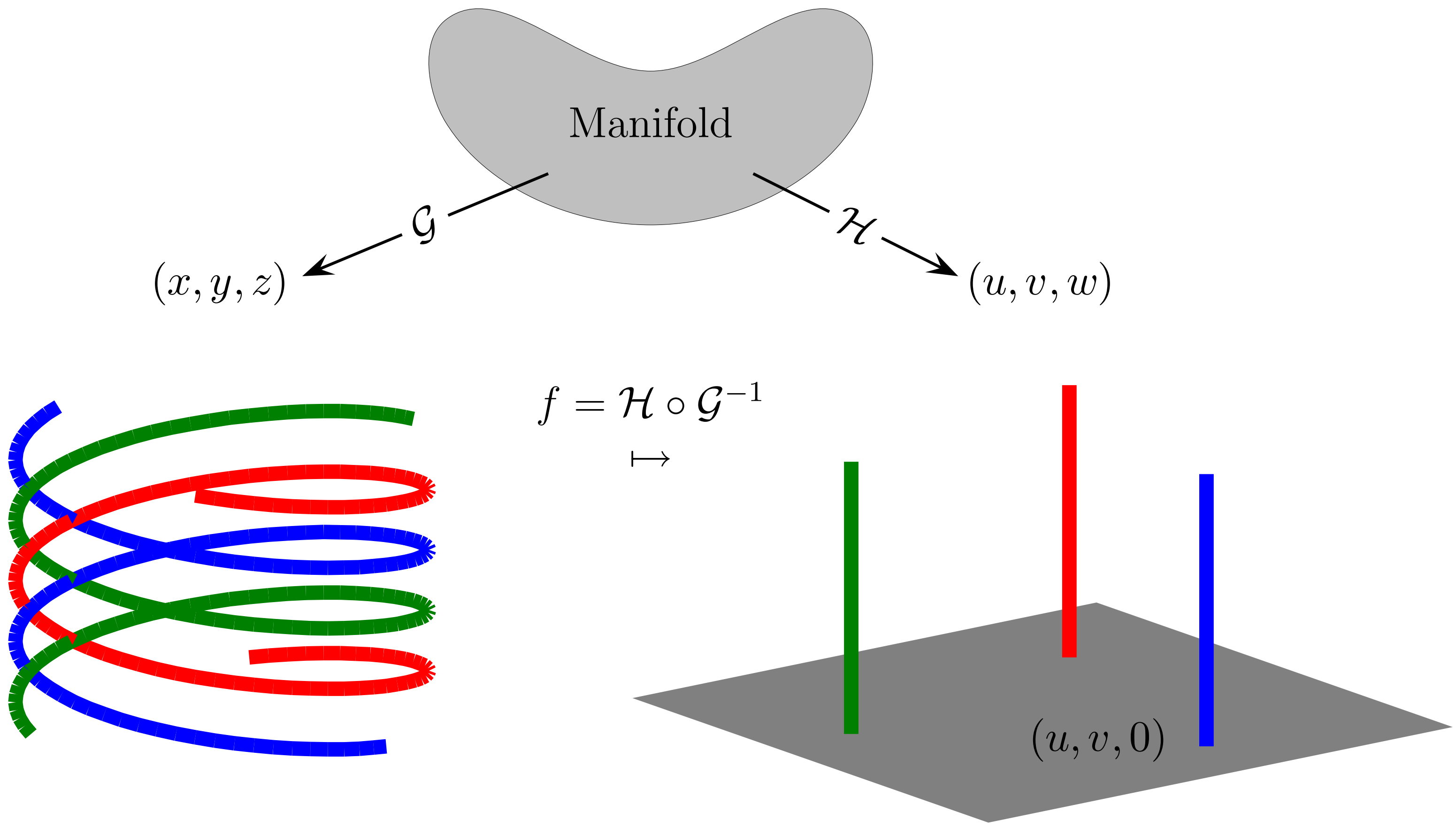
Examples of using manifold originating modelling

- In general: modelling is not restricted to what we see and as we see it.
- Simulation of movement with rigid meshes.



Examples of using manifold originating modelling

- In general: modelling is not restricted to what we see and as we see it.
- Being helicoidal is not absolute (compare being a doughnut or a mug).



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- Example: translation symmetry
  $F(x,y,z) = F(x,y,z+a) \quad \forall a \in \mathbb{R}$

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- Example: reflection symmetry
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- Example: combination of translation and rotation symmetry in a helicoidal structure in Cartesian coordinates
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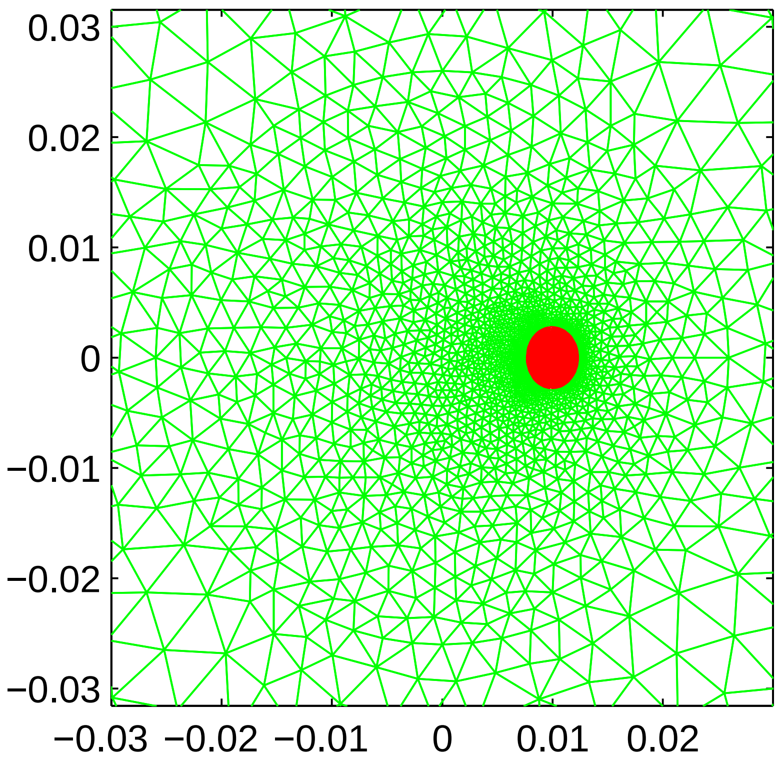
- Presentation of the domain in another coordinates than Cartesian can be more convenient for computations $\rightarrow$ we get rid of rotation and work only with translation symmetry.
  Theory of Riemannian manifolds tells us how to express with numbers the material properties in this new coordinate system.

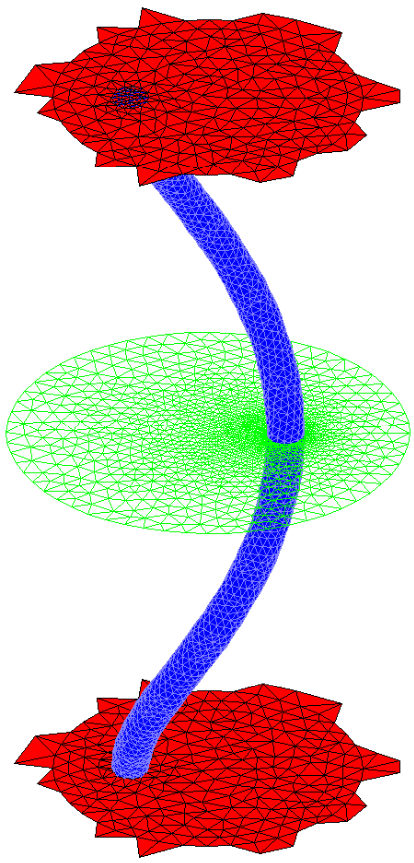
SYMMETRY ANIMATION

Proof of concept: magnetostatics and helicoidal current

Twist pitch = 10 cm, filament/conductor radius = 2.5 mm

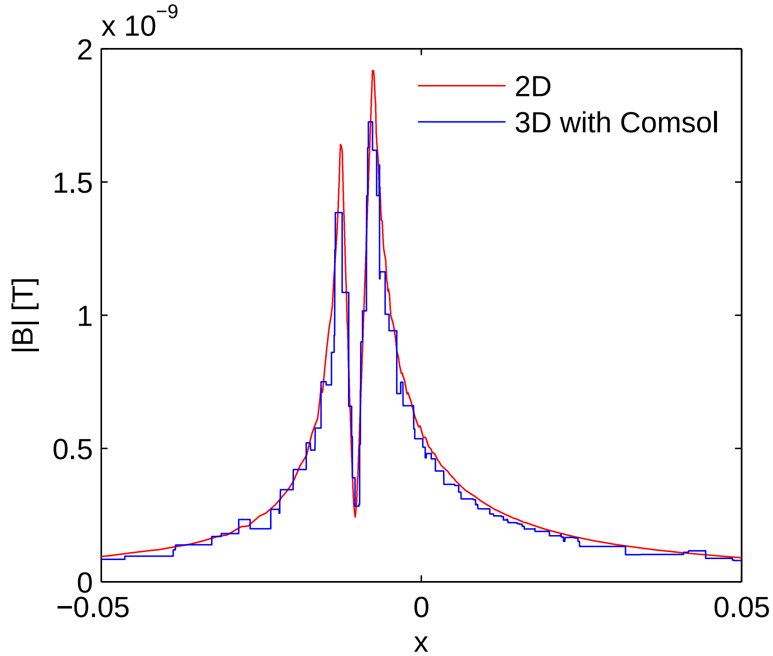


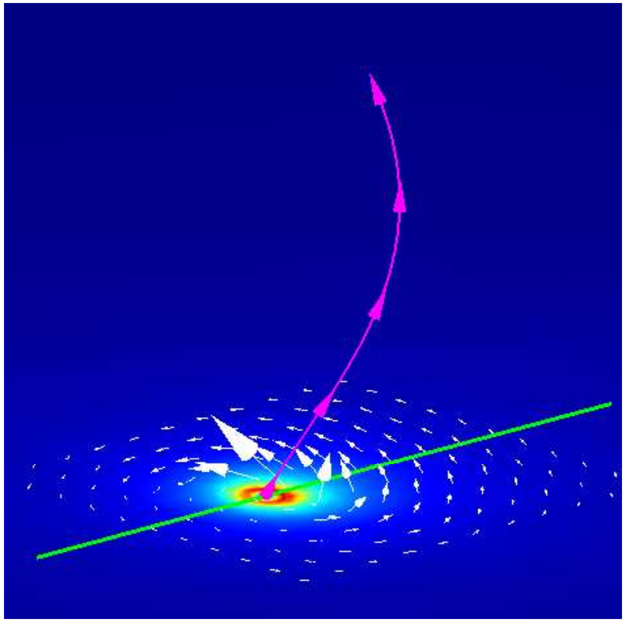
2D



3D (generated with Comsol)

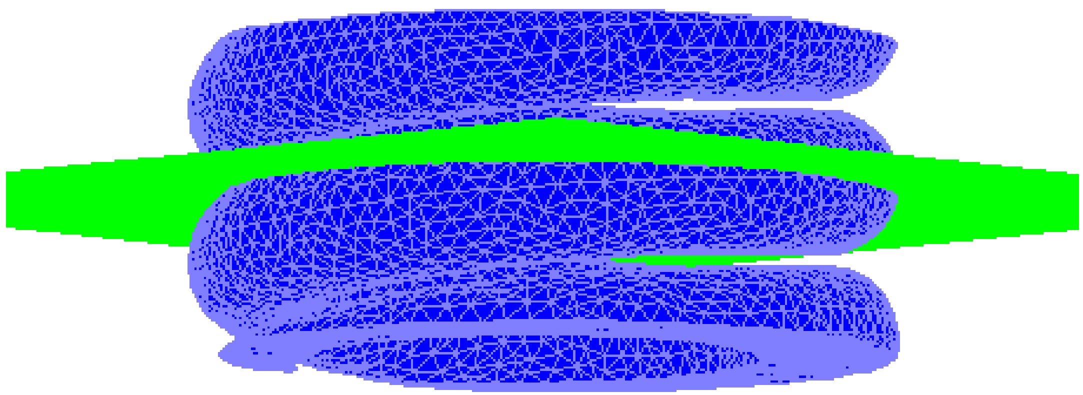
Proof of concept: magnetostatics and helicoidal current





Proof of concept: magnetostatics and helicoidal current

Twist pitch = 5.5 mm



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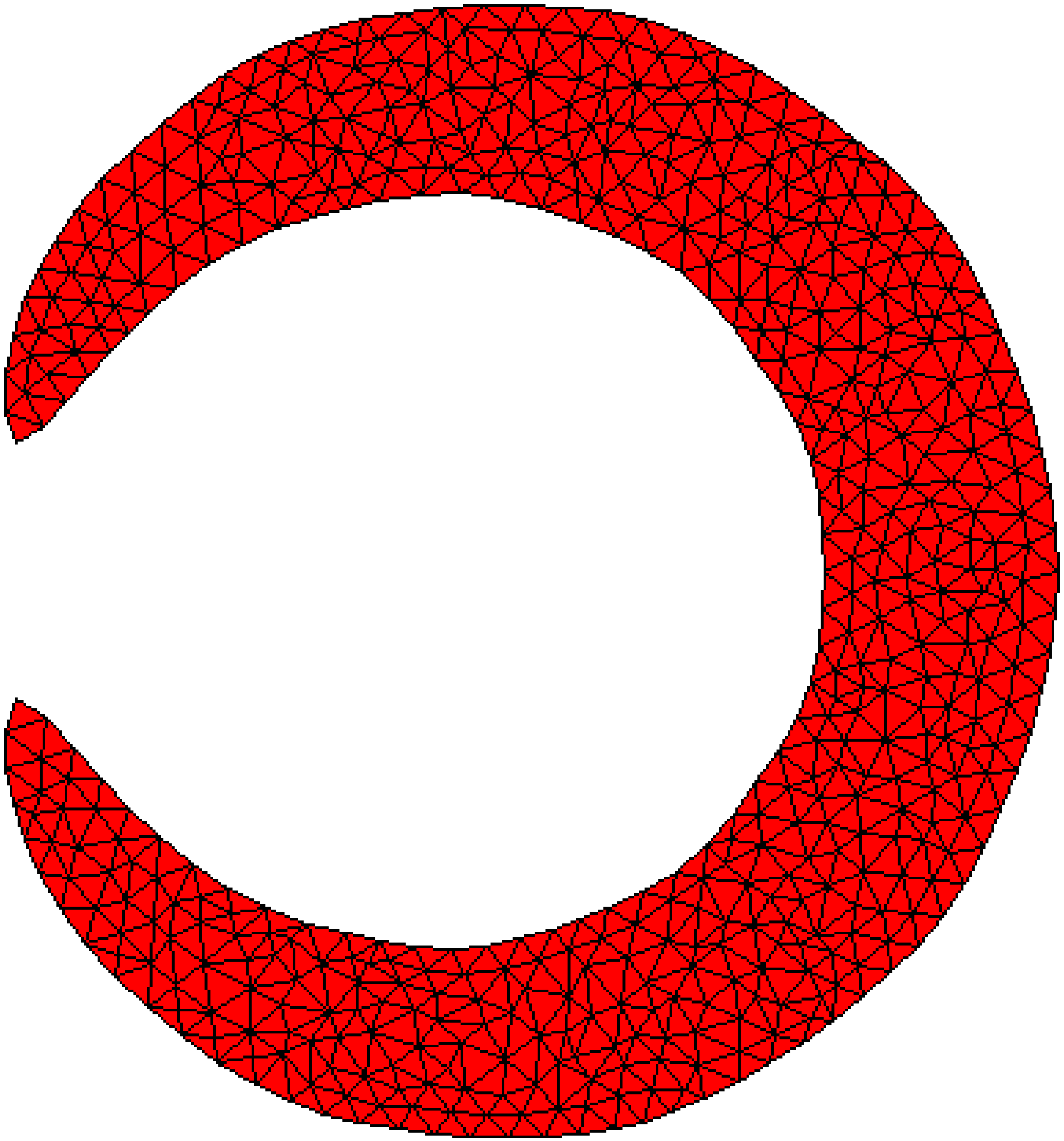

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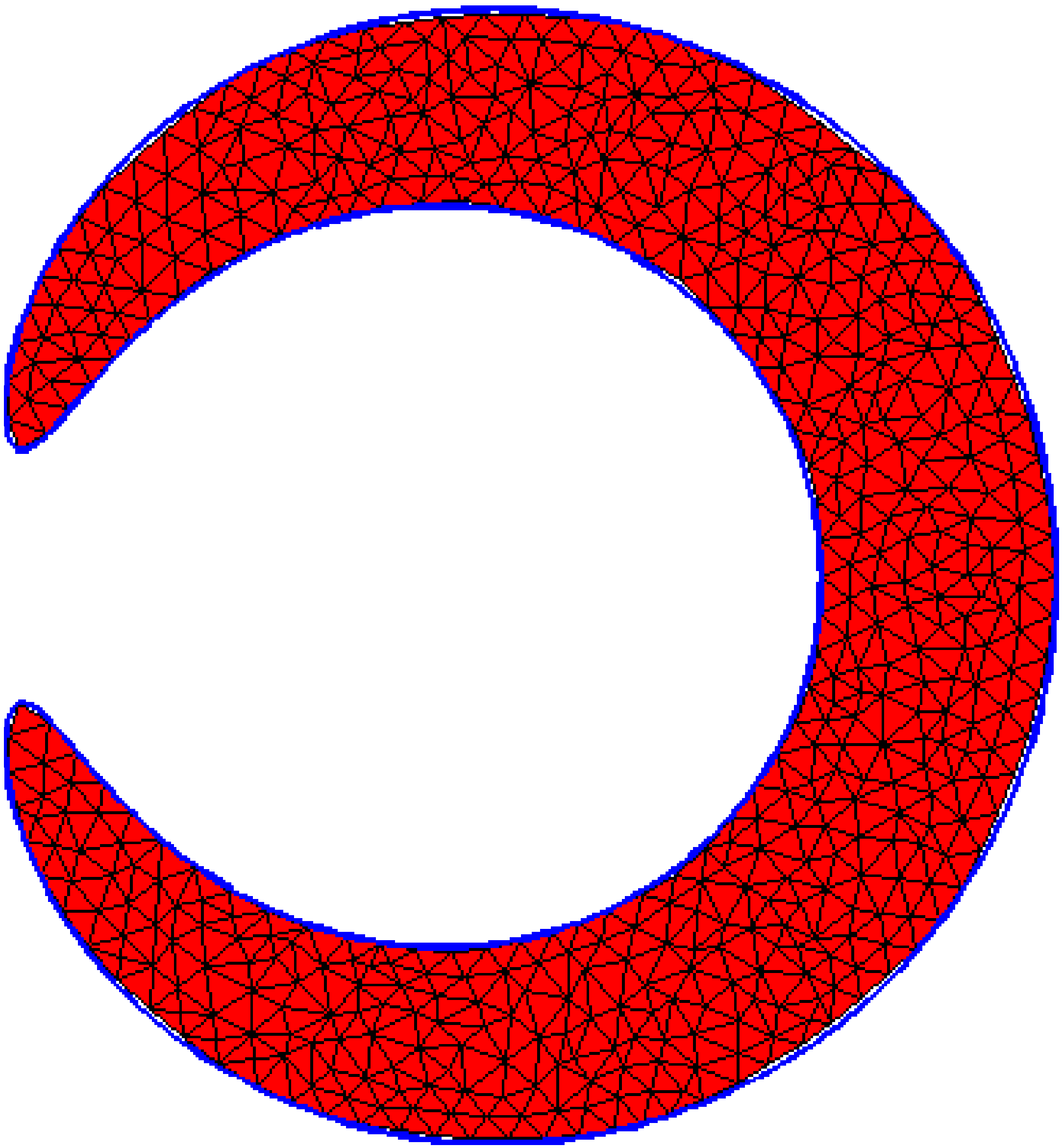

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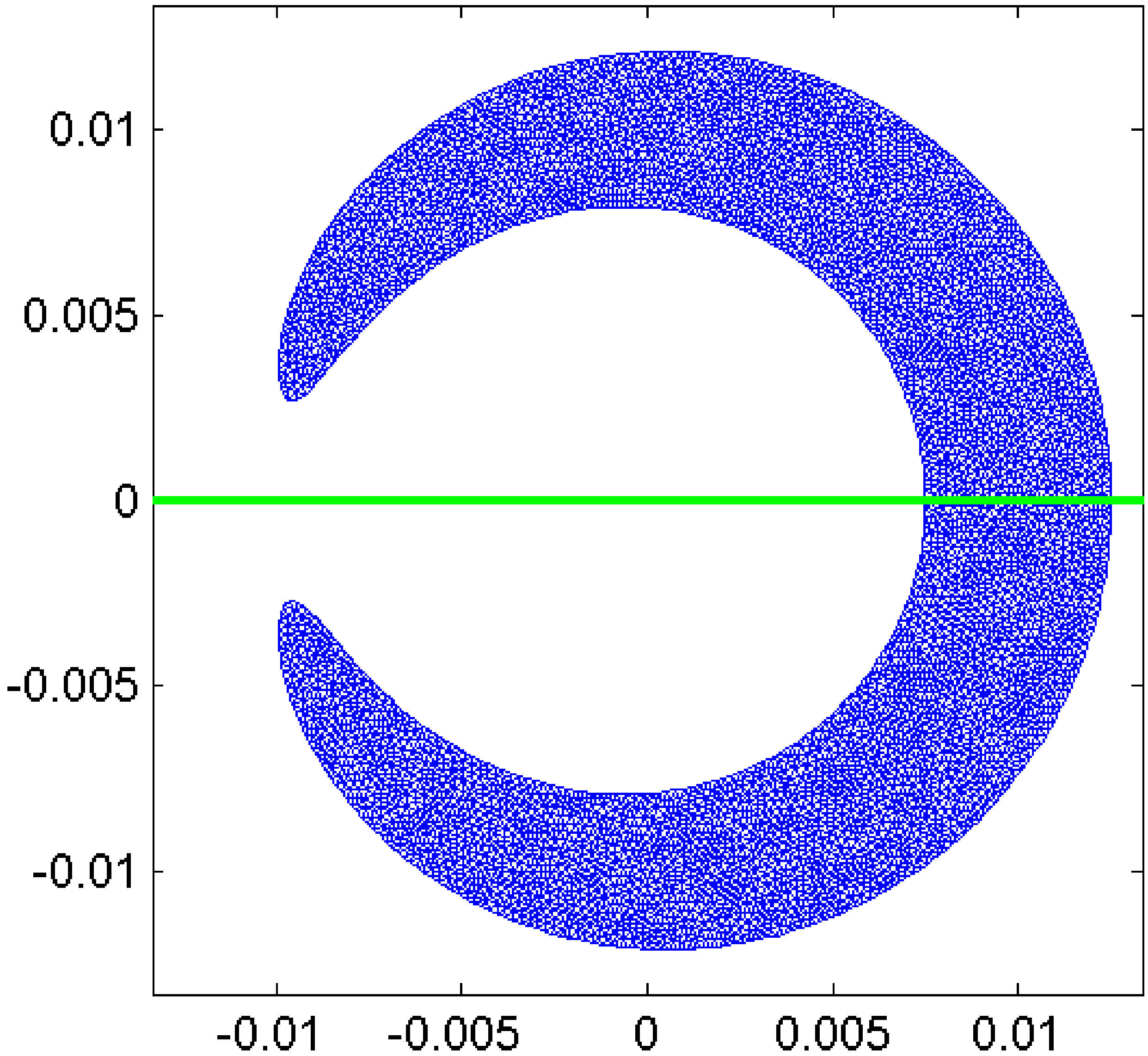

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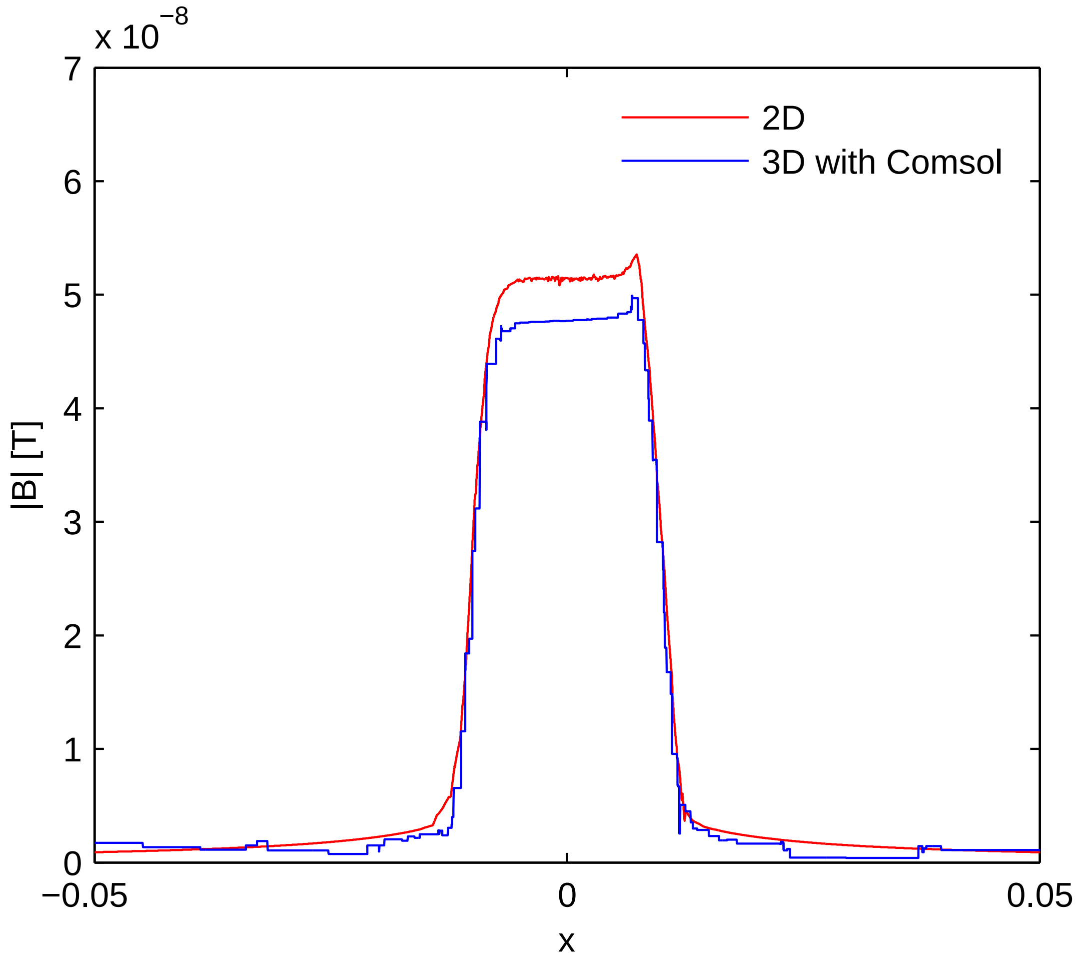
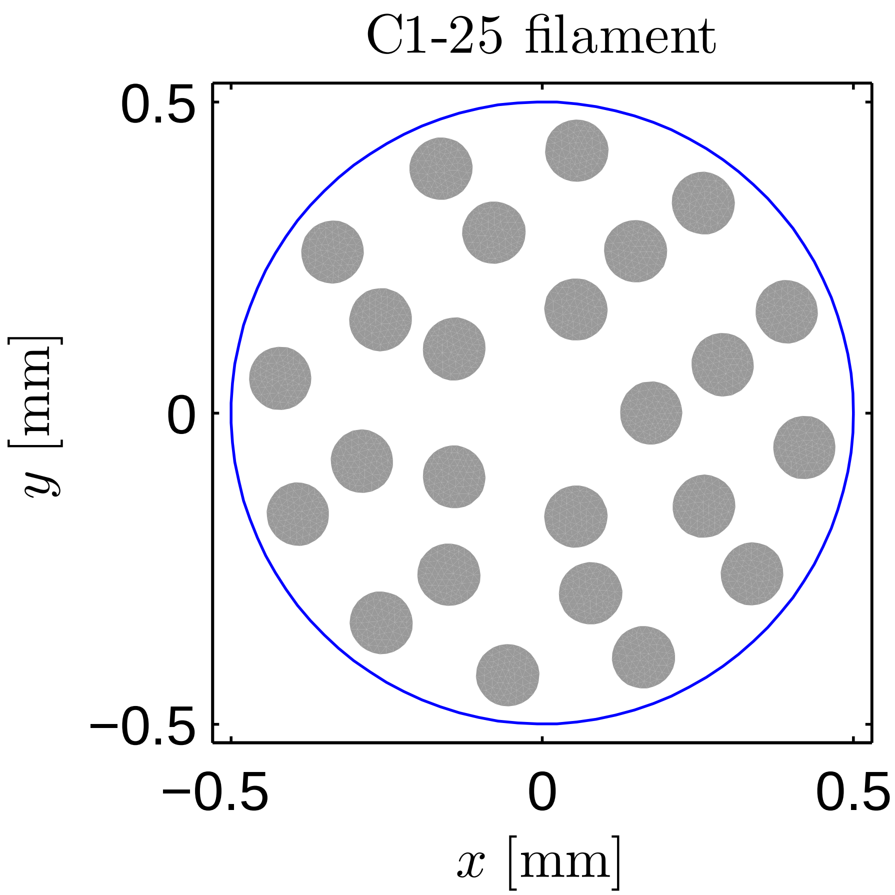
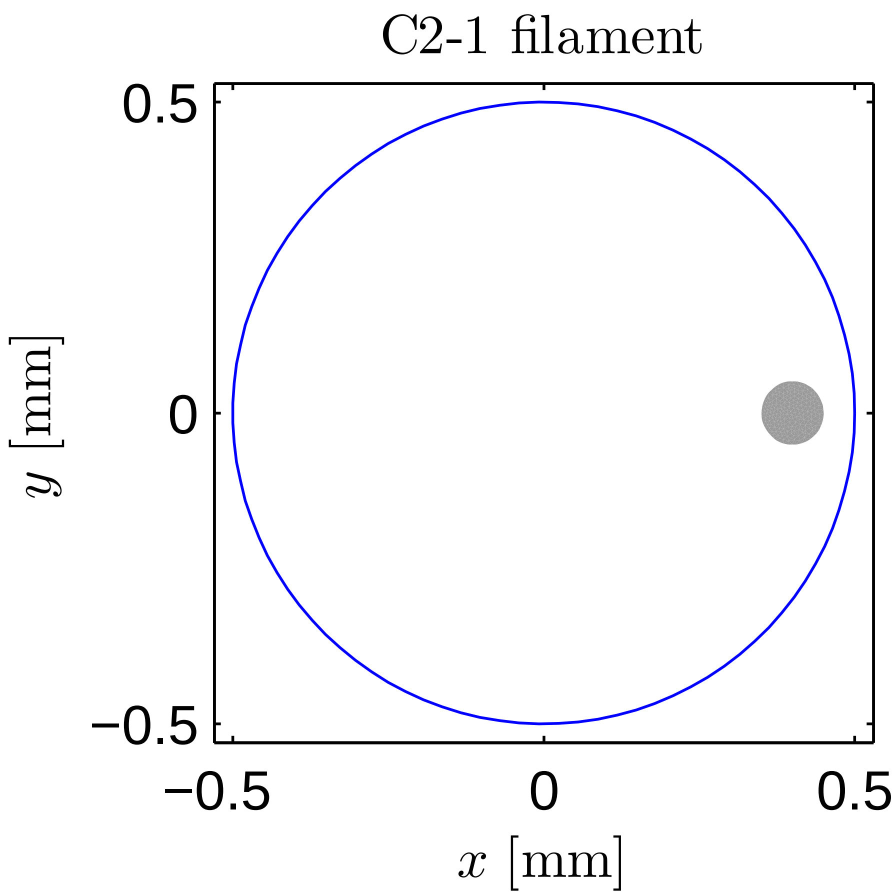


Application to superconductors: I_c computations





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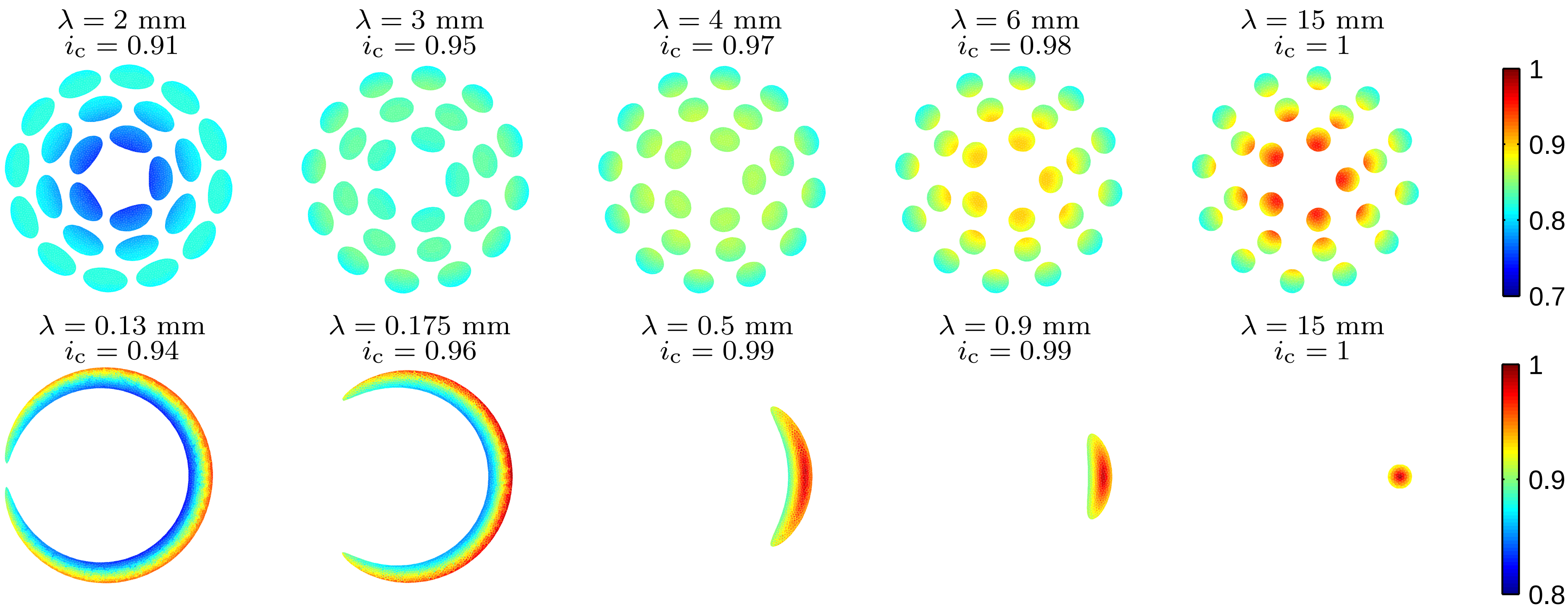
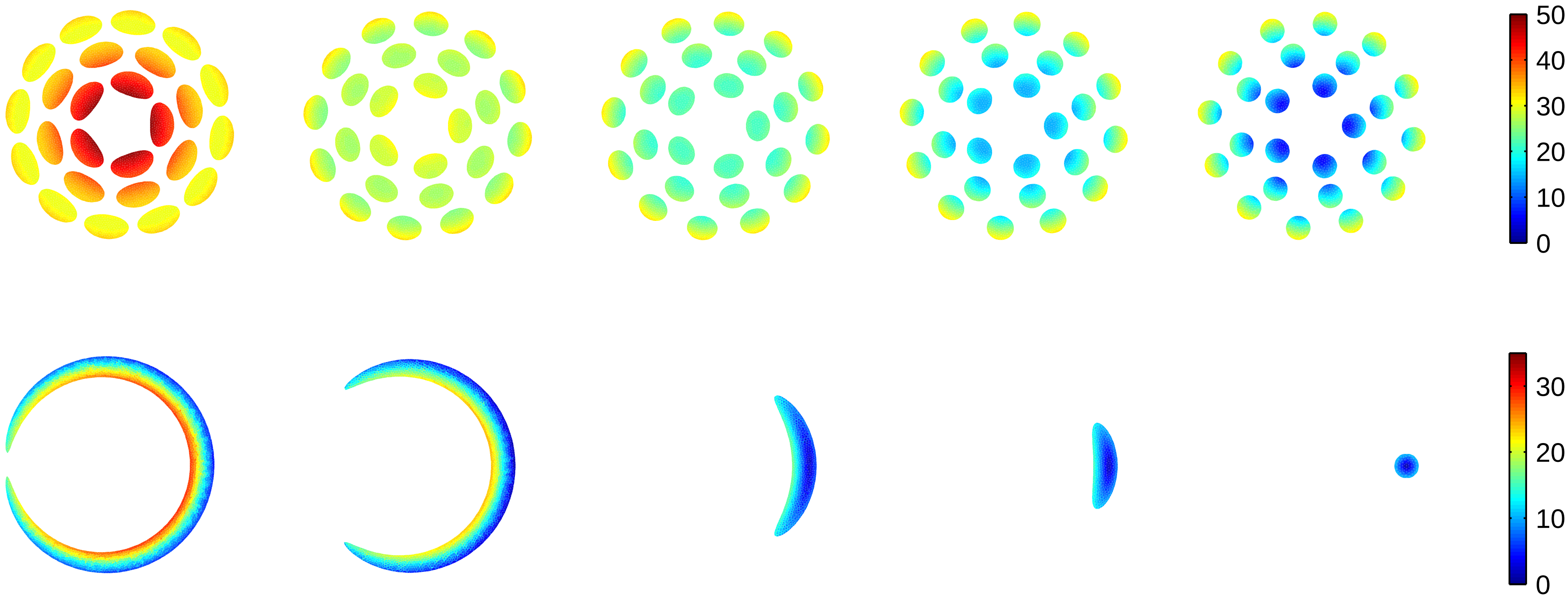


Local critical current density normalized to zero field J_c at saturation state.

Application to superconductors: I_c computations



$\|B\|$ [mT]

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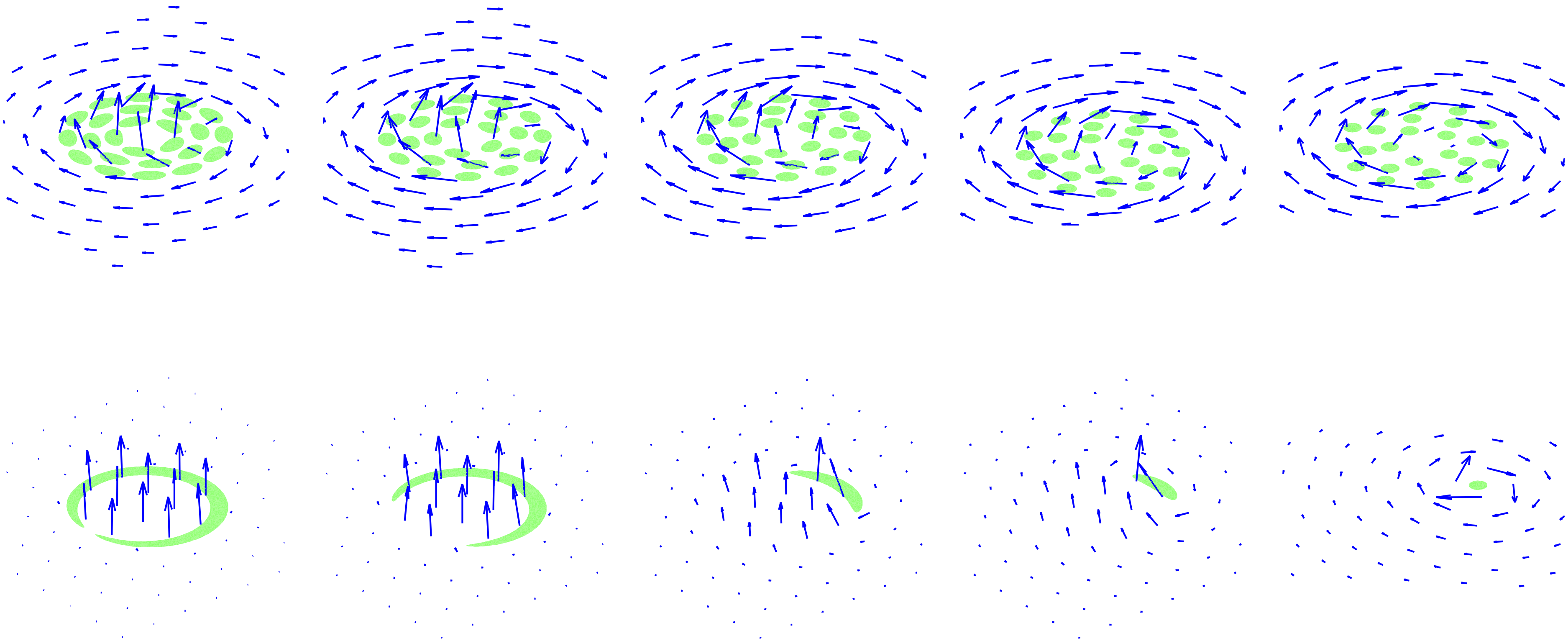
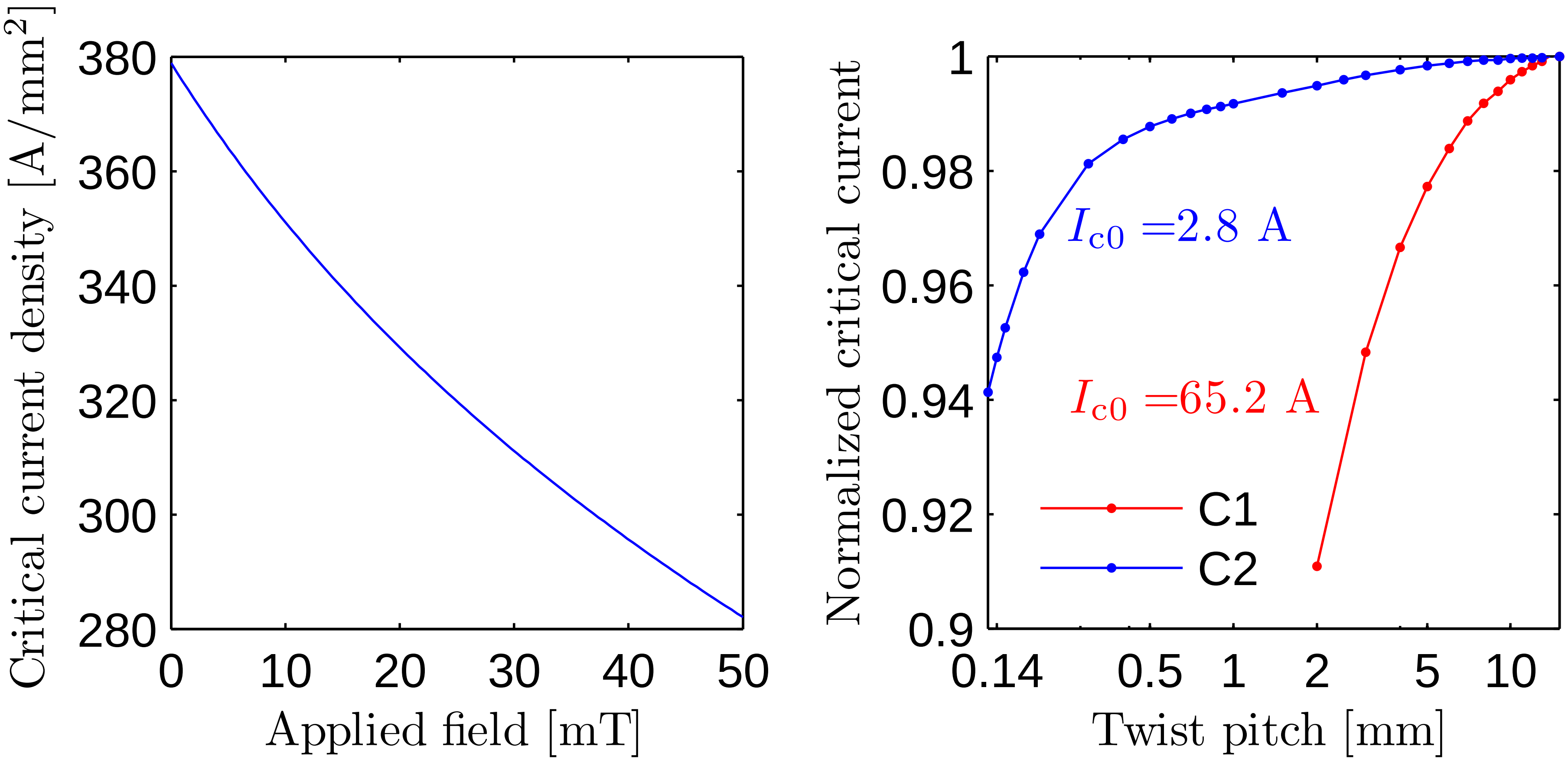


Magnetic flux density vectors

Application to superconductors: I_c computations



Application to superconductors: hysteresis loss simulation

- Starting point: eddy current formulation and non-linear power-law resistivity in superconducting region.
- Two approaches: H-formulation in 3D (previous talk) and mixed $A-V-J-H$-formulation in 2D utilizing the symmetry.
- First assumption in 2D model: currents flow only helicoidally.
- Same geometry as earlier (twist pitch = 10 cm),
  - $J_\mathrm{c} = 100\ \mathrm{A/mm^2}$
  - n−value = 19
  - operation current $0.5 I_\mathrm{c}$, $0.7 I_\mathrm{c}$, $0.9 I_\mathrm{c}$ ($I_\mathrm{c} = 1965$ A)

Application to superconductors: hysteresis loss simulation

- Comparison of computed losses (mJ) per twist per cycle

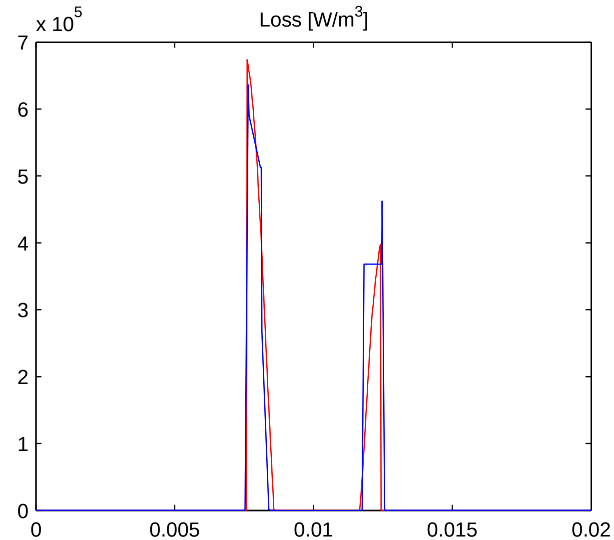
i	3D	2D	Norris loss*
0.5	3.85#	5.19#	4.41
0.7	14.2	15.3	14.5
0.9	35.2	36.0	41.0

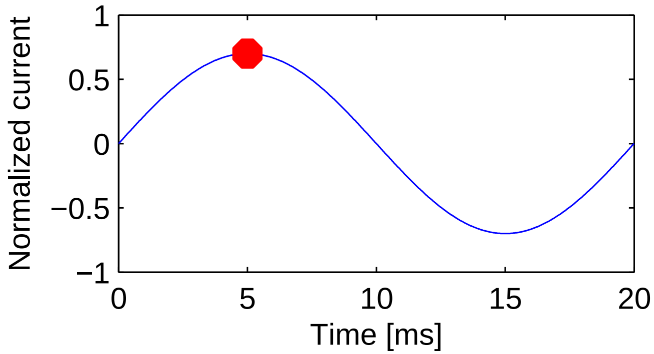
*Norris loss computed using Norris ellipse formula and multiplied by conductor length in one twist pitch (slightly over 10 cm). This is not the analytical solution of the problem!

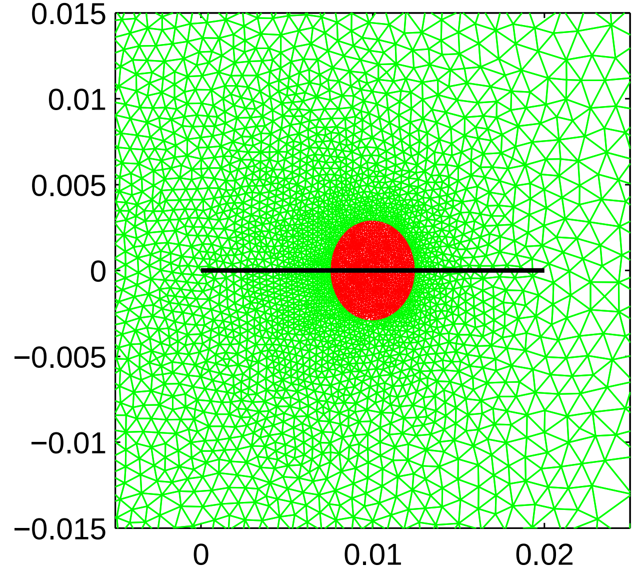
It is (well) known that clearly below I_c $A-V$-based FEM formulations produce too high loss values. Eddy current based methods result into too low losses when mesh is sparse and current amplitude is low.

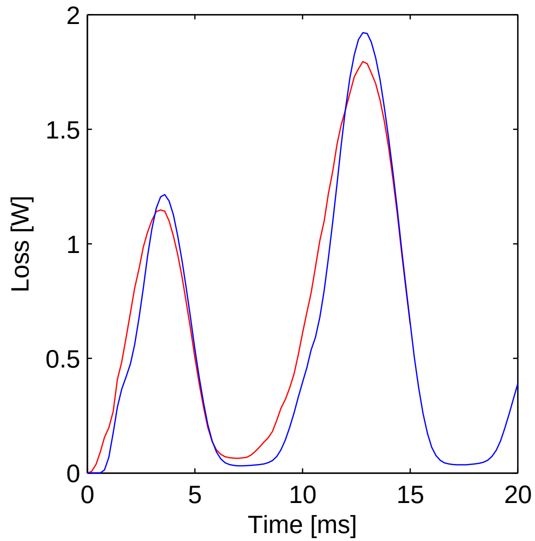
Application to superconductors: hysteresis loss simulation

- Comparison of field profiles solved in **2D** and **3D** programs (i=0.7)









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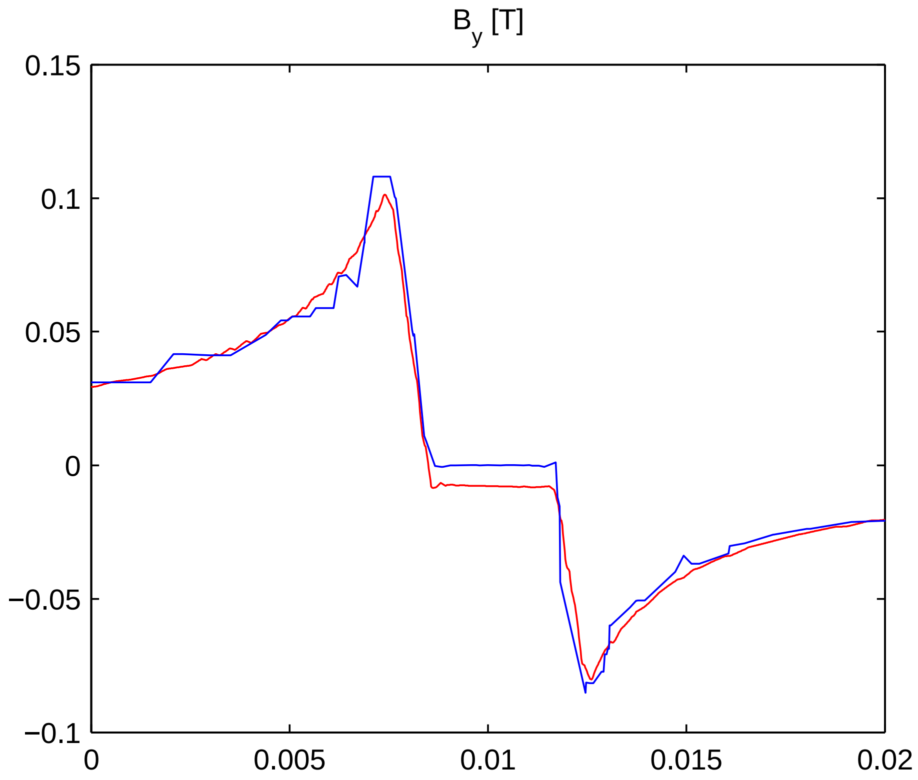

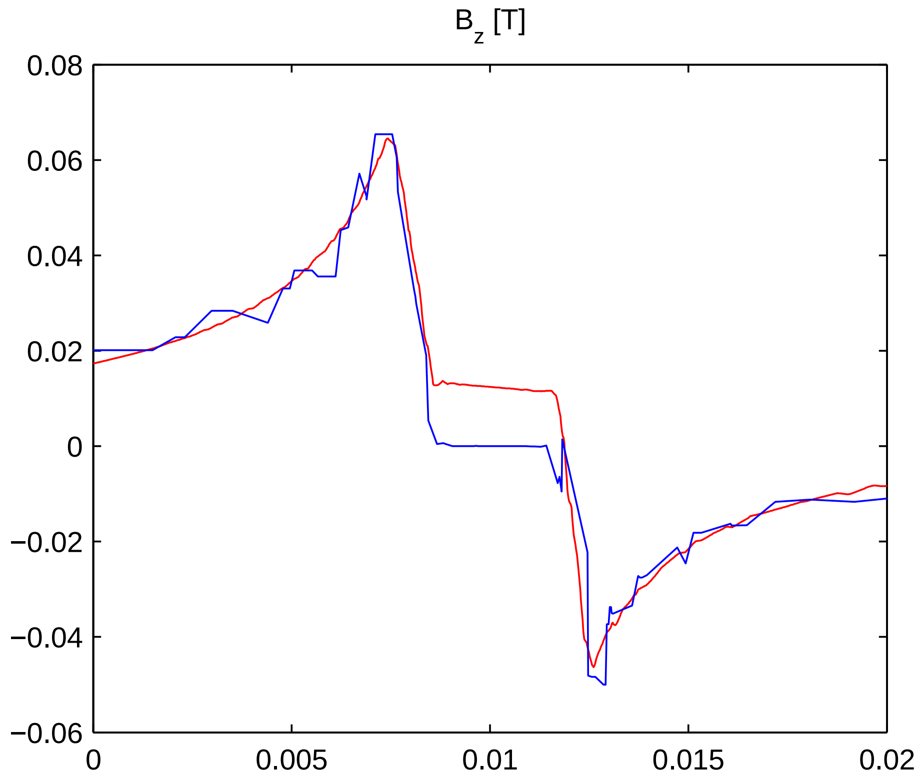

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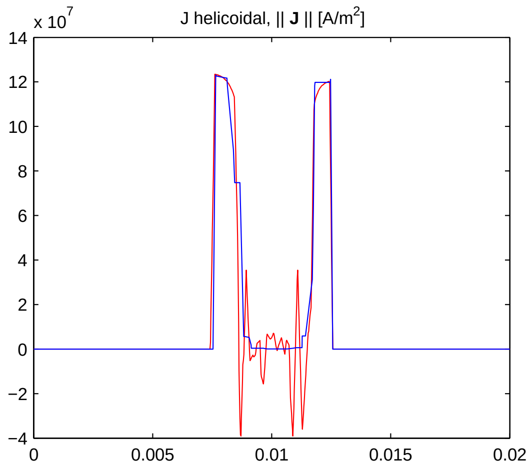

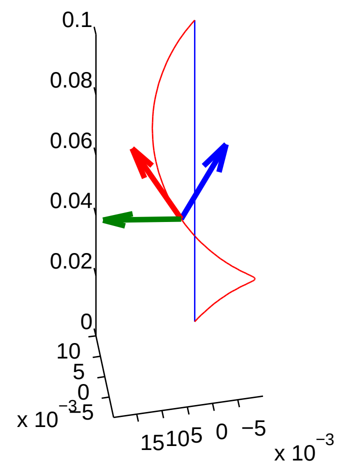

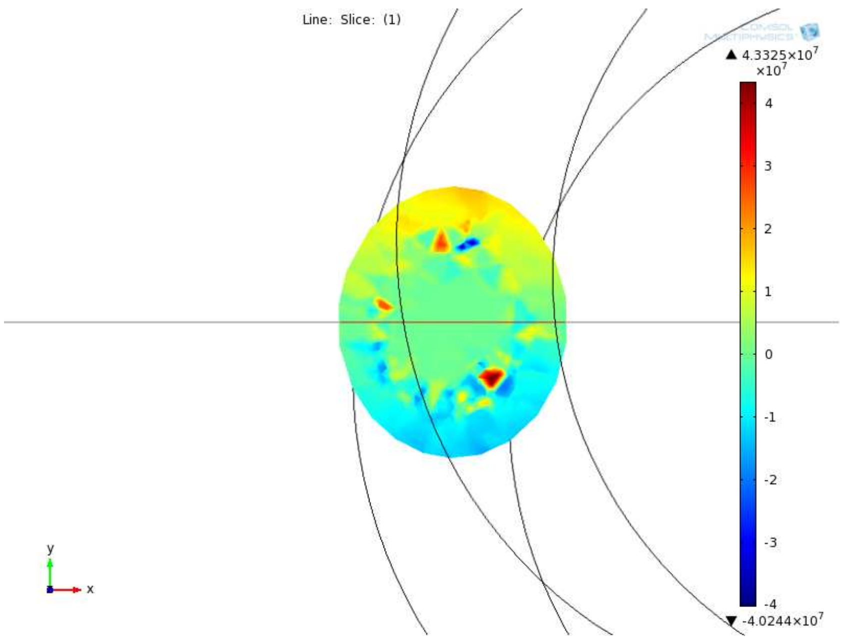

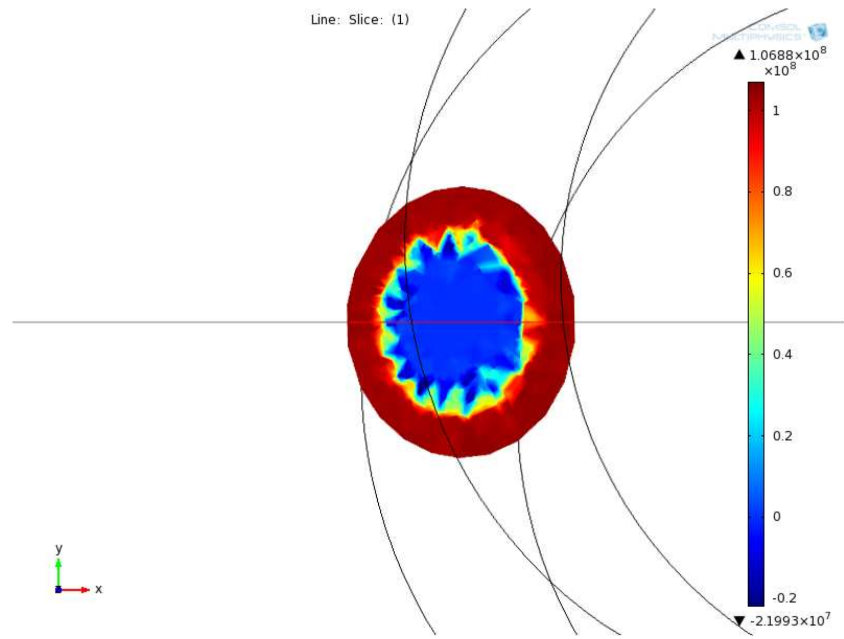

Application to superconductors: hysteresis loss simulation

- In a twisted conductor, current does not flow helicoidally, otherwise center of the conductor is not field free below I_c.
- It should be possible to also include on plane currents to 2D model, this work is underway.
- Unknown issue:
  - When $\mathbf{B} \perp \mathbf{J}$ does not hold, how constitutive law or J_c should be addressed?

Summary

- Real world physics do not depend on our modelling results – or on measurement results.
- In modelling, everything does not need to be like we see.
- Models can be done in many practical equivalent ways.
- Thinking via manifolds can increase one's toolbox of implementing intuitive concepts which seem difficult at first clance.
- Finally, we used some of these ideas to simulate critical current and AC-losses in a helicoidal structures.

Thank you!

This presentation is also available in my home page:

`http://antti.stenvall.fi`

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