

Determinations of $\alpha_{\text{QED}}(m_Z)$ and future perspectives

*~ Based on the DHMZ methodology for the data-driven HVP determination
with realistic uncertainties ~*

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Based (mainly) on: [1908.00921](#) (DHMZ) and [2008.08107](#) (BM, MS)

ECFA Higgs Factory: WG1-PREC "Precision in theory and experiment"

14/07/2022



Content of the talk

- Introduction: the HVP contribution to $\alpha_{\text{QED}}(m_Z)$ and $(g-2)_\mu$
- Data on $e^+e^- \rightarrow \text{hadrons}$
- Combination of all e^+e^- data:
focus on the combination procedure
(HVPTools and fit based on analyticity & unitarity)
- Indications of uncertainties on uncertainties and on correlations & their implications for combinations
- Results on $\alpha_{\text{QED}}(m_Z)$ and a_μ
- Impact of correlations between $\alpha_{\text{QED}}(m_Z)$ and a_μ on the EW fit
- Conclusions

Hadronic Vacuum Polarization: α_{QED} & Muon $(g-2)_\mu$

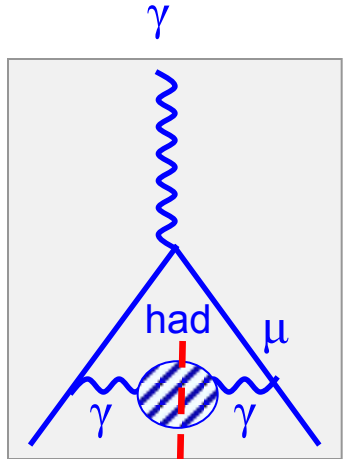
Dominant uncertainty for the theoretical prediction: from lowest-order HVP piece

Cannot be calculated from pQCD (low mass scale), but one can use experimental data on $e^+e^- \rightarrow \text{hadrons}$ cross section

Born: $\sigma^{(0)}(s) = \sigma(s)(\alpha/\alpha(s))^2$

$$12\pi \text{Im}\Pi_\gamma(s) = \frac{\sigma^0 [e^+e^- \rightarrow \text{hadrons} (\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

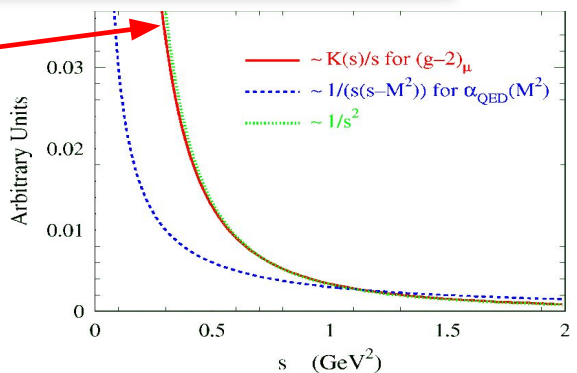
$\text{Im}[\text{diagram}] \propto |\text{diagram hadrons}|^2$



$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

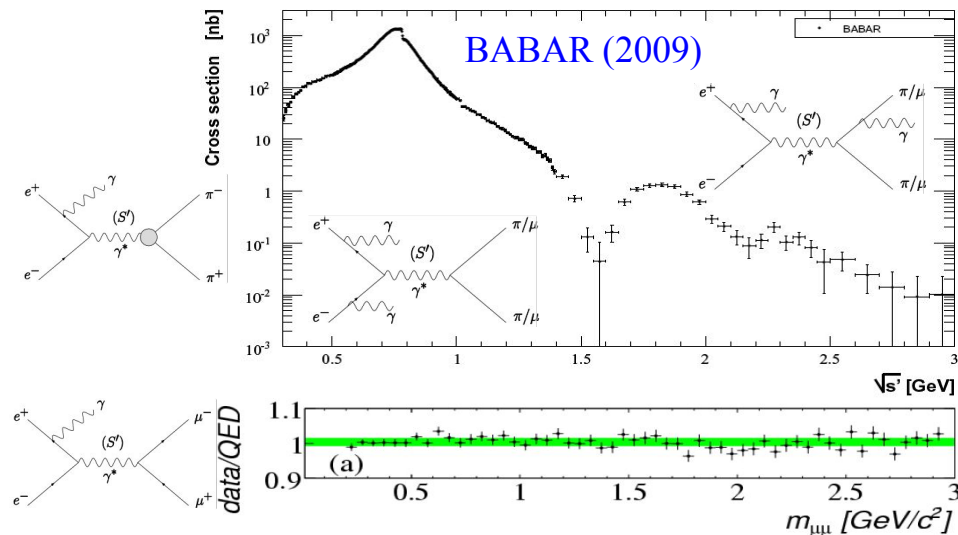
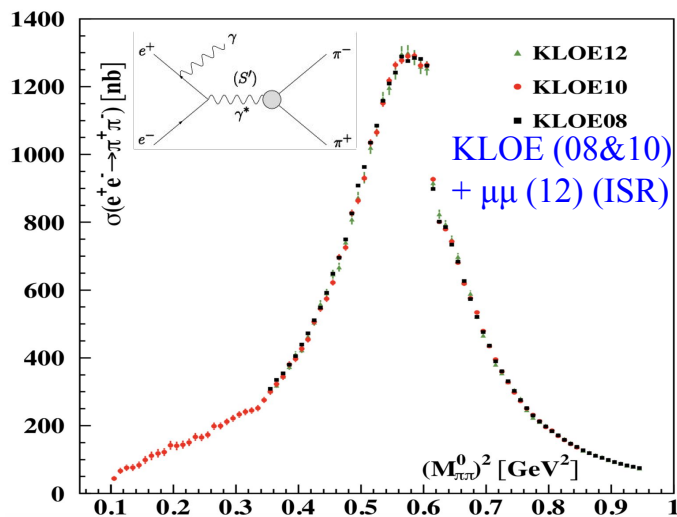
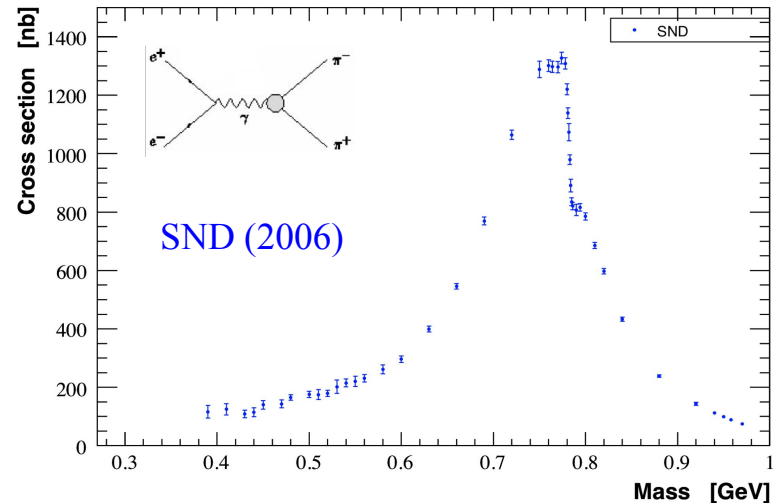
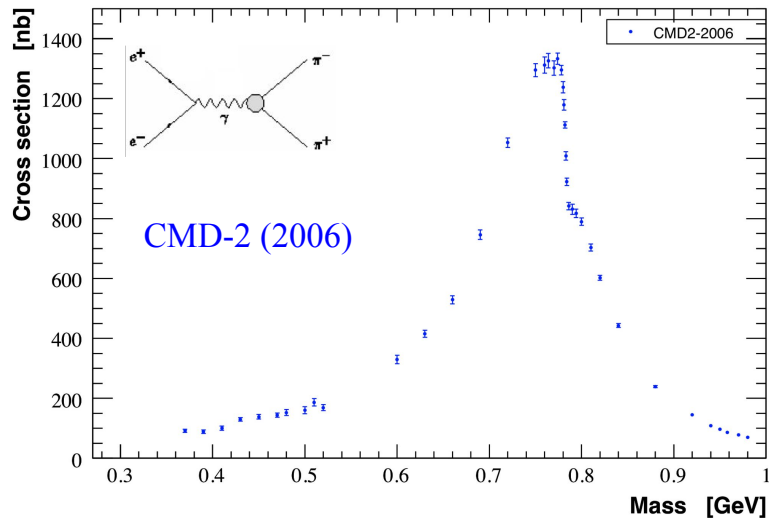
Dispersion relation

Bouchiat and Michel, 1961

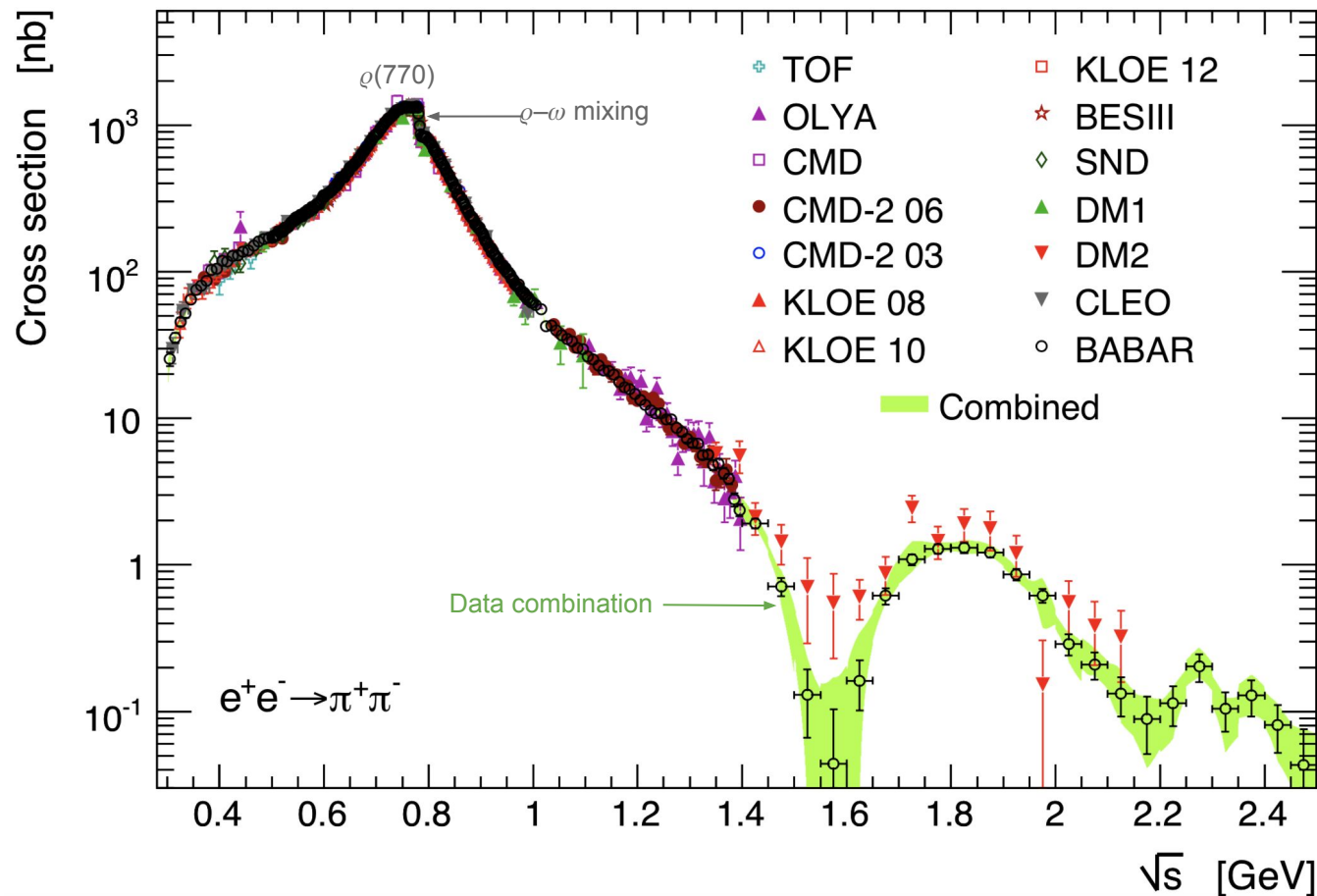


- Precise $\sigma(e^+e^- \rightarrow \text{hadrons})$ measurements at low energy are very important for $(g-2)_\mu$
- For $\alpha_{\text{QED}}(m_Z)$ the kernel induces relatively larger contributions from the higher mass regions
- Current precision for $\Delta\alpha_{\text{had}}(m_Z^2)$: $1.0 \cdot 10^{-4}$; need improvement by factor 2-4 to match direct determination of $\alpha_{\text{QED}}(m_Z)$ from $A_{\text{FB}}^{\mu\mu}$ @ FCC-ee (arXiv:1512.05544 - P. Janot)

HVP: Data on $e^+e^- \rightarrow \text{hadrons}$



Combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel



Procedure and software (*HVPTools* - Since 2009) for combining cross section data with arbitrary point spacing/binning

Combine cross section data: goal and requirements

→ Goal: combine experimental spectra with arbitrary point spacing / binning

→ Requirements:

- Properly propagate uncertainties and correlations
 - *Between measurements (data points/bins) of a given experiment*
(covariance matrices and/or detailed split of uncertainties in sub-components)
 - *Between experiments (common systematic uncertainties, e.g. VP)*
based on detailed information provided in publications
 - *Between different channels* – motivated by understanding of the meaning of systematic uncertainties and **identifying the common ones**

BABAR luminosity (ISR or Bhabha), efficiencies (photon, Ks, Kl, modeling);

BABAR radiative corrections; $4\pi^2\pi^0-\eta\omega$

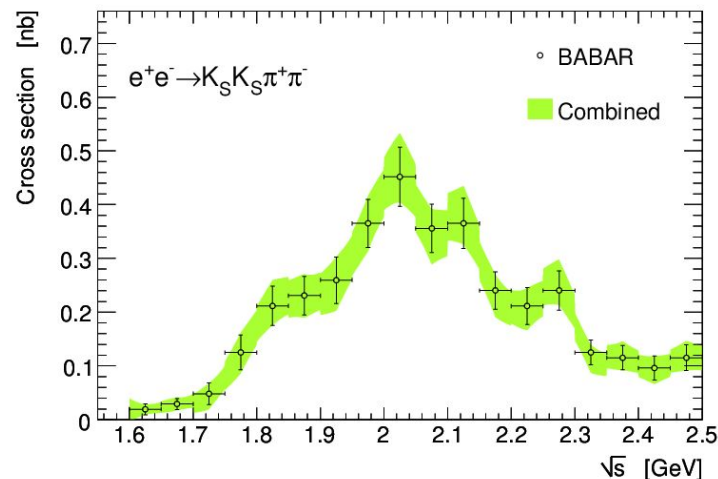
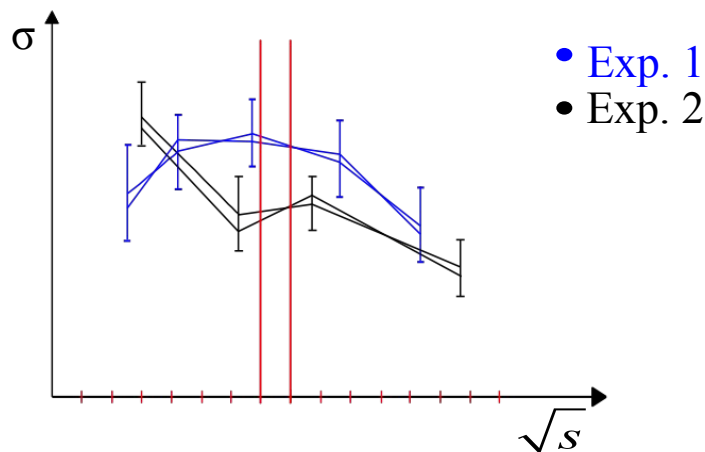
CMD2 $\eta\gamma - \pi^0\gamma$; CMD2/3 luminosity; SND luminosity;

FSR; hadronic VP (old experiments)

(1st motivation for using DHMZ uncertainties as “baseline” in the g-2 TI White Paper 2006.04822)

- Minimize biases
- Optimize $\Delta\alpha_{\text{had}}(m_Z^2)$ and g-2 integral uncertainties
(without overestimating the precision with which the uncertainties of the measurements are known)

Combination procedure implemented in HVPTools software



- Define a (fine) final binning (to be filled and used for integrals etc.)
- Linear/quadratic splines to interpolate between the points/bins of each experiment
 - for binned measurements: preserve integral inside each bin
 - closure test: replace nominal values of data points by Gounaris-Sakurai model and re-do the combination
 - (non-)negligible bias for (linear)quadratic interpolation
- Fluctuate data points taking into account correlations & re-do the splines for each (pseudo-)experiment
 - each uncertainty fluctuated coherently for all the points/bins that it impacts
 - eigenvector decomposition for (statistical) covariance matrices

Combination procedure implemented in HVPTools software

For each final bin:

- Compute an average value for each measurement and its uncertainty
- Compute correlation matrix between experiments
- Minimize χ^2 and get average coefficients (weights)
- Compute average between experiments and its uncertainty

Evaluation of integrals and propagation of uncertainties:

- Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys
- The pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections → Integral evaluation
- Uncertainties also propagated through $\pm 1\sigma$ shifts of each uncertainty:
 - allows to account for correlations between different channels (for integrals and spectra)
- *Checked consistency between the different approaches*

Combination procedure: weights of various measurements

For each final bin:

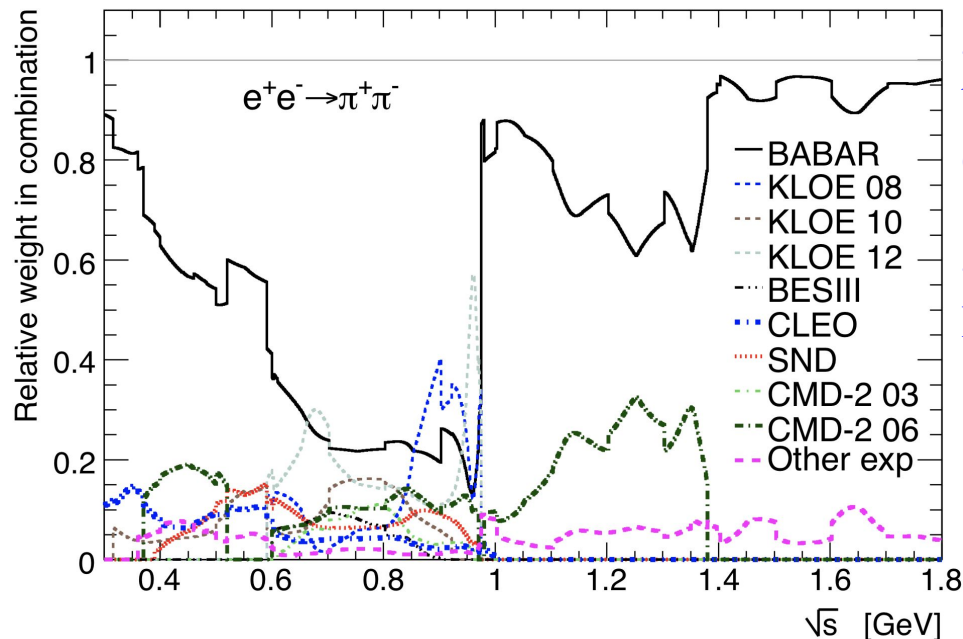
→ Minimize χ^2 and get average coefficients

Note: average weights must account for bin sizes / point spacing of measurements

(do not over-estimate the weight of experiments with large bins)

→ weights in fine bins evaluated using a common (large) binning for measurements + interpolation

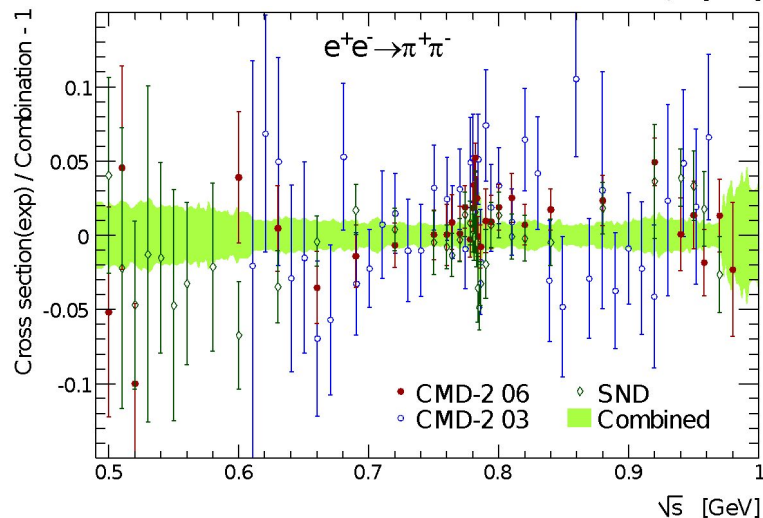
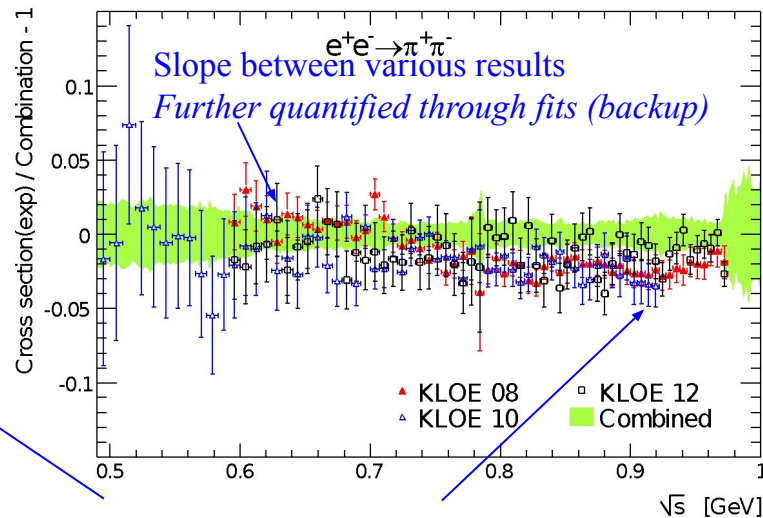
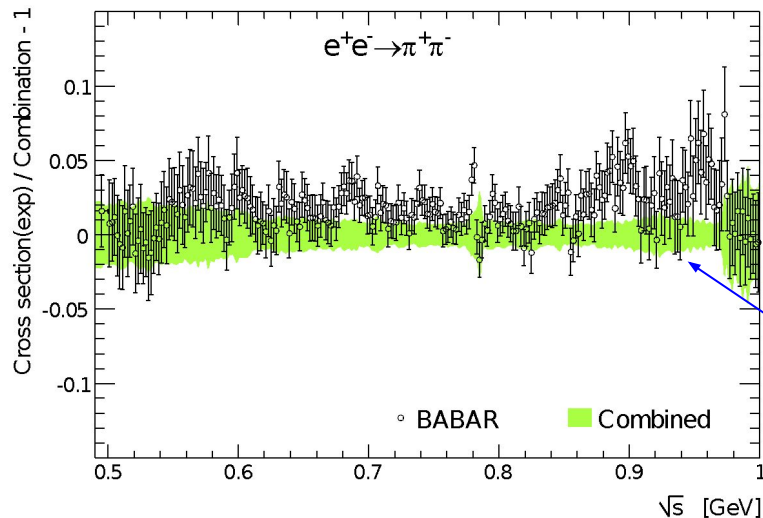
→ compare the precisions on the same footing



→ Bins used by KLOE larger than the ones by BABAR in ρ - ω interference region (factor ~ 3)

→ Average dominated by BaBar and KLOE, BaBar covering full range

More on the combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel



Local tension & systematic trends
 Indication of “uncertainties on uncertainties”
 (i.e. unaccounted biases)

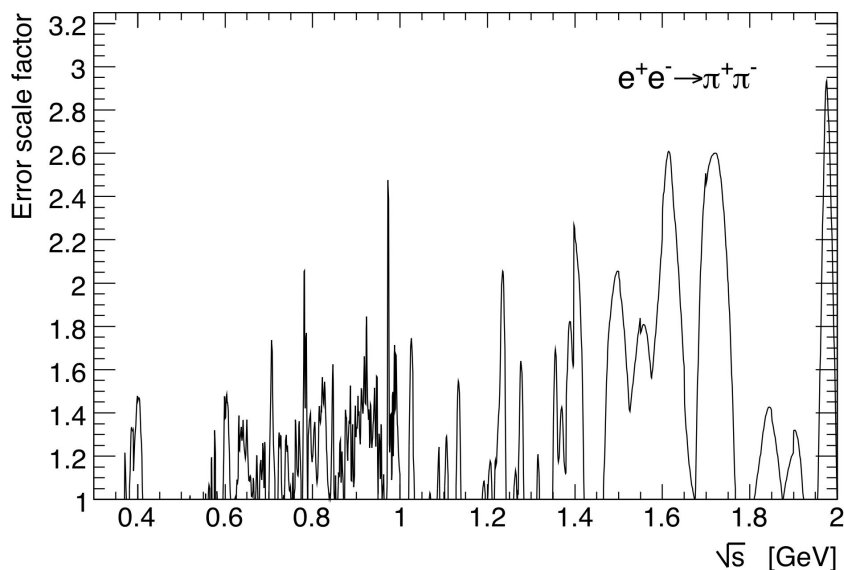
Other experiments not yet precise enough
 to discriminate
 (see however recent update from SND (backup):
 ~significant tension with KLOE above 720 MeV)

Combination procedure: compatibility between measurements

For each final bin:

→ χ^2/ndof : test locally the level of agreement between input measurements, *taking into account the correlations*

→ Scale uncertainties in bins with $\chi^2/\text{ndof} > 1$ (PDG): *locally conservative*; Adopted by KNT since '17



→ Tension between measurements:
indication of underestimated uncertainties
Motivates conservative uncertainty treatment
in combination fit (evaluation of weights)

→ Observed (systematic) tension between BABAR and KLOE measurements

→ (*Since 2019*) Included extra (dominant) uncertainty: 1/2 difference between integrals w/o either BABAR or KLOE (*2nd motivation for using DHMZ uncertainties as “baseline” in the TI White Paper*)

Extra uncertainty starts to be adopted in other studies (2205.12963)

Improving a_μ through fits for the $e^+e^- \rightarrow \pi^+\pi^-$ channel (*Since 2019*)

→ Fit bare form-factor using 6 param. model based on *analyticity* and *unitarity*

$$|F_\pi^0|^2 = |R(s) \times J(s)|^2$$

$$R(s) = 1 + \alpha_V s + \frac{\kappa s}{m_\omega^2 - s - im_\omega \Gamma_\omega} \quad (1611.09359, \text{C. Hanhart et al.})$$

$$J(s) = e^{1 - \frac{\delta_1(s_0)}{\pi}} \left(1 - \frac{s}{s_0}\right)^{\left[1 - \frac{\delta_1(s_0)}{\pi}\right] \frac{s_0}{s}} \left(1 - \frac{s}{s_0}\right)^{-1} e^{\frac{s}{\pi} \int_{4m_\pi^2}^{s_0} dt \frac{\delta_1(t)}{t(t-s)}} \quad \text{Omnès integral}$$

(hep-ph/0402285, F.J. Yndurain et al.)

$$\cot \delta_1(s) = \frac{\sqrt{s}}{2k^3} (m_\rho^2 - s) \left[\frac{2m_\pi^3}{m_\rho^2 \sqrt{s}} + B_0 + B_1 \omega(s) \right]$$

$$k = \frac{\sqrt{s - 4m_\pi^2}}{2} \quad (1102.2183, \text{F.J. Yndurain et al.})$$

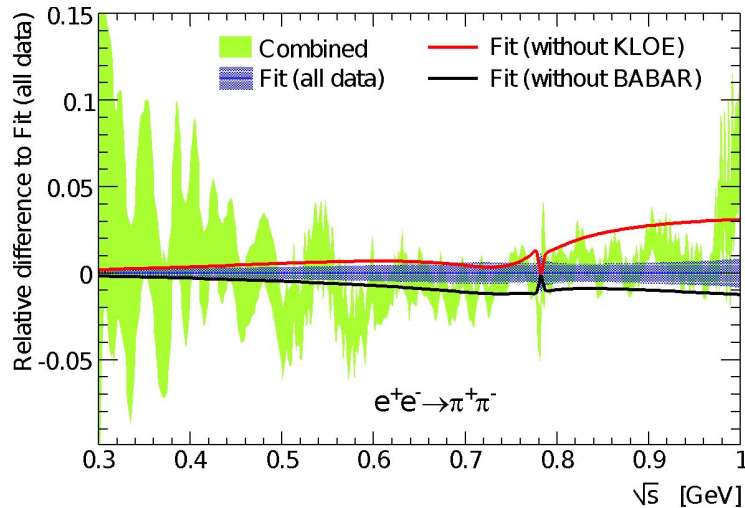
$$\omega(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}} \quad \sqrt{s_0} = 1.05 \text{ GeV}$$

→ Conservative χ^2 (diagonal matrix) & local rescaling of input uncertainties

→ Full propagation of uncertainties & correlations using pseudo-experiments

DHMZ - 1908.00921

Fit performed up to 1 GeV, Result used up to 0.6 GeV



→ Use fit only below 0.6 GeV for a_μ integral:

- where data is less precise and scarce
- less impacted by potential uncertainties of inelastic effects

\sqrt{s} range [GeV]	$a_\mu^{\text{had}} [10^{-10}]$ Fit	$a_\mu^{\text{had}} [10^{-10}]$ Data Integration
0.3 - 0.6	$109.80 \pm 0.37_{\text{exp}} \pm 0.36_{\text{para}^*}$	$109.6 \pm 1.0_{\text{exp}}$

→ The difference 0.2 ± 0.9
(72% correlation accounted for)

→ The fit improves the precision by a factor ~ 2

(*) Parameter uncertainty corresponds to variations with/without the B_1 term in the phase shift formula and \sqrt{s}_0 varied from 1.05 GeV to 1.3 GeV (absolute values summed linearly), *checked to be statistically significant*

Combined results: Fit [$<0.6\text{GeV}$] + Data[$0.6-1.8\text{GeV}$]

→ Full uncertainty propagation using the same pseudo-experiments as for the spline-based combination: 62% correlation among the two contributions

\sqrt{s} range [GeV]	a_μ^{had} [10^{-10}] All data	a_μ^{had} [10^{-10}] All but BABAR	a_μ^{had} [10^{-10}] All but KLOE
threshold - 1.8	$506.9 \pm 1.9_{\text{total}}$	$505.0 \pm 2.1_{\text{total}}$	$510.6 \pm 2.2_{\text{total}}$

→ The difference “All but BABAR” and “All but KLOE” = 5.6, to be compared with 1.9 uncertainty with “All data”

- The local error inflation is not sufficient to amplify the uncertainty
- Global tension (normalisation/shape) not previously accounted for
- Potential underestimated uncertainty in at least one of the measurements?
- Other measurements not precise enough to discriminate BABAR / KLOE

→ Given the fact we do not know which dataset is problematic, we decide to:

- Add half of the discrepancy (2.8×10^{-10}) as an uncertainty (corrected local PDG inflation to avoid double counting)
- Take (“All but BABAR” + “All but KLOE”) / 2 as central value

Channel	$a_\mu^{\text{had,LO}}$ [10^{-10}]	$\Delta\alpha_{\text{had}}(m_Z^2)$ [10^{-4}]
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm 3.23 \pm 0.55$	$34.50 \pm 0.06 \pm 0.20 \pm 0.04$

→ *Potential precision improvement* for a_μ ; less important for $\Delta\alpha_{\text{had}}(m_Z^2)$, where the BABAR-KLOE syst. is $0.16 \cdot 10^{-4}$

Uncertainties on uncertainties and on correlations

Topic relevant in other fields too (see backup)

[1908.00921\(DHMZ\)](#), [2006.04822\(WP g-2 Theory Initiative\)](#)

Two different approaches for combining (e^+e^-) data

DHMZ:

- χ^2 computed locally (in each fine bin), taking into account correlations between measurements (see previous slides)
- Used to determine the weights on the measurements in the combination and their level of agreement
- Uncertainties and correlations propagated using pseudo-experiments or $\pm 1\sigma$ shifts of each uncertainty component

KNT:

- χ^2 computed globally (for full mass range)

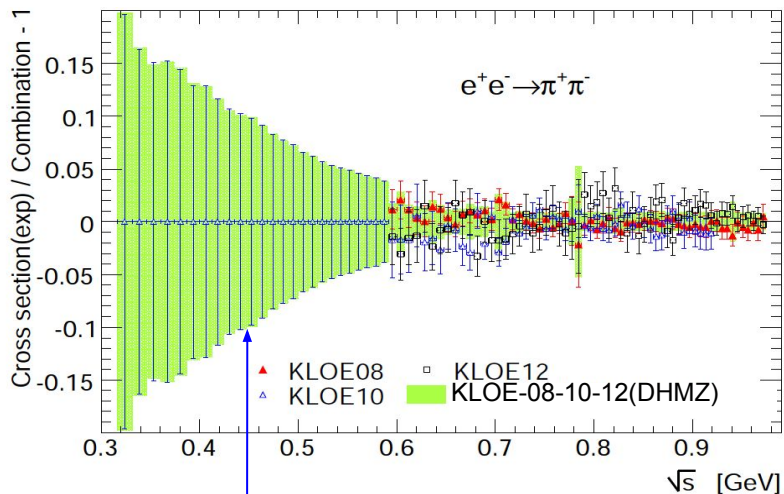
$$\chi_I^2 = \sum_{i=1}^{N_{\text{tot}}} \sum_{j=1}^{N_{\text{tot}}} (R_i^{(m)} - \mathcal{R}_m^{i,I}) \mathbf{C}_I^{-1}(i^{(m)}, j^{(n)}) (R_j^{(n)} - \mathcal{R}_n^{j,I}) \quad \text{KNT (1802.02995)}$$

$$\chi^2 = \sum_{i=1}^{195} \sum_{j=1}^{195} (\sigma_{\pi\pi(\gamma)}^0(i) - \bar{\sigma}_{\pi\pi(\gamma)}^0(m)) \mathbf{C}^{-1}(i^{(m)}, j^{(n)}) (\sigma_{\pi\pi(\gamma)}^0(j) - \bar{\sigma}_{\pi\pi(\gamma)}^0(n)) \quad \text{KLOE-KMT (1711.03085)}$$

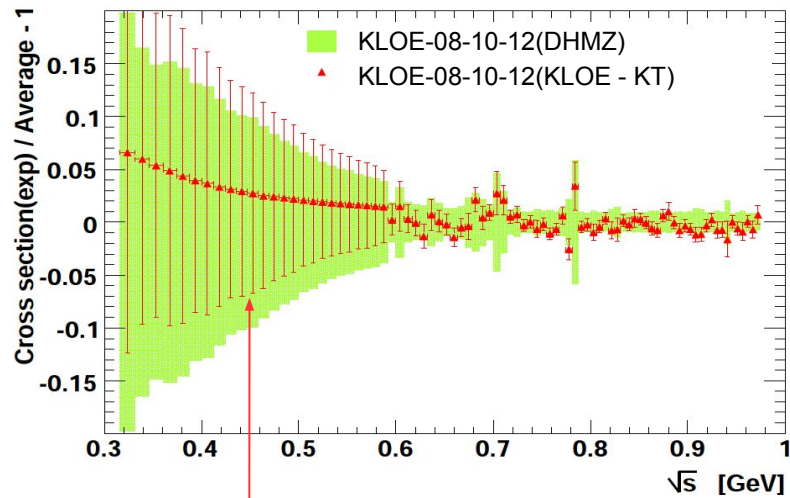
- Relies on description of correlations on long ranges

- *One of the main sources of differences for the uncertainty on a_μ*

Combining the 3 KLOE measurements



Local combination (DHMZ)



Information propagated between mass regions, through shifts of systematics - relying on correlations, amplitudes and shapes of systematics (KLOE-KT)

- KLOE-08-10-12(DHMZ) - $a_\mu[0.6 ; 0.9] : 366.5 \pm 2.8$ (Without χ^2 rescaling: ± 2.2)
- Conservative treatment of uncertainties and correlations (*not perfectly known*) in weight determination
- KLOE-08-10-12(KLOE-KT) - $a_\mu[0.6 ; 0.9]\text{GeV} : 366.9 \pm 2.2$ (Includes χ^2 rescaling)
- Assuming perfect knowledge of the correlations to minimize average uncertainty

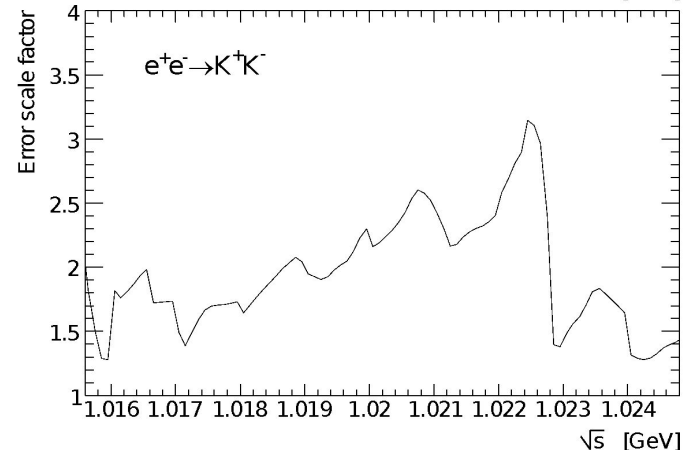
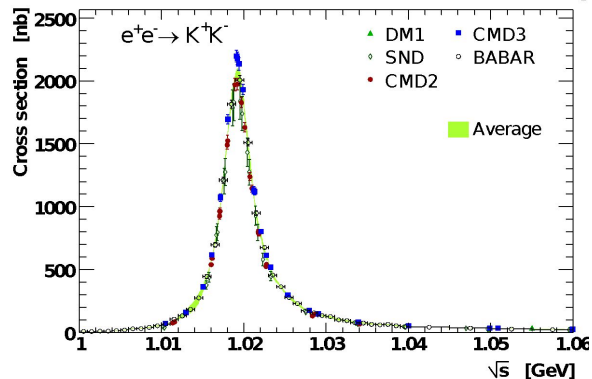
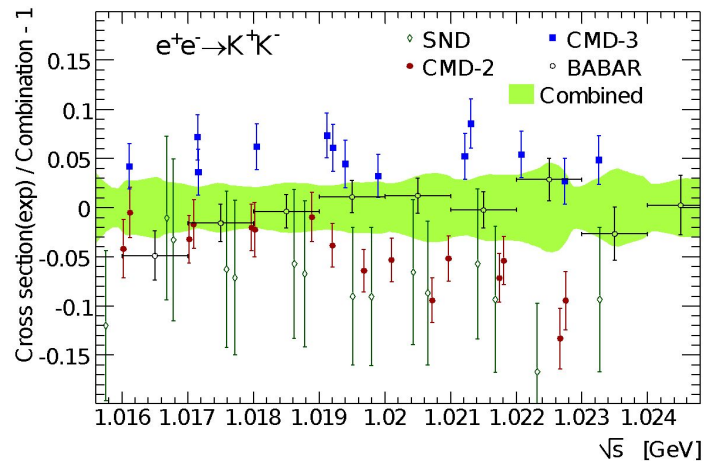
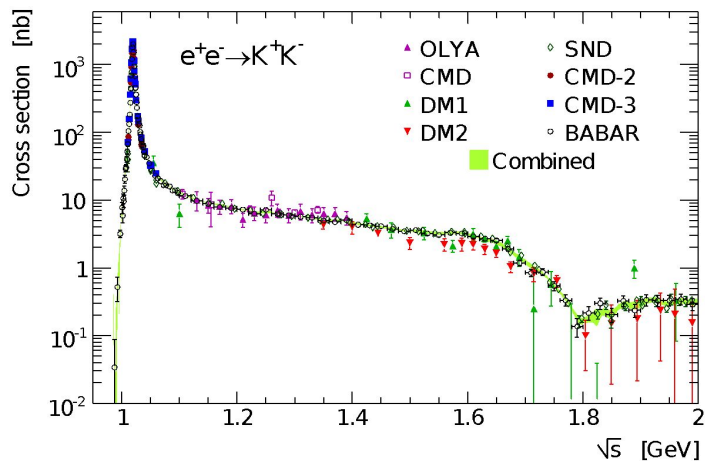
Uncertainties on uncertainties and correlations

Numerous indications of uncertainties on uncertainties and on correlations, with a direct impact on combination fits

- Shapes of systematic uncertainties *evaluated* in \sim -wide mass ranges with sharp transitions (*backup*)
- One standard deviation is statistically not well defined for systematic uncertainties
- Systematic uncertainties like acceptance, tracking efficiency, background etc. not necessarily fully correlated between low and high mass
- Are all systematic uncertainty components fully independent between each-other? (e.g. tracking and trigger)
- *Yield uncertainties on uncertainties and on correlations*
- Tensions between measurements (BABAR/KLOE; 3 KLOE results etc.): *experimental indications of underestimated uncertainties*
- *Statistical methods* (χ^2 with correlations, likelihood fits, ratios of measured quantities etc.) *should not over-exploit the information on the amplitude and correlations of uncertainties*

Combination of measurements for various channels and total HVP contribution

Combination for the $e^+e^- \rightarrow K^+K^-$ channel

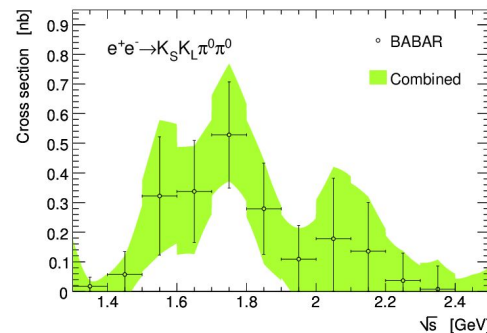
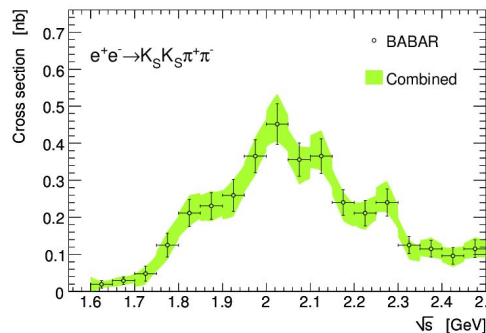
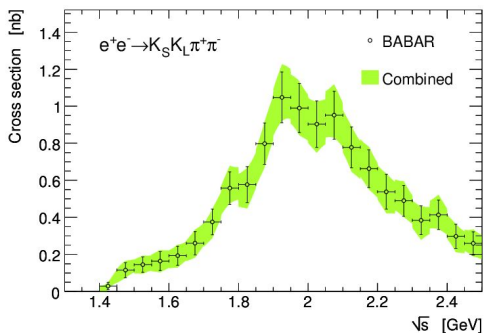
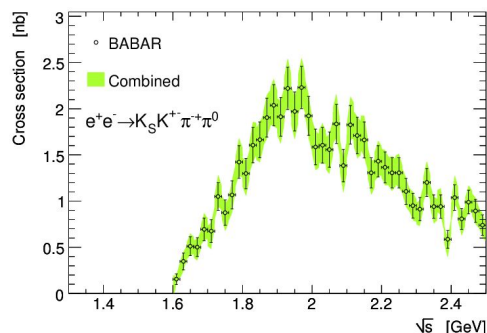
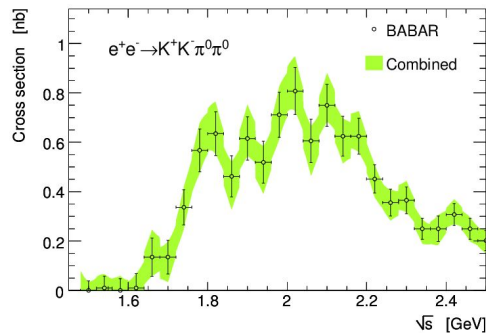
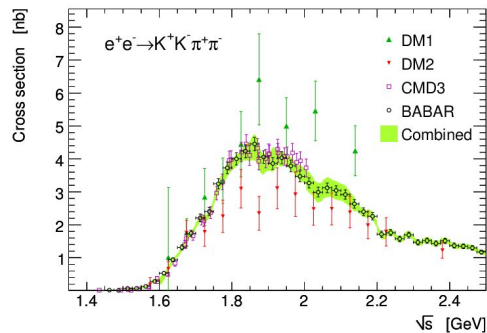
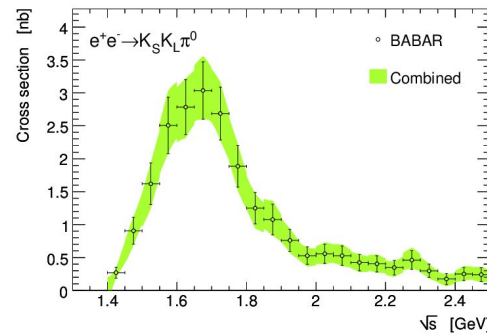
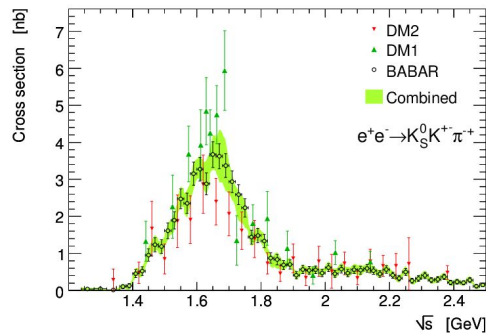
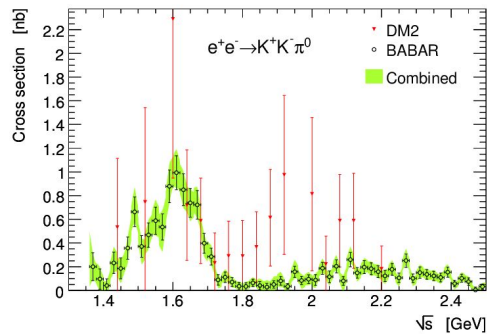


→ Tension between measurements, *to be understood*

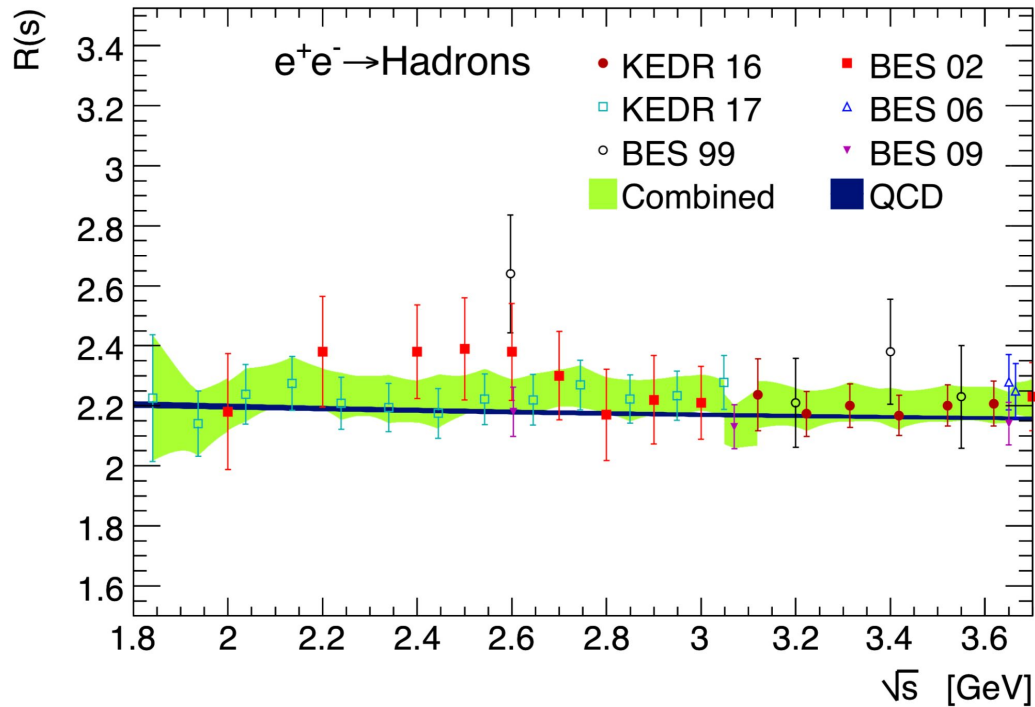
→ $a_\mu[\rightarrow 1.8\text{GeV}]$: 23.08 ± 0.20 (stat.) ± 0.40 (syst.) [10^{-10}] (currently enhancement $\times 2.2$)

→ $\Delta\alpha_{\text{had}}(m_Z^2)$: 3.35 ± 0.03 (stat.) ± 0.06 (syst.) [10^{-4}]

Combination for the $e^+e^- \rightarrow KK\pi$ and $KK2\pi$ channels



Contributions from the 1.8 – 3.7 GeV region



→ Contribution evaluated from pQCD (4 loops) + $O(\alpha_s^2)$ quark mass corrections

→ Uncertainties: α_s , truncation of perturbative series, CIPT/FOPT, m_q

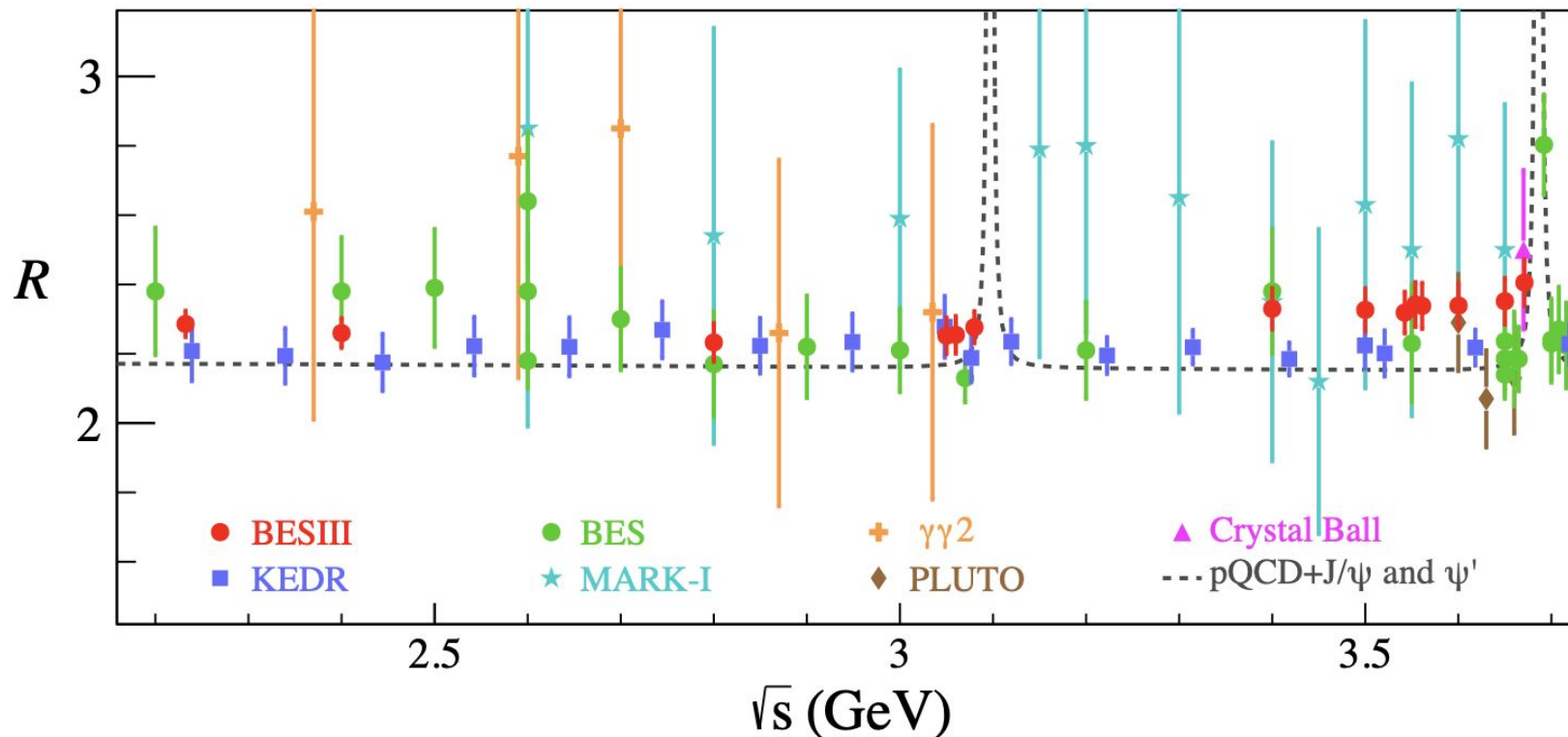
→ 1.8-2.0 GeV: 7.65 ± 0.31 (data excl.); 8.30 ± 0.09 (QCD); added syst. $0.65 [10^{-10}]$ for $a_\mu^{\text{had,LO}}$

Syst. for $\Delta\alpha_{\text{had}}(m_Z^2)$: $0.28 [10^{-4}]$ → *Potential for uncertainty reduction* with more high-multiplicity modes measured

→ 2.0-3.7 GeV: 25.82 ± 0.61 (data); 25.15 ± 0.19 (QCD) [10^{-10}]; agreement within 1σ

Comparison of inclusive measurements with pQCD

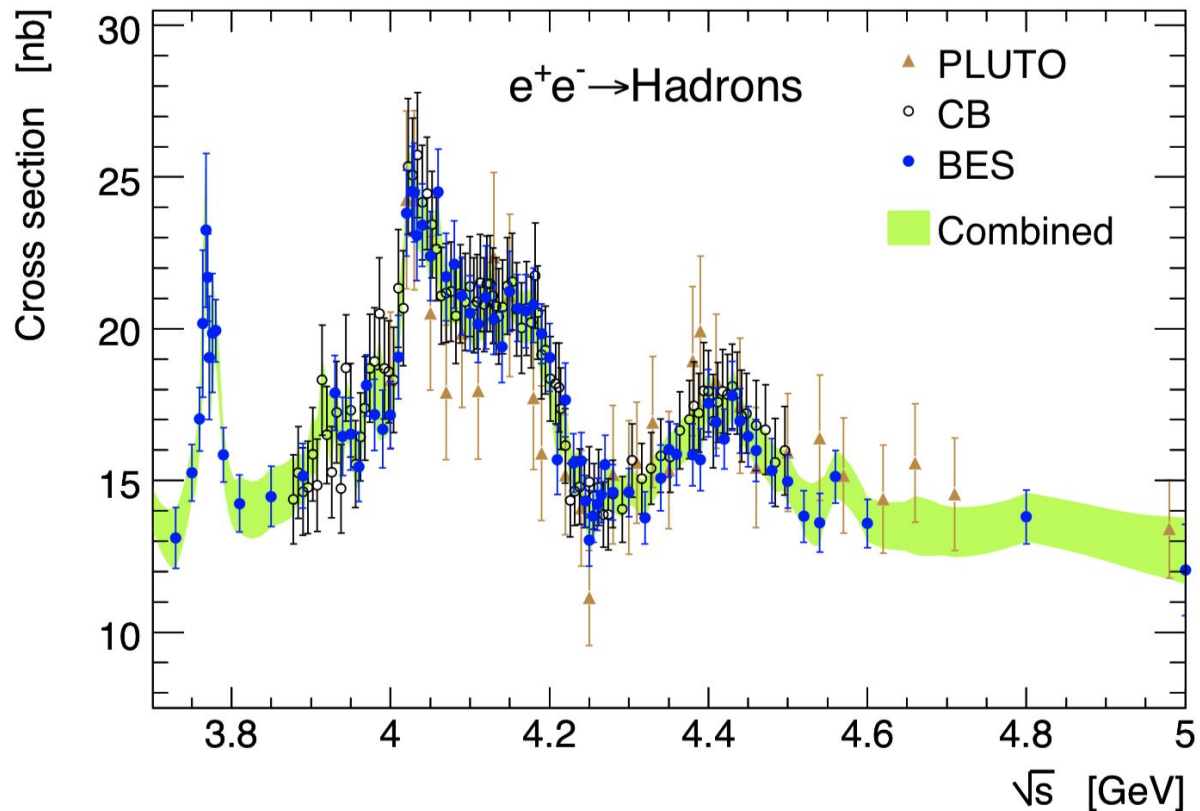
arXiv:2112.11728



→ BES III results to be included: \sim tension with pQCD and with KEDR 16

→ Another example of “*uncertainties on the uncertainties*” / *systematic effects to be understood* at the level of precision that is claimed

Contributions from the charm resonance region



Channel	$a_\mu^{\text{had,LO}} [10^{-10}]$	$\Delta\alpha_{\text{had}}(m_Z^2) [10^{-4}]$
R data [3.7 – 5.0] GeV	$7.29 \pm 0.05 \pm \boxed{0.30} \pm 0.00$	$15.79 \pm 0.12 \pm \boxed{0.66} \pm 0.00$

→ Potential precision improvement, especially important for $\Delta\alpha_{\text{had}}(m_Z^2)$

Situation in arXiv:1908.00921 (EPJC)

Channel	$a_\mu^{\text{had,LO}} [10^{-10}]$	$\Delta\alpha_{\text{had}}(m_Z^2) [10^{-4}]$
$\pi^0\gamma$	$4.41 \pm 0.06 \pm 0.04 \pm 0.07$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.85 \pm 0.83 \pm \boxed{3.23} \pm 0.55$	$34.50 \pm 0.06 \pm \boxed{0.20} \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.21 \pm 0.40 \pm 1.10 \pm 0.86$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$\eta\pi^+\pi^-\pi^0$ (non- ω , ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \rightarrow \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega 2\pi$ ($\omega \rightarrow \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non- 3π , $\pi\gamma$, $\eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
$K_S K_L$	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
ϕ (non- $K\bar{K}$, 3π , $\pi\gamma$, $\eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\bar{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
$K\bar{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
$K\bar{K}\omega$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta K\bar{K}$ (non- ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega 3\pi$ ($\omega \rightarrow \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (BW integral)	6.20 ± 0.11	7.00 ± 0.13
$\psi(2S)$ (BW integral)	1.56 ± 0.05	2.48 ± 0.08
R data [3.7 – 5.0] GeV	$7.29 \pm 0.05 \pm \boxed{0.30} \pm 0.00$	$15.79 \pm 0.12 \pm \boxed{0.66} \pm 0.00$
$R_{\text{QCD}} [1.8 - 3.7 \text{ GeV}]_{uds}$	$33.45 \pm 0.28 \pm \boxed{0.65_{\text{dual}}}$	$24.27 \pm 0.18 \pm \boxed{0.28_{\text{dual}}}$
$R_{\text{QCD}} [5.0 - 9.3 \text{ GeV}]_{udsc}$	6.86 ± 0.04	34.89 ± 0.18
$R_{\text{QCD}} [9.3 - 12.0 \text{ GeV}]_{udscb}$	1.20 ± 0.01	15.53 ± 0.04
$R_{\text{QCD}} [12.0 - 40.0 \text{ GeV}]_{udscb}$	1.64 ± 0.00	77.94 ± 0.13
$R_{\text{QCD}} [> 40.0 \text{ GeV}]_{udscb}$	0.16 ± 0.00	42.70 ± 0.05
$R_{\text{QCD}} [> 40.0 \text{ GeV}]_t$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$694.0 \pm 1.0 \pm 3.5 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$	$275.29 \pm 0.15 \pm \boxed{0.72} \pm 0.23 \pm 0.15_\psi \pm \boxed{0.55_{\text{QCD}}}$

→ 32 exclusive channels are integrated up to 1.8 GeV

Relative contributions to a_μ from missing channels (estimated based on isospin symmetry)

→ $0.87 \pm 0.15 \%$ (DEHZ 2003)

→ $0.69 \pm 0.07 \%$ (DHMZ 2010)

→ $0.09 \pm 0.02 \%$ (DHMZ 2017)

→ $0.016 \pm 0.016 \%$ (DHMZ 2019)

(Nearly complete set of exclusive measurements from BABAR)

Estimation procedures also adopted by KNT

pQCD (4 loops) + $O(\alpha_s^2)$ quark mass corrections

Uncertainties $[10^{-4}]$:

α_s : 0.34 → Strongly reduced @ FCC-ee

Truncation of perturbative series: 0.25 &

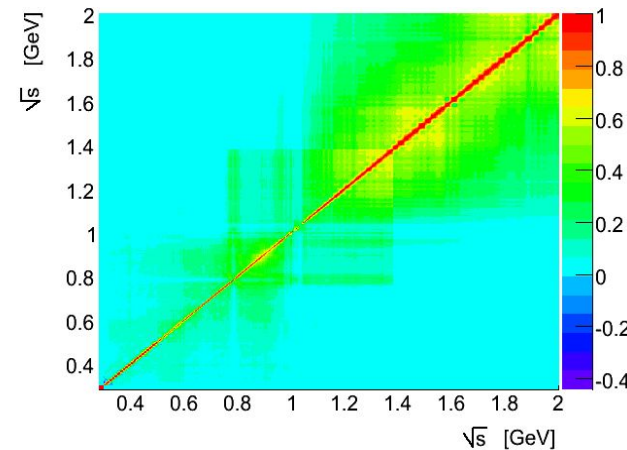
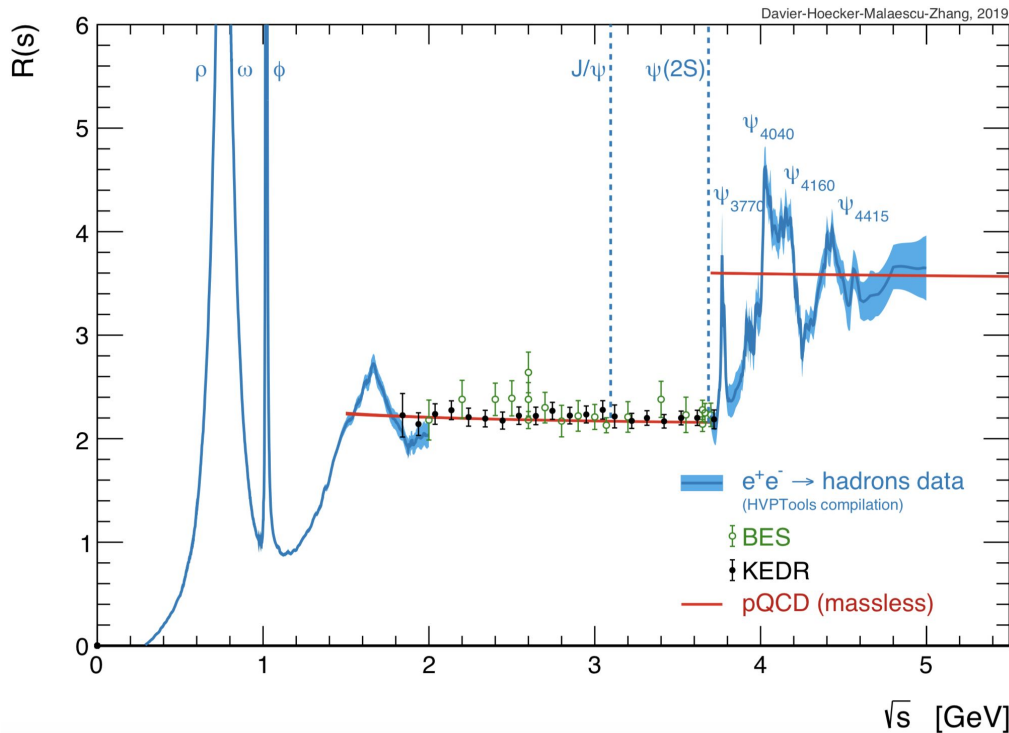
CIPT/FOPT: 0.16 → Reduced including higher orders

m_q : 0.14

DVs: 0.28 → Expected reduction with more data

$$a_\mu^{\text{had,LO}} = 694.0 \pm 4.0 \text{ (693.1} \pm 3.2 \text{ w/o BABAR-KLOE)}; \Delta\alpha_{\text{had}}(m_Z^2) = (275.3 \pm 1.0) \cdot 10^{-4} \text{ (275.2} \pm 0.9 \text{ w/o BABAR-KLOE)}$$

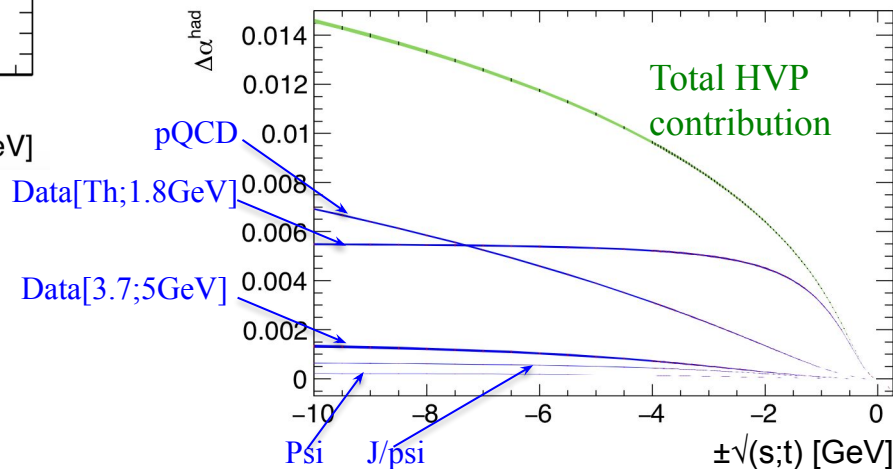
$R_{e^+e^-} \rightarrow \text{Hadrons}$



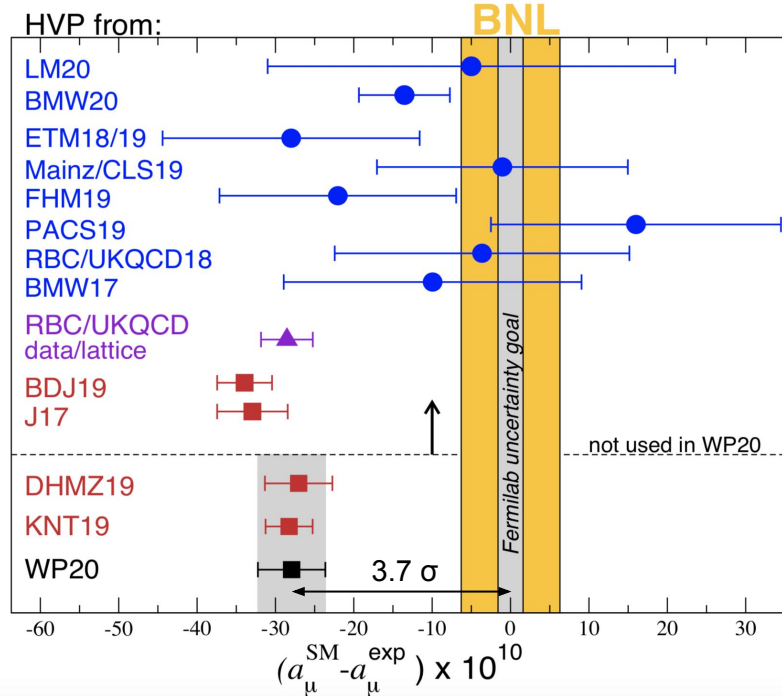
Sum of 32 *exclusive channels* with *full propagation of correlations*

→ Performed non-trivial check: a_μ and $\Delta\alpha_{\text{had}}$ from sum of individual channels and from R_{ee} integral $< 1.8 \text{ GeV}$

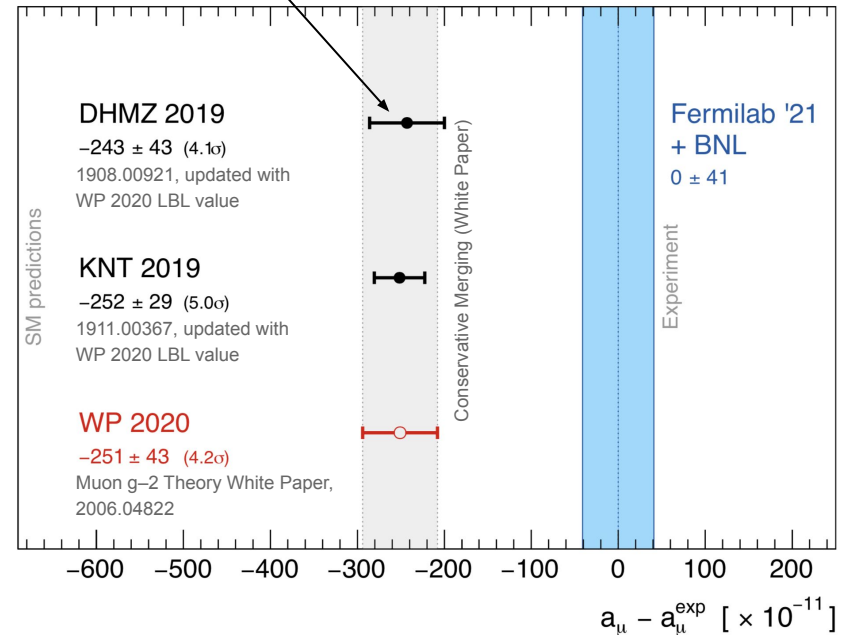
→ Enables the determination of the various HVP contributions to the “running” of α_{QED}



Status of a_μ before/with Fermilab result



Important to account for BABAR-KLOE diff. & inter-channel correlations

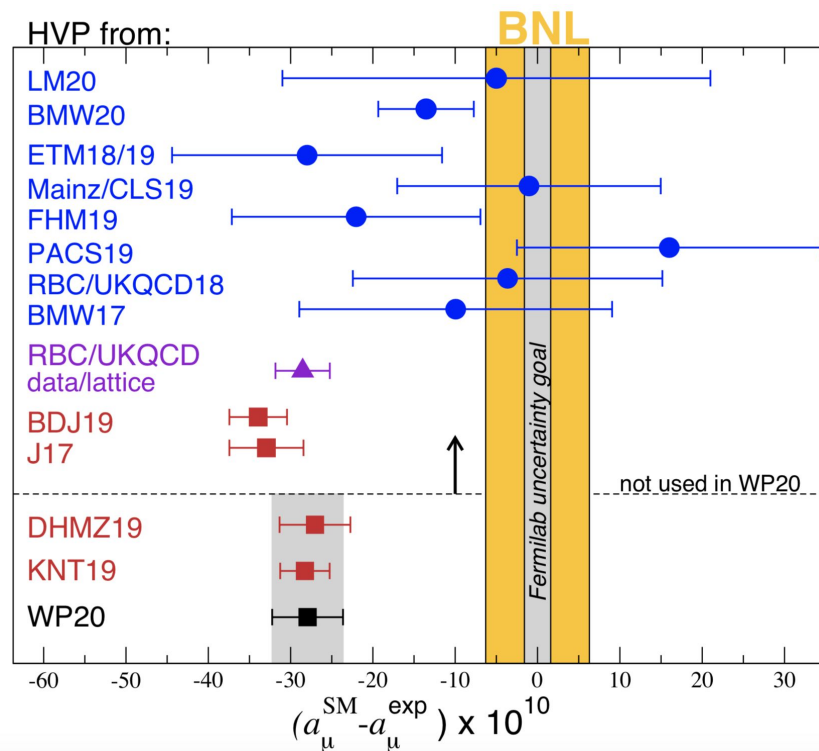


- Caution about significance: statistics-dominated measurement; prediction uncertainty limited by non-Gaussian systematic effects
- Nevertheless, large discrepancy between measurement and reference SM prediction (to be significantly improved in view of the forthcoming updates of the Fermilab measurement)
- Tension significantly smaller when using BMW20 for the LO HVP (TBC by other lattice groups), *not* incompatible with the EW fit (see next slides)

Impact of correlations between a_μ and α_{QED} on the EW fit

[2008.08107](#)(BM, Matthias Schott)

See also: Crivellin et al, 2003.04886;
Keshavarzi et al., 2006.12666 ;de Rafael,
2006.13880; Colangelo et al, 2010.07943



Approaches considered for treating the $a_\mu - \alpha_{\text{QED}}$ correlations

Studied approaches probing different hypotheses concerning the possible source(s) of the a_μ tension(s) :

(0) Scaling factor applied to the HVP contribution from some energy range of the hadronic spectrum

→ Approaches taking into account (*for the first time*) the full correlations between the uncertainties of the HVP contributions to a_μ and α_{QED} , based on input from DHMZ 19 (arXiv:1908.00921):

correlations between points/bins of a measurement in a given channel, between different measurements in the same channel, between different channels; full treatment of the BABAR-KLOE tension in the $\pi^+\pi^-$ channel

Computation (Energy range)	$a_\mu^{\text{HVP, LO}} [10^{-10}]$	$\Delta\alpha_{\text{had}}(M_Z^2) [10^{-4}]$	ρ
Phenomenology (Full HVP)	694.0 ± 4.0	275.3 ± 1.0	44%
Phenomenology ([Th.; 1.8 GeV])	635.5 ± 3.9	55.4 ± 0.4	86%
Phenomenology ([Th.; 1 GeV])	539.8 ± 3.8	36.3 ± 0.3	99.5%
Lattice (Full HVP)BMW 20 (v1)	712.4 ± 4.5	-	-

(1) Cov. matrix of a_μ and α_{QED} (Pheno) described by a nuisance parameter (NP_1) impacting both quantities (used to shift a_μ to some “target” value - coherent shift applied to α_{QED}) and another one (NP_2) impacting only α_{QED} (used in the EW fit)

Note: “target” values chosen in order to reach agreement with the BMW 20 prediction / Experimental $a_\mu (\pm 1\sigma)$

Uncertainty components	$a_\mu^{\text{HVP, LO}}$	$\Delta\alpha_{\text{had}}(M_Z^2)$
NP_1	$\sigma(a_\mu^{\text{HVP, LO}})$	$\sigma(\Delta\alpha_{\text{had}}(M_Z^2)) \cdot \rho$
NP_2	0	$\sigma(\Delta\alpha_{\text{had}}(M_Z^2)) \cdot \sqrt{1 - \rho^2}$

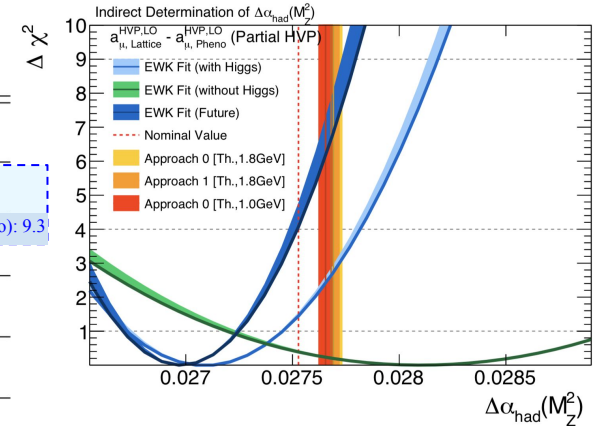
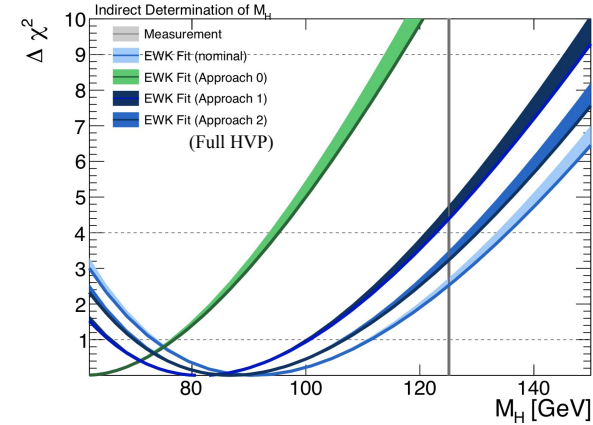
(2) Include the HVP contribution to a_μ as extra parameter in the EW fit, constrained by the Pheno & BMW 20 values

Note: Also accounted for the coherent impact of α_s on the HVP contribution and on the EW fit

Results: comparing the Phenomenology & BMW 20 values

$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	Approach 0		Approach 1		
	Scaling factor	$\Delta' \alpha_{\text{had}}(M_Z^2)$	Shift NP ₁	$\sigma'(\Delta \alpha_{\text{had}}(M_Z^2))$	$\Delta' \alpha_{\text{had}}(M_Z^2)$
$a_\mu^{\text{HVP, LO}}(\text{Lattice}) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ (Full HVP)	1.027	0.02826	4.6	$9.0 \cdot 10^{-5}$	0.02774
$(a_\mu^{\text{HVP, LO}}(\text{Lattice}) - 1\sigma) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ (Full HVP)	1.020	0.02808	3.5	$9.0 \cdot 10^{-5}$	0.02769
$a_\mu^{\text{HVP, LO}}(\text{Lattice}) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1.8 GeV])	1.029	0.02769	4.7	$9.5 \cdot 10^{-5}$	0.02768
$(a_\mu^{\text{HVP, LO}}(\text{Lattice}) - 1\sigma) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1.8 GeV])	1.022	0.02765	3.5	$9.5 \cdot 10^{-5}$	0.02764
$a_\mu^{\text{HVP, LO}}(\text{Lattice}) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1 GeV])	1.034	0.02765	-	-	-
$(a_\mu^{\text{HVP, LO}}(\text{Lattice}) - 1\sigma) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1 GeV])	1.026	0.02762	-	-	-

→ Large scaling factors (w.r.t. exp. uncertainties) & significant shifts of NP₁



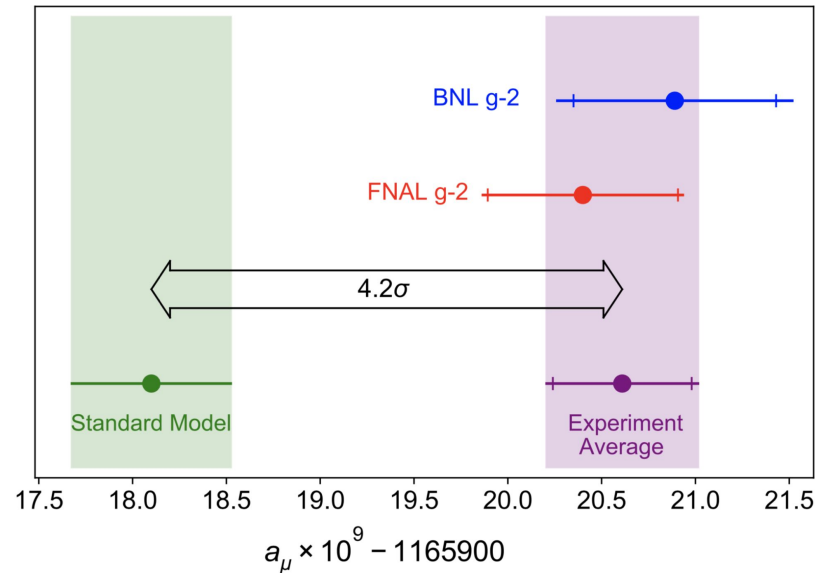
$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	Nominal		Approach 0		Approach 1		Approach 2	
	$\Delta' \alpha_{\text{had}}(M_Z^2)$	χ^2/ndf	$\Delta' \alpha_{\text{had}}(M_Z^2)$	χ^2/ndf	$\Delta' \alpha_{\text{had}}(M_Z^2)$	χ^2/ndf	$\Delta' \alpha_{\text{had}}(M_Z^2)$	χ^2/ndf
	0.02753	18.6/16 (p=0.29)	-	-	-	-	0.02753	28.1/17 (p=0.04)
$a_\mu^{\text{HVP, LO}}(\text{Lattice}) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ (Full HVP)	-	-	0.02826	27.6/16 (p=0.04)	0.02774	20.3/16 (p=0.21)	-	$\chi^2(\text{BMW20-Pheno}): 9.3$
$a_\mu^{\text{HVP, LO}}(\text{Lattice}) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1.8 GeV])	-	-	0.02769	19.9/16 (p=0.22)	0.02768	19.8/16 (p=0.23)	-	-
$a_\mu^{\text{HVP, LO}}(\text{Lattice}) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1.0 GeV])	-	-	0.02765	19.6/16 (p=0.24)	-	-	-	-

→ Addressing the BMW 20 - Pheno difference for a_μ has little impact on the EW fit, except for the unrealistic scenario rescaling the full HVP contribution

Note: Similar conclusions for the comparison with the Experimental a_μ value (see backup)

Conclusion

We have an interesting, long standing, multifaceted problem to solve...



???

Guiding ideas:

- Need *rigorous and realistic* treatment of uncertainties and correlations at all levels
(Underestimated uncertainties do not bring scientific progress & can put studies on wrong path)
- Studies for understanding differences between data-driven and Lattice QCD approaches need to follow similar standards as the g-2 experiment: *double-blinding*

Theoretical improvements for $\alpha_{\text{QED}}(m_Z)$ and a_μ are strongly connected through HVP

Backup

Lepton Magnetic Anomaly: from Dirac to QED

- Magnetic dipole moment of a charged lepton: $\vec{\mu} = g \frac{e}{2m} \vec{s}$
Dirac (1928) $g_e=2$ $a_e=0$
- “anomaly” = deviation w.r.t. Dirac’s prediction: $a = \frac{g-2}{2}$

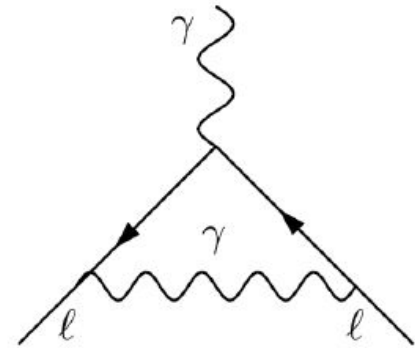
anomaly discovered:

Kusch-Foley (1948) $a_e = (1.19 \pm 0.05) 10^{-3}$

and explained by $O(\alpha)$ QED contribution:

Schwinger (1948) $a_e = \alpha/2\pi = 1.16 10^{-3}$

first triumph of QED



⇒ a_e sensitive to quantum fluctuations of fields

More Quantum Fluctuations

Why is it (so) complicated to compute one number ? (*very precisely*)

0.001 ppm

0.01 ppm

+ Many other diagrams at higher orders...
QED up to $O(\alpha^5)$ (Kinoshita et al.)

0.34 ppm

0.15 ppm

had

had

Dominant uncertainties: non-perturbative...

$\tilde{\chi}$

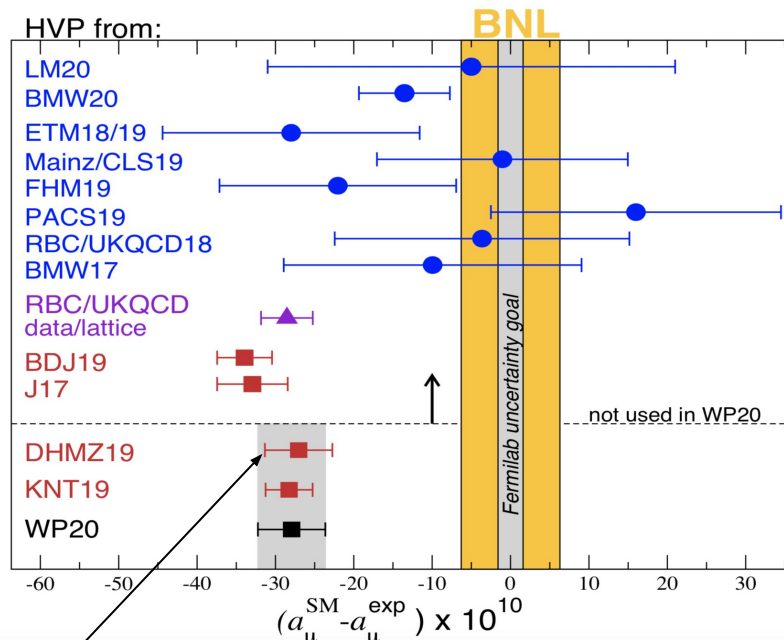
$\tilde{\nu}$

$\tilde{\mu}$

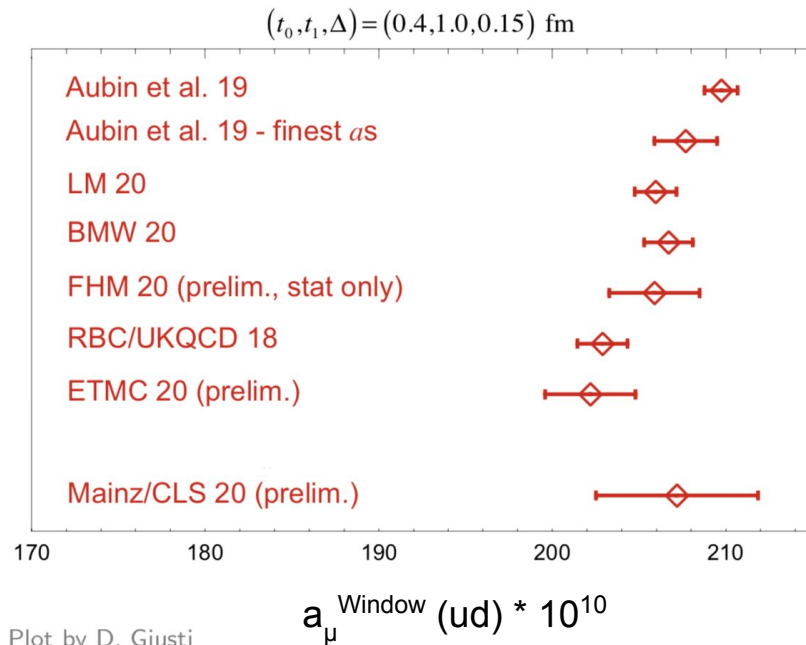
$\tilde{\chi}^0$

???

Status of a_μ (HVP)



Important to account for BABAR-KLOE diff. & inter-channel correlations



→ *HVP(WP20): Merging of model independent results: DHMZ and KNT (and CHKS for $\pi^+\pi^-$ & $\pi^+\pi^-\pi^0$)* Central value from simple average; BABAR-KLOE tension & correlations between channels from DHMZ; Max(DHMZ & KNT uncertainties) in each channel

→ *Excellent progress on the Lattice QCD (+QED) calculations; Precision of BMW20 (to be cross-checked by other lattice groups) became similar to the one of dispersive approaches; Ongoing cross-checks using Euclidean time windows (related to HVP with suppression of very low and high energies) for which various groups achieved similar precision; If BMW20 result is confirmed, the difference w.r.t. dispersive results to be understood.*

Theory initiative white paper executive summary & new results

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^- , LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

→ Dominant uncertainty: HVP LO → Based on *merging of model-independent methods*

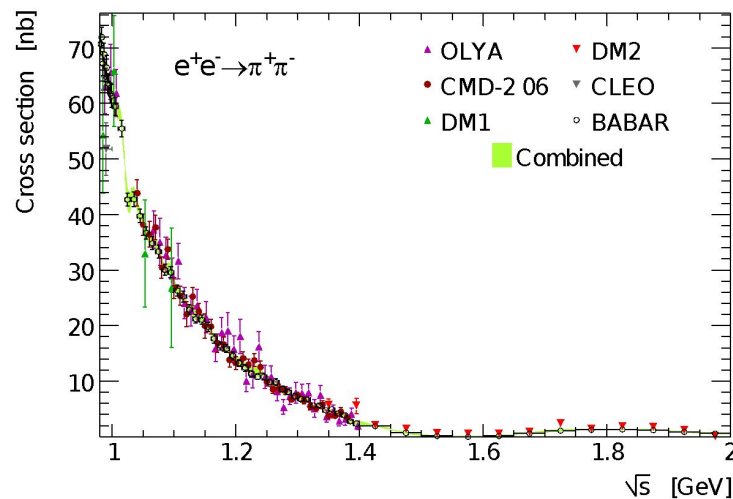
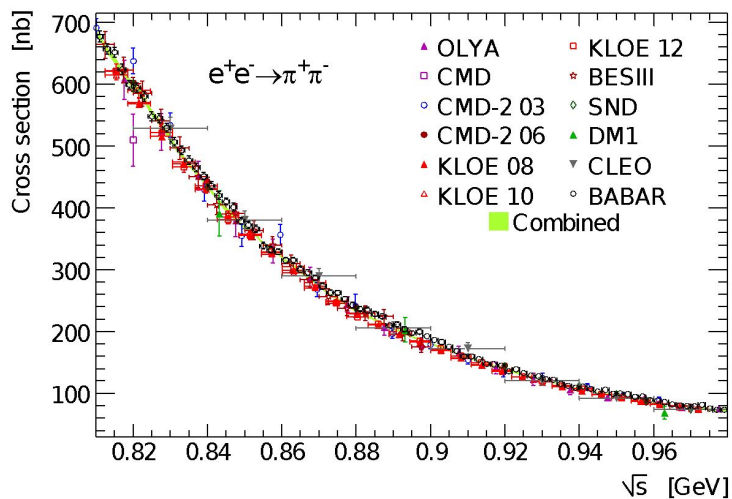
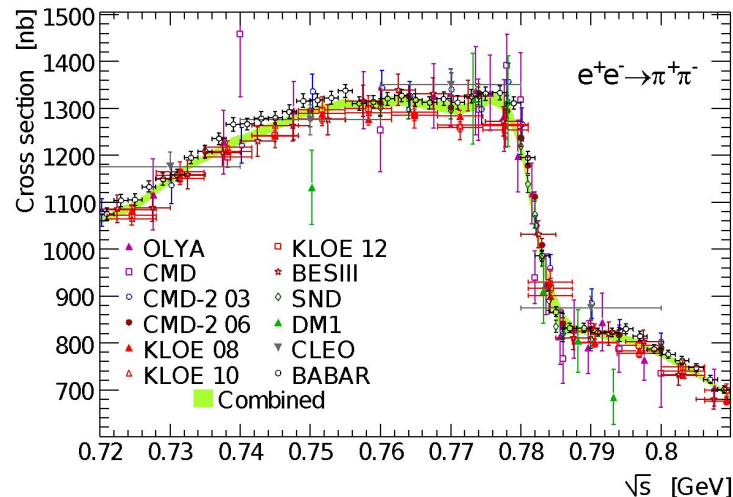
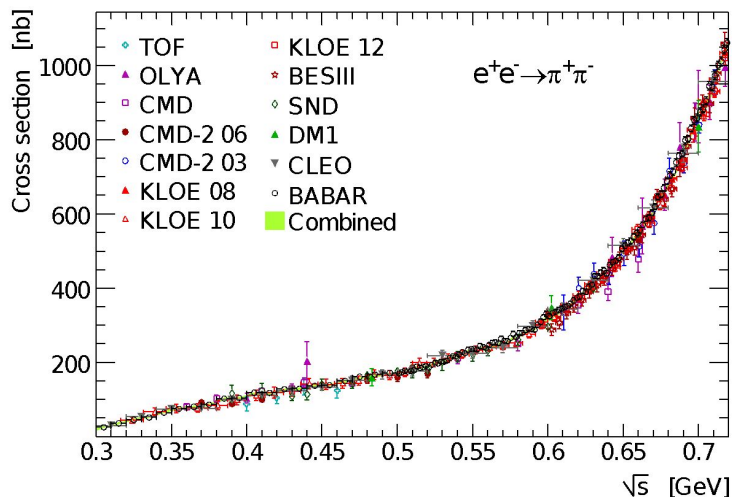
→ HLbL also has an important uncertainty

→ Lattice results become more and more interesting

→ A tension between the BNL measurement and the reference SM prediction: $\sim 3.7 \sigma$ ($\sim 4.2 \sigma$ including FNAL)

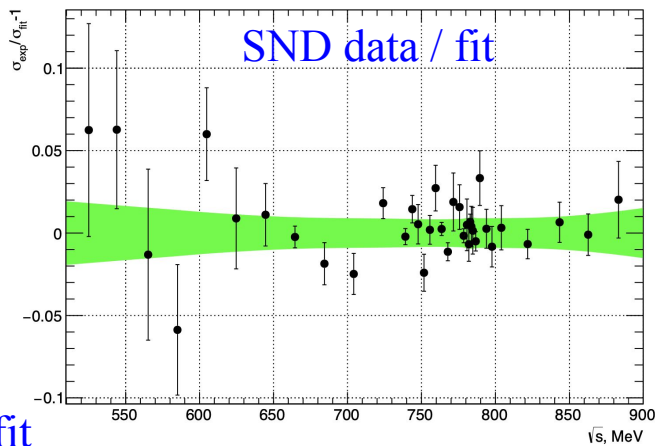
→ Tension significantly smaller when using BMW20 for the LO HVP (TBC by other lattice groups)

More on the combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel



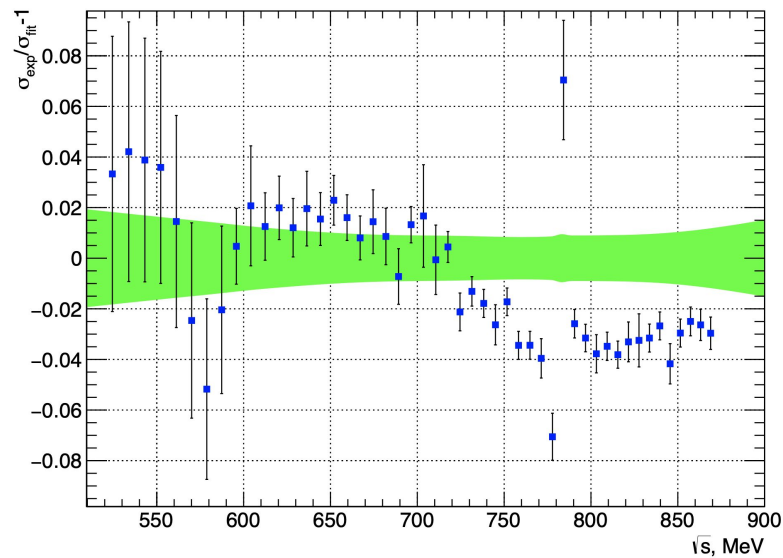
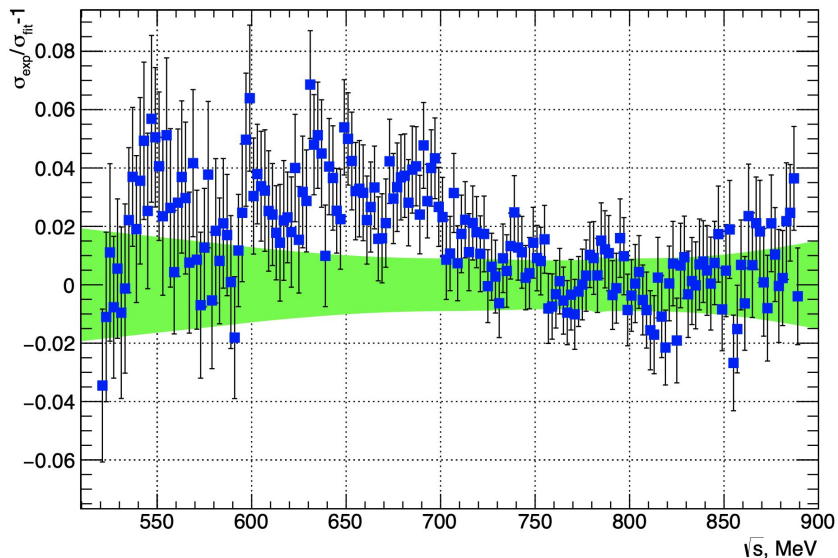
Comparison of SND measurement with BABAR and KLOE

2004.00263

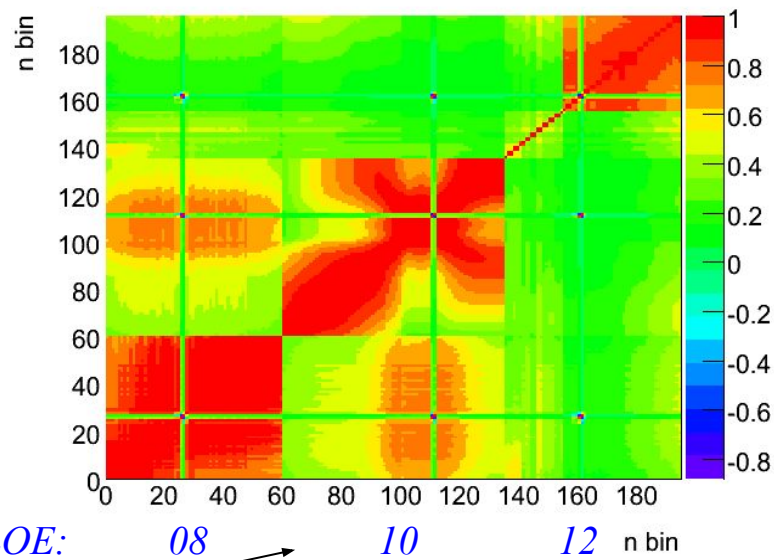
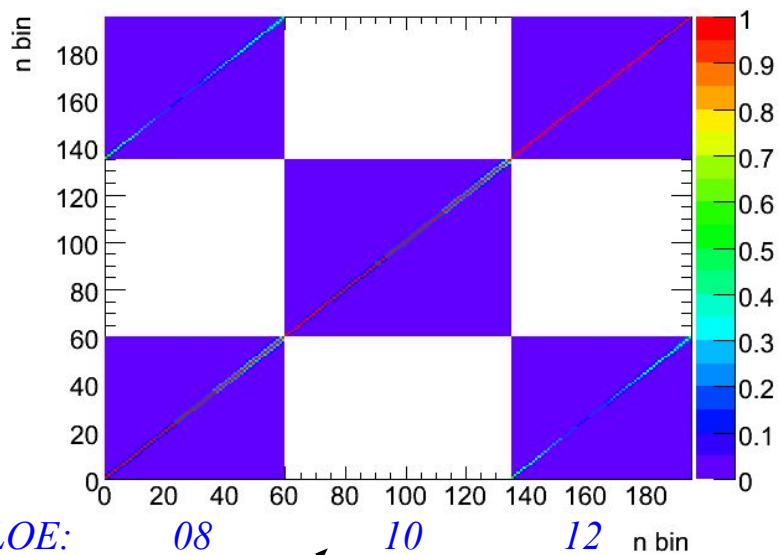


BABAR / SND fit

KLOE / SND fit

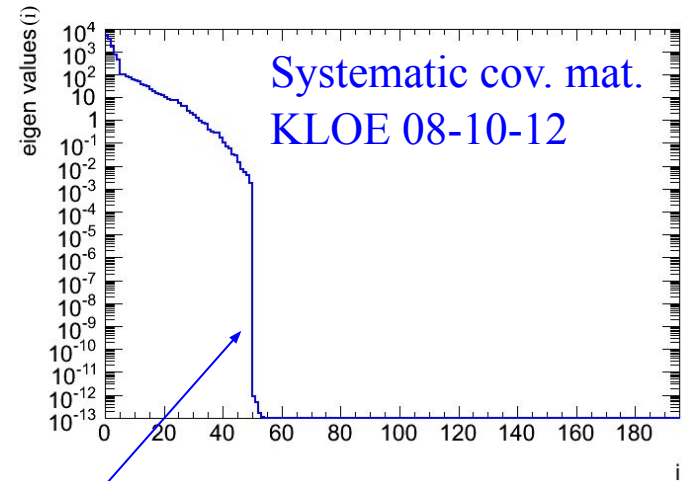
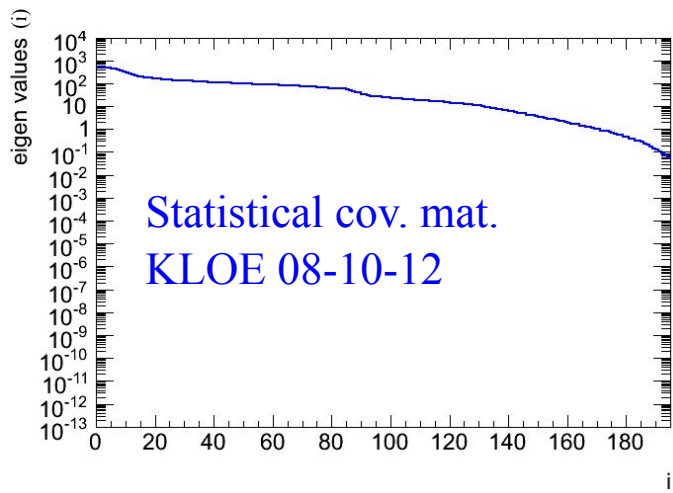


Treatment of the KLOE correlation matrices



→ Statistical and systematic correlation matrices among the 3 measurements

Treatment of the KLOE data – eigenvector decomposition

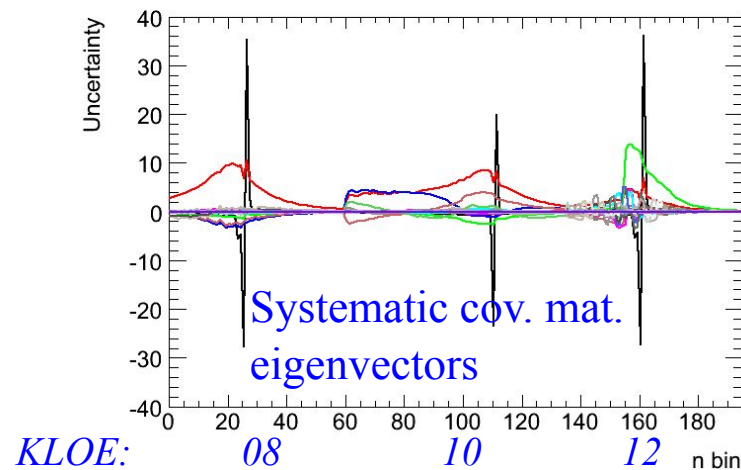
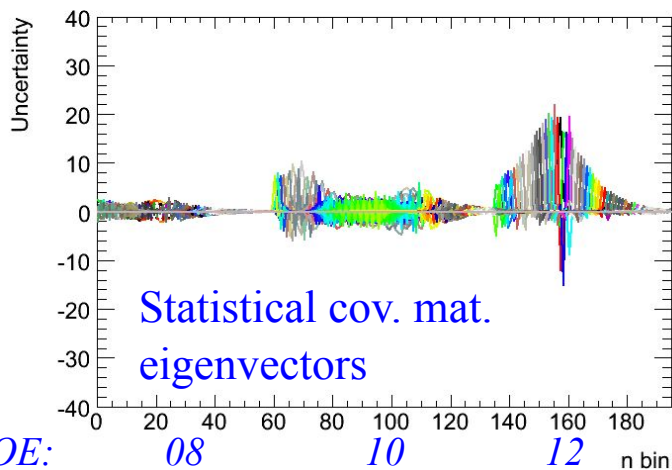


→ “counting” the number of independent components (50) used to build the covariance matrix

$$C = S \cdot D \cdot S^T$$
$$D = \begin{pmatrix} \diagdown & 0 & 0 \\ 0 & \sigma_i^2 & 0 \\ 0 & 0 & \diagdown \end{pmatrix}$$
$$S = \begin{pmatrix} V_1 & \dots & V_n \\ \vdots & & \vdots \end{pmatrix}$$

→ Problem of negative eigenvalues for previous systematic covariance matrix solved (informed KLOE collaboration about the problem in summer 2016)

Treatment of the KLOE data – eigenvector decomposition

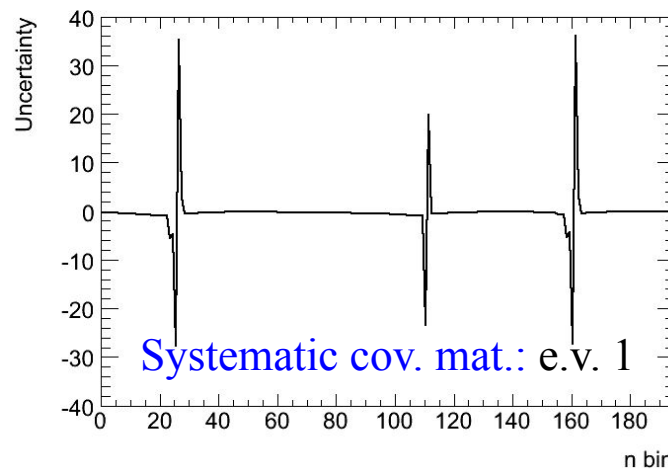
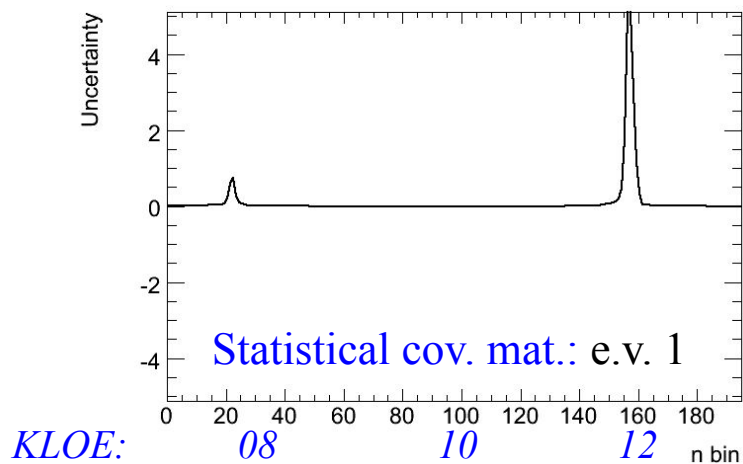


- Each normalized eigenvector ($\sigma_i \cdot V_i$) treated as an uncertainty fully correlated between the bins
- All these uncertainties are independent between each-other

$$C = \sum_{i=1}^{N_{bins}} \sigma_i^2 \cdot C(V_i)$$

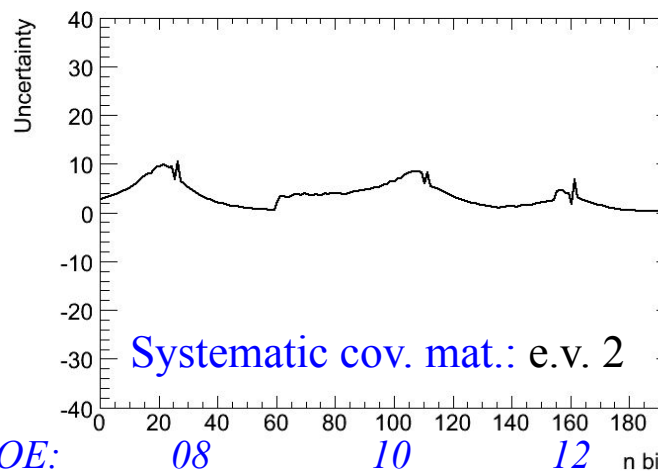
- Checked exact matching with the original matrices + with all a_μ integrals and uncertainties published by KLOE

Treatment of the KLOE data – eigenvector decomposition

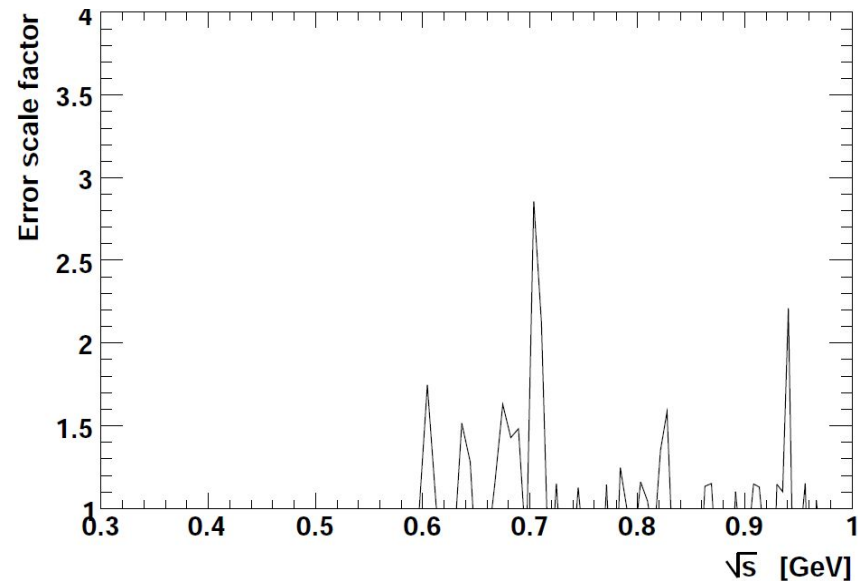


→ Eigenvectors carry the general features of the correlations:

- long-range for systematics
- ~short-range for statistical uncertainties + correlations between KLOE 08 & 12



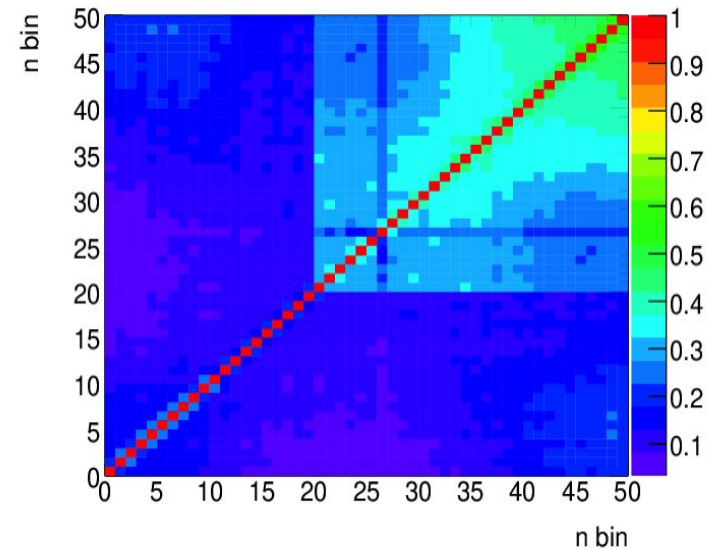
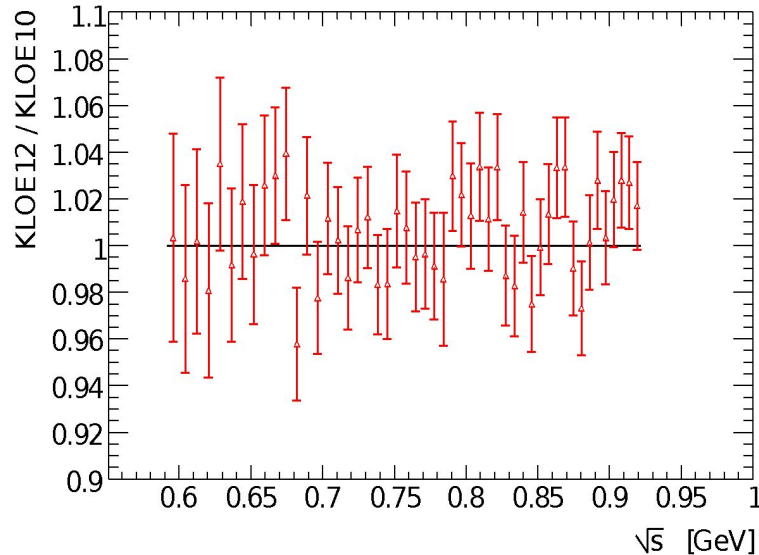
Local comparison of the 3 KLOE measurements



- Local χ^2 /ndof test of the local compatibility between KLOE 08 & 10 & 12, taking into account the correlations: some tensions observed
- Does not probe general trends of the difference between the measurements (e.g. slopes in the ratio)

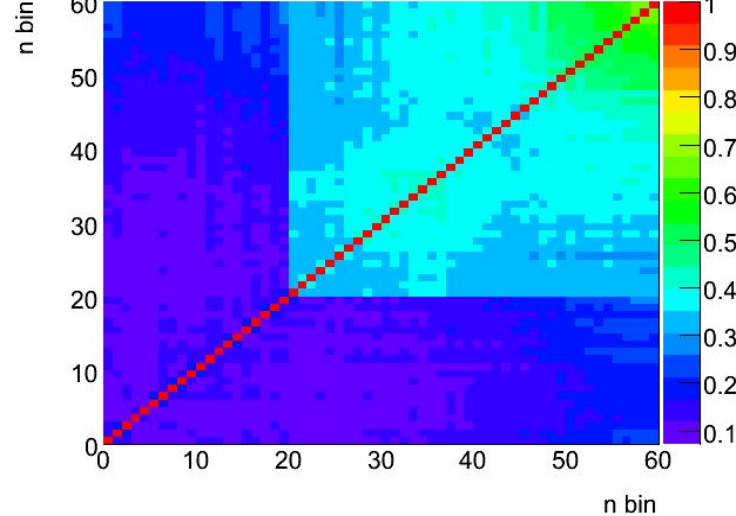
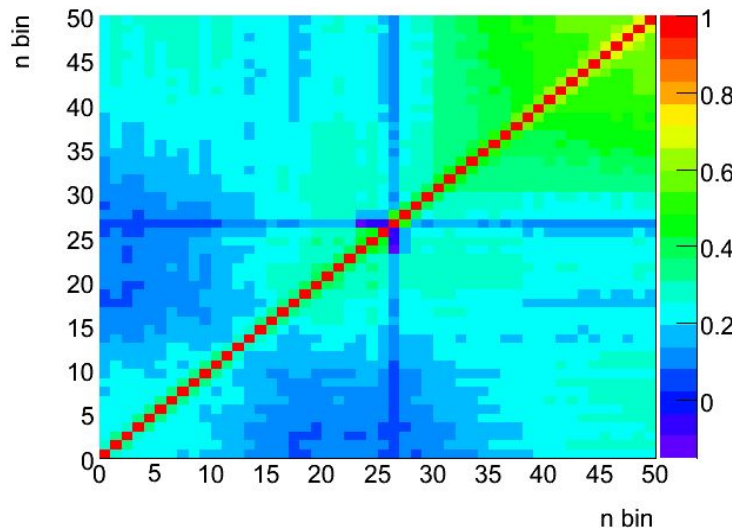
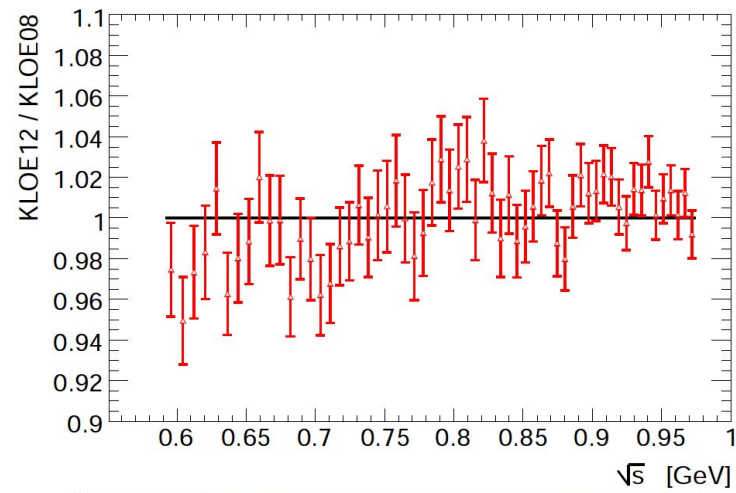
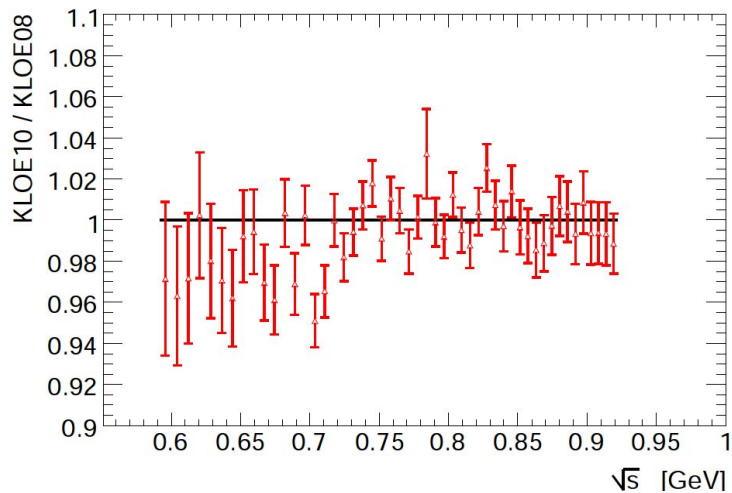
Ratios between measurements

- Compute ratio between pairs of KLOE measurements
- Full propagation of uncertainties and correlations using pseudo-experiments (agreement with analytical linear uncertainty propagation)



- Good agreement between KLOE 10 and KLOE 12

Ratios between measurements



Direct comparison of the 3 KLOE measurements

→ Quantitative comparison between the ratios and unity, taking into account correlations

KLOE 10 / KLOE 08

$\chi^2 [0.35;0.85] \text{ GeV}^2 : 79.0 / 50(\text{DOF})$
p-value= 0.0056

$\chi^2 [0.35;0.58] \text{ GeV}^2 : 46.2 / 23(\text{DOF})$
p-value= 0.0028

$\chi^2 [0.58;0.85] \text{ GeV}^2 : 29.7 / 27(\text{DOF})$
p-value= 0.33

$\chi^2 [0.64;0.85] \text{ GeV}^2 : 20.7 / 21(\text{DOF})$
p-value= 0.47

KLOE 12 / KLOE 08

$\chi^2 [0.35;0.95] \text{ GeV}^2 : 73.7 / 60(\text{DOF})$
p-value= 0.11

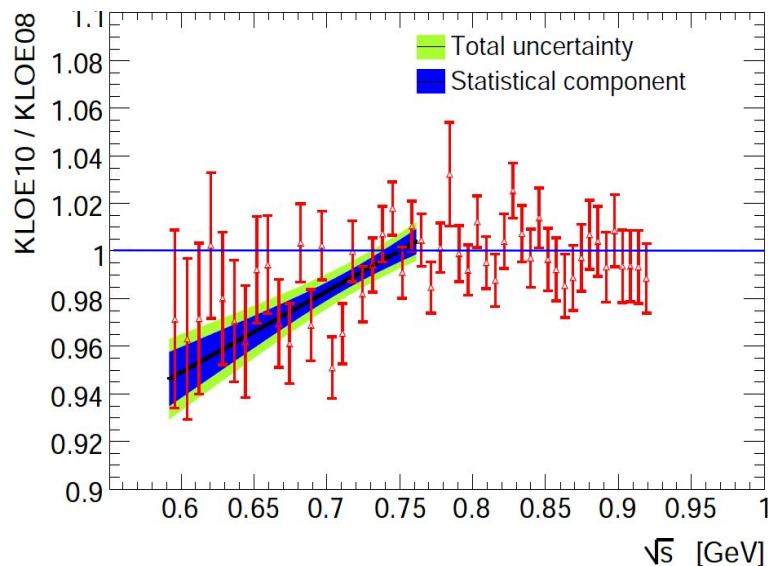
$\chi^2 [0.35;0.58] \text{ GeV}^2 : 21.8 / 23(\text{DOF})$
p-value= 0.53

$\chi^2 [0.35;0.64] \text{ GeV}^2 : 27.5 / 29(\text{DOF})$
p-value= 0.55

$\chi^2 [0.64;0.95] \text{ GeV}^2 : 39.4 / 31(\text{DOF})$
p-value= 0.14

Quantitative comparisons of the KLOE measurements

- Quantitative comparison between the ratios and unity, taking into account correlations
- Fitting the ratio taking into account correlations
- Full propagation of uncertainties and correlations – 3 methods yielding consistent results: $\pm 1\sigma$ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit



Comparison with Unity:

$\chi^2 [0.35;0.85] \text{ GeV}^2 : 79.0 / 50(\text{DOF})$

p-value= 0.0056

$\chi^2 [0.35;0.58] \text{ GeV}^2 : 46.2 / 23(\text{DOF})$

p-value= 0.0028

$\chi^2 [p0 + p1\sqrt{s}] : 36.1 / 21(\text{DOF})$

p-value= 0.02

$p0 : 0.745 \pm 0.085$

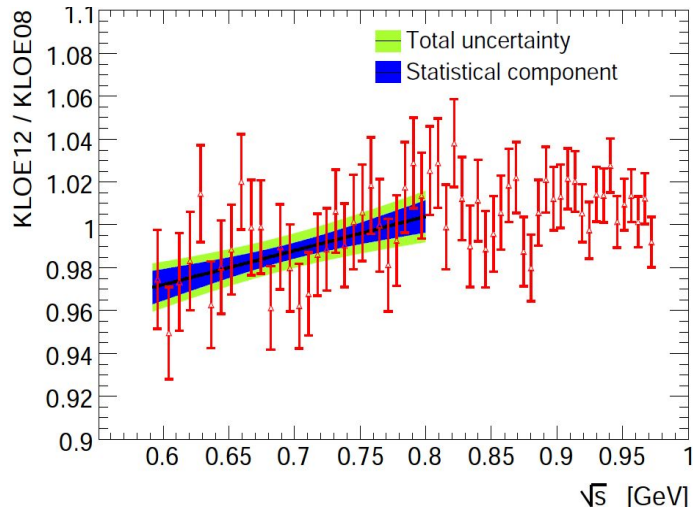
$p1 : 0.341 \pm 0.117$

- Significant shift & slope ($\sim 2.5\text{-}3\sigma$) at low \sqrt{s} , no significant shift at high \sqrt{s}
Similar shift & slope for KLOE 12 / KLOE 08 (*see below*)
- Should motivate conservative treatment of uncertainties and correlations in combination

Direct comparison of the 3 KLOE measurements

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results:
 $\pm 1\sigma$ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit

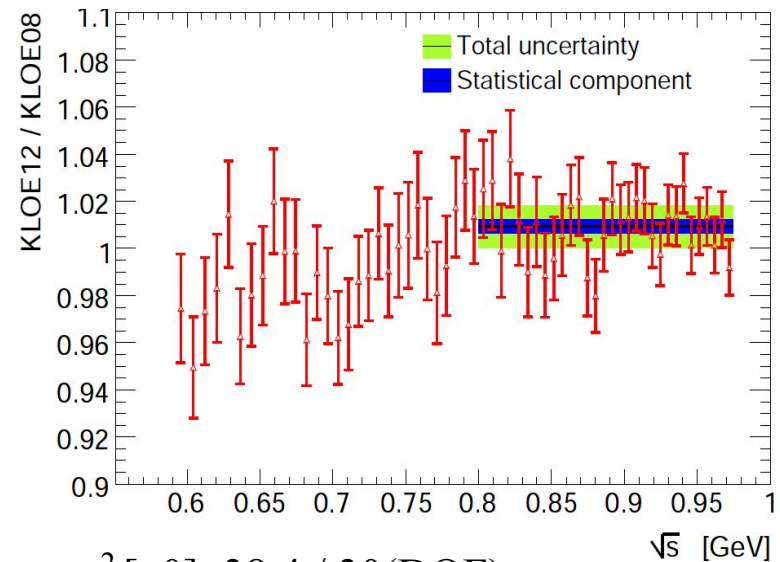


$$\chi^2 [p0 + p1\sqrt{s}]: 20.7 / 27(\text{DOF})$$

$$p\text{-value} = 0.80$$

$$p0 : 0.876 \pm 0.056$$

$$p1 : 0.159 \pm 0.081$$



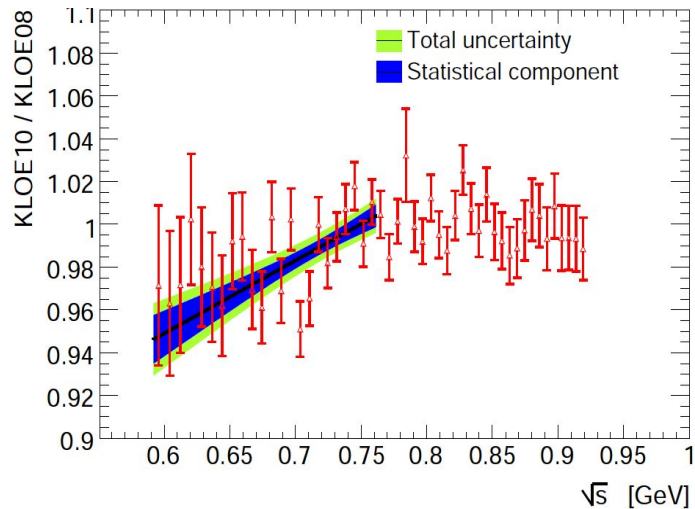
$$\chi^2 [p0]: 38.4 / 30(\text{DOF})$$

$$p\text{-value} = 0.14$$

$$p0 : 1.009 \pm 0.009$$

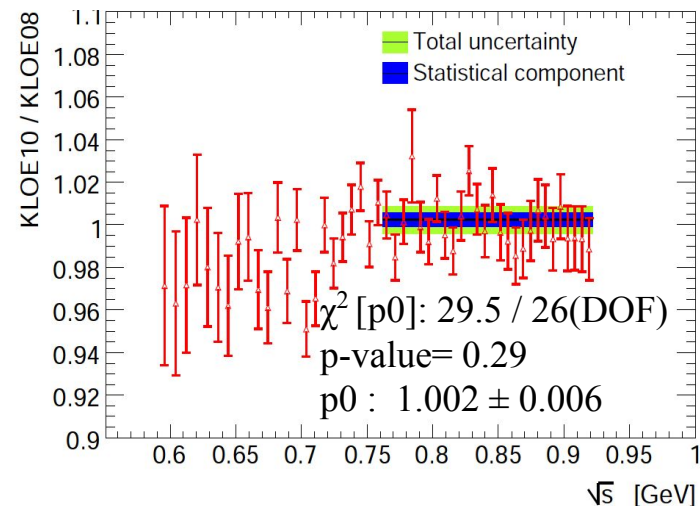
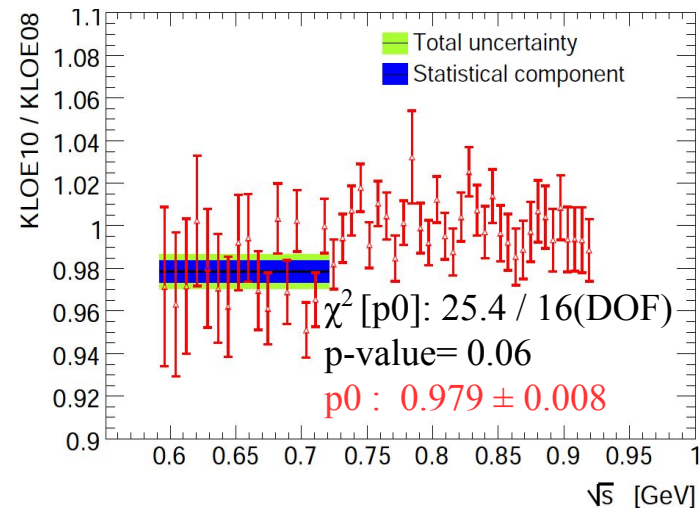
→ Significant shift and slope ($\sim 2\sigma$) at low \sqrt{s} , no significant shift at high \sqrt{s}

Direct comparison of the 3 KLOE measurements

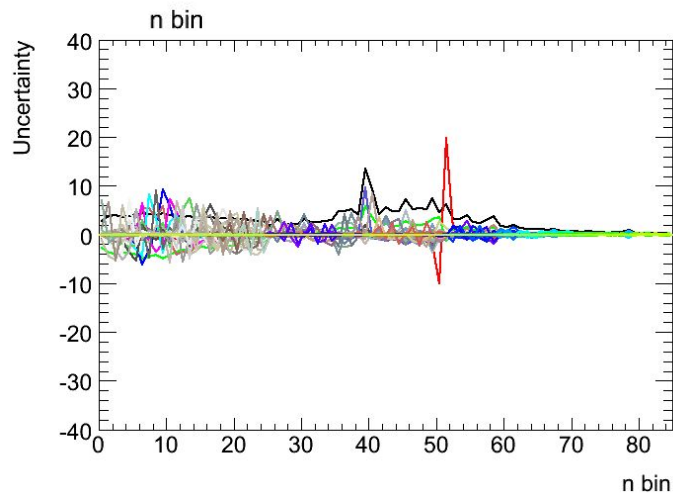
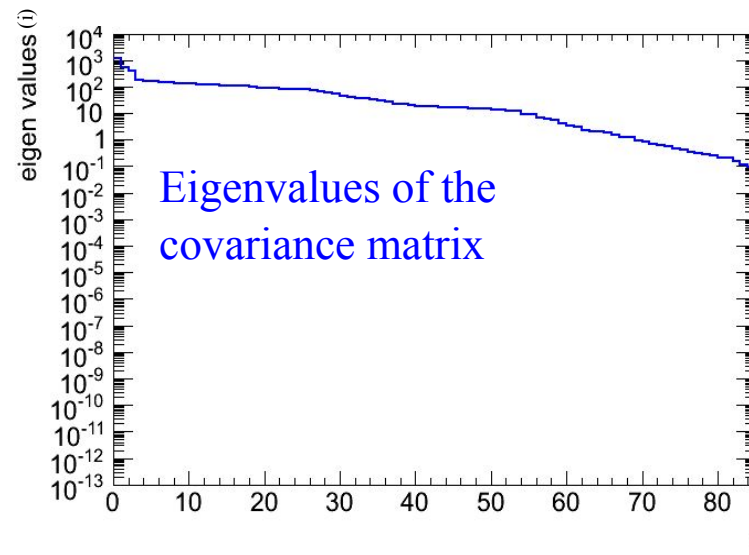
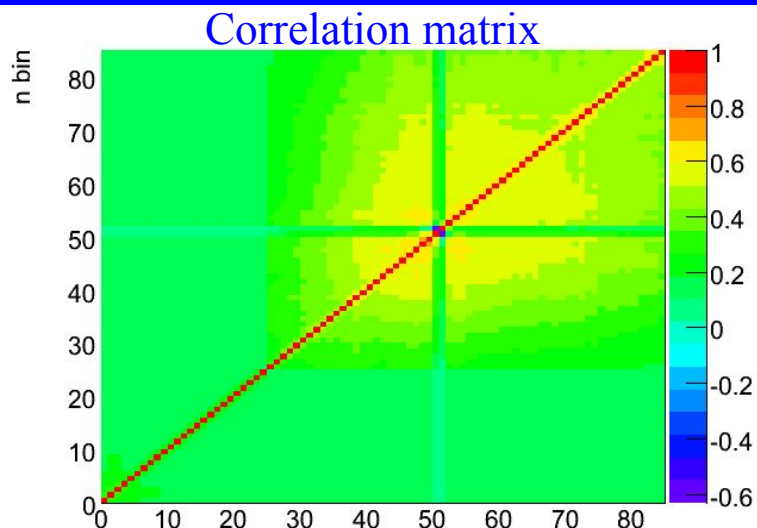


χ^2 [p0 + p1 \sqrt{s}]: 36.1 / 21(DOF)
 p-value= 0.02
 p0 : 0.745 \pm 0.085
 p1 : 0.341 \pm 0.117

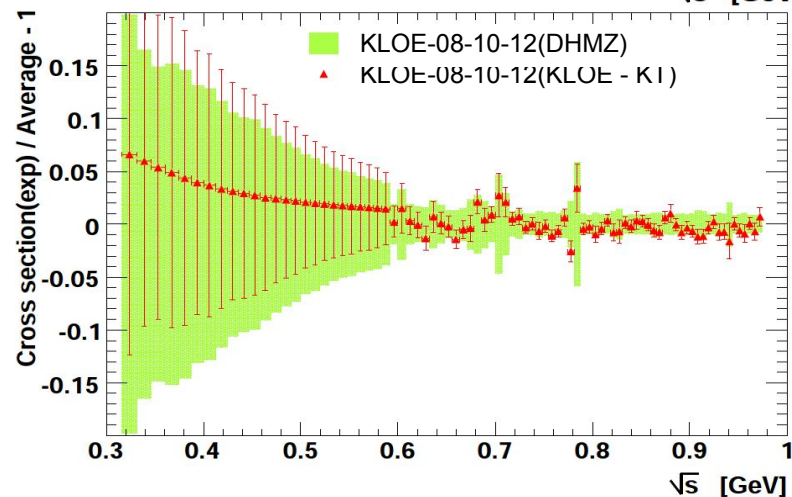
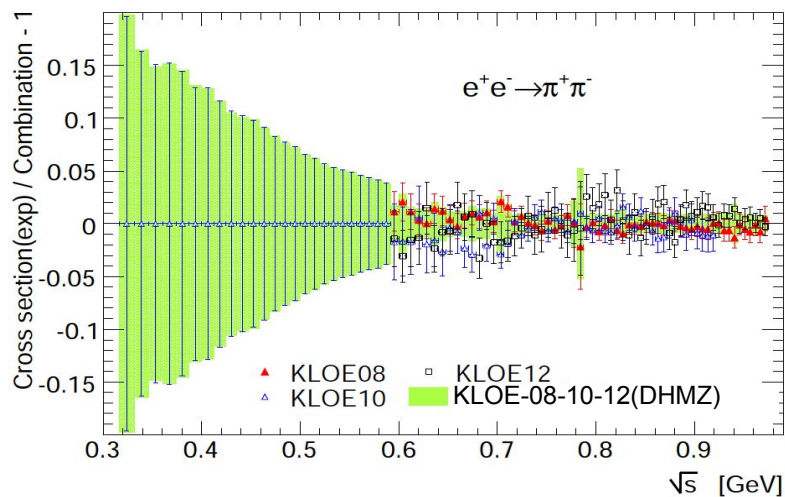
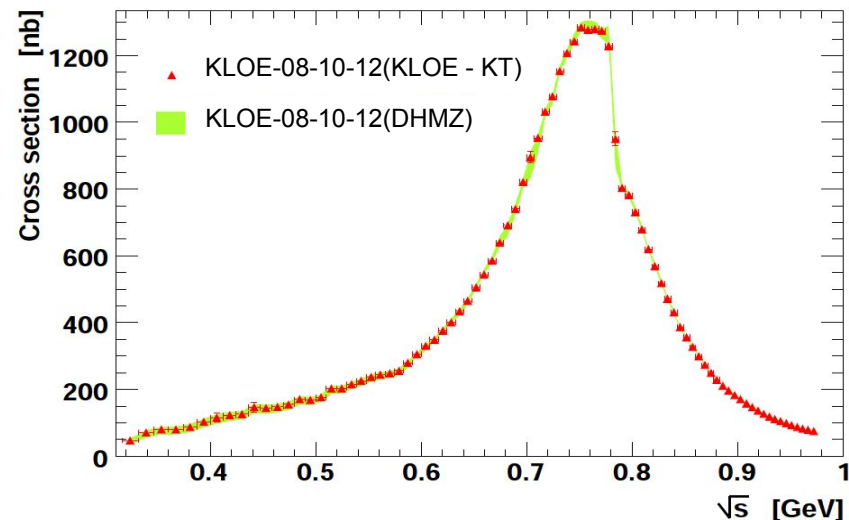
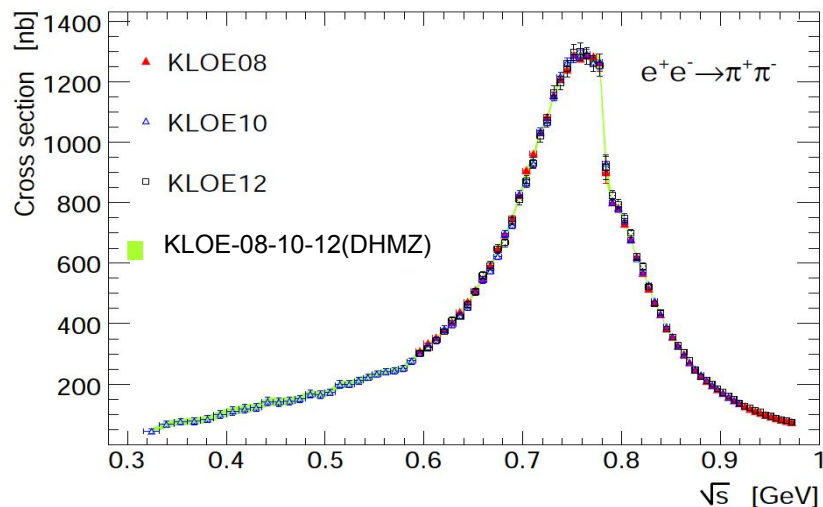
→ Significant shift and slope (~ 2.5 - 3σ) at low \sqrt{s} ,
 no significant shift at high \sqrt{s}



Treatment of the combined KLOE data



Combining the 3 KLOE measurements



Combining the 3 KLOE measurements - $a_{\mu}^{\pi\pi}$ contribution

KLOE08 $a_{\mu}[0.6 ; 0.9] : 368.3 \pm 3.2 [10^{-10}]$

KLOE10 $a_{\mu}[0.6 ; 0.9] : 365.6 \pm 3.3$

KLOE12 $a_{\mu}[0.6 ; 0.9] : 366.8 \pm 2.5$

→ Correlation matrix:

	08	10	12
08	1	0.70	0.35
10	0.70	1	0.19
12	0.35	0.19	1

→ Amount of independent information provided by each measurement

→ KLOE-08-10-12(DHMZ) - $a_{\mu}[0.6 ; 0.9] : 366.5 \pm 2.8$ (Without χ^2 rescaling: ± 2.2)

→ Conservative treatment of uncertainties and correlations (*not perfectly known*) in weight determination

→ KLOE-08-10-12(KLOE-KT) - $a_{\mu}[0.6 ; 0.9]\text{GeV} : 366.9 \pm 2.2$ (Includes χ^2 rescaling)

→ Assuming perfect knowledge of the correlations to minimize average uncertainty

Evaluation of uncertainties and correlations (e^+e^-)

	$\sigma_{\pi\pi\gamma}$	$\sigma_{\pi\pi}^0$	F_π	$\Delta^{\pi\pi} a_\mu$
Reconstruction Filter	negligible			
Background subtraction	Tab. 1		0.3%	
Trackmass	0.2%			
Pion cluster ID	negligible			
Tracking efficiency	0.3%			
Trigger efficiency	0.1%			
Acceptance	Tab. 2		0.2%	
Unfolding	Tab. 3		negligible	
L3 filter	0.1%			
\sqrt{s} dependence of H	-	Tab. 4		0.2%
Luminosity	0.3%			
Experimental systematics	0.6%			
FSR resummation	-	0.3%		
Radiator function H	-	0.5%		
Vacuum Polarization	-	0.1%	-	0.1%
Theory systematics	0.6%			

→ Systematics *evaluated* in \sim wide mass ranges with sharp transitions

$M_{\pi\pi}^2$ range (GeV ²)	Systematic error (%)
$0.35 \leq M_{\pi\pi}^2 < 0.39$	0.6
$0.39 \leq M_{\pi\pi}^2 < 0.43$	0.5
$0.43 \leq M_{\pi\pi}^2 < 0.45$	0.4
$0.45 \leq M_{\pi\pi}^2 < 0.49$	0.3
$0.49 \leq M_{\pi\pi}^2 < 0.51$	0.2
$0.51 \leq M_{\pi\pi}^2 < 0.64$	0.1
$0.64 \leq M_{\pi\pi}^2 < 0.95$	-

KLOE 08 (0809.3950)

KLOE 10 (1006.5313)

	$\sigma_{\pi\pi\gamma}$	$\sigma_{\pi\pi}^{\text{bare}}$	$ F_\pi ^2$	$\Delta a_\mu^{\pi\pi}$
	threshold ; ρ -peak			(0.1 - 0.85 GeV ²)
Background Filter	0.5% ; 0.1%			negligible
Background subtraction	3.4% ; 0.1%			0.5%
$f_0 + \rho\pi$ bkg.	6.5% ; negl.			0.4%
Ω cut	1.4% ; negl.			0.2%
Trackmass cut	3.0% ; 0.2%			0.5%
π -e PID	0.3% ; negl.			negligible
Trigger	0.3% ; 0.2%			0.2%
Acceptance	1.9% ; 0.3%			0.5%
Unfolding	negl. ; 2.0%			negligible
Tracking	0.3%			
Software Trigger (L3)	0.1%			
Luminosity	0.3%			
Experimental syst.				1.0%
FSR treatment	-	7% ; negl.		0.8%
Radiator function H	-	0.5%		
Vacuum Polarization	-	Ref. 34	-	0.1%
Theory syst.				0.9%

Evaluation of uncertainties and correlations (e^+e^-)

Sources	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.9	0.9-1.2	1.2-1.4	1.4-2.0	2.0-3.0
trigger/ filter	5.3	2.7	1.9	1.0	0.7	0.6	0.4	0.4
tracking	3.8	2.1	2.1	1.1	1.7	3.1	3.1	3.1
π -ID	10.1	2.5	6.2	2.4	4.2	10.1	10.1	10.1
background	3.5	4.3	5.2	1.0	3.0	7.0	12.0	50.0
acceptance	1.6	1.6	1.0	1.0	1.6	1.6	1.6	1.6
kinematic fit (χ^2)	0.9	0.9	0.3	0.3	0.9	0.9	0.9	0.9
correl $\mu\mu$ ID loss	3.0	2.0	3.0	1.3	2.0	3.0	10.0	10.0
$\pi\pi/\mu\mu$ non-cancel.	2.7	1.4	1.6	1.1	1.3	2.7	5.1	5.1
unfolding	1.0	2.7	2.7	1.0	1.3	1.0	1.0	1.0
ISR luminosity	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.4
sum (cross section)	13.8	8.1	10.2	5.0	6.5	13.9	19.8	52.4

BABAR (1205.2228)

→ Systematics *evaluated* in \sim wide mass ranges with sharp transitions
(statistics limitations when going to narrow ranges)

$a_{\mu}^{\pi\pi}$ contribution [0.28; 1.8] GeV – spline-based (2018)

→ Updated result:

$$506.70 \pm 2.32 (\pm 1.01 \text{ (stat.)} \pm 2.08 \text{ (syst.)}) [10^{-10}]$$

(after uncertainty enhancement by $\sim 14\%$ caused by the tension between inputs, taken into account through a local rescaling)

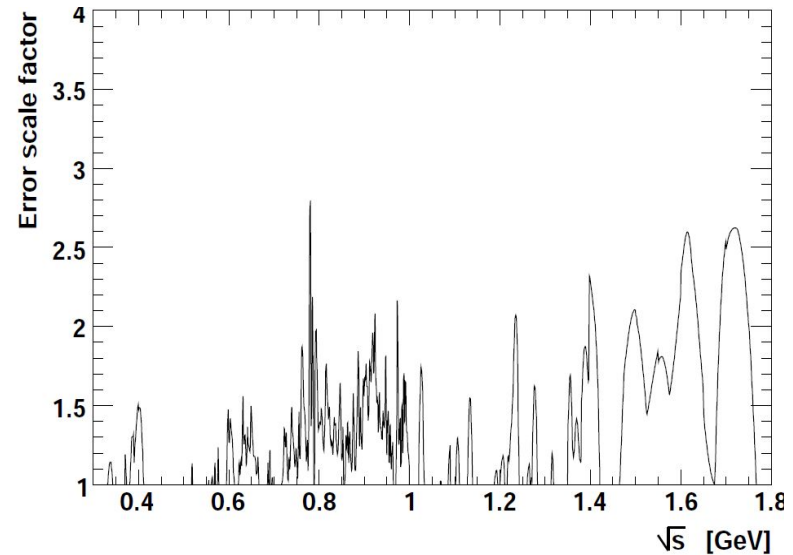
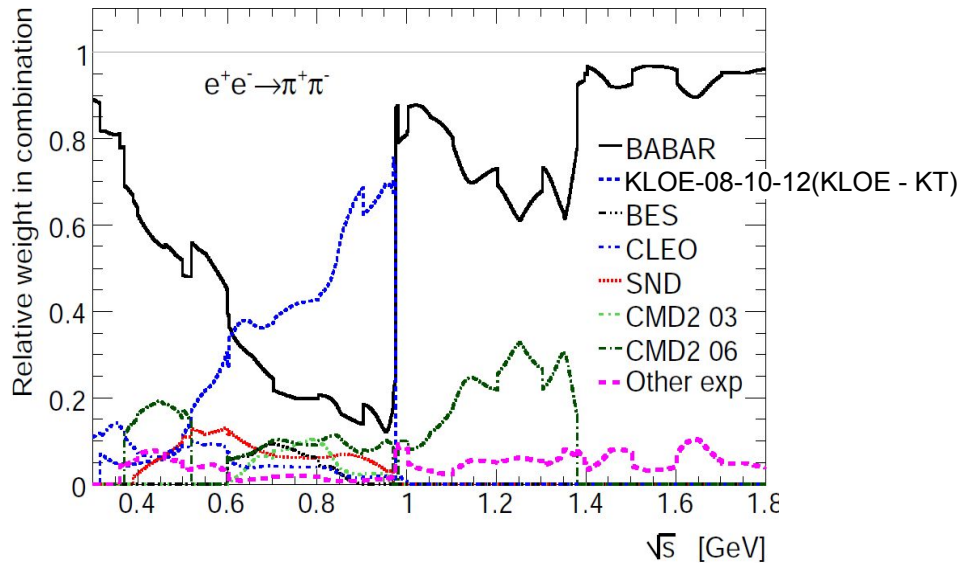
Total uncertainty: 5.9 (2003) \rightarrow 2.8 (2011) \rightarrow 2.6 (2017) \rightarrow 2.3 (2018)

$a_{\mu}^{\pi\pi}$ contribution [0.28; 1.8] GeV – spline-based (2018)

→ with KLOE-08-10-12 (KLOE-KT) used as input: $506.55 \pm 2.38 [10^{-10}]$

(after uncertainty enhancement by 18% caused by the tension between inputs, taken into account through a local rescaling)

→ Compensation between uncertainty reduction for KLOE-08-10-12 (KLOE-KT), inducing a change of weights in DHMZ combination, and tension enhancement



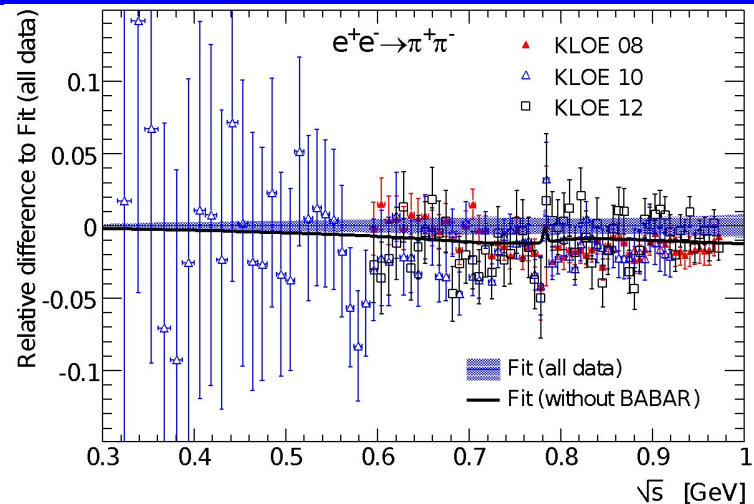
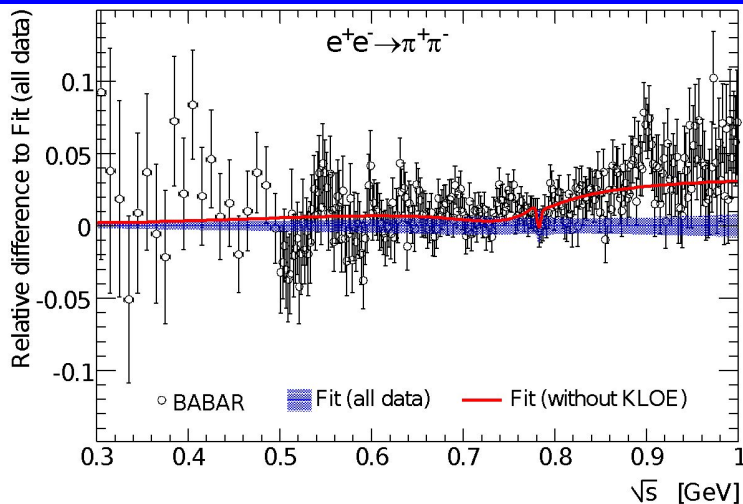
Fit parameters, uncertainties and correlations $e^+e^- \rightarrow \pi^+\pi^-$

	α_V	$\kappa[10^{-4}]$	B_0	B_1	m_ρ [MeV]	m_ω [MeV]
α_V	0.133 ± 0.020	0.52	-0.45	-0.97	0.90	-0.25
$\kappa[10^{-4}]$		21.6 ± 0.5	-0.33	-0.57	0.64	-0.08
B_0			1.040 ± 0.003	0.40	-0.40	0.29
B_1				-0.13 ± 0.11	-0.96	0.20
m_ρ [MeV]					774.5 ± 0.8	-0.17
m_ω [MeV]						782.0 ± 0.1

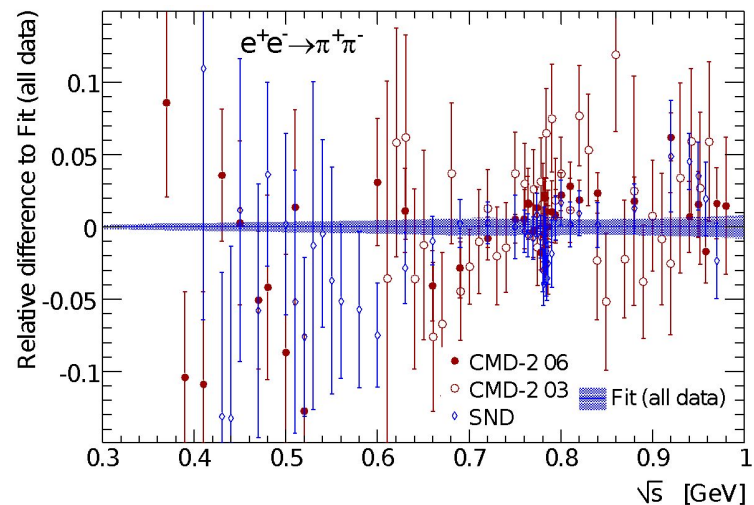
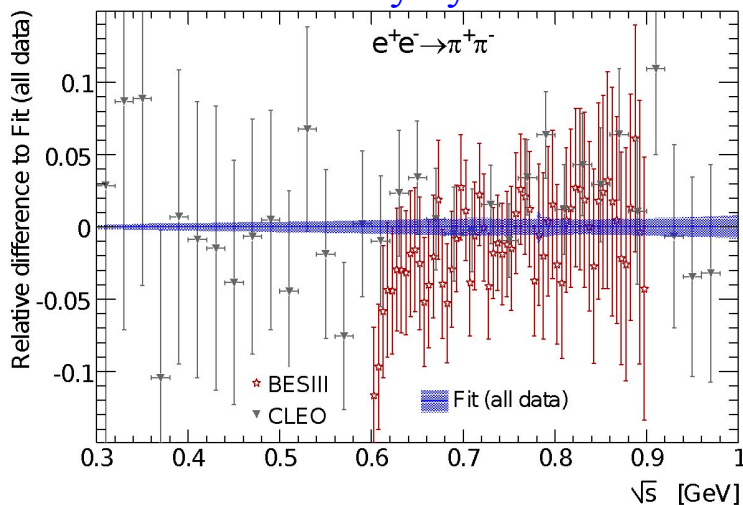
→ κ corresponds to a Br ($\omega \rightarrow \pi^+\pi^-$) of $(2.09 \pm 0.09) \cdot 10^{-2}$, in agreement with the result extracted from the fit of arXiv:1810.00007, $(1.95 \pm 0.08) \cdot 10^{-2}$. Both values disagree with the PDG average $(1.51 \pm 0.12) \cdot 10^{-2}$, dominated by the result of arXiv:1611.09359 which uses fits to essentially the same data.

→ The fitted ω mass is found to be lower than the PDG average obtained from 3π decays by $(0.65 \pm 0.12 \pm 0.12_{\text{PDG}})$ MeV, in agreement with previous fits of the $\rho - \omega$ interference in the 2π spectrum (see e.g. arXiv:1205.2228 and arXiv:1810.00007).

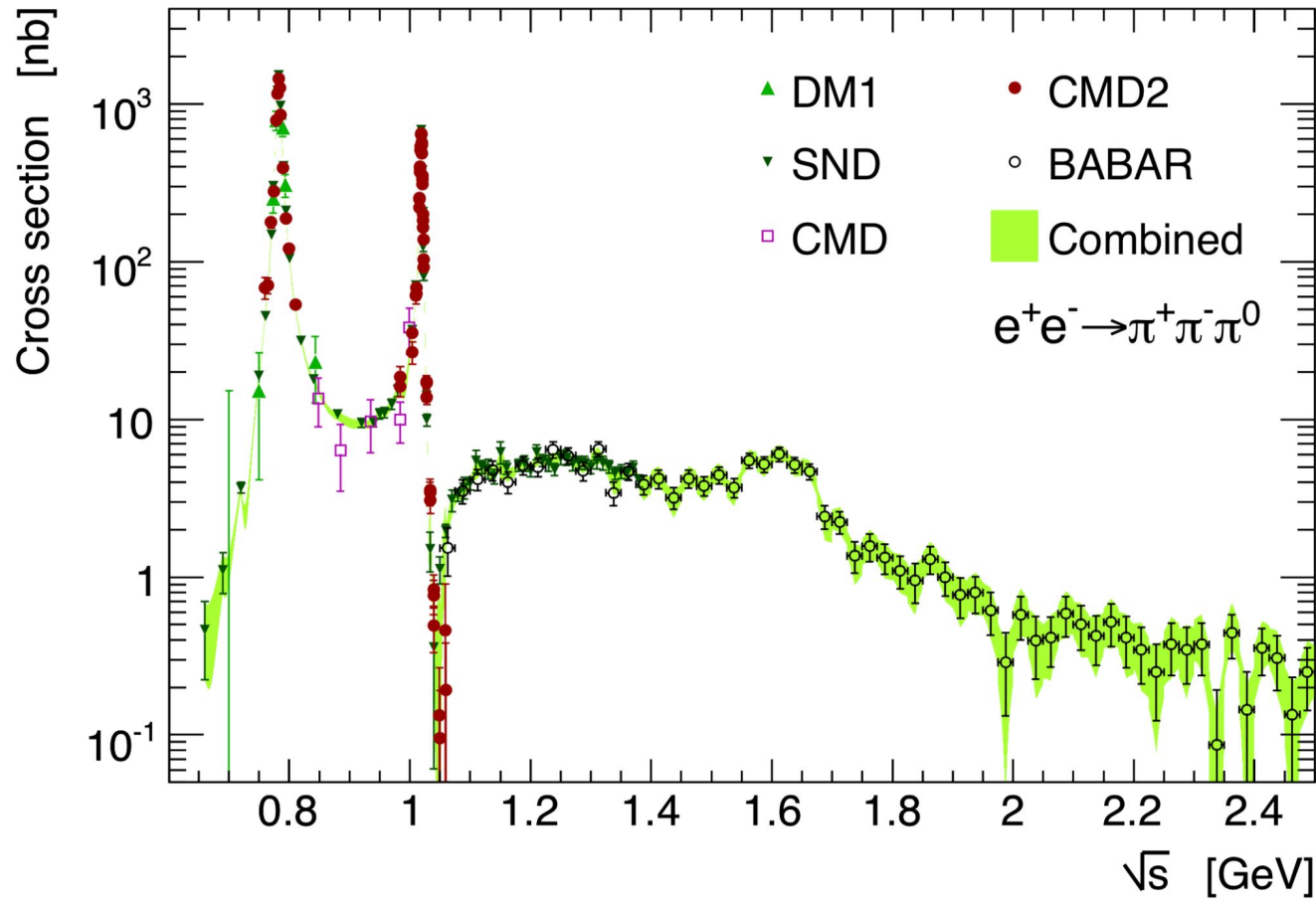
Fit performed up to 1 GeV: comparison with data



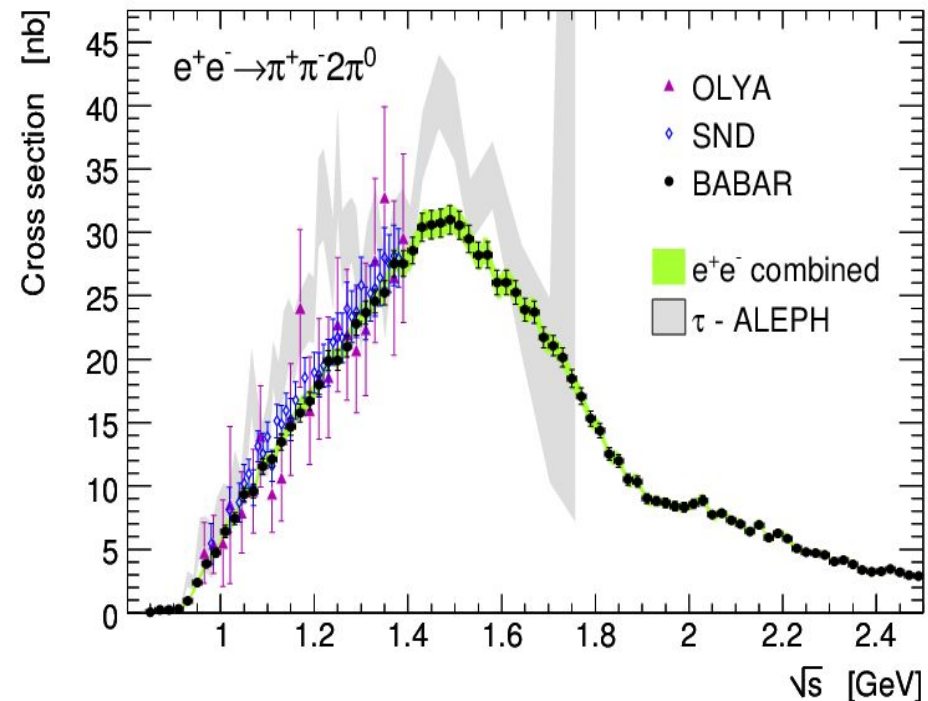
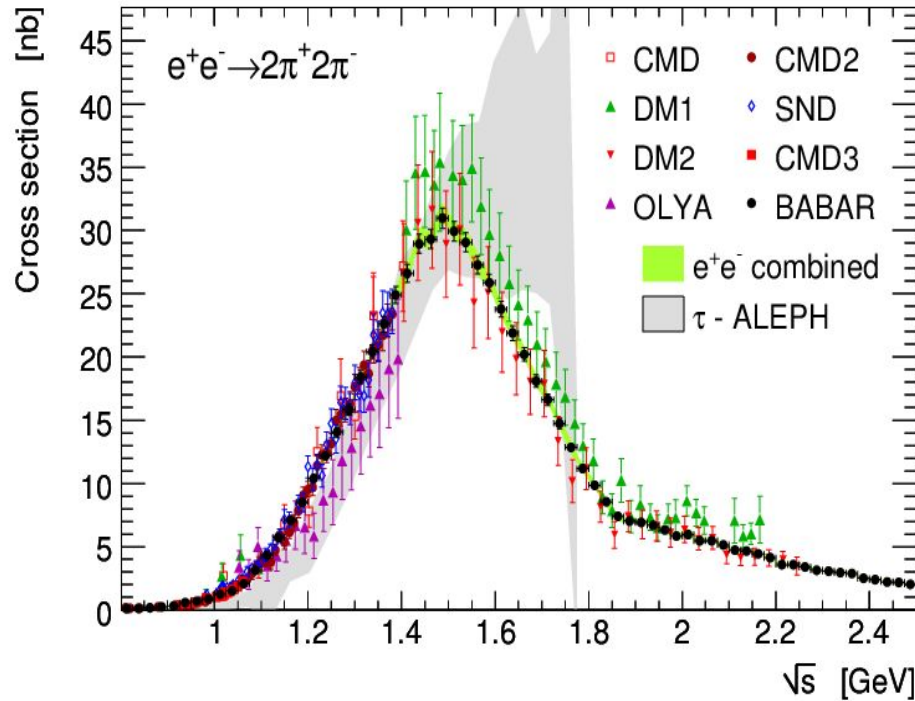
→ Fit constrained mainly by BABAR and KLOE measurements



Combination for the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ channel



$$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-, e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$$



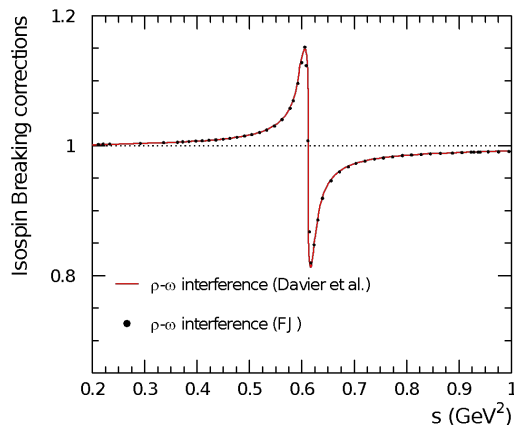
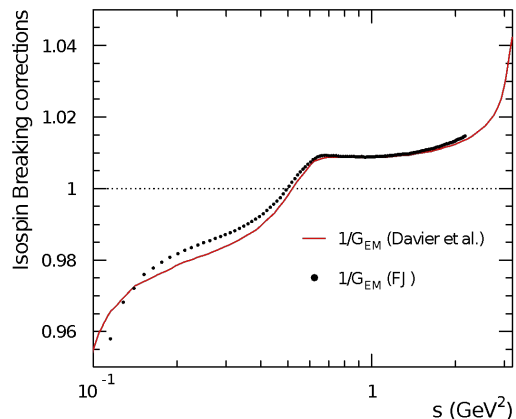
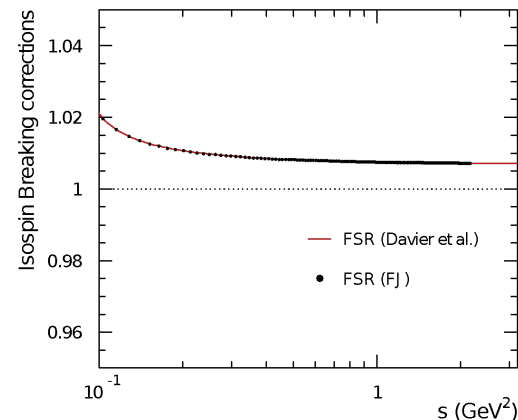
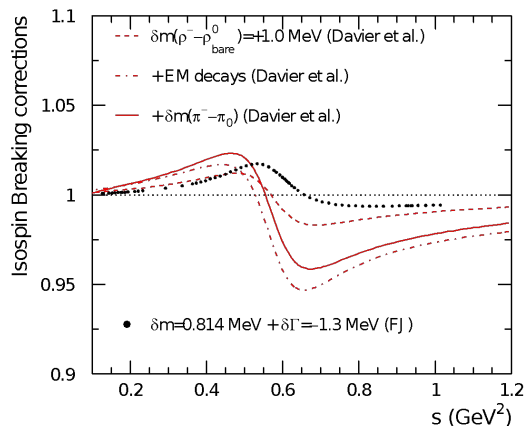
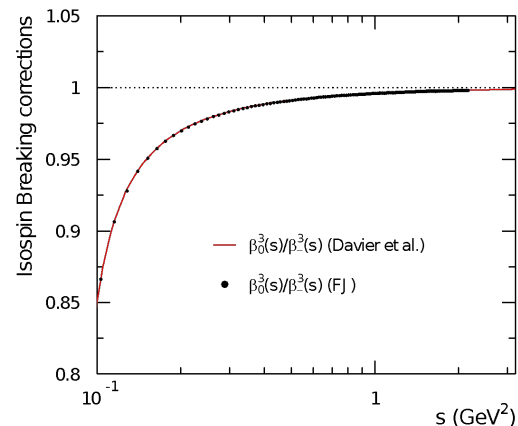
→ Essentially normalization differences w.r.t. τ data: *cross-checks very desirable*

Comparison with IB-corrected τ data

$$v_{1,X^-}(s) = \frac{m_\tau^2}{6|V_{ud}|^2} \frac{\mathcal{B}_{X^-}}{\mathcal{B}_e} \frac{1}{N_X} \frac{dN_X}{ds} \times \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)^{-1} \frac{R_{\text{IB}}(s)}{S_{\text{EW}}}$$

$$R_{\text{IB}}(s) = \frac{\text{FSR}(s) \beta_0^3(s)}{G_{\text{EM}}(s) \beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

→ Comparing corrections used by Davier et al. with the ones by F. Jegerlehner

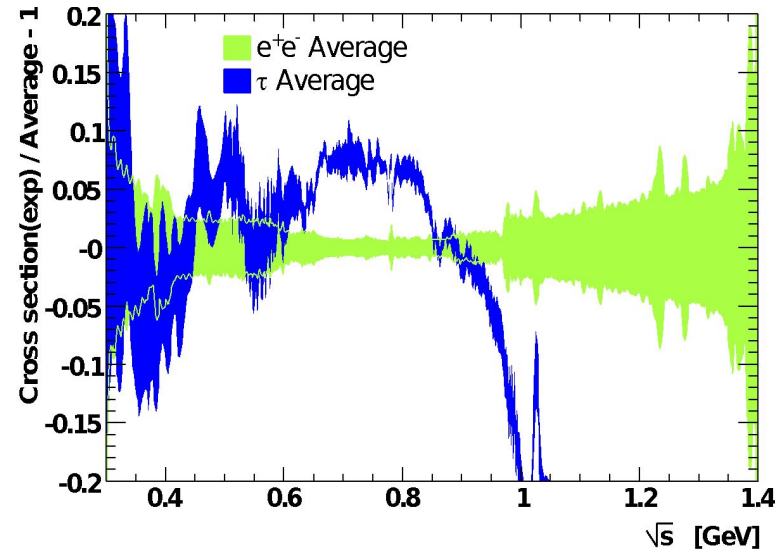
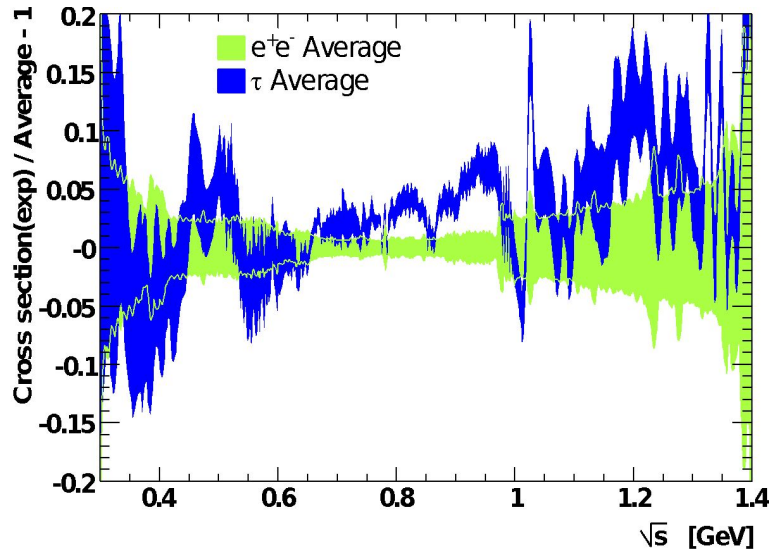
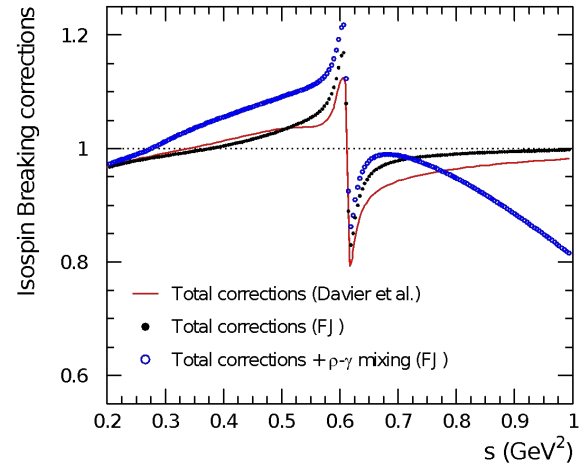


Plots by Z. Zhang, based on private communication with F. Jegerlehner

Comparison with IB-corrected τ data

→ for a μ^2 , $e^+e^- - \tau$ difference of 2.2σ
(Davier et al.)

→ the ρ - γ mixing correction proposed in
arXiv:1101.2872 (FJ) seems to over-estimate
the $e^+e^- - \tau$ difference



χ^2 definitions and properties

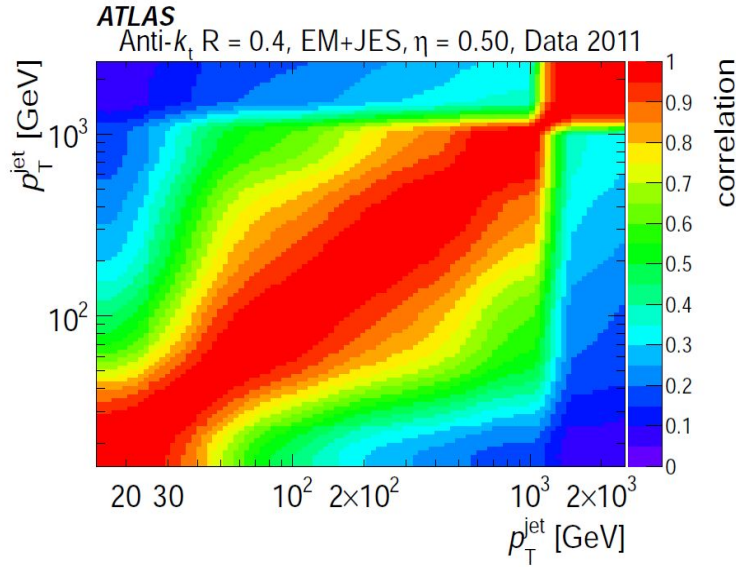
$$\chi^2(\mathbf{d}; \mathbf{t}) = \sum_{i,j} (d_i - t_i) \cdot [C^{-1}(\mathbf{t})]_{ij} \cdot (d_j - t_j) \quad C_{ij} = C_{ij}^{stat} + \sum_k s_i^k \cdot s_j^k$$

$$\chi^2(\mathbf{d}; \mathbf{t}) = \min_{\beta_a} \left\{ \sum_{i,j} \left[d_i - \left(1 + \sum_a \beta_a \cdot (\epsilon_a^\pm(\beta_a))_i \right) t_i \right] \cdot [C_{su}^{-1}(\mathbf{t})]_{ij} \cdot \left[d_j - \left(1 + \sum_a \beta_a \cdot (\epsilon_a^\pm(\beta_a))_j \right) t_j \right] + \sum_a \beta_a^2 \right\},$$

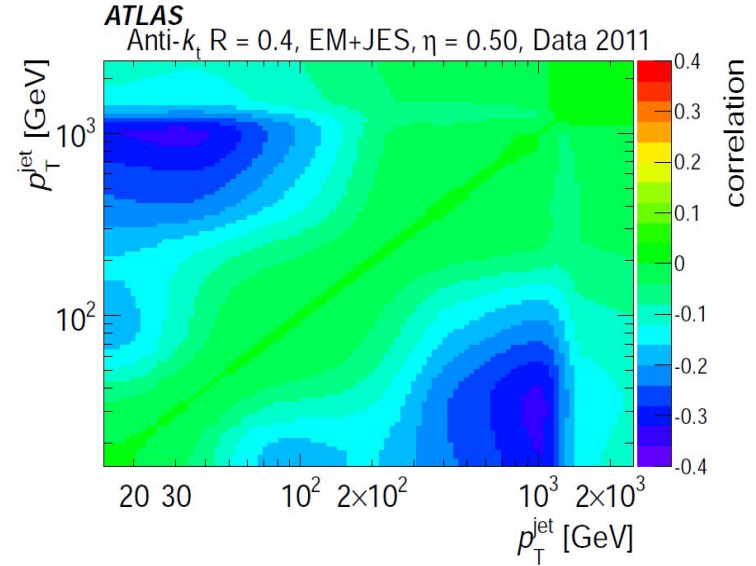
- Two χ^2 definitions, with systematic uncertainties included in covariance matrix or treated as fitted “nuisance parameters”
- Equivalent for symmetric Gaussian uncertainties
(1312.3524 - ATLAS)
- *Both approaches assume the knowledge of the amplitude, shape (phase-space dependence) and correlations of systematic uncertainties*

Example: published uncertainties on correlations

1406.0076 – ATLAS jet energy scale uncertainties



Nominal correlation scenario



Weaker - stronger correlation scenarios

Scaling factors and NP shifts

$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	Approach 0		Approach 1		
	Scaling factor	$\Delta' \alpha_{\text{had}}(M_Z^2)$	Shift NP ₁	$\sigma' (\Delta \alpha_{\text{had}}(M_Z^2))$	$\Delta' \alpha_{\text{had}}(M_Z^2)$
$a_\mu^{\text{HVP, LO}} - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ (Full HVP)	1.027	0.02826	4.6	$9.0 \cdot 10^{-5}$	0.02774
$(a_\mu^{\text{HVP, LO}} - 1\sigma) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ (Full HVP)	1.020	0.02808	3.5	$9.0 \cdot 10^{-5}$	0.02769
$a_\mu^{\text{HVP, LO}} - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1.8 GeV])	1.029	0.02769	4.7	$9.5 \cdot 10^{-5}$	0.02768
$(a_\mu^{\text{HVP, LO}} - 1\sigma) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1.8 GeV])	1.022	0.02765	3.5	$9.5 \cdot 10^{-5}$	0.02764
$a_\mu^{\text{HVP, LO}} - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1 GeV])	1.034	0.02765	-	-	-
$(a_\mu^{\text{HVP, LO}} - 1\sigma) - a_\mu^{\text{HVP, LO}}(\text{Pheno})$ ([Th.; 1 GeV])	1.026	0.02762	-	-	-
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM}}(\text{Pheno})$ (Full HVP)	1.037	0.02856	6.6	$9.0 \cdot 10^{-5}$	0.02782
$(a_\mu^{\text{Exp}} - 1\sigma) - a_\mu^{\text{SM}}(\text{Pheno})$ (Full HVP)	1.028	0.02831	5.0	$9.0 \cdot 10^{-5}$	0.02775
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM}}(\text{Pheno})$ ([Th.; 1.8 GeV])	1.041	0.02776	6.6	$9.5 \cdot 10^{-5}$	0.02774
$(a_\mu^{\text{Exp}} - 1\sigma) - a_\mu^{\text{SM}}(\text{Pheno})$ ([Th.; 1.8 GeV])	1.031	0.02770	5.0	$9.5 \cdot 10^{-5}$	0.02769
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM}}(\text{Pheno})$ ([Th.; 1 GeV])	1.048	0.02771	-	-	-
$(a_\mu^{\text{Exp}} - 1\sigma) - a_\mu^{\text{SM}}(\text{Pheno})$ ([Th.; 1 GeV])	1.036	0.02766	-	-	-

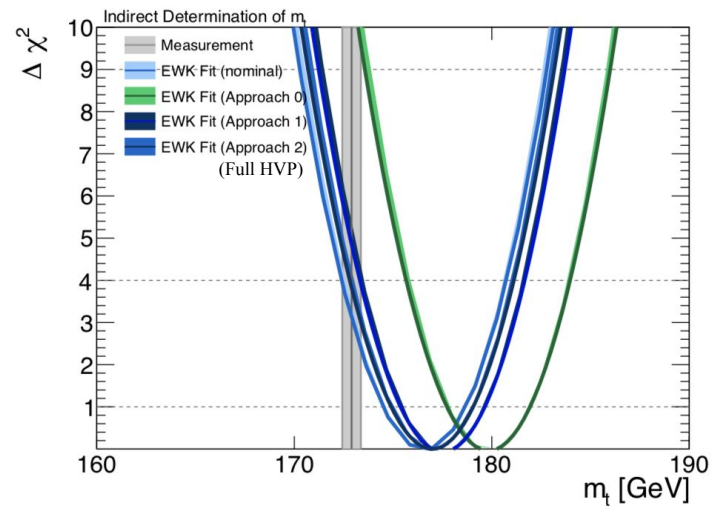
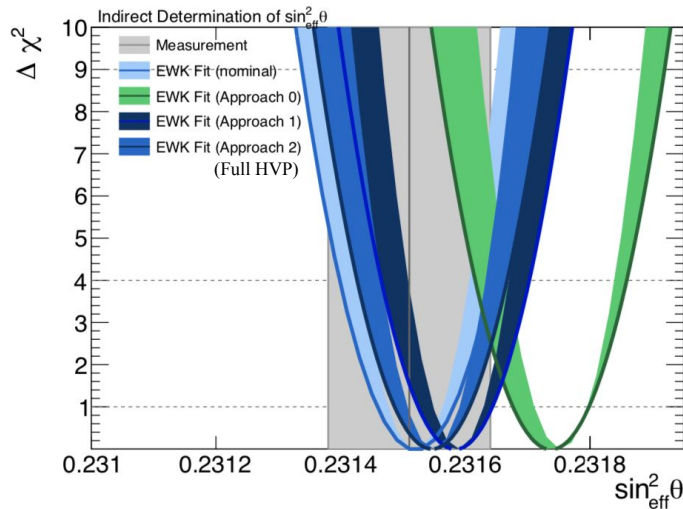
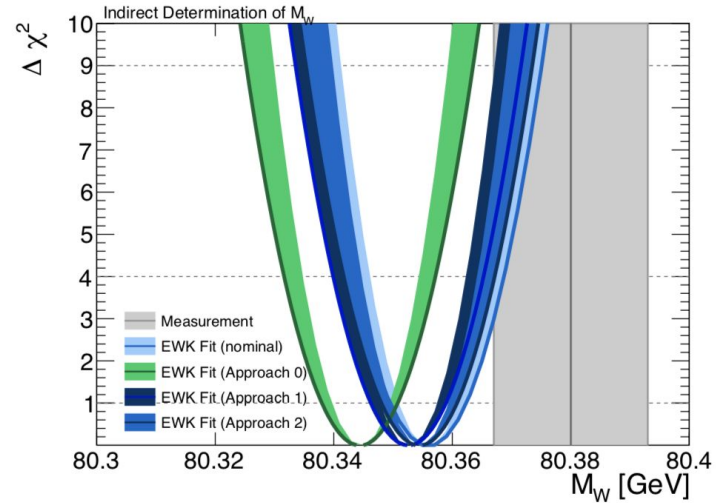
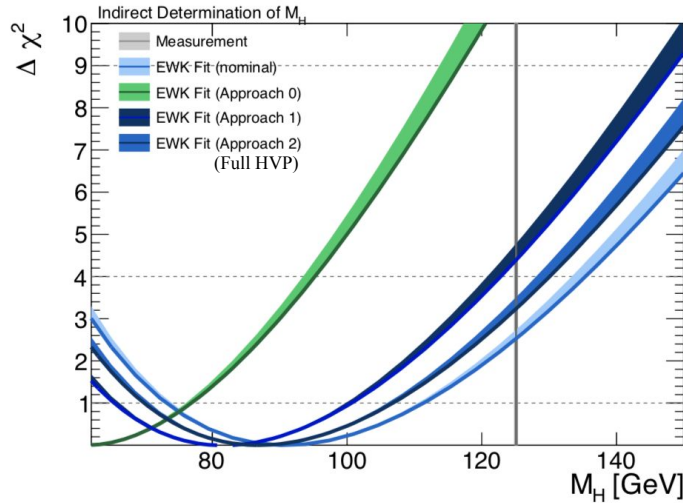
→ Large scaling factors (w.r.t. uncertainties) & significant shifts of NP₁

EW fit inputs and χ^2 results

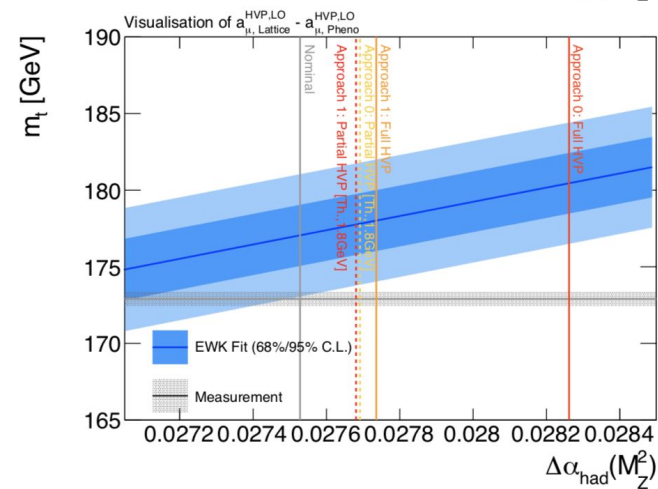
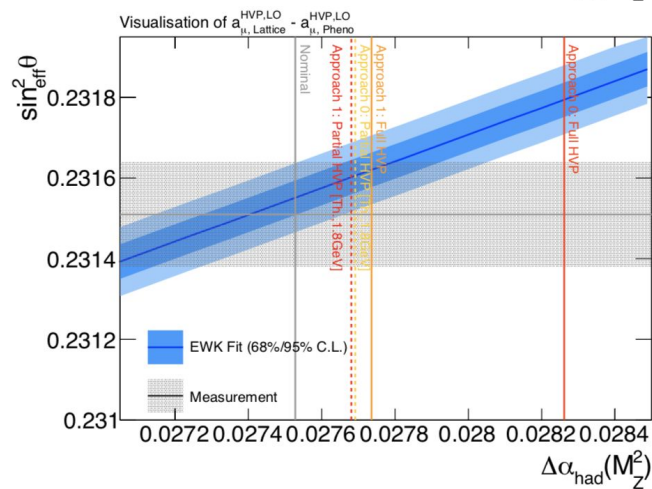
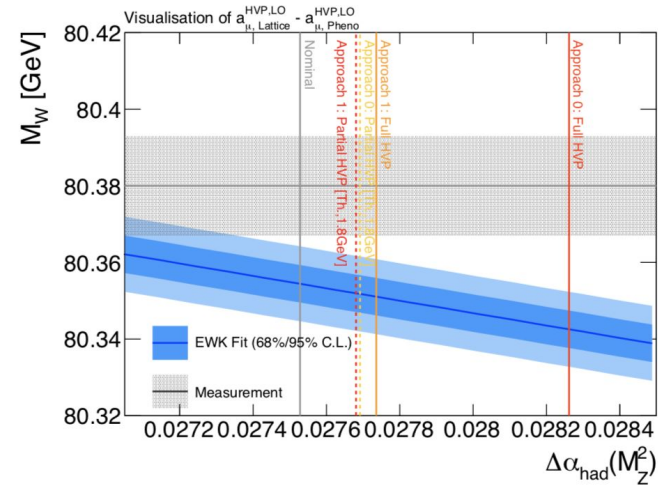
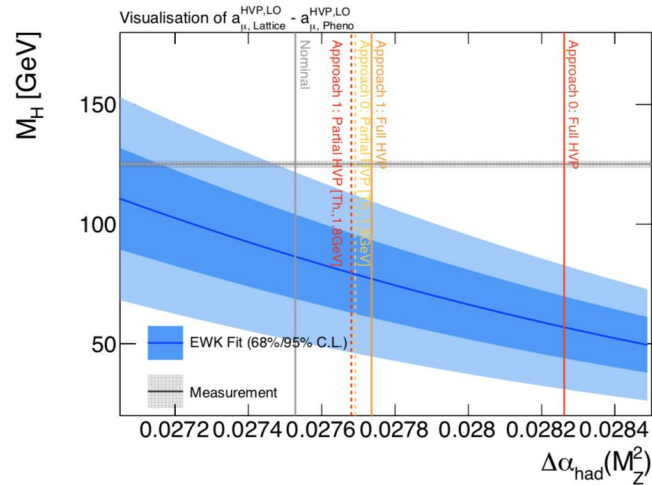
LEP/LHC/Tevatron					
M_Z [GeV]	91.188 ± 0.002	R_c^0	0.1721 ± 0.003	M_H [GeV]	125.09 ± 0.15
σ_{had}^0 [nb]	41.54 ± 0.037	R_b^0	0.21629 ± 0.00066	M_W [GeV]	80.380 ± 0.013
Γ_Z [GeV]	2.495 ± 0.002	A_c	0.67 ± 0.027	m_t [GeV]	172.9 ± 0.5
A_l (SLD)	0.1513 ± 0.00207	A_l (LEP)	0.1465 ± 0.0033	$\sin^2 \theta_{\text{eff}}^l$	0.2314 ± 0.00023
A_{FB}^l	0.0171 ± 0.001	m_c [GeV]	$1.27_{-0.11}^{+0.07}$ GeV	After HL-LHC	
A_{FB}^c	0.0707 ± 0.0035	m_b [GeV]	$4.20_{-0.07}^{+0.17}$ GeV	M_W [GeV]	80.380 ± 0.008
A_{FB}^b	0.0992 ± 0.0016	$\alpha_s(M_Z)$	0.1198 ± 0.003	$\sin^2 \theta_{\text{eff}}^l$	0.2314 ± 0.00012
R_l^0	20.767 ± 0.025	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ [10^{-5}]	2760 ± 9	m_t [GeV]	172.9 ± 0.3

$a_\mu^{\text{HVP, LO}}$ shift (Energy range)	Nominal		Approach 0		Approach 1		Approach 2	
	$\Delta' \alpha_{\text{had}}(M_Z^2)$	χ^2/ndf	$\Delta' \alpha_{\text{had}}(M_Z^2)$	χ^2/ndf	$\Delta' \alpha_{\text{had}}(M_Z^2)$	χ^2/ndf	$\Delta' \alpha_{\text{had}}(M_Z^2)$	χ^2/ndf
	0.02753	18.6/16 (p=0.29)	-	-	-	-	0.02753	28.1/17 (p=0.04)
$a_\mu^{\text{HVP, LO}} - a_\mu^{\text{HVP, LO}}$ (Lattice) (Pheno) (Full HVP)	-	-	0.02826	27.6/16 (p=0.04)	0.02774	20.3/16 (p=0.21)	-	$\chi^2(\text{BMW20-Pheno}): 9.3$
$a_\mu^{\text{HVP, LO}} - a_\mu^{\text{HVP, LO}}$ (Lattice) (Pheno) ([Th.; 1.8 GeV])	-	-	0.02769	19.9/16 (p=0.22)	0.02768	19.8/16 (p=0.23)	-	-
$a_\mu^{\text{HVP, LO}} - a_\mu^{\text{HVP, LO}}$ (Lattice) (Pheno) ([Th.; 1.0 GeV])	-	-	0.02765	19.6/16 (p=0.24)	-	-	-	-
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM}}$ (Pheno) (Full HVP)	-	-	0.02856	33.6/16 (p=0.01)	0.02782	21.2/16 (p=0.17)	-	-
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM}}$ (Pheno) ([Th.; 1.8 GeV])	-	-	0.02776	20.6/16 (p=0.19)	0.02774	20.4/16 (p=0.20)	-	-
$a_\mu^{\text{Exp}} - a_\mu^{\text{SM}}$ (Pheno) ([Th.; 1.0 GeV])	-	-	0.02771	20.1/16 (p=0.22)	-	-	-	-

EW fit results: χ^2 scans



EW fit results: parameter scans for varying $\Delta\alpha_{\text{had}}(M_Z^2)$



EW fit results: indirect determination of $\Delta\alpha_{\text{had}}(M_Z^2)$

