Lattice QCD based determinations of $\alpha_{\mathrm{QED}}(m_Z)$ and future perspectives

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Basic relations

Let $\alpha = 1/137.036\ldots$ be the fine-structure constant.

Effective coupling in the on-shell scheme:

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}$$

Hadronic contribution to $\Delta\alpha(q^2)$:

$$\Delta\alpha_{\rm had}(q^2) = 4\pi\alpha \mathrm{Re}\, \overline{\Pi}(q^2)\,, \qquad \overline{\Pi}(q^2) = \Pi(q^2) - \Pi(0)\,.$$

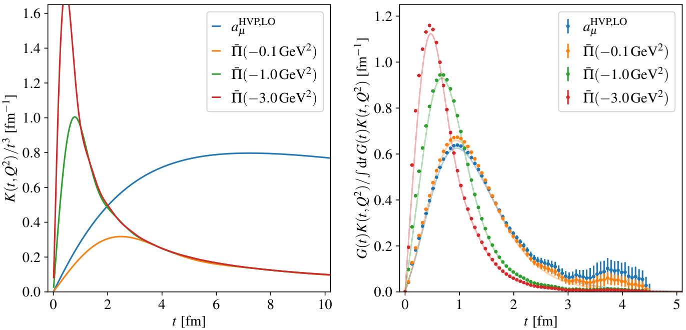
Dispersive representation in terms of the R-ratio $\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$:

$$\overline{\Pi}(-Q^2) = \frac{Q^2}{12\pi^2}\int_0^\infty ds\, \frac{R(s)}{s(s+Q^2)}.$$

Or use lattice QCD for $\overline{\Pi}(q^2)$... many figures in the following are from the recent Cè, Gérardin, von Hippel, HM, Miura, Ottnad, Risch, San José, Wilhelm, Wittig 2203.08676 (accepted in JHEP).

$\Pi(-Q^2) - \Pi(0)$ from lattice-QCD current-current correlators $G(t)$

$$G(t) \equiv \int d^3x \langle j_3(t,\vec{x})\, j_3^\dagger(0)\rangle = \int_0^\infty d\omega\, \omega^2 \frac{R_{e^+e^-\to\mathrm{hadrons}}(\omega^2)}{12\pi^2}\, e^{-\omega t}.$$

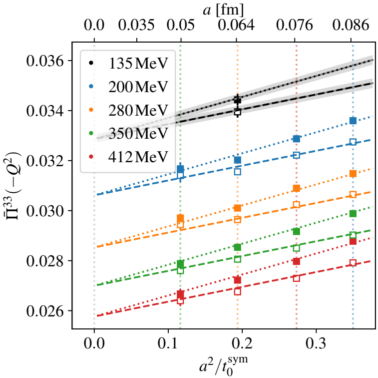


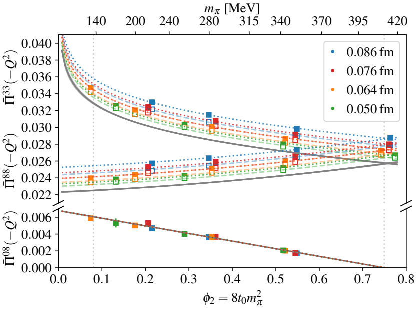
$$\Pi(-Q^2) - \Pi(0) = \int_0^\infty dt\; G(t)\; \underbrace{\left[t^2 - \frac{4}{Q^2}\sin^2\!\left(\frac{Qt}{2}\right)\right]}_{=K(t,Q^2)}.$$

Mainz/CLS, 2203.08676; time-momentum representation Bernecker & HM, 1107.4388.

Chiral and continuum extrapolation in a^2 and m_π^2

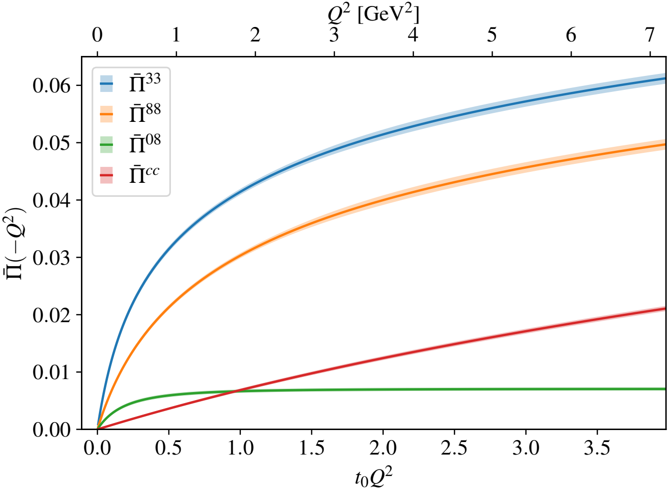
$Q^2 = 1$ GeV^2





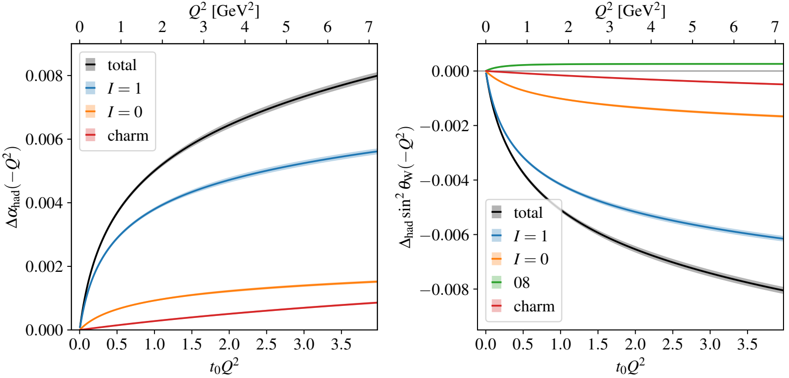
$\sqrt{8t_0} = (0.415 \pm 0.004 \pm 0.002)$ fm. CLS, 1608.08900.

Flavour-decomposition of vacuum polarization



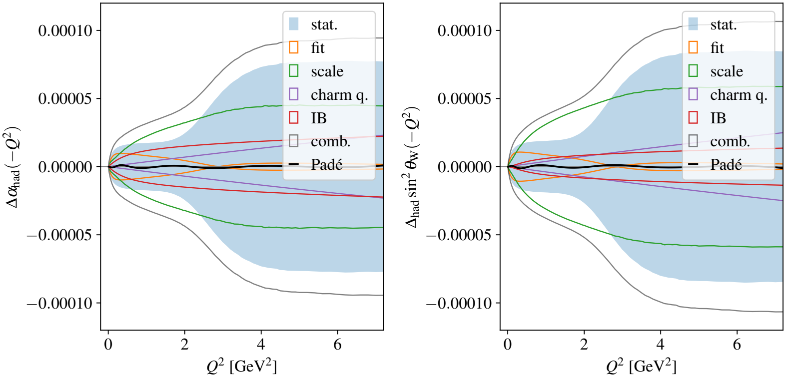
Mainz/CLS, 2203.08676

Hadronic contribution to running of α and $\sin^2\theta_W$



Mainz/CLS, 2203.08676

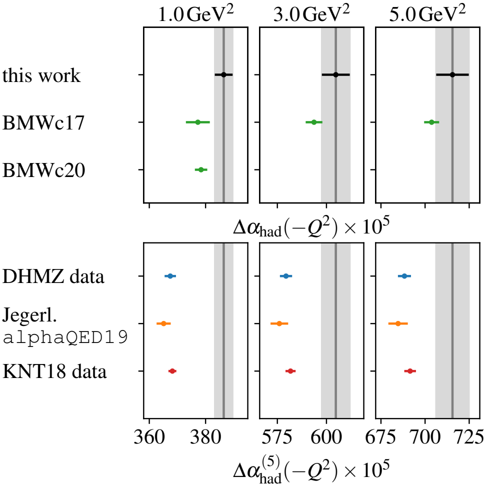
Running of α and $\sin^2\theta_W$: error budget



At large Q^2, error increases due to the more difficult continuum exptrapolation.

Mainz/CLS, 2203.08676

Comparison btw lattice and dispersive determinations of $\Delta\alpha_{\mathrm{had}}(-Q^2)$



Tension of up to 3.5 standard deviation between our lattice calculation and phenomenological estimates for space-like virtualities between 3 and 7 GeV^2.

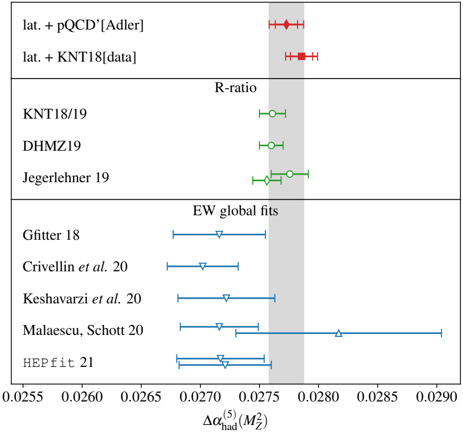
$Q^2 = 3\,\mathrm{GeV}^2$: Mainz CLS result about 2% larger than BMWc17 result, but only a 1.3σ effect.

Running of α up to Z-pole, $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$

Use lattice data in the 'Euclidean split' technique:
[Eidelman, Jegerlehner, Kataev, Veretin hep-ph/9812521]

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = \Delta\alpha_{\mathrm{had}}^{(5)}(-Q_0^2) + \left[\Delta\alpha_{\mathrm{had}}^{(5)}(-M_Z^2) - \Delta\alpha_{\mathrm{had}}^{(5)}(-Q_0^2)\right] + \left[\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) - \Delta\alpha_{\mathrm{had}}^{(5)}(-M_Z^2)\right]_{\mathrm{pQCD}}$$

The second term is handled either as an integral over the perturbative Adler function $Q^2 \frac{d\Pi}{dQ^2}$, or dispersively using e^+e^- cross-section data.



Final result: $\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = 0.02773(9)_{\mathrm{lat}}(2)_{\mathrm{bottom}}(12)_{\mathrm{pQCD}}$

Mainz CLS, 2203.08676

Recent progress on understanding cutoff effects from short distances

At short distances in massless lattice QCD:

$$G(t,a) = G_{\rm cont}(t)(1 + \mathrm{O}((a/t)^2))$$

Therefore, since $G_{\rm cont}(t) \sim 1/t^3$,

$$\int_0^t dt'\, t'^4\, G(t',a) = \int_0^t dt'\, t'^4\, G_{\rm cont}(t') + \mathrm{const} \times a^2 \int_a^t dt'\, t'^4\, \frac{1}{t'^5} + \mathrm{O}(a^2)$$

one obtains a logarithmically enhanced cutoff effect from short distances.

In leading order of Wilson lattice perturbation theory, one finds

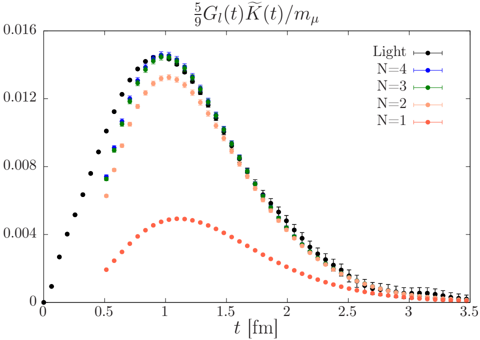
$$\int_0^t dt'\, t'^4\, G(t',a) = \int_0^t dt'\, t'^4\, G_{\rm cont}(t') + \frac{7N_c \sum_f \mathcal{Q}_f^2}{60\pi^2} a^2 \log(1/a) + \mathrm{O}(a^2).$$

Also, developed strategy to reach higher virtualities: at high Q^2,
$\Pi(-Q^2) - \Pi(-(Q/2)^2)$ is not very sensitive to the volume
$\rightsquigarrow$ for the same cost, simulate finer lattices.

Cè, Harris, HM, Toniato, Török [2106.15293] (JHEP).

Possible strategies to improve control over the long-distance tail

1. Auxiliary calculation of the (discrete, finite-volume) spectrum of $\pi\pi$ states and their coupling to the e.m. current.
   The low-lying states saturate the correlator at long distances.



D200:
$a = 0.064$ fm
$m_\pi = 200$ MeV
Fig. from 1904.03120.

See also RBC/UKQCD 2019; Fermilab-HPQCD-MILC 2021.

2. 'all-to-all' propagators using the low eigenmodes of the Dirac operator
   RBC collaboration 1801.07224 (PRL); BMW collaboration 2002.12347 (Nature).
3. approximate factorization of the QCD path integral with bias correction.
   Dalla Brida et al., 2007.02973.

Conclusion

- Lattice calculations for hadronic contribution to running of $\Delta\alpha_{\mathrm{had}}(-3\ \mathrm{GeV}^2)$ are in reasonbly good agreement, but exhibit a tension of $3+$ standard deviations with dispersive determination.

- Similar tension seen in the intermediate 'window' contribution to a_μ^{hvp}: resolving this tension currently has the highest priority.

  BMW 2002.12347; Mainz-CLS 2206.06582; ETMC 2206.15084; Fermilab-HPQCD-MILC 2207.04765.

- Future improvement of lattice-based $\Delta\alpha_{\mathrm{had}}(M_Z^2)$ will require reaching higher Q^2 in lattice QCD $\rightsquigarrow$ dedicated treatment of discretization effects required.

- Reduction of current uncertainty on $\Delta\alpha_{\mathrm{had}}(M_Z^2)$ based on lattice-QCD & perturbation theory by a factor of two to three is realistic within the next five years.