$\Delta\alpha_{had}$: the KNT evaluation and the big picture

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ECFA WG1 Higgs Factory
Parametric uncertainties at future e^+e^- colliders: a_{EM}

14th July 2022

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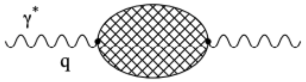
What is $\Delta\alpha_{\text{had}}$?

- Dyson summation of Real part of one-particle irreducible blobs $\boldsymbol{\Pi}$ into the effective, real running coupling $\boldsymbol{\alpha}_{\mathbf{QED}}$:

$\Pi =$ 

Full photon propagator $\sim 1 + \Pi + \Pi \cdot \Pi + \Pi \cdot \Pi \cdot \Pi + \ldots$

$$\rightsquigarrow \quad \alpha(q^2) = \frac{\alpha}{1 - \mathrm{Re}\Pi(q^2)} = \alpha \,/\, \left(1 - \Delta\alpha_{\text{lep}}(q^2) - \Delta\alpha_{\text{had}}(q^2)\right)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^- \to hadrons)$:

$$\Delta\alpha^{(5)}_{\text{had}}(q^2) = -\frac{q^2}{4\pi^2\alpha} \, \mathrm{P} \int_{m_\pi^2}^{\infty} \frac{\sigma^0_{\text{had}}(s)\,\mathrm{d}s}{s - q^2} \,, \quad \sigma_{\text{had}}(s) = \frac{\sigma^0_{\text{had}}(s)}{|1 - \Pi|^2}$$

Identical input as for a_μ^{HVP}!

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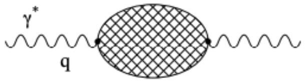

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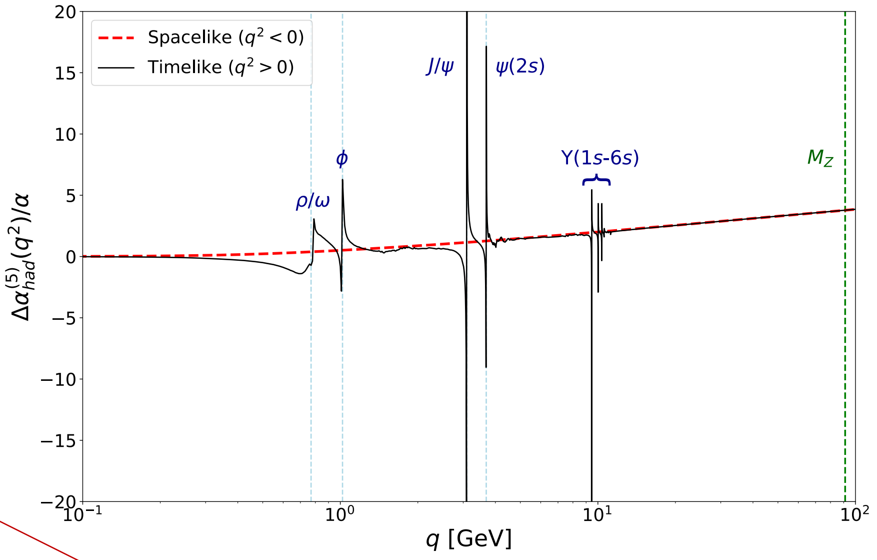

q^2 can be:
- Timelike ($q^2 = s > 0$), rapidly changing function.
  - Data-driven only.
- Spacelike ($q^2 = t < 0$), smooth function.
  - Data-driven & Lattice.

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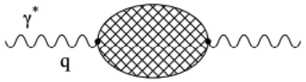
$\Pi =$ 

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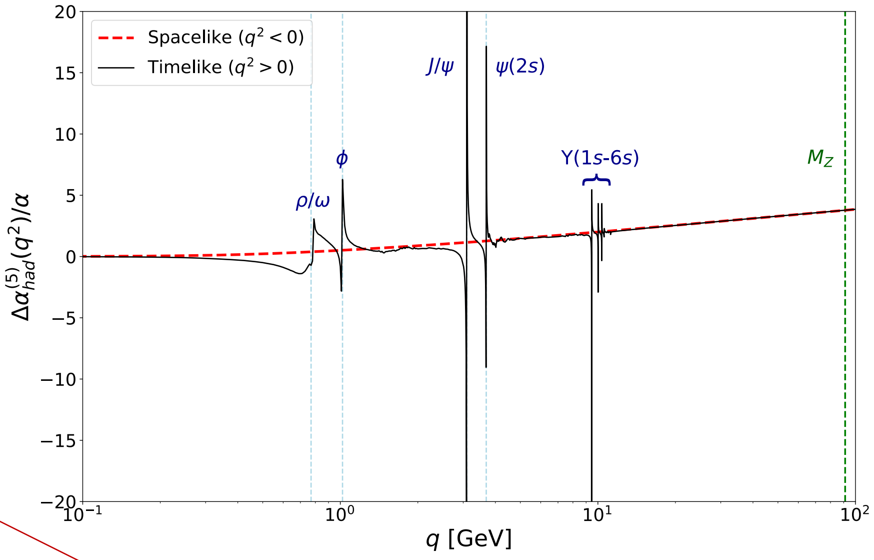

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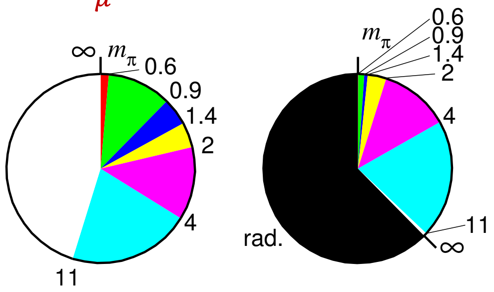
Identical input as for a_μ^{HVP}!

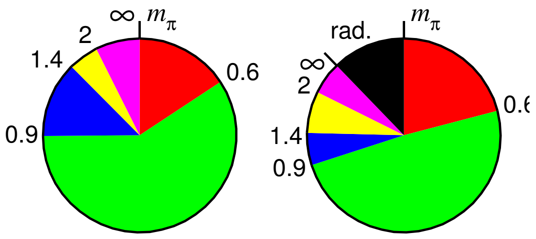
Most commonly evaluated at Z-pole: $q^2 = \pm M_Z^2$.

→ Principal component for global EW precision fits and limits their precision.

KNT19: $\Delta\alpha^{(5)}_{had}(M_Z^2) = 0.02761(12)$

(Note that timelike $\Delta\alpha^{(5)}_{had}(q^2)$ is smooth at $s = M_Z^2$)

$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ 

$a_\mu^{\text{had,LO VP}}$ 

Why is $\Delta\alpha_{\mathrm{had}}$ important?

- Limits precision of EW precision fits and so the effectiveness of high-precision EW measurements.
- Can draw a direct parallel with evaluation of the Muon g-2 and probe the muon g-2 discrepancy.
- Is a test of low-energy hadronic theory, e.g. Lattice QCD vs dispersive e^+e^- data.

Uncertainty from e^+e^- data $\sim 0.5\%$

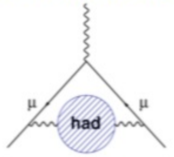
Parameter	Input value	Fit result	Result w/o input value
M_W (GeV)	80.379(12)	80.359(3)	80.357(4)(5)
M_H (GeV)	125.10(14)	125.10(14)	94^{+20+6}_{-18-6}
$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)\times 10^4$	276.1(1.1)	275.8(1.1)	272.2(3.9)(1.2)
m_t (GeV)	172.9(4)	173.0(4)	...
$\alpha_s(M_Z^2)$	0.1179(10)	0.1180(7)	...
M_Z (GeV)	91.1876(21)	91.1883(20)	...
Γ_Z (GeV)	2.4952(23)	2.4940(4)	...
Γ_W (GeV)	2.085(42)	2.0903(4)	...
σ^0_{had} (nb)	41.541(37)	41.490(4)	...
R^0_l	20.767(25)	20.732(4)	...
R^0_c	0.1721(30)	0.17222(8)	...
R^0_b	0.21629(66)	0.21581(8)	...
$\bar{m}_c$ (GeV)	1.27(2)	1.27(2)	...
$\bar{m}_b$ (GeV)	$4.18^{+0.03}_{-0.02}$	$4.18^{+0.03}_{-0.02}$	...
$A^{0,l}_{\mathrm{FB}}$	0.0171(10)	0.01622(7)	...
$A^{0,c}_{\mathrm{FB}}$	0.0707(35)	0.0737(2)	...
$A^{0,b}_{\mathrm{FB}}$	0.0992(16)	0.1031(2)	...
A_ℓ	0.1499(18)	0.1471(3)	...
A_c	0.670(27)	0.6679(2)	...
A_b	0.923(20)	0.93462(7)	...
$\sin^2\theta^{\mathrm{lep}}_{\mathrm{eff}}(Q_{\mathrm{FB}})$	0.2324(12)	0.23152(4)	0.23152(4)(4)
$\sin^2\theta^{\mathrm{lep}}_{\mathrm{eff}}$(Had Coll)	0.23140(23)	0.23152(4)	0.23152(4)(4)

Keshavarzi, Marciano, Passera and Sirlin, *Phys.Rev.D* 102 (2020) 3, 033002

Experimentally measured hadronic cross section:

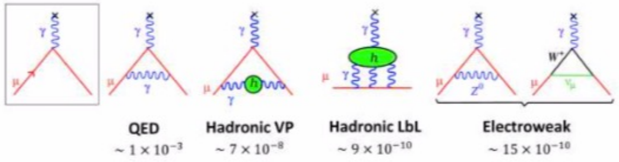
σ_{had}

Muon g-2:
hadronic vacuum polarisation contribution



$$a_\mu^{\mathrm{had,\,VP}} = \frac{1}{4\pi^3}\int_{m_\pi}^{\infty} ds\ \sigma_{\mathrm{had}}(s)\,K(s)$$

... sum with other SM contributions...



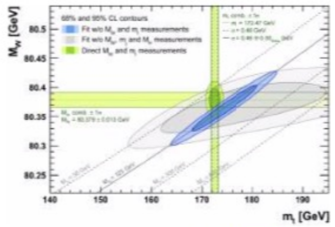
→ Determines a_μ^{SM} and $\Delta a_\mu = 3.7\sigma$

Increase cross section so that $\Delta a_\mu = 0$?
→ Solves muon g-2 discrepancy

Running QED coupling:
hadronic contribution to running

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) = \frac{q^2}{4\pi\alpha^2}\int_{m_\pi}^{\infty} ds\ \sigma_{\mathrm{had}}(s)\frac{q^2}{(q^2-s)}$$

... evaluate at $q^2 = M_Z^2$ and input into **global EW fit**...

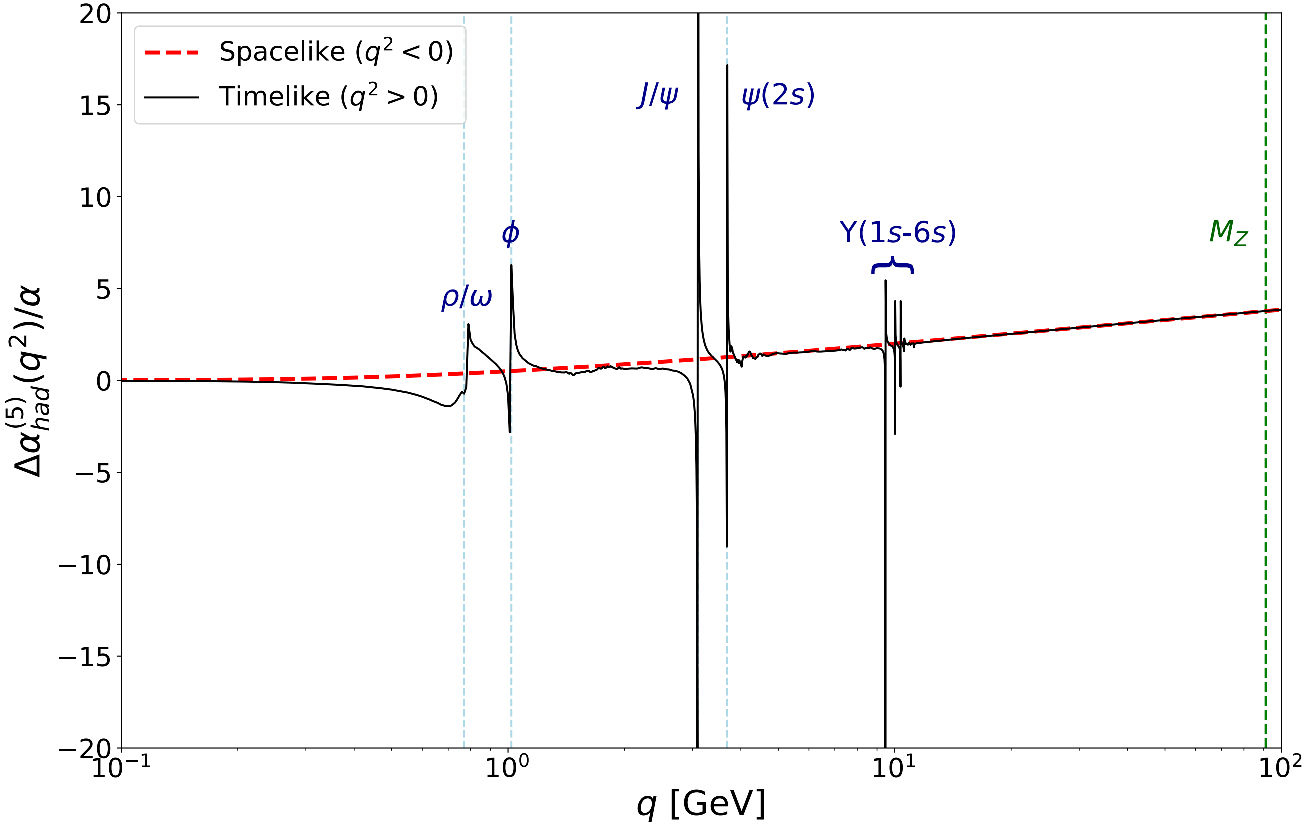


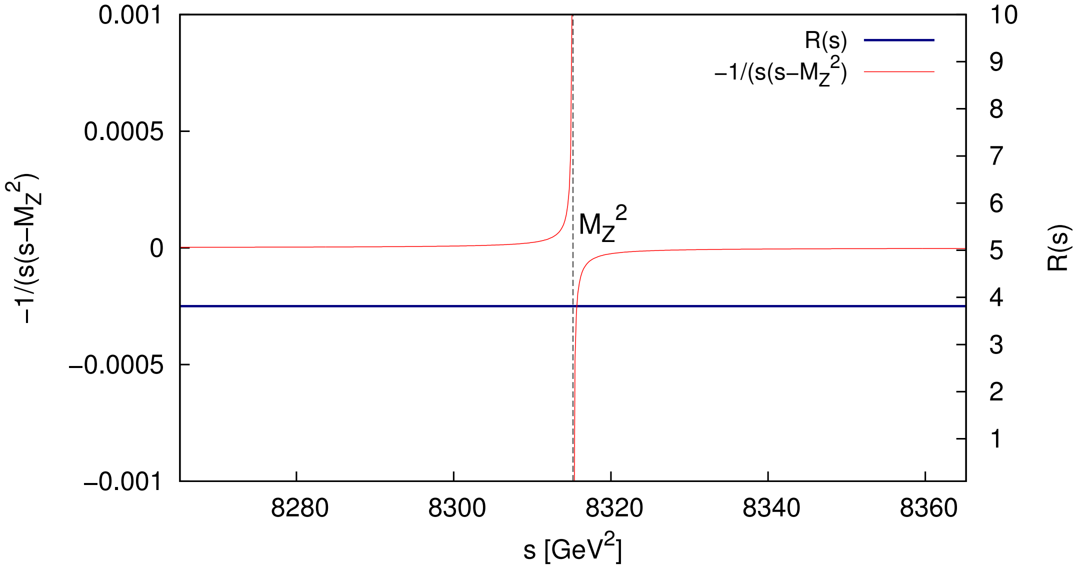
→ Predicts $M_W, M_H, \sin^2\theta^{lep}_{eff}$ and more...

Increase cross section so that $\Delta a_\mu = 0$?
→ What happens to precision EW parameters?

$\Delta\alpha_{\text{had}}$: M_Z^2 vs. low Q^2

$$\Delta\alpha_{\text{had}}^{(5)}(q^2) = -\frac{q^2}{4\pi^2\alpha} \text{P} \int_{m_\pi^2}^{\infty} \frac{\sigma_{\text{had}}^0(s)\,\text{d}s}{s - q^2}$$



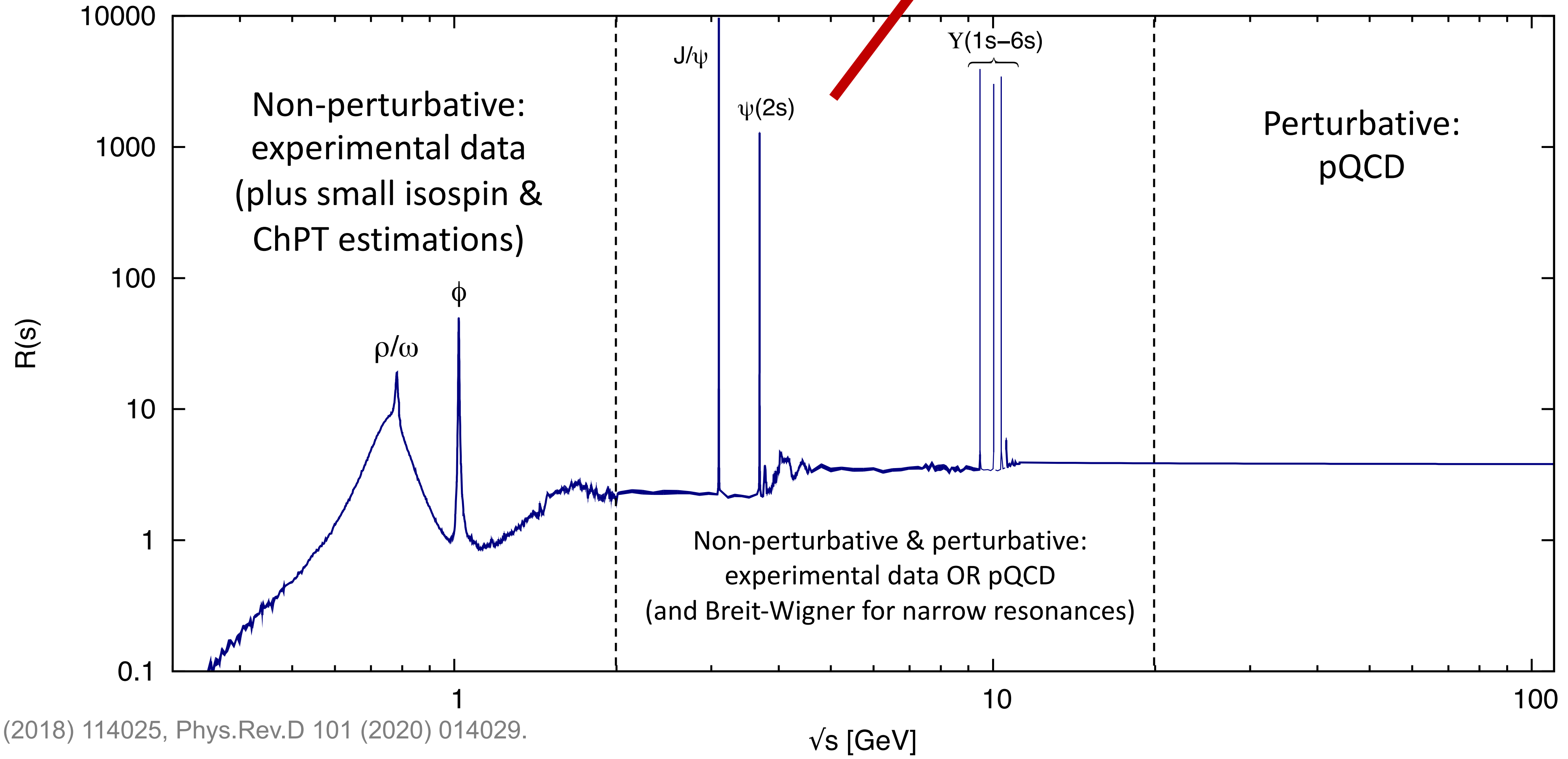


- Both functions smooth at $Q^2 = \pm M_Z^2$. Not true for low-energy timelike resonances.
- Timelike principal value integral over combined (non-smooth) data evaluated at low Q^2 can result in fluctuations.
  → Improved KNT VP evaluation planned to address this.
- Differences for $Q^2 = \pm M_Z^2$ are:
  KNT19: $\Delta\alpha_{had}^{(5)}(+M_Z^2) - \Delta\alpha_{had}^{(5)}(-M_Z^2) = 0.40(6)\times10^{-4}$.
  F. Jegerlehner, 1905.05078 - pQCD: $= 0.45(2)\times10^{-4}$.

The KNT evaluation of $\sigma^0_{had}(s)$

$$R(s) = \sigma^0_{had}(s)/(\frac{4\pi\alpha^2}{3s})$$

$$\Delta\alpha^{(5)}_{\rm had}(q^2) = -\frac{q^2}{4\pi^2\alpha}\,{\rm P}\int_{m_\pi^2}^{\infty}\frac{\sigma^0_{\rm had}(s)\,{\rm d}s}{s-q^2}$$



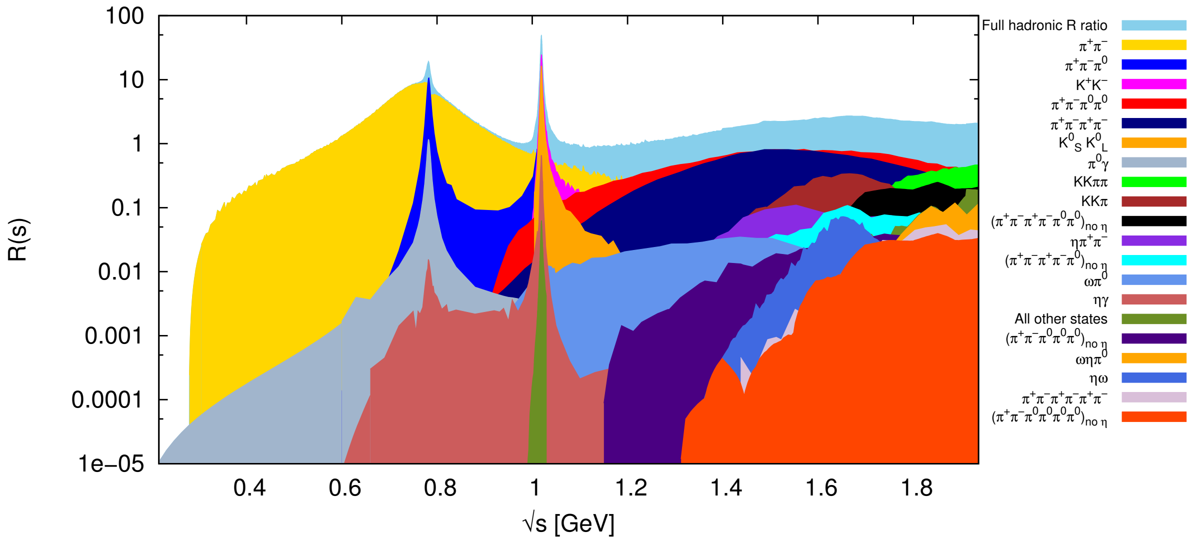
Phys.Rev.D 97 (2018) 114025, Phys.Rev.D 101 (2020) 014029.

The KNT evaluation of $\sigma^0_{had}(s)$ & $\Delta\alpha_{had}(q^2)$

Must calculate $\sigma^0_{had}(s)$ for each hadronic in the low-energy region, and combine many measurements for each final state:

- Radiative corrections to account for VP and FSR contributions, with corresponding systematic uncertainties.
- Re-binning of data, with scans to optimise e.g., resonance regions.
- Combine measurements via linear χ^2 minimisation in given channel, avoiding fitting biases and incorporating all available experimental uncertainty and correlation information.
- Inflation of uncertainties in local regions of tension in data.
- Sum all channels (+ isospin/ChPT estimates + BW narrow resonances + pQCD) to get the full $\sigma^0_{had}(s)$.
- Integrate over $\sigma^0_{had}(s)$ for all $\pm q^2$ to get $\Delta\alpha^{(5)}_{had}(\pm q^2)$:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) = -\frac{q^2}{4\pi^2\alpha} \mathrm{P} \int_{m_\pi^2}^{\infty} \frac{\sigma^0_{\mathrm{had}}(s)\,\mathrm{d}s}{s-q^2}$$



To arrive at the final KNT software routine containing full VP function $\Pi(q^2)$:

- Note that $\sigma^0_{had}(s)$ requires a VP correction from $\Pi(q^2)$ which contains the calculated $\Delta\alpha^{(5)}_{had}(q^2)$…
  → Iterate between producing $\Pi(q^2)$ and correcting data until convergence on results.

The self-consistent KNT vacuum polarisation software routine, `vp_knt_v3_1`, is available by contacting us directly

(and is due to be updated shortly…)

Phys.Rev.D 97 (2018) 114025, Phys.Rev.D 101 (2020) 014029.

Results for data-driven evaluations of $\Delta\alpha_{had}^{(5)}(M_Z^2)$ and $\alpha(M_Z^2)$

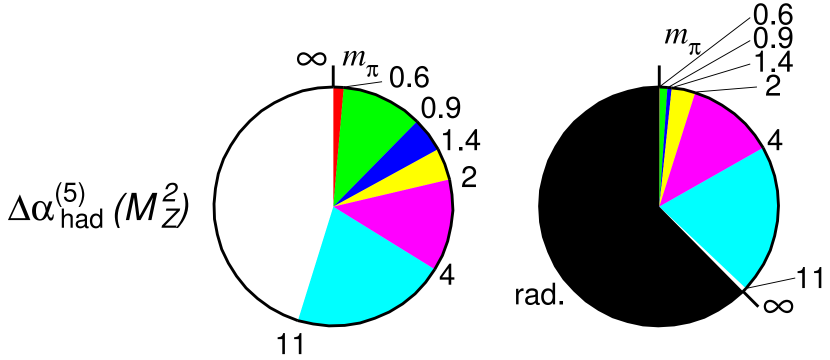
KNT19: $\Delta\alpha_{had}^{(5)}(M_Z^2) = 276.09(1.12) \times 10^{-4}$

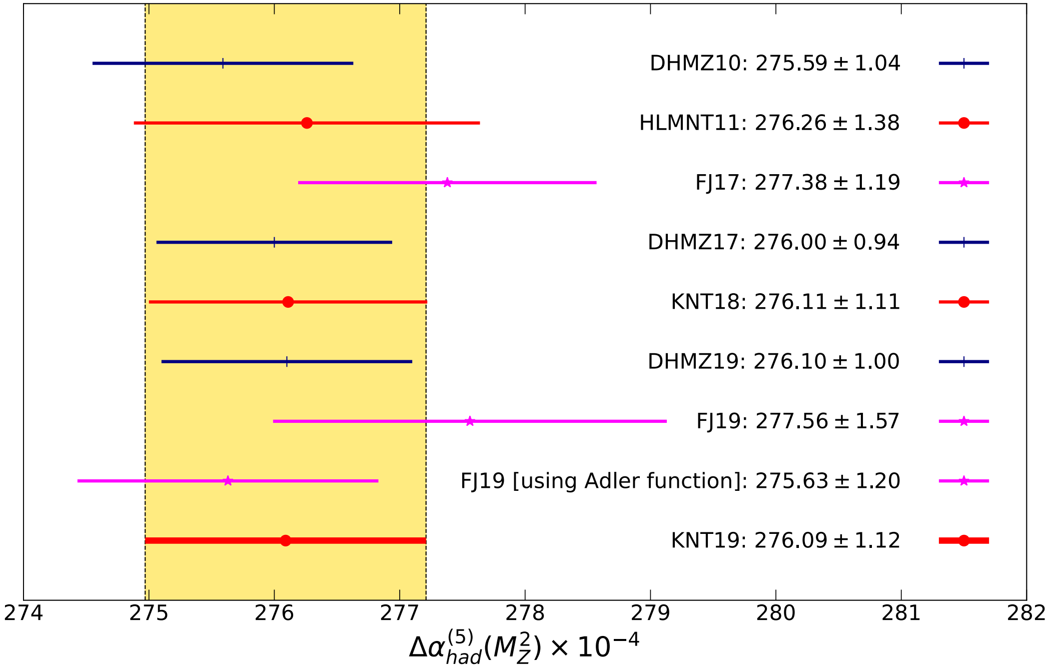
$$\rightarrow \quad \alpha^{-1}(M_Z^2) = \left(1 - \Delta\alpha_{lep}(M_Z^2) - \Delta\alpha_{had}^{(5)}(M_Z^2) - \Delta\alpha_{top}(M_Z^2)\right)\alpha^{-1}$$

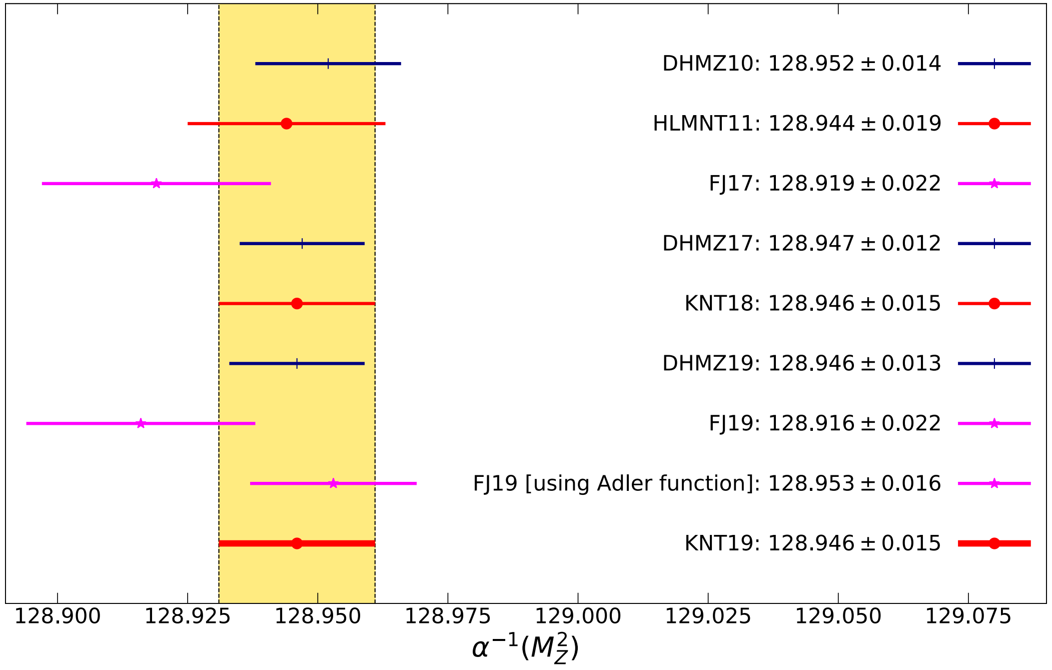
$\Delta\alpha_{lep}(M_Z^2) = 314.979(2) \times 10^{-4}$

$\Delta\alpha_{top}(M_Z^2) = -0.7201(37) \times 10^{-4}$

$$= 128.946(15)$$





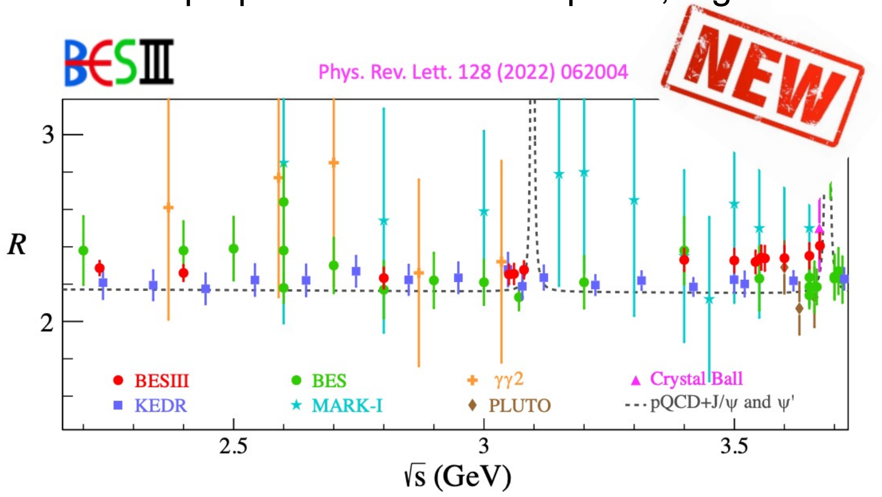


Phys.Rev.D 97 (2018) 114025, Phys.Rev.D 101 (2020) 014029.

Prospects and motivation for improvement

Areas/plans for improvement from KNT:

- New data
  - New cross section measurements are currently being included in preparation for a new update, e.g. BESIII:



  - More cross section measurements due to be released.
  - Updated data analysis from KNT in the next year(s), including updated VP routine.
  - Future plans include a new evaluation of VP with significant improvements and a specific VP-dedicated publication.

Motivation for improvement: Future measurements

- FCC/FCC-ee (for example) would probe new physics at the precision of non-perturbative hadronic corrections to the running coupling for the first time.
  → Order(s) of magnitude improvement expected in e.g., $\sin^2\theta_{eff}$ and M_W.
  World average: $\sin^2\theta_{eff} = 0.23151(14)$

  Erler and Schott, Prog. Part. Nucl. Phys. 2019

  EW fit prediction: $\sin^2\theta_{eff} = 0.23152(4)_{parametric}(4)_{th}$

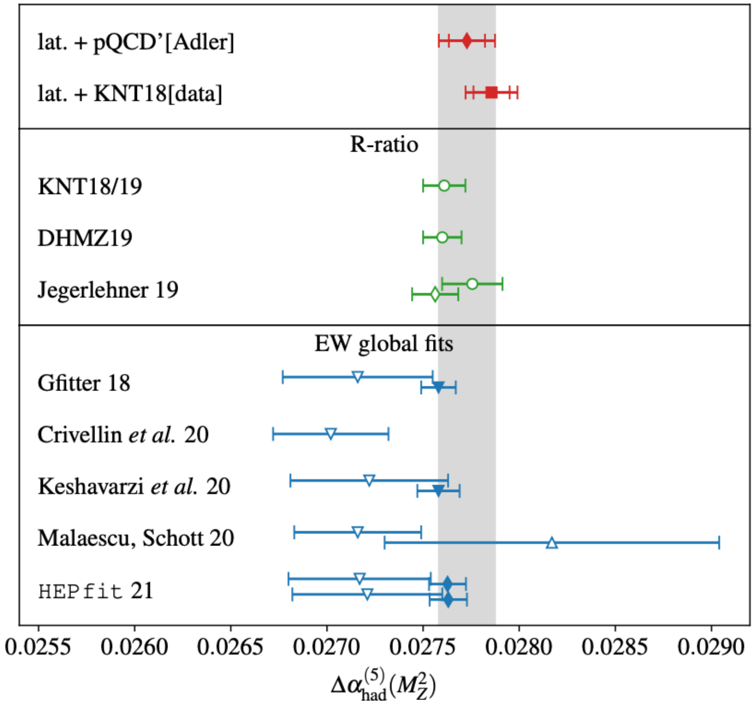
  Keshavarzi, Marciano, Passera and Sirlin, *Phys.Rev.D* 102 (2020) 033002, using Gfitter

  Parametric error $4{\times}10^{-5}$ on $\sin^2\theta_{eff}$ is dominated by $\Delta\alpha_{had}^{(5)}\,(M_Z^2)$ uncertainty.

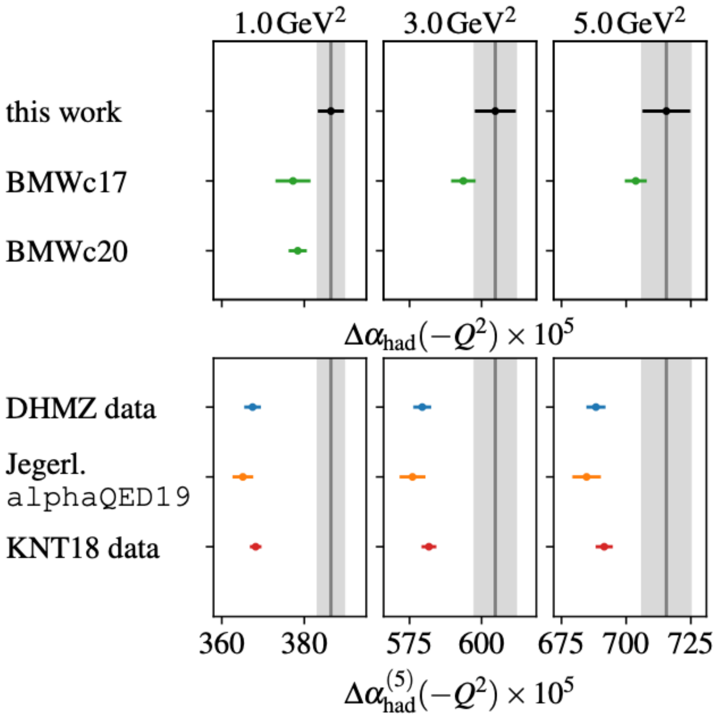
- Without an improvement in the precision of $\Delta\alpha_{had}^{(5)}$, the precision of the EW fit prediction will become more precise than the current best determination!
- Need an improvement $\sim\times 3$ in $\Delta\alpha_{had}^{(5)}$ precision to make it compatible with such measurements (e.g. $\sin^2\theta_{eff}$ precision $\lesssim 1\times10^{-5}$).

Prospects and motivation for improvement

Motivation for improvement: tensions with lattice QCD



Tension with data-driven results washed out at the Z pole.



Up to 3.5σ tension with data-driven results between 1 and 7 GeV²
(comparable to g-2 discrepancy…).

Other prospects for improvement:

- New low-energy data for $\sigma_{had}^{0}(s)$ (CMD-3, SND, KEDR, BESIII, Belle-2, …).
- Direct determination of $\Delta\alpha_{had}^{(5)}\,(M_Z^2)$ measuring the muon asymmetry $A_{FB}^{\mu\mu}$(s) in the vicinity of the Z-pole (see Patrick Janot's talk in this workshop).
- Euclidean split method (Adler function). Needs spacelike offset $\Delta\alpha_{had}^{(5)}\,(-M_0^2)$ with $-M_0^2 \sim 2$ GeV and pQCD (see Fred Jegerlehner's talk in this workshop).
- Direct measurement of $\Delta\alpha_{had}^{(5)}\,(q^2)$ from MUonE muon-electron scattering experiment.
- More lattice QCD evaluations..

Simon Kuberski, Mainz Lattice, SchwingerFest 2022

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The muon g-2 and $\Delta\alpha$ connection

Keshavarzi, Marciano, Passera and Sirlin, *Phys.Rev.D* 102 (2020) 3, 033002

- Shift KNT hadronic cross section in fully energy-dependent (point-like and binned) analysis to account for Δa_μ.
- Input new values of $\Delta\alpha$ into Gfitter to predict EW observables.
- Analysis greatly constrained from more precise EW observables measurements and more comprehensive hadronic cross section.

- **Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?**
- **An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.**
- **Consider:**

a_μ^{HLO} → $$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)}$ → $$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

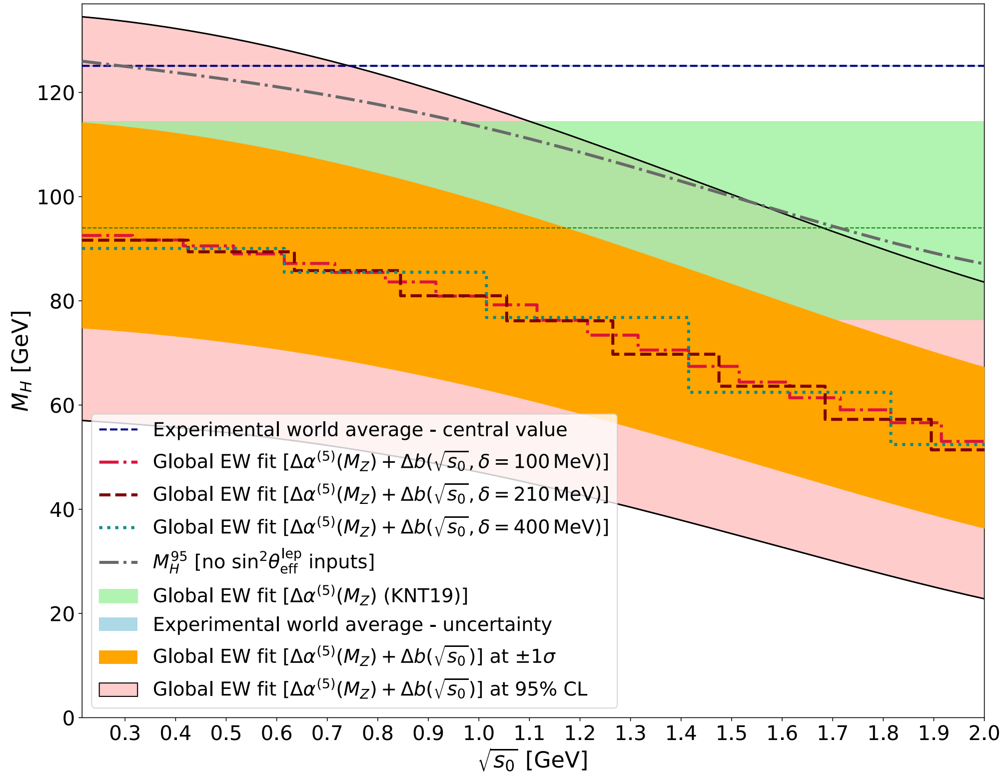
$$\Delta\sigma(s) = \epsilon\sigma(s)$$

Note the very different energy-dependent weighting of the integrands…

ϵ>0, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$

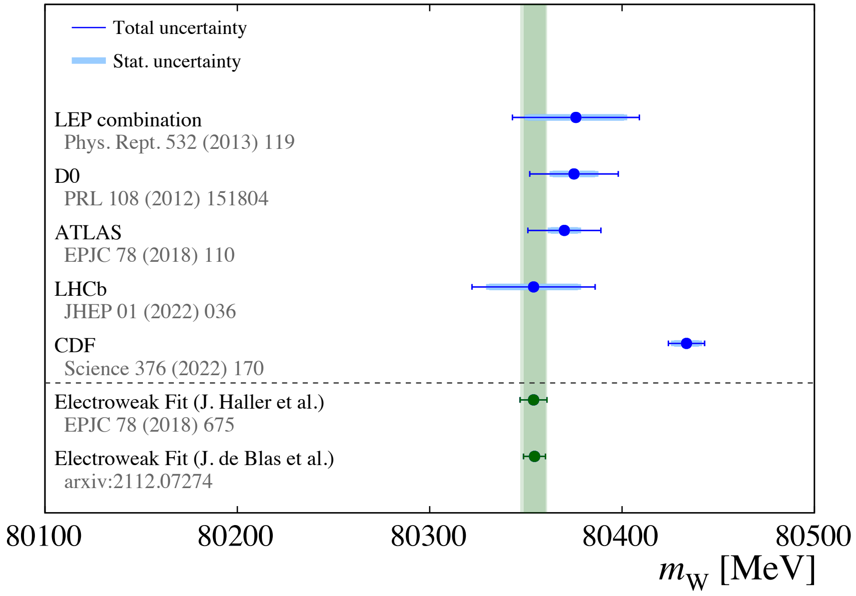
Use Gfitter and precise and up-to-date compilation of total hadronic cross section from KNT, Keshavarzi, Nomura and Teubner, Phys.Rev.D 101 (2020) 014029.



Shifting $\Delta\sigma(s)$ to fix Δa_μ is possible but excluded above ~ 1 GeV.

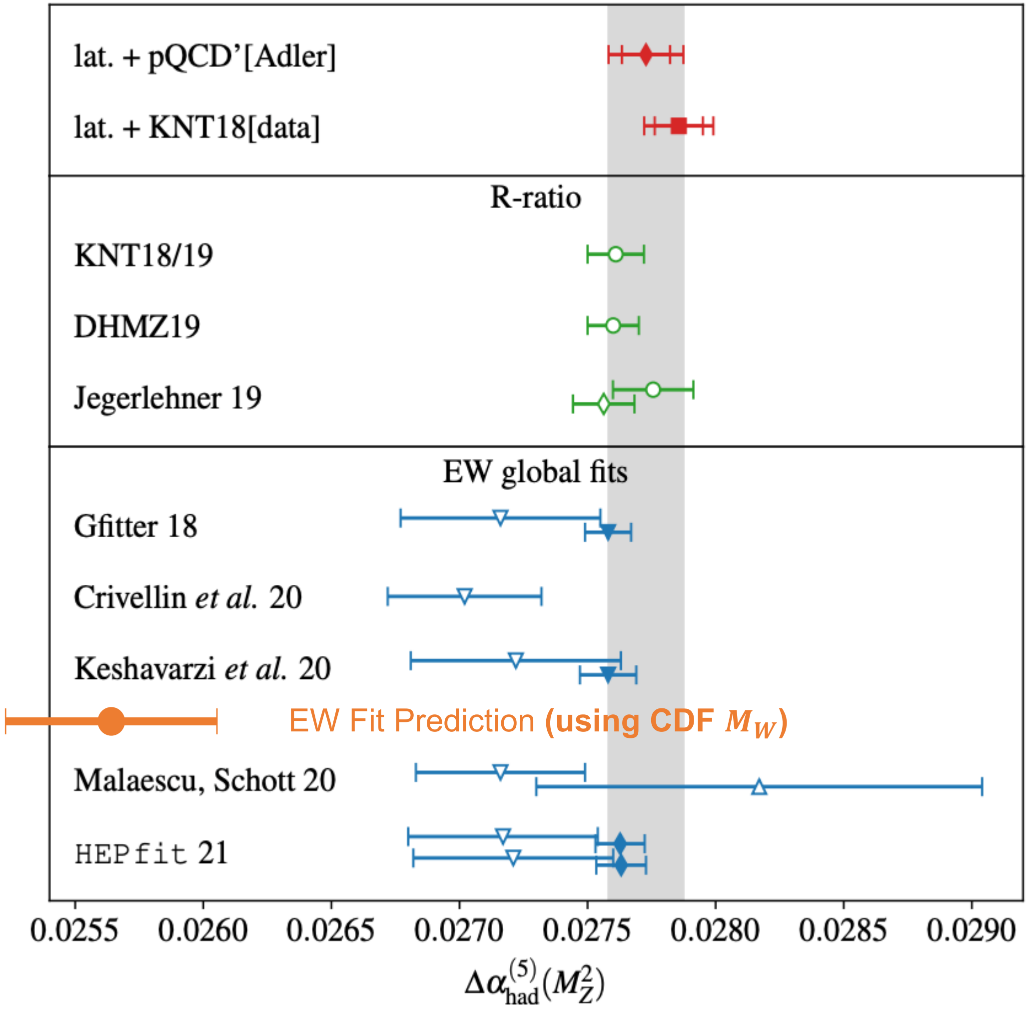
Aside: What about the new CDF M_W?

Exhibits complete inconsistency with EW fit / SM…



Parameter	Input Value (Experiment)	EW Fit Prediction (No CDF M_W)	EW Fit Prediction **(using CDF M_W)**
$\Delta\alpha_{had}^{(5)}(M_Z^2)\times 10^4$	276.1(1.1)	272.2(4.0) [0.9σ]	256.4(3.9) **[4.9σ]**
M_H (GeV)	125.10(14)	94(21) [1.5σ]	40(9) **[9.5σ]**

Taken largest asymmetric M_H error as symmetric error



And bear in mind that comparisons of the lattice vs data-driven $\Delta\alpha_{had}^{(5)}(q^2)$ are worse evaluated at low-energies…

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Conclusions

- $\Delta\alpha_{\text{had}}$ can be evaluated dispersively from $e^+e^- \rightarrow$ hadrons cross section data.
- $\Delta\alpha_{\text{had}}$ limits precision of EW precision fits and so the effectiveness of high-precision EW measurements.
- Can draw a direct parallel with evaluation of the Muon g-2 and probe the muon g-2 discrepancy.
- Is a test of low-energy hadronic theory, e.g. Lattice QCD vs $e^+e^- \rightarrow$ hadrons data used in dispersive approach.
- $\Delta\alpha_{\text{had}}(q^2)$ is different for spacelike (smooth) and timelike (resonant) q^2.
- KNT and other data-driven evaluations (see talks in the workshop) are in good agreement.
- Prospects for improvement in data-driven evaluation and from direct measurement or Lattice QCD.
- Improvements are crucial for effectiveness of future precision EW measurements and for understanding tension with Lattice QCD.
- $\Delta\alpha_{\text{had}}(M_Z^2)$ has been used to constrain new physics in hadronic muon g-2 to < 1 GeV.
- CDF M_W results in low prediction of $\Delta\alpha_{\text{had}}(M_Z^2)$ that is inconsistent with the SM.

Thank you.