A critical assessment of $\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(M_Z^2)$ and the prospects for improvements.

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MiniWorkshop: parametric uncertainties: α_{em},
Thursday, Jul 14, 2022,
subgroup of ECFA e^+e^- Higgs/EW/Top Factory working group WG1

Outline of Talk:

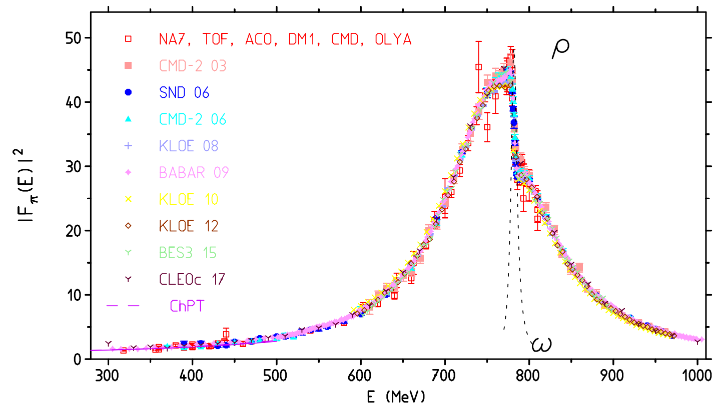
- 1. Reducing uncertainties via the Euclidean split trick: exploiting Adler function controlled pQCD
- 2. $\Delta\alpha^{\rm had}_{\rm QED}(M_Z^2)$ issues
- 3. Prospects for future improvements
- 4. Need for space-like $\alpha_{\rm QED,eff}(t)$
- 5. Conclusions

An update of
"$\alpha_{\rm QED,\,eff}(s)$ for precision physics at the FCC-ee/ILC"
Talk at 11th FCC-ee workshop, 8-11 January 2019, CERN Geneva.
Report CERN-TH-2019-061, A. Blondel et al., arXiv:1905.05078

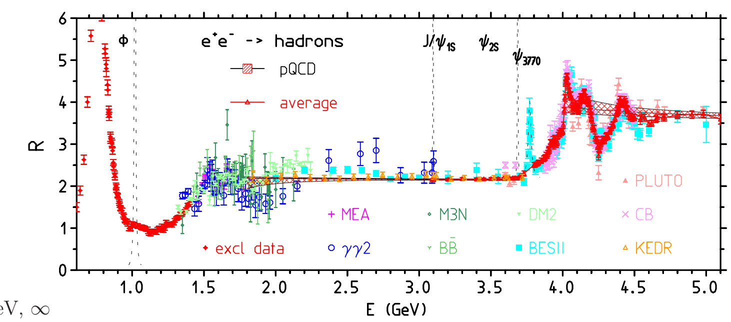
❐ R-data based dispersive $\alpha(M_Z^2)$ determination

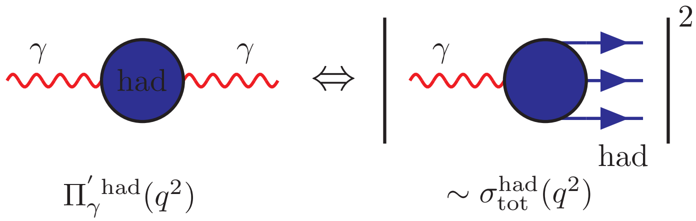
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)$ **can be evaluated in terms of** $\sigma(e^+e^- \to \text{hadrons})$ **data via dispersion integral:**

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\oint\!\!\!\!\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \oint\!\!\!\!\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

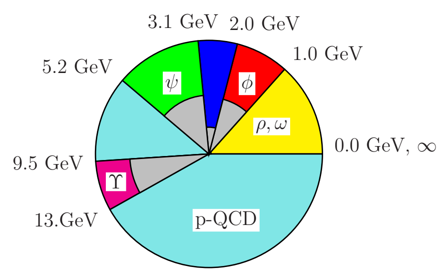


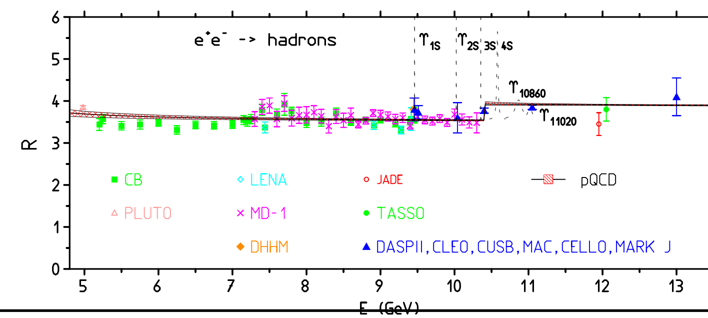
where $R_\gamma(s) \equiv \dfrac{\sigma^{(0)}(e^+e^-\to\gamma^*\to\text{hadrons})}{\frac{4\pi\alpha^2}{3s}}$





hadronic vacuum polarization





$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)$$

Present situation: (after KLOE, BaBar and BESIII results)

$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)$	=	0.027756 ± 0.000157	
		0.027563 ± 0.000120	Adler
$\alpha^{-1}(M_Z^2)$	=	128.916 ± 0.022	
		128.953 ± 0.016	Adler

Talks Bogdan Malaescu, Alex Keshavarzi

Possible complementary improvements:

- direct dispersion integral requires reducing error of $R(s)$ to 1% up to above Υ resonances
  (likely nobody will do that)

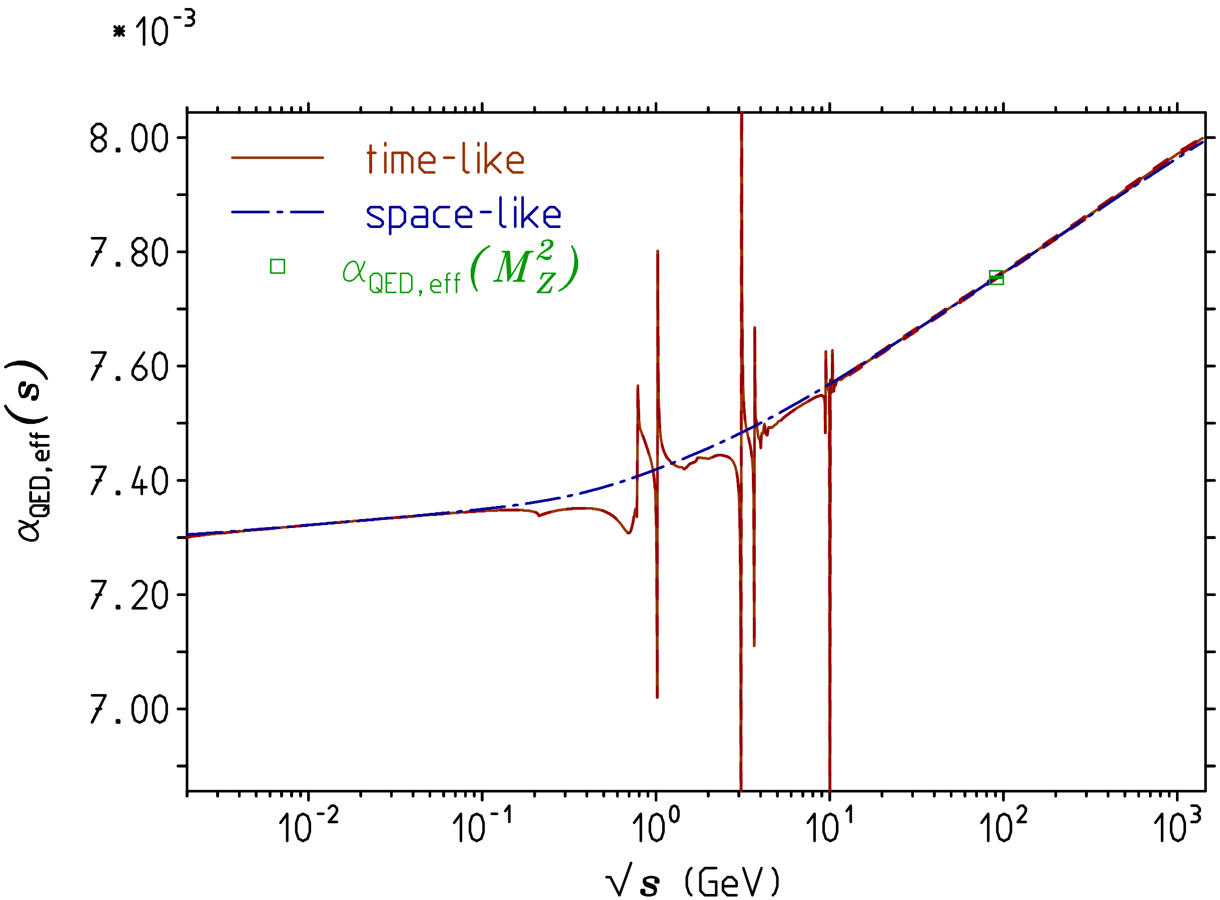
- Euclidean split method (Adler) requires
  - ❏ improvement of 1 to 2 GeV exclusive region (NSK,Belle II can top what BaBar has achieved)
  - ❏ improved pQCD Adler function massive 4-loop, better parameters m_c and m_b besides α_s

  (profiting from ongoing activities, ECFA e^+e^- Higgs/EW/Top Factory project strong motivation)

- direct $\alpha_{\rm QED}(M_Z^2)$ determination via forward backward asymmetry in $e^+e^- \to \mu^+\mu^-$ at and off the Z-resonance

Talk Patrick Janot

$\alpha_{\mathrm{QED,eff}}$: time-like vs. space-like



$\alpha_{\mathrm{QED,eff}}$ duality: $\alpha_{\mathrm{QED,eff}}(s)$ is varying dramatically near resonances, but agrees quite well in average with space-like version. Locally ill-defined near OZI suppressed meson decays: $J/\psi, \psi_1, \Upsilon_{1,2,3}$! Dyson series not convergent.

2. Reducing uncertainties via the Euclidean split trick: Adler function controlled pQCD

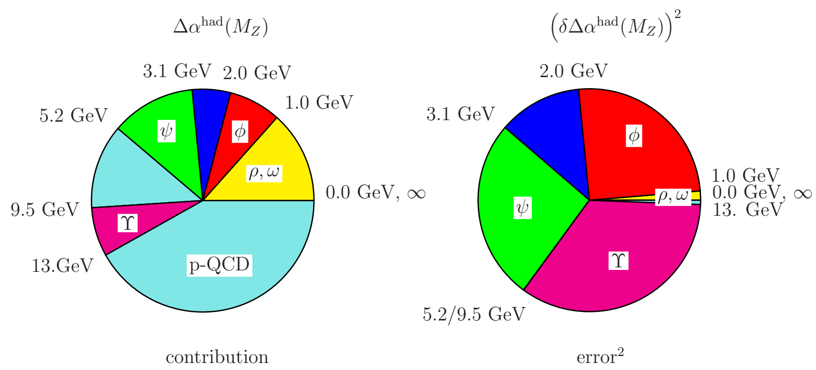
- data side: more precise measurements of $R(s)$ and $\Pi'_\gamma(-s)$ (LQCD, MUonE)
- theory side: $\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach

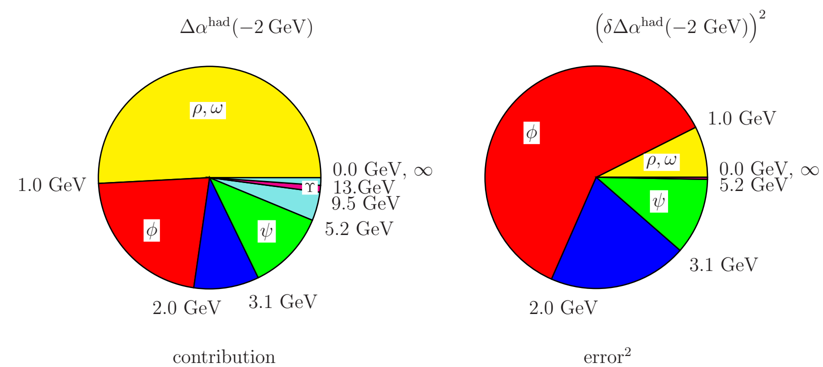
$$\alpha(M_Z^2) \quad = \quad \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$$

↗ data ↗ pQCD Adler ↗ pQCD HVP

- the space-like $-s_0$ is chosen such that pQCD is well under control for $-s < -s_0$; offset $\alpha^{\rm data}(-s_0)$ integrated $R(s)$ data or measured $\Pi'_\gamma(-s)$
- the Adler function is i) the monitor to control the applicability of pQCD and ii) pQCD part $\left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD}$ by integrated Adler function $D(Q^2)$
- small remainder $\left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$ by calculation of VP function $\Pi'_\gamma(s)$; non-perturbative part essentially cancels out.

Profiles contributions and errors $\Delta\alpha^{\rm had\,(5)}(M_Z)$ vs. $\Delta\alpha^{\rm had\,(5)}(-2\ \mathrm{GeV})$





Note the very different profile between $\Delta\alpha^{\rm had\,(5)}(M_Z)$ and $\Delta\alpha^{\rm had\,(5)}(-2\ \mathrm{GeV})$!

Ongoing projects attempting to scrutinize $a_\mu^{\rm had}$ improve $\Delta\alpha^{\rm had\,(5)}(-2\ \mathrm{GeV})$!

❒ $\Delta\alpha^{\rm had}$ Adler function controlled

✓ use old idea: Adler function: Monitor for comparing theory and data

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

$$\Rightarrow \quad D(Q^2) = Q^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} {\rm d}s\, \frac{R(s)^{\rm data}}{(s+Q^2)^2} + \int_{E^2_{\rm cut}}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2}\, {\rm d}s \right) .$$

pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory precisely	smooth simple function in Euclidean region

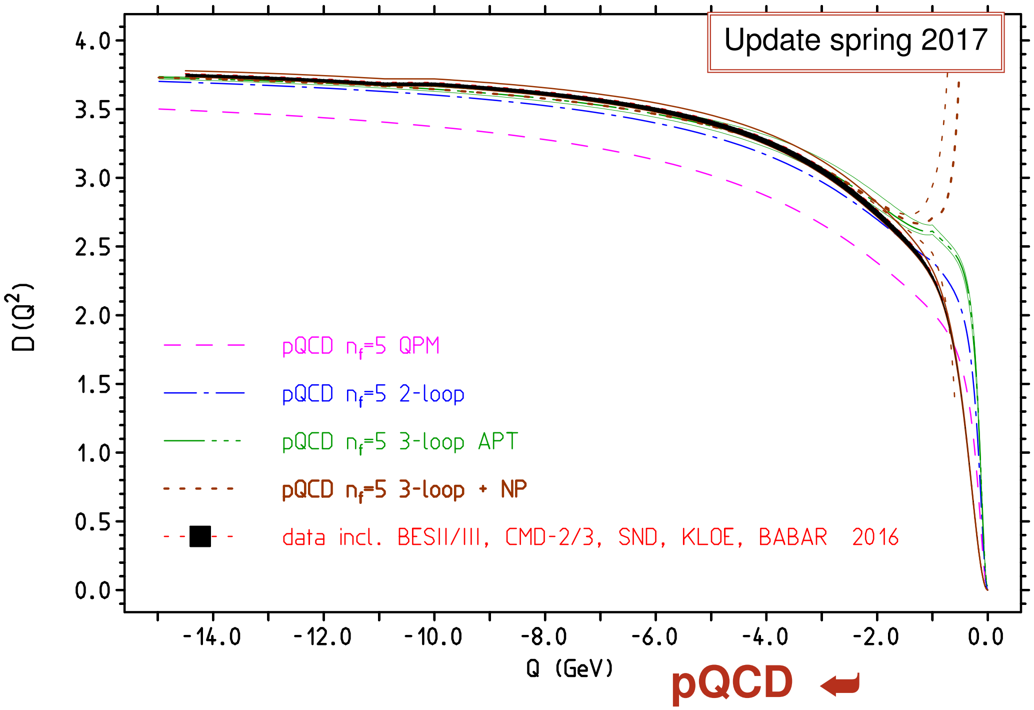
Conclusion:

❖time-like approach: pQCD works well in “perturbative windows”
3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞ Kühn,Harlander,Steinhauser

❖space-like approach: pQCD works well for $\sqrt{Q^2=-q^2} > 2.0$ GeV (see plot)

“Experimental” Adler–function versus theory (pQCD + NP)

Error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only)!



(Eidelman, F. J., Kataev, Veretin 98, FJ 08/17 updates)

theory based on results by Chetyrkin, Kühn et al.

Why to utilize Adler-function monitoring?

- **time-like region: pQCD fails in resonance regions which account for a large part of HVP**
- **space-like regime: comparing smooth monotonically increasing functions Adler 1973**

Euclidean split trick: non-perturbative part more strongly correlated to HVP part of Muon anomaly a_μ and LQCD $\Pi(Q^2)$

$\Rightarrow$ **pQCD work well monitored to predict $D(Q^2)$ down to $s_0 = (2.0\,\mathbf{GeV})^2$; use this to calculate**

$$\left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} = \frac{\alpha}{3\pi}\int_{s_0}^{M_Z^2} {\rm d}Q'^2 \frac{D^{\rm pQCD}(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.0\,\mathbf{GeV})^2$: **(FJ 98/17)**

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} &= 0.006409 \pm 0.000063 \\
\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) &= 0.027483 \pm 0.000118 \\
\Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027523 \pm 0.000119
\end{aligned}$$

❖**shift** $+0.000008$ **from the 5-loop** α_s **contribution**

❖**error** ± 0.000100 **added in quadrature form perturbative part**

QCD parameters:

- $\alpha_s(M_Z) = 0.1189(20)$**,**
- $m_c(m_c) = 1.286(13)$ $[M_c = 1.666(17)]$ **GeV** , $m_b(m_c) = 4.164(25)$ $[M_b = 4.800(29)]$ **GeV**

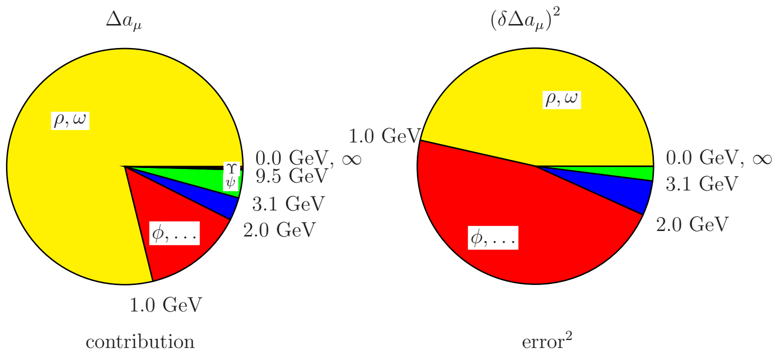
- **based on a complete 3–loop massive QCD analysis Kühn et al 2007**
  4–loop Padés available Maier&Marquard 2017

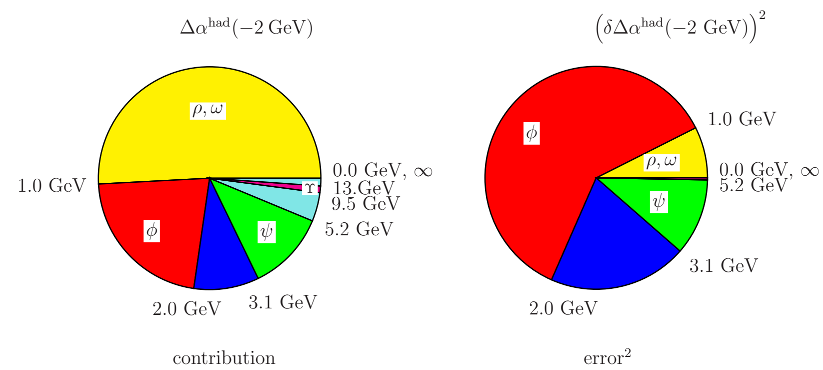
see also F. J., Nucl. Phys. Proc. Suppl. **181-182** (2008) 135

Note: the Adler function monitored Euclidean data vs pQCD split approach
is only moderately more pQCD-driven,
than the time-like approach adopted by Davier et al. and others.

All efforts to improve HVP in Muon $g-2$ also has substantial impact on $\alpha^{\mathrm{had}}_{\mathrm{QED}}(-s_0)$ and hence on $\alpha_{\mathrm{QED}}(M_Z^2)$!

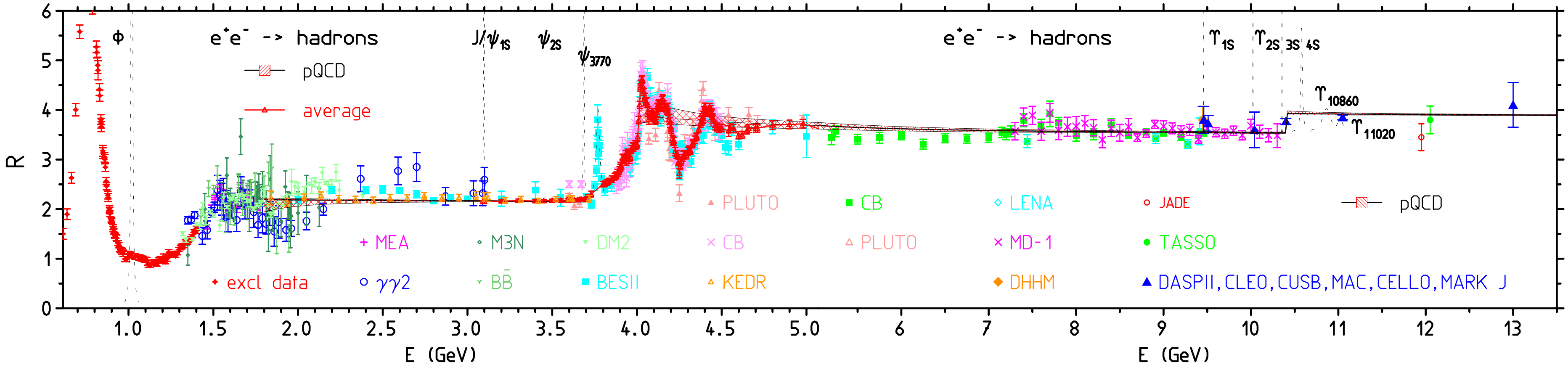
Correlation between different contributions to $a_\mu^{\rm had}$ and $\Delta\alpha^{\rm had\,(5)}(-2\ \text{GeV})$



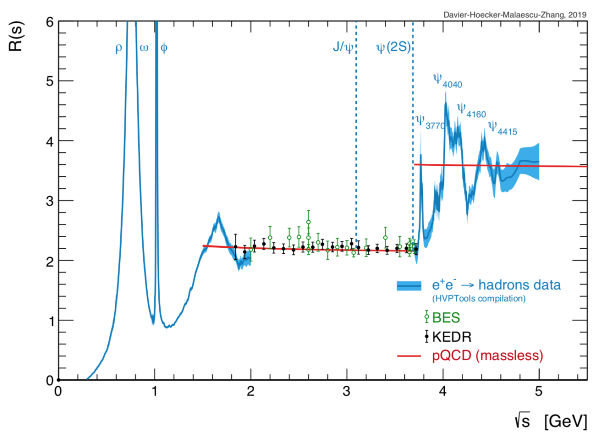


Ongoing Muon HVP progress affects closely $\Delta\alpha^{\rm had\,(5)}(-2\ \text{GeV})$!

2. R-data issues



The compilation of $R(s)$–data utilized.



$R(s)$ compilation by Davier et al.

Data time-like $R(s)$-type :

- ❒ Scan of $e^+e^- \to$ hadrons; NSK, BESII, KEDR, $\cdots$
- ❒ ISR (radiative return at meson factories) $e^+e^- \to \gamma +$ hadrons; KLOE ,BaBar, BESIII
- ❒ τ-decay spectra $\tau \to \nu_\tau +$ hadrons; ALEPH, CLEO, Belle

Alternative methods: space-like HVP $\Pi'_\gamma(t=-s)$

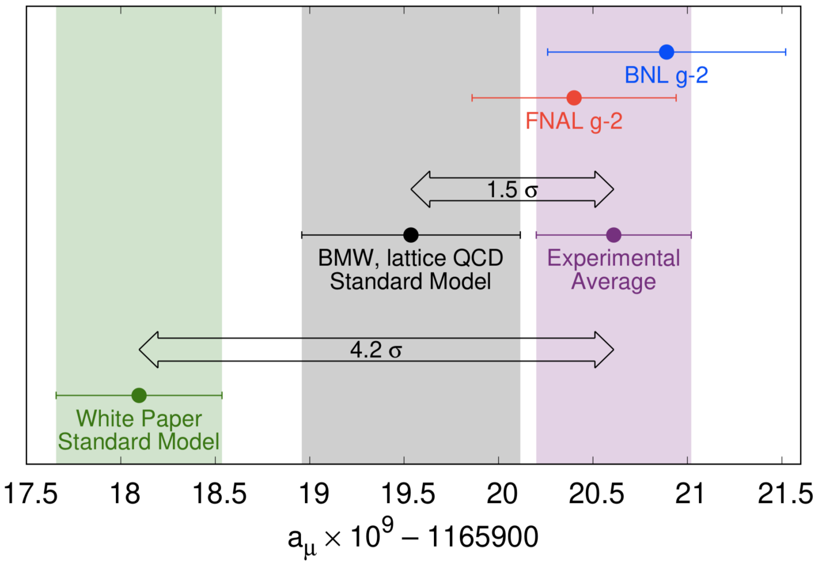
- ❒ Lattice QCD
- ❒ Dedicated MUonE: elastic $\mu + e^- \to \mu + e^-$; $\sigma \propto \alpha^2_{\mathrm{em}}(t)$ direct

Data related issues not understood and clarified:

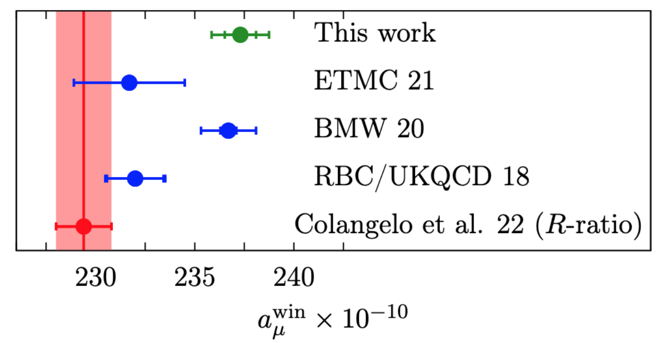
- ❑ dispersive approach vs. lattice QCD results
- ❑ relation between NC e^+e^- and CC τ-decay spectra
- ❑ exclusive measurements vs. inclusive measurements in 1.4 to 2.2 GeV region

HVP in Muon $g-2$

New LQCD result $a_\mu^{\rm win} = 237.30(1.46) \times 10^{-10}$ from Mainz confirms BMW result!



Talk Harvey Meyer



Muon g-2 plot incl. lattice QCD results. Left: BMW, right: Mainz plots

White paper HVP gives 4.2 σ;
BMW lattice HVP gives 1.6 σ;
Mainz lattice HVP 1.6 σ;
RBC/UKQCD 2.1 σ;

$$a_\mu^{\rm HVP-LO}[{\rm THE}] = 693.1(4.0) \times 10^{-10}$$
$$a_\mu^{\rm HVP-LO}[{\rm BMW}] = 707.5(5.5) \times 10^{-10}$$
$$a_\mu^{\rm HVP-LO}[{\rm Mainz}] = 707.8(5.5) \times 10^{-10}$$
$$a_\mu^{\rm HVP-LO}[{\rm RBC}] = 703.2(5.5) \times 10^{-10}$$

Theory leading uncertainty: HVP (and HLbL); DR+Data vs lattice QCD

Theory: all of SM counts

$$a_\mu^{\rm SM} = (g_\mu/2-1) \;=\; a_\mu^{\rm (QED+EW+HVP_{LO}+HVP_{NLO}+HVP_{NNLO}+HLbL_{LO}+HLbL_{NLO})}$$

		QED	+HVP	+EW	
$a_\mu^{\rm the}$	=	0.00 1165	91	810	(43)
$a_\mu^{\rm exp}$	=	0.00 1165	92	061	(41)

agree to 8 digits! Would need 9 digits SM precision?

$$\delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 25.1 \pm 5.9 \times 10^{-10} = \Delta a_\mu^{\rm BSM} ???$$

Gap $\Delta a_\mu^{\rm BSM}$ is 1.6 times EW contribution! Big effect missing!

SM theory incl. τ-decay spectra:

Davier et al., arXiv:0906.5443: $a_\mu^{\rm HVP-LO}[\tau[ee]] = 705.3 \pm 4.5\,[689.8 \pm 5.2] \times 10^{-10}$

Miranda&Roig, arXiv:2007.11019: $a_\mu^{\rm HVP-LO}[\tau] = 705.7^{+4.0}_{-4.1} \times 10^{-10}$

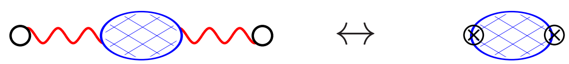
$\Leftrightarrow$ $a_\mu^{\rm had}[{\rm BMW, Mainz}] = 707.5 \pm 5.5 \times 10^{-10}$

If “BSM = missing HVP” need $a_\mu^{\rm HVP-LO} = 718.2 \times 10^{-10}$

The new puzzle:

$\delta a_\mu^{\rm HVP-LO}[\rm LQCD - DR] = 14.4(6.8) \times 10^{-10}$ vs $a_\mu^{\rm EW} = 15.36 \pm 0.11 \times 10^{-10}$

Commonly forgotten: mixing of ρ^0, ω, ϕ with the photon [$\rho^0 - \gamma$ mixing] i.e. effect concerning relation



e^+e^- measurement $\Leftrightarrow$ LQCD calculation

- how to disentangle QED from QCD in e^+e^--data ?
- $\rho^0 - \gamma$ absent in CC $\tau \to \nu_\tau \pi\pi$ data,
  but QED-QCD interference part incl. in $e^+e^- \to \pi^+\pi^-$ data,
- for getting had blob in e^+e^- the $\gamma - \rho^0$ mixing has to be removed!

- for the I=1 part of $a_\mu^{\rm had}[\pi\pi]$ results in

$$\delta a_\mu^{\rm had}[\rho\gamma] \simeq (5.1 \pm 0.5) \times 10^{-10} \,,$$

as a correction applied for the range [0.63,0.96] GeV. The correction is not too large, but at the level of $1\,\sigma$ and thus non-negligible. Davier et al. got $\delta a_\mu^{\rm had}[\tau - ee] \simeq 15.5 \pm 6.9 \times 10^{-10}$ later reduced to $9.2 \pm 6.3 \times 10^{-10}$ a factor 2 larger, what data tell us, moves in the right direction!

To be clarified by QED supplemented LQCD!

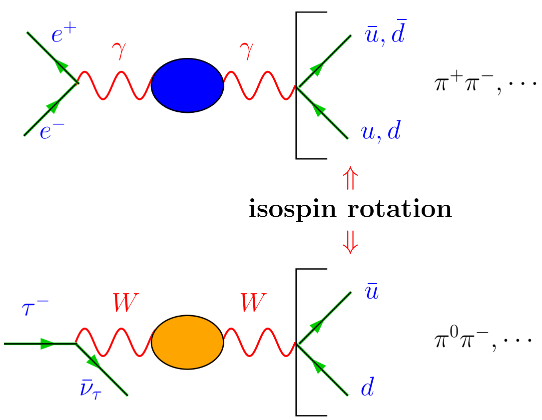
Need to compute photon-propagator to compare with current-correlator!

What is needed in hadronic blob expansion of a_μ is the current-correlator (what LQCD provides) not the partially undressed photon-propagator.

The role of τ decay spectra

What about τ-decay spectra ALEPH, CLEO, Belle: completely different set-up. Not mediated via photon propagator $\rho^\pm$ no mixing like ρ^0 mixing with γ, ω and ϕ. Corrected from QED corrections no VP subtraction! A pure I=1 Breit-Wigner shape. Strong IB, pion masses and meson mixing (in $\pi\pi$ channel the ω) to be added. On the e^+e^- side $\rho^0 - \gamma$ mixing (a QCD-QED interference) to be removed! Additional data besides e^+e^- ones providing improvements:

❶ **τ–decay spectra:** good idea, use isospin symmetry to include existing high quality τ–data (applying isospin breaking corrections) Alemany, Davier, Höcker 1996



bare $\pi^+\pi^-$ spectrum after photon VP, ISR and FSR subtraction

CVC $\leftrightarrow$ isospin rotation symmetry

bare $\pi^0\pi^-$ spectrum after EW radiative and phase space correction

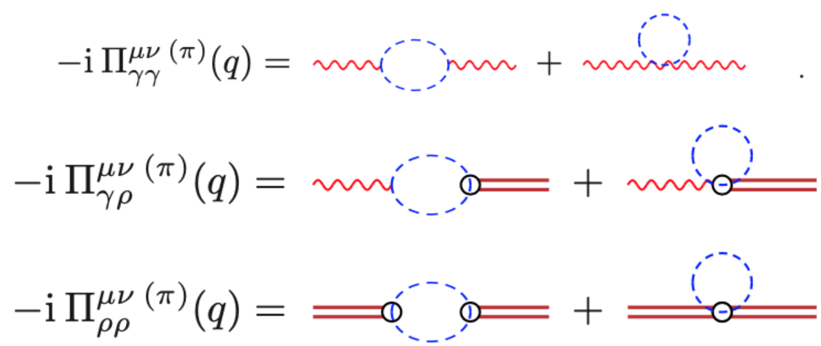
Standard IB corrected data: large discrepancy [$\sim$ 10%] persists! τ vs. e^+e^- puzzle! [manifest since 2002 Davier et al, resolved 2011 taking into account $\rho^0 - \gamma$ interference]

Why is it still a good idea to include τ CC spectra? Can shed light on $e^+e^- \to \pi^+\pi^-$ data clashes (e.g. BaBar vs KLOE)!

Taking into account $\rho - \gamma$ interference resolves τ (charged channel) vs. e^+e^- (neutral channel) puzzle, F.J.& R. Szafron [JS11], M. Benayoun et al.. However, not accepted by WP as a possible effect, which is analogous to $Z - \gamma$ interference established at LEP in the 90's.

$\rho - \gamma$ interference
(absent in charged channel)
often mimicked by large shifts
in M_ρ and Γ_ρ
ρ^0 is mixing with γ:
propagators are obtained by
inverting the symmetric 2×2
self–energy matrix

$$\hat{D}^{-1} = \begin{pmatrix} q^2 + \Pi_{\gamma\gamma}(q^2) & \Pi_{\gamma\rho}(q^2) \\ \Pi_{\gamma\rho}(q^2) & q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) \end{pmatrix}.$$



Irreducible self-energy contribution at one-loop

Taking $q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)$ term only $\Rightarrow$Gounaris-Sakurai; $\Pi_{\gamma\rho}(q^2)$ $\gamma - \rho$ interference

Effect well known from LEP Z resonance physics: $Z-\gamma$ mixing affects Z lineshape.

Self-energies: pion loops to photon-ρ vacuum polarization (VMDII+sQED)

$$\Pi_{\gamma\gamma} = \frac{e^2}{48\pi^2} f(q^2) \,, \quad \Pi_{\gamma\rho} = \frac{e g_{\rho\pi\pi}}{48\pi^2} f(q^2) \quad \text{and} \quad \Pi_{\rho\rho} = \frac{g^2_{\rho\pi\pi}}{48\pi^2} f(q^2) \,,$$

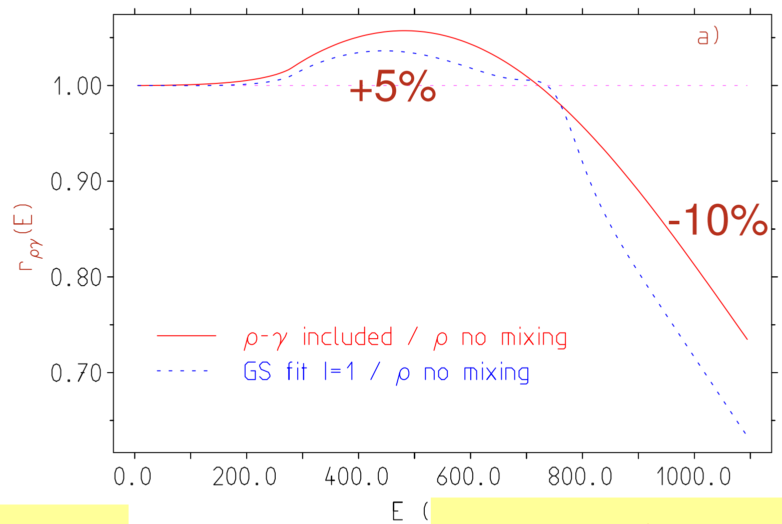
No unknown adjustable parameters: e, $g_{\rho\pi\pi}$ and $M_\rho \Rightarrow \Gamma_\rho$, mixing etc are predictions!

Usually IB correction applied to τ spectra:

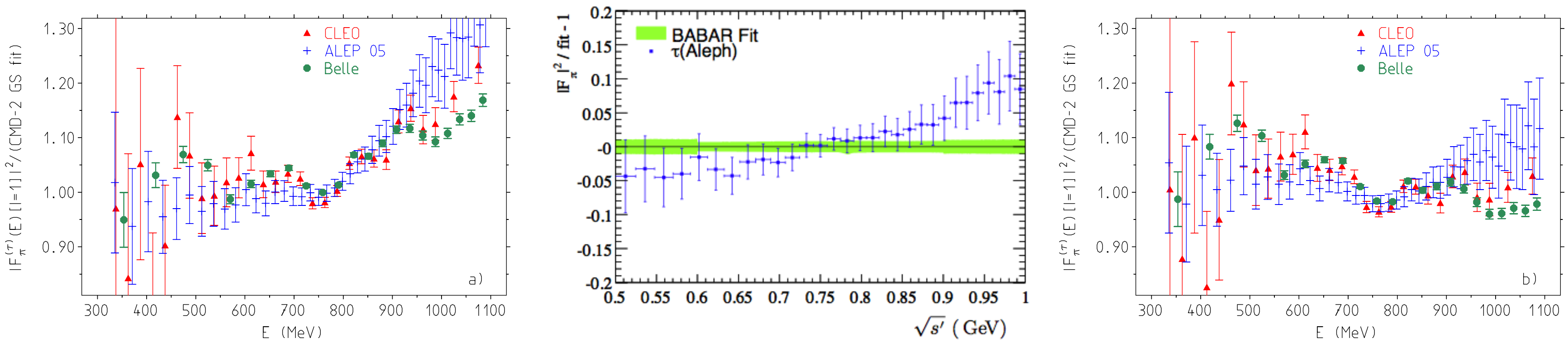
- ❒ τ require to be corrected for missing $\rho-\gamma$ mixing!
- ❒ results obtained from e^+e^- data is what goes into a_μ

In fact, lattice QCD results reveal:

- ● to get required pure QCD 1PI "blob" $\Rightarrow \rho-\gamma$ mixing should be removed from usual "bare" $R_\gamma(s)$ data



Correct implementation: $v_0(s) \Rightarrow v_h(s)$ $v_h(s) = R_{\rm IB}(s)\, v_-(s)$ from τ-spectra $v_h(s) = r^{-1}_{\rho\gamma}(s)\, v_0(s)$ from e^+e^--data



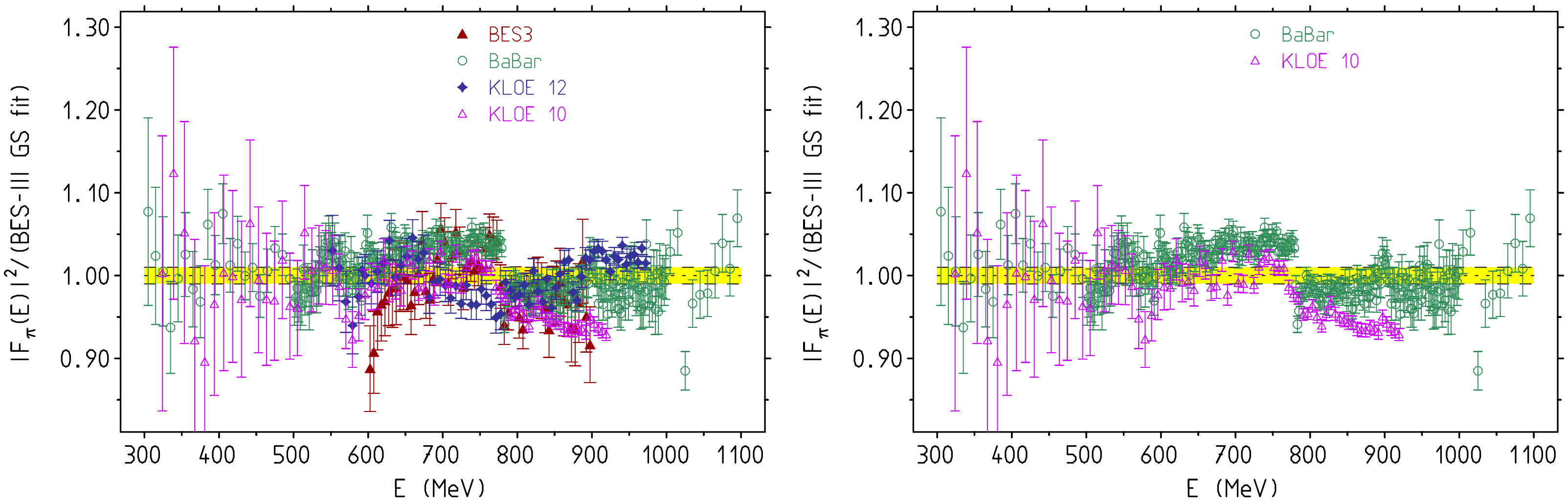
$|F_\pi(E)|^2$ in units of e^+e^- $I=1$ (CMD-2 GS fit): Left: τ data uncorrected for $\rho-\gamma$ mixing [Szafron, F.J. 11, Benayoun et al 11]. Center: ALEPH τ vs BaBar e^+e^- [Davier et al. 2009]. Right: after correcting for mixing. $\pi\pi$ [+5% −10%].

After LQCD results: apply correction in opposite direction

Shift: $\delta a_\mu^{\rm had}[\rho\gamma] \simeq (5.1 \pm 0.5) \times 10^{-10}$,
thus $693.46(3.94) \times 10^{-10} \Longrightarrow 698.56(3.97) \times 10^{-10}$! closer to LQCD (BMW&Mainz)

❐ Still unclear $\pi\pi$ below 1 GeV

Experimental input for HVP: NSK, KLOE, BaBar, BESIII, CLEO-c, VEPP-2000



Recent BES-III vs BaBar and KLOE

KLOE vs. BaBar in conflict: e.g. Resonance Lagrangian Approach global fit:

$a_\mu^{\mathrm{HVP-LO}}(KLOE) = (687.31 \pm 2.93) \times 10^{-10}$ 90% fit NSK, BESSIII,CLEO-c +KLOE

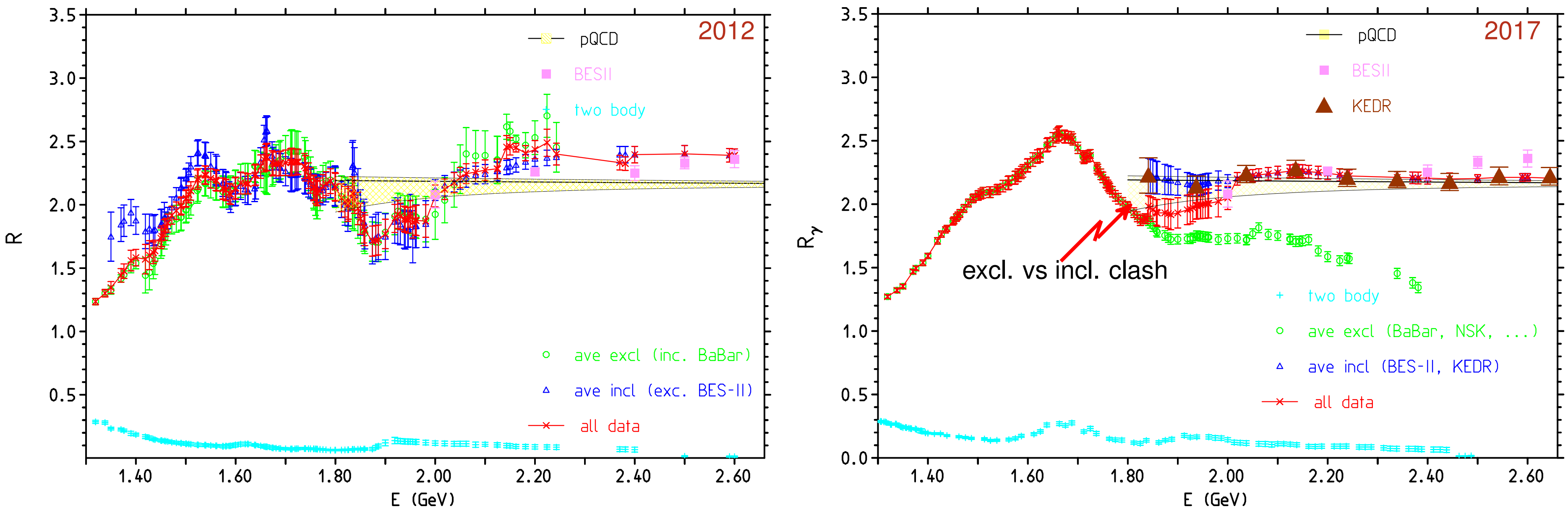
$a_\mu^{\mathrm{HVP-LO}}(BaBar) = (692.36 \pm 2.95) \times 10^{-10}$ 40% fit NSK, BESSIII,CLEO-c +BaBar

with BaBar close to WP $693.1(4.0) \times 10^{-10}$

M. Benayoun, L. Del Buono, F. J. 2021 arXiv:2105.13018 differ by $1.7\ \sigma$. Should be understood!

❒ Still an issue in HVP

❒ region 1.2 to 2 GeV data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 40% of uncertainty



● illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

3. Prospects for future improvements

Note: new muon $g-2$ experiments at Fermilab and JPARC trigger continuation of $e^+e^- \to$ hadrons cross section measurements in low energy region by VEPP 2000 at Novosibirsk, BES III Beijing, Belle II at KEK. This automatically helps improving split trick approach (Adler function controlled)

direct DR approach requires precise data up to much higher energies or heavy reliance on pQCD calculation of time-like $R(s)$!

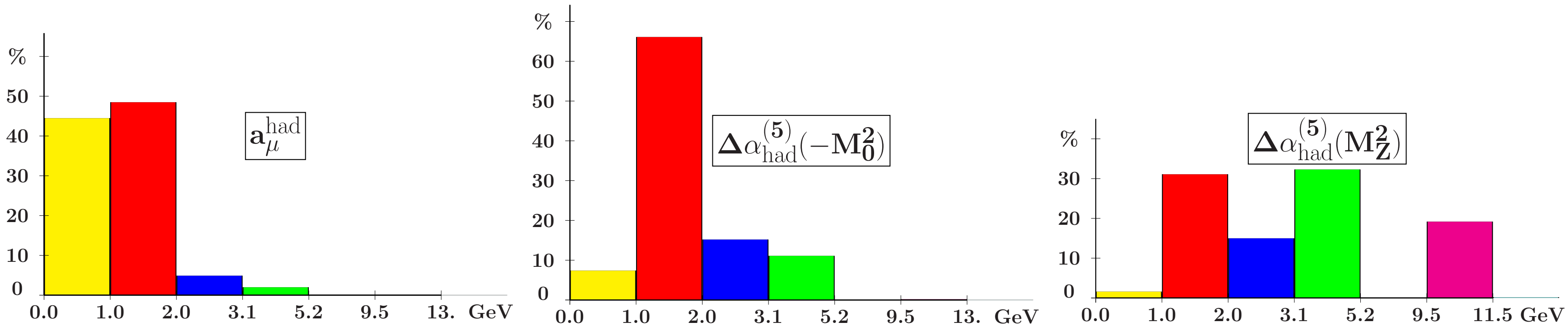
Mandatory pQCD improvements required are:

- **4–loop massive pQCD calculation of Adler function;**
  required are a number of terms in the low and high momentum series expansions which allow for the appropriate Padé improvements
  [essentially equivalent to a massive 4–loop calculation of $R(s)$];
  few moments already available Maier&Marquard 2017
- **m_c, m_b improvements by sum rule and/or lattice QCD evaluations;**
- **improved α_s in low Q^2 region above the τ mass.**

Theory: (QCD parameters) has to improve by factor 10 ! $\to \pm 0.20$

Settling the HVP issue for a_μ settles it largely for $\Delta\alpha(-M_0^2)$

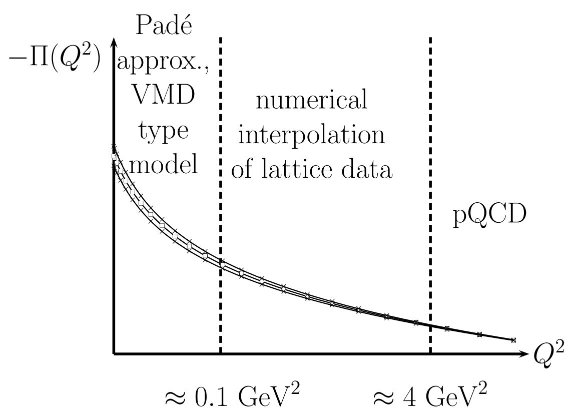
Error profiles (standard approach):

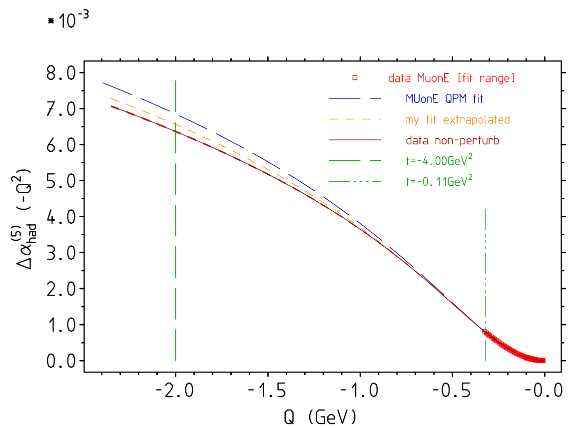


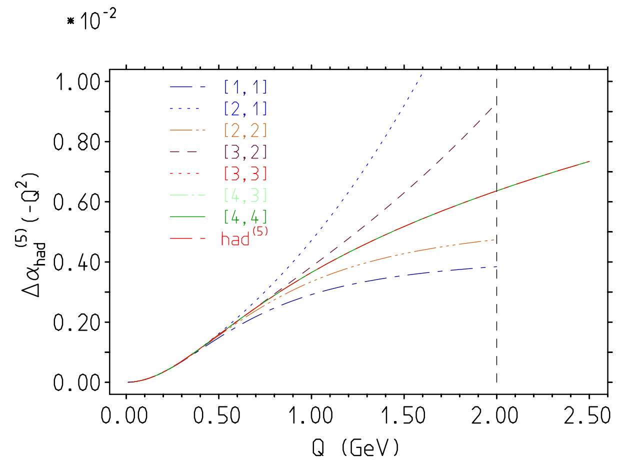
Contributions to the total error from different energy regions to the hadronic lowest order vacuum polarization contribution to a_μ, $\Delta\alpha(M_Z^2)$ and $\Delta\alpha(-M_0^2)$ for $M_0 = 2$ GeV in percent. These errors are to be added in quadrature to get the total uncertainty. The graph illustrates where experimental effort is needed in order to get a better precision.

The virtues of Adler function approach are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.0 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!**
- **$\Delta\alpha(-M_0^2)$ would be directly accessible in MUonE experiment (project) and lattice QCD.**



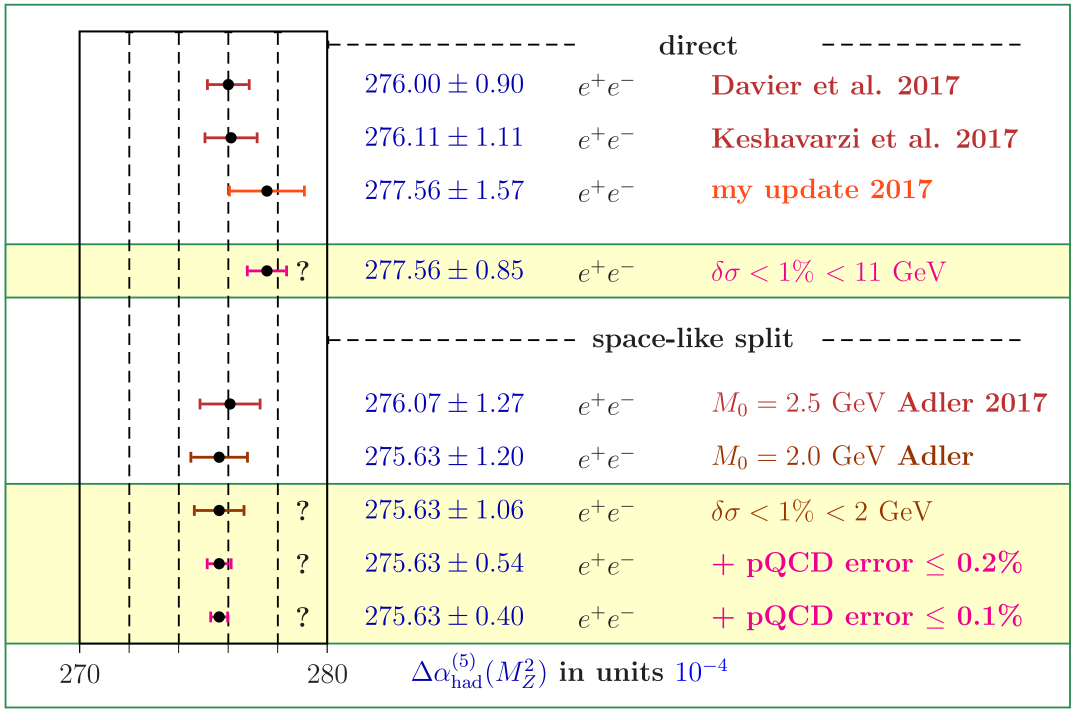




Complementarity of LQCD and MUonE

MUonE perfect at low Q^2 where LQCD only accessible by extrapolation; LQCD perfect at "intermediate" Q^2 which are accessible to MUonE by extrapolation only.

What can we achieve:



Davier et al. 2011: use pQCD above 1.8 GeV

- **no improvement by remeasuring cross sections above 1.8 GeV**
- **no proof that pQCD works at 0.04% precision as adopted**

My analysis is data driven: pQCD $5.2 - 9.5$ and > 11.5 GeV

- × **pQCD at 0.2% Adler function: pQCD error = ½ × present error**
- × **pQCD at 0.1% Adler function: pQCD error = data error ± 0.28**

Note: theory-driven standard analyses ($R(s)$ integral) using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

precision in α:	present	direct	1.7×10^{-4}
		Adler	1.2×10^{-4}
	future	Adler QCD 0.2%	5.4×10^{-5}
		Adler QCD 0.1%	3.9×10^{-5}
	future	via $A_{\rm FB}^{\mu\mu}$ off Z	3×10^{-5}

- Adler function method is competitive with Patrick Janot's direct near Z pole determination via forward backward asymmetry in $e^+e^- \to \mu^+\mu^-$

$$A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3\,a^2}{4\,v^2}\frac{\mathcal{I}}{\mathcal{Z}+\mathcal{G}}$$

where

$\gamma - Z$ interference term	$\mathcal{I} \propto \alpha(s)\,G_\mu$
Z alone	$\mathcal{Z} \propto G_\mu^2$
γ only	$\mathcal{G} \propto \alpha^2(s)$
v vector Z coupling	also depends on $\alpha(s \sim M_Z^2)$ and $\sin^2\Theta_f(s \sim M_Z^2)$
a axial Z coupling	sensitive to ρ-parameter (strong M_t dependence)

❏ using v, a as measured at Z-peak

Talk Patrick Janot

Challenges for direct measurement:

❑ **radiative corrections*** ❑ **needs dedicated off-Z peak running**

* **under way see e.g. Gluza et al. arXiv:1804.10236**

- **Adler function method is much cheaper to get, I think!**

Requirement look to be realistic:

❖ **pin down experimental errors to 1% level in all non-perturbative regions up to 2.0 GeV**

❖ **switch to Euclidean approach, monitored by the Adler function**

❖ **improve on QCD parameters, mainly on m_c and m_b**

4. Need for space-like $\alpha_{\rm QED,eff}(t)$

❐ LQCD vs data driven HVP

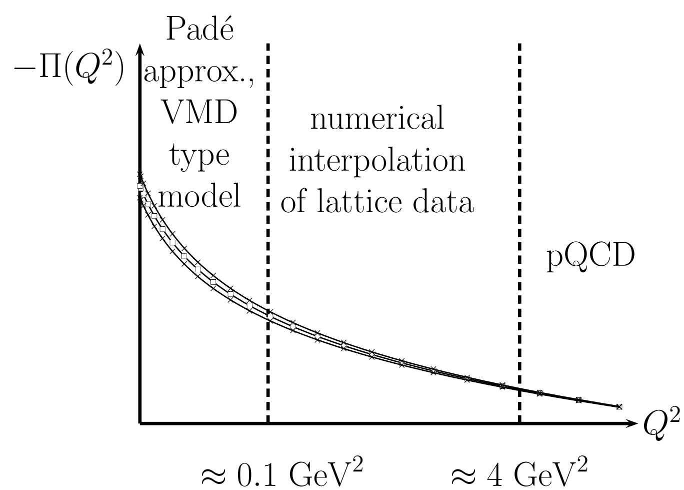
Primary object for HVP in LQCD: e.m. current correlator in configuration space

$$\langle J_\mu(\vec{x},t)\, J_\nu(\vec{0},0)\rangle\,,\quad J_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \cdots$$

A Fourier transform yields the bare vacuum polarization function $\Pi(Q^2)$

$$\Pi_{\mu\nu}(Q) = \int d^4x e^{i\,Qx}\,\langle J_\mu(x)\, J_\nu(0)\rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu}\, Q^2\right)\,\Pi(Q^2)$$

$$a_\mu^{\rm HVP} = 4\alpha^2 \int_0^\infty dQ^2\, f(Q^2)\,\left\{\Pi(Q^2) - \Pi(0)\right\}$$



- ❖ **lattice data:** $Q^2 > (2\pi/L)^2$
- ❖ **extrapolate to** $Q^2 = 0$ **via Padé's**
- ❖ **Note: need** $\Pi(0)$ **! a potential problem**
- ❖ **required accuracy: need LQCD data down to** $Q^2_{\rm min} \approx 0.1$ **GeV**^{2}

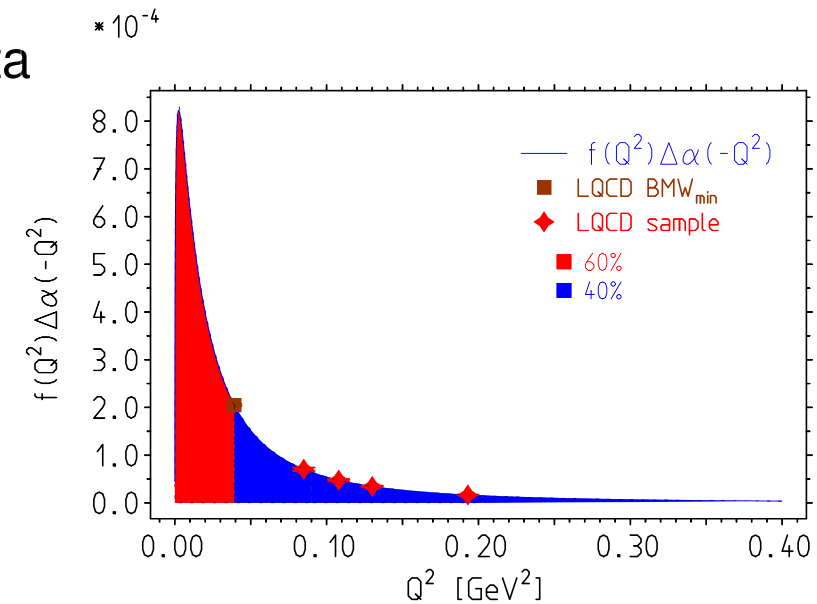
- ❐ **LQCD lattice in finite box: momenta are quantized** $Q_{\rm min} = 2\pi/L$ **where** L **is the lattice box length.** $Q_{\rm min} \to 0 \Leftrightarrow L \to \infty$ **infinite volume limit**
- ❐ $Q_{\rm min} = 2\pi/L$ **with** $m_\pi L \gtrsim 4$ **for** $m_\pi \sim 200$ **MeV, such that** $Q_{\rm min} \sim 314$ **MeV**
- ❐ **about 44% of the low** x **contribution to** $a_\mu^{\rm had}$ **is not covered by data yet**

BMW about $Q_{\min}$: $L_{\rm ref}$ =6.272 fm and M_π =110 MeV, using $\hbar c = 197.33$ MeV fm and $Q_{\min} = 2\pi/L \simeq 1\mathrm{fm}^{-1}$ ≃198 MeV or $Q^2_{\min} \simeq 0.0392$ GeV^2.

$Q_{\min}$=116 MeV [1 Lat]		198 MeV [27 Lat]	
$Q_1 = Q_{\min}$	31.12%	59.75%	part to be estimated by extrapolation
$Q_2 = 0.15$	48.40%	19.77%	
$Q_3 = 0.30$	11.94%	11.94%	lattice data
$Q_4 = 0.45$	7.61%	7.61%	
$Q_5 = 1.00$	0.92%	0.92%	pQCD

very sensitive to $Q_{\min}$ i.e. to box size L !

Lattice-QCD BMW arXiv:2002.12347
$a_\mu^{\rm HVP-LO}[\mathrm{lat}] = 707.5 \pm 5.5$
Davier et al. arXiv:0906.5443
$A_\mu^{\rm HVP-LO}[\tau[ee]] = 705.3 \pm 4.5\ [689.8 \pm 5.2]$
Miranda&Roig arXiv:2007.11019
$a_\mu^{\rm HVP-LO}[\tau] = 705.7^{+4.0}_{-4.1}$

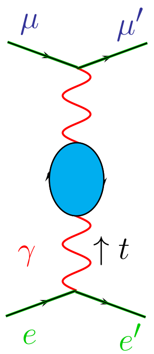


40% Data; 60% extrapolation

[a_μ in units 10^{-10}] Note: τ vs ee differ by 15.5 units

New project: measuring directly low energy $\alpha_{\rm QED}(t)$

- very different paradigm: no VP subtraction issue!
- no exclusive channel collection
- even 1% level measurement can provide important independent information
- use $\mu^- e^-$ scattering   MUonE projects   G. Abbiendi et al. , arXiv:1609.08987



$$\frac{d\sigma^{\rm unpol.}_{\mu^- e^- \to \mu^- e^-}}{dt} = 4\pi\,\alpha(t)^2\,\frac{1}{\lambda(s,m_e^2,m_\mu^2)}\left\{\frac{\left(s-m_\mu^2-m_e^2\right)^2}{t^2}+\frac{s}{t}+\frac{1}{2}\right\}$$

- The primary goal determining $a_\mu^{\rm had}$ in an alternative way

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}\left(-Q^2(x)\right)$$

where $Q^2(x) \equiv \frac{x^2}{1-x} m_\mu^2$ is the space–like square momentum–transfer

- $\Delta\alpha_{\rm had}(-Q^2) = \frac{\alpha}{\alpha(-Q^2)} + \Delta\alpha^{\rm lep}(-Q^2) - 1$ directly compares with lattice QCD data

- My proposal here: determine very accurately

$$\Delta\alpha_{\rm had}\left(-Q^2\right) \text{ at } Q \approx 2.5 \text{ GeV}$$

by this method (one single number!) as the non-perturbative part of $\Delta\alpha_{\rm had}\left(M_Z^2\right)$ as in "Adler function" approach.

- direct comparison with LQCD but is complementary!

  LQCD low Q^2 extrapolation problems (see above)

- direct comparison with LQCD

- directly useful for small angle Bhabha luminosity-meter!

MUonE best at low Q^2 (below 10 MeV) extrapolation up to 2 GeV challenging.

see the Aide-mémoire on MUonE extrapolation

`http://www-com.physik.hu-berlin.de/~fjeger/HVPNoteJan2020.pdf`

5. Conclusions

- Muon $g-2$ theory uncertainty remains the key issue and strongly motivates more precise measurements of low energy $e^+e^- \to \mathrm{hadrons}$ cross sections (Novosibirsk VEPP 2000/CMD3,SND, Beijing BEPCII/BESIII, Tsukuba SuperKEKB/BelleII).

- helps to improve $\alpha_{\mathrm{QED}}(t)$ in region relevant for small angle Bhabha process and in calculating $\alpha_{\mathrm{QED}}(s)$ at FCC-ee/ILC energies via Euclidean split trick (Adler function controlled data vs pQCD split)

- the latter method requires pQCD prediction of the Adler-function to improve by a factor 2 (improved parameters mainly m_c and m_b)

- Are presently estimated (essentially agreed) evaluations in terms of R-data reliable? Alternative methods important!

- Patrick Janot's approach certainly is an important alternative method directly accessing $\alpha_{\mathrm{QED}}(M_Z^2)$ with very different systematics. Challenging!

- Another interesting option is an improved radiative return measurement of $\sigma(e^+e^- \to \mathrm{hadrons})$ at the GigaZ (directly improves dispersion integral incl all resonances and thresholds in one experiment!)
- In any case on paper $e^-\mu^+ \to e^-\mu^+$ looks to be the ideal process to perform an unambiguous measurement of $\alpha(-Q^2)$, which determines the LO HVP to a_μ as well as the non-perturbative part of $\alpha_{\mathrm{QED}}(s)$!
- Lattice QCD results competitive now, and raising new qustions! LQCD is the only method to disentangle QCD against QED effects!

❒ at the end we have alternatives available allowing for important crosschecks.

Thanks you for your attention!

For more datails see
http://www-com.physik.hu-berlin.de/~fjeger/SMalphaFCCee19.pdf