Direct $\alpha_{QED}(m_Z)$ measurement at FCC-ee

- **Outline**
  - **Motivation** — NEW! Based on (very) recent estimates from A. Blondel (July 2022)
  - **Method**
  - **Projected uncertainties**
    (Method and Projected uncertainties: Based on arXiv:1512.05544)
    - Parametric
      - → m_Z, Γ_Z, $\sin^2\theta_{W,eff}$, G_F — UPDATED — Based on arXiv:2106.13885
    - Experimental
      - → √s calibration — UPDATED
      - → √s spread — NEW!
      - → Other
      (Based on arXiv:1909.12245)
    - Theoretical
      - → EW higher orders
      - → QED: ISR, FSR, IFI — NEW! — Based on work by S. Jadach and P. Janot in July/August 2018 — CONFIDENTIAL TOP SECRET CONFIDENTIAL — Unpublished
    - Summary
  - **Outlook – shopping list for interested newcomers**

FCC

Introduction (see all other talks today)

- **Today, $\alpha_{QED}(m_Z)$ is determined from $\alpha_{QED}(0)$ with the well-known running formula**

$$\alpha_{\rm QED}(m_Z^2) = \frac{\alpha_{\rm QED}(0)}{1 - \Delta\alpha_\ell(m_Z^2) - \Delta\alpha_{\rm had}^{(5)}(m_Z^2)}$$

  - **Its uncertainty is dominated by the determination of the hadronic vacuum polarization**

$$\Delta\alpha_{\rm had}^{(5)}(m_Z^2) = \frac{\alpha m_Z^2}{3\pi}\int_{4m_\pi^2}^{\infty}\frac{R_\gamma(s)}{s(m_Z^2 - s)}ds$$

    - **$R\gamma(s)$ is the hadronic cross section normalized to the dimuon cross section in e^+e^- collisions**
      - ➔ **Relies on measurements at low centre-of-mass energies $\sqrt{s}$**
    - **Most recent/precise evaluation (DHMZ'19) gives $\Delta\alpha_{had}^{(5)}(m_Z^2) = (276.10 \pm 1.0)\cdot 10^{-4}$**

$$\alpha_{\rm QED}^{-1}(m_Z^2) = 128.946 \pm 0.013 \quad \Longrightarrow \quad \frac{\Delta\alpha}{\alpha} \simeq 1.0 \times 10^{-4}$$

See presentations from B. Malaescu, A. Keshavarzi, F. Jegerlehner, H. Meyer

Motivation for a direct measurement at $\sqrt{s} = m_Z$?

- **Systematic uncertainties are entirely different**
  - No running from low energy to m_Z
  - No need for low-energy measurements

- **$\alpha_{QED}(m_Z^2)$ dominate the parametric uncertainties on the SM prediction for $\sin^2\theta_{W,eff}$**
  - Today's precision

Direct measurement

$$\sin^2\theta_W^{\rm eff} = 0.23153 \pm 0.00016$$

EWPO Fit to the SM (and nothing else)

$$\sin^2\theta_W^{\rm eff} = 0.231488 \pm 0.000029_{m_{\rm top}} \pm 0.000015_{m_Z} \pm 0.000035_{\alpha_{\rm QED}} \pm 0.000010_{\alpha_S} \pm 0.000001_{m_H} \pm 0.000047_{\rm theory} = 0.23149 \pm 0.00007_{\rm total},$$

  - Direct measurement will be improved by two orders of magnitude at FCC-ee

Direct measurement

$$\sin^2\theta_W^{\rm eff} = 0.23153 \pm 0.0000015$$

Parametric uncertainties will need to drop accordingly

FCC-ee prospects for m_Z, m_{top}, $\alpha_S(m_Z)$, m_H are good

Need to improve $\alpha_{QED}(m_Z^2)$ as much as possible

Spoiler alert – FCC-ee prospects

- **Parametric uncertainties on other FCC-ee observables**

NEW!

FCC note in preparation
Uploaded to the agenda

Observable	present value ± error	FCC-ee Stat.	from $\alpha_{QED}(m_Z^2)$	from $\sin^2\theta_W^{eff}$	Comments
m_Z (keV)	91186700 ± 2200	**4**	N.A.	N.A.	Input
$G_F (\times 10^{-5})$	1.166378 ± 0.000006	N.A	N.A.	N.A	Input
$1/\alpha_{QED}(m_Z^2)(\times 10^3)$	128952 ± 14	**3**	N.A.	N.A.	Input; from $A_{FB}^{\mu\mu}$ off peak
$\sin^2\theta_W^{eff} (\times 10^6)$	231480 ± 160	**1.5**	**10.5**	N.A.	from $A_{FB}^{\mu\mu}$ and $A_{FB}^{pol,\tau}$ at Z peak **Possible alternative input**
m_W (MeV)	80350 ± 15	**0.250**	**0.547**	**0.078**	
Γ_ℓ (keV)	83985 ± 86	**0.2**	**0.53**	**0.076**	stat. based on muon pair statistics NB cleanest determination of ρ parameter
R_ℓ^Z $(\times 10^3)$	20767 ± 25	**0.06**	**0.17**	**0.025**	ratio of hadrons to leptons determination of strong coupling constant
R_b $(\times 10^6)$	216290 ± 660	**0.3**	**0.42**	**0.06**	ratio of $b\bar{b}$ to hadrons test of N.P. coupled to 3d generation
Γ_Z (keV)	2495200 ± 2300	**4**	**28**	**4**	From Z line shape scan Beam energy calibration

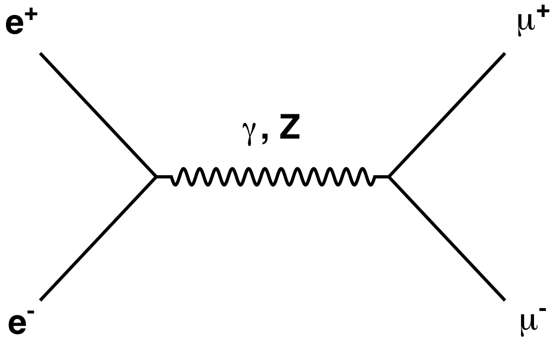
Must measure $\alpha_{QED}(m_Z)$ with the best possible precision

For the other EW observables, use $\sin^2\theta_{W,eff}$ as alternative input, much better measured at FCC-ee

A. Blondel, 07/22

First attempt: the $e^+e^- \to \mu^+\mu^-$ process

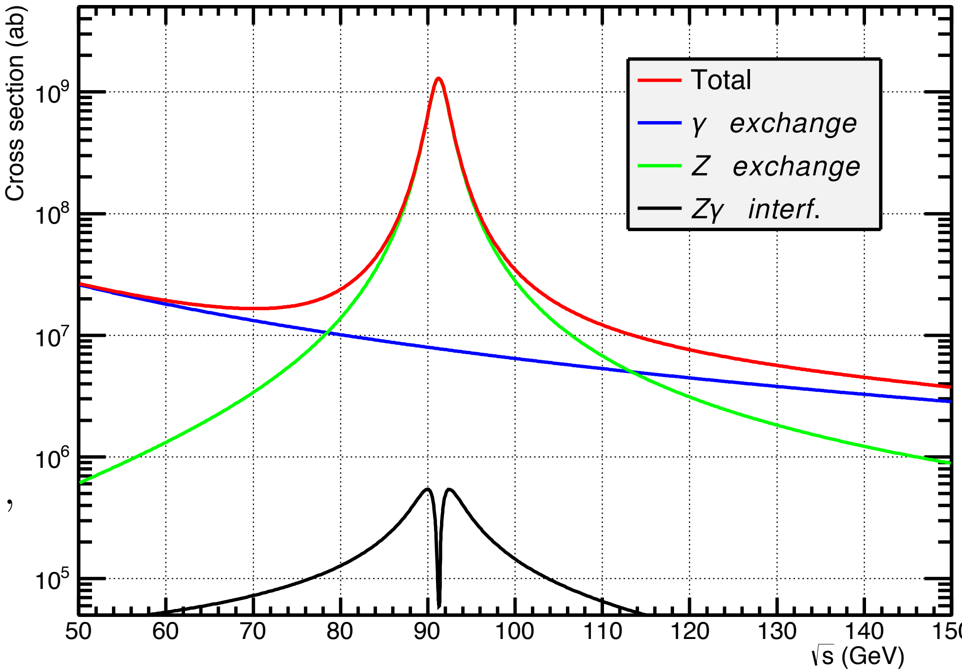
- **Production cross section (improved Born approx.)**
-



$$\mathcal{G} = \frac{c_\gamma^2}{s},$$

$$\mathcal{Z} = \frac{c_Z^2 (v^2 + a^2)^2 \times s}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2},$$

$$\mathcal{I} = \frac{2 c_\gamma c_Z v^2 \times (s - m_Z^2)}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2},$$



$$c_\gamma = \sqrt{\frac{4\pi}{3}} \alpha_{\mathrm{QED}}(s), \quad c_Z = \sqrt{\frac{4\pi}{3} \frac{m_Z^2}{2\pi} \frac{G_F}{\sqrt{2}}}, \quad a = -\frac{1}{2}, \quad v = a \times (1 - 4 \sin^2 \theta_W),$$

(~ -0.037)

- Z exchange proportional to G_F^2
- γ exchange proportional to $\alpha^2(s)$
- Interference term proportional to $\alpha(s)\ G_F$

} Sensitivity to $\alpha_{\mathrm{QED}}(s)$

  - Largest sensitivity below the Z pole with the γ exchange term
    - ➔ Extrapolation issue, not a "direct" measurement, not (yet) in the baseline programme
  - Absolute cross section measurements challenging to the required precision
    - ➔ Absolute luminosity determination, absolute selection efficiency / acceptance simulation

First attempt: the $e^+e^- \longrightarrow \mu^+\mu^-$ process

Angular distribution and forward-backward asymmetry

Tree level:

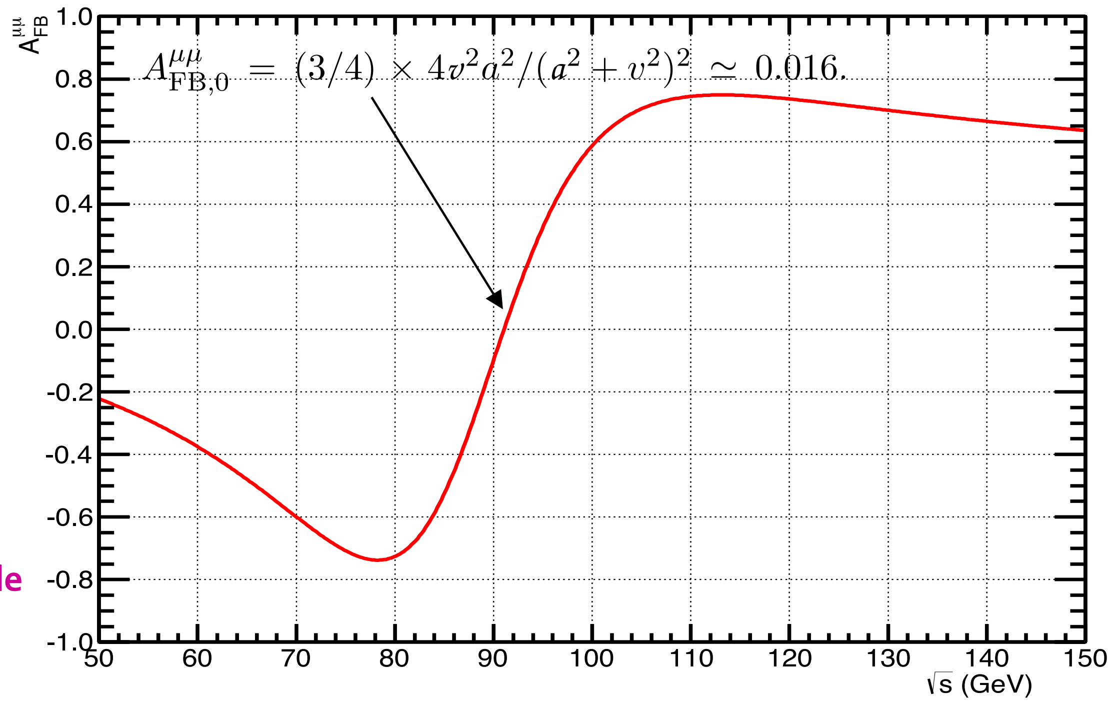
$$\frac{d\sigma_{\mu\mu}}{d\cos\theta}(s) \propto G_1(s) \times (1+\cos^2\theta) + G_3(s) \times 2\cos\theta,$$

$$G_1(s) = \mathcal{G} + \mathcal{I} + \mathcal{Z} \quad \text{and} \quad G_3(s) = \frac{a^2}{v^2}\left\{\mathcal{I} + \frac{4v^4/a^4}{(1+v^2/a^2)^2}\mathcal{Z}\right\}$$

$$A_{\rm FB}^{\mu\mu} = \frac{\sigma_{\mu\mu}^{\rm F} - \sigma_{\mu\mu}^{\rm B}}{\sigma_{\mu\mu}^{\rm F} + \sigma_{\mu\mu}^{\rm B}}, \quad \Longrightarrow \quad A_{\rm FB}^{\mu\mu}(s) = \frac{3}{4}\frac{G_3(s)}{G_1(s)}.$$

$$A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3}{4}\frac{a^2}{v^2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}}.$$

Maximal close to the Z pole
Proportional to $\alpha_{\rm QED}(s)$



- **In the vicinity of the Z pole, the slope of $A_{\rm FB}$ is proportional to $\alpha_{\rm QED}(m_Z)$**
  - Direct measurement of $\alpha_{\rm QED}(m_Z)$
  - $A_{\rm FB}$ is a self-normalizing quantity (no uncertainty from luminosity measurement)
  - If properly handled, the selection efficiency also disappears in the ratio (see later)

FCC-ee target : Statistical uncertainty

- **Statistical uncertainty on A_{FB}**

$$\sigma\left(A_{\rm FB}^{\mu\mu}\right) = \sqrt{\frac{1 - A_{\rm FB}^{\mu\mu\,2}}{\mathcal{L}\sigma_{\mu\mu}}}.$$

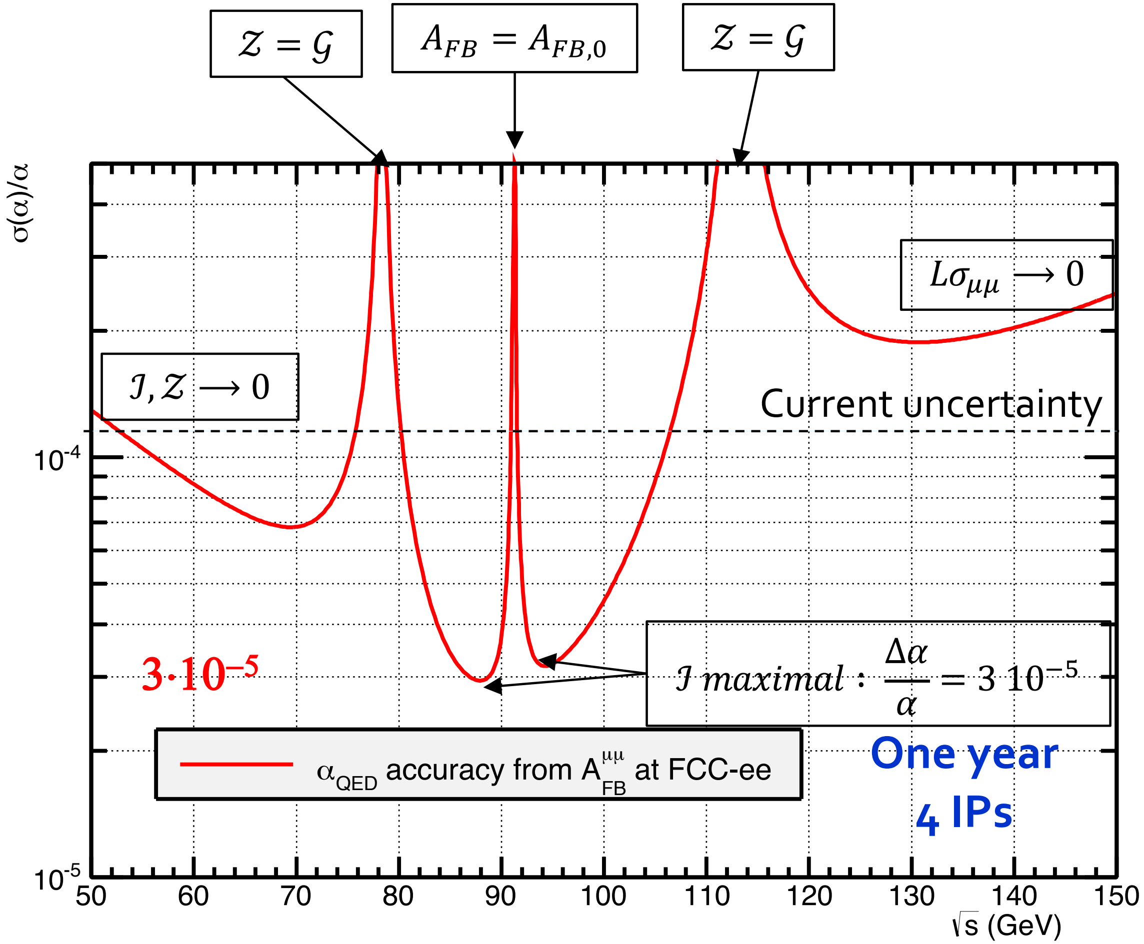
- **Variation of A_{FB} with α**

$$A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3}{4}\frac{a^2}{v^2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}}.$$

$$\frac{d\mathcal{Z}}{d\alpha} = 0;\ \frac{d\mathcal{I}}{d\alpha} = \frac{\mathcal{I}}{\alpha};\ \frac{d\mathcal{G}}{d\alpha} = \frac{2\mathcal{G}}{\alpha}$$

$$\frac{\Delta A_{FB}}{\Delta\alpha} = \frac{1}{\alpha}\frac{3}{4}\frac{a^2}{v^2}\frac{\mathcal{I}(\mathcal{Z}-\mathcal{G})}{(\mathcal{G}+\mathcal{Z})^2} = (A_{FB} - A_{FB,0})\times\frac{\mathcal{Z}-\mathcal{G}}{\mathcal{Z}+\mathcal{G}}\times\frac{1}{\alpha}$$

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu} - A_{\rm FB,0}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}} \simeq \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}},$$



$\sqrt{s_-}$=87.9 GeV

$\sqrt{s_+}$=94.3 GeV

Extracting $\alpha(m_Z)$ from A_{FB}: two methods

- **From two measurements $A_{FB}(s_-)$ and $A_{FB}(s_+)$**
  - Extract two values of α

$$\alpha_- \equiv \alpha_{\rm QED}(s_-) \text{ and } \alpha_+ \equiv \alpha_{\rm QED}(s_+),$$

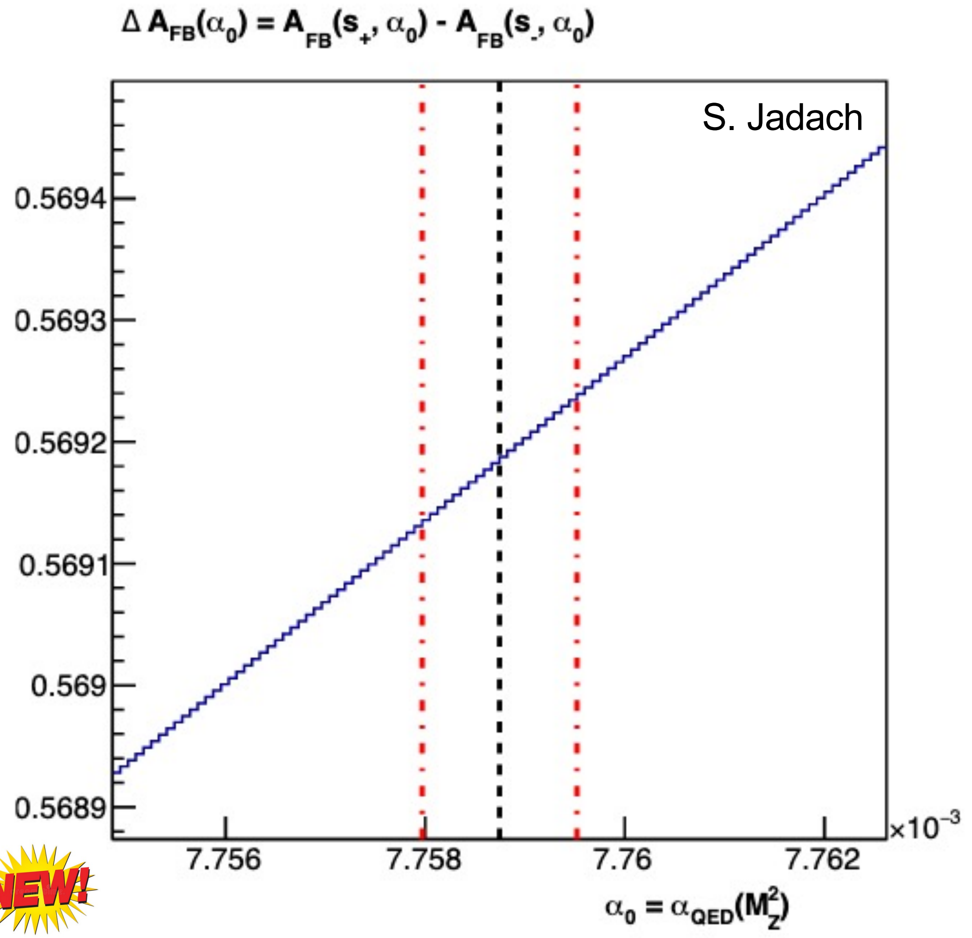
  - Run from $s_\pm$ to m_Z : two determinations of α_0

$$\frac{1}{\alpha_0} = \frac{1}{\alpha_\pm} + \beta \log \frac{s_\pm}{m_Z^2},$$

  - Solve for $\alpha_0 = \alpha(m_Z)$ exactly

$$\frac{1}{\alpha_0} = \frac{1}{2}\left(\frac{1-\xi}{\alpha_-} + \frac{1+\xi}{\alpha_+}\right), \quad \text{where} \quad \xi = \frac{\log s_- s_+/m_Z^4}{\log s_-/s_+} \simeq 0.045,$$

- **Solve directly for α_0 from $\Delta A_{FB} = A_{FB}(s_-) - A_{FB}(s_+)$**
  - Quasi-linear dependence, solve iteratively (exactly) NEW!



- **In both cases, almost exact cancellations for correlated effects at $s_\pm$**

Parametric uncertainties

- At (improved) born level, the asymmetry depends on m_Z, Γ_Z, $\sin^2\theta_{W,eff}$, G_F (and $\sqrt{s}$)
  - These parameters will be measured precisely at FCC-ee
    - Except G_F, which is already very precisely measured
  - The corresponding parametric uncertainties are negligible wrt the statistical error ($3\cdot10^{-5}$)
    - The Z mass is entirely correlated with the absolute $\sqrt{s}$ determination (next slide)
      - ➔ Negligible effect

Observables	Present value	FCC-ee stat.	FCC-ee exp. syst.	$\Delta\alpha/\alpha$
m_Z (keV)	91 187 500 ± 2100	4	100	–
Γ_Z (keV)	2 495 500 ± 2300 [*]	4	25	$5\cdot10^{-7}$
$\sin^2\theta_{W,eff}$ ($\times10^6$)	231 530 ± 160	1.5	?	10^{-6}
G_F ($\times10^6$)	1 166 378. 76 ± 0.51	–	–	$5\cdot10^{-7}$

UPDATED

UPDATED

Experimental uncertainties

- **Centre-of-mass energy calibration (with resonant depolarisation)**
  - At s = $s_{\pm}$, the energy dependence of AFB can be approximated as

$$A_{\rm FB}^{\mu\mu}(s, m_{\rm Z}) \propto (s - m_{\rm Z}^2)/(s m_{\rm Z}^2)$$

    - Error propagation

$$\frac{\sigma(A_{\rm FB}^{\mu\mu})}{A_{\rm FB}^{\mu\mu}} \simeq \frac{1}{\sqrt{s}m_{\rm Z}} \sqrt{\left(s + m_{\rm Z}^2 - \sqrt{s}m_{\rm Z}\right)^2 \frac{\sigma_D^2}{D^2} + \left(s + m_{\rm Z}^2 + \sqrt{s}m_{\rm Z}\right)^2 \frac{\sigma_\Sigma^2}{\Sigma^2}}$$

$$D = \sqrt{s} - m_{\rm Z}$$

$$\Sigma = (\sqrt{s} + m_{\rm Z})/2.$$

    - Dominant term : point-to-point calibration uncertainty D

$$\frac{\sigma(\alpha_\pm)}{\alpha_\pm} \simeq \frac{\sigma_{D_\pm}}{D_\pm}, \quad \text{with} \quad D_\pm = \sqrt{s_\pm} - m_{\rm Z}$$

Method 1

or

UPDATED

$$\frac{\sigma(\alpha_0)}{\alpha_0} \approx \frac{\sigma_{\sqrt{s_+}-\sqrt{s_-}}}{\sqrt{s_+} - \sqrt{s_-}}$$

40 keV

6 GeV

NEW!

Method 2

  - Related uncertainty : $6{\cdot}10^{-6}$ UPDATED

Experimental uncertainties

- Centre-of-mass energy spread (~0.12% around the Z pole)
  - On average, modify the asymmetry as follows

$$\Delta A_{\rm FB}^{\mu\mu}(s_\pm) = \frac{\int A_{\rm FB}^{\mu\mu}(s)\sigma_{\mu\mu}(s)\exp -\frac{\left(\sqrt{s}-\sqrt{s_\pm}\right)^2}{2s_\pm\delta^2}d\sqrt{s}}{\int \sigma_{\mu\mu}(s)\exp -\frac{\left(\sqrt{s}-\sqrt{s_\pm}\right)^2}{2s_\pm\delta^2}d\sqrt{s}} - A_{\rm FB}^{\mu\mu}(s_\pm)$$

NEW!

    - The steep slope of $\sigma_{\mu\mu}$ tends to increase (decrease) the average $\sqrt{s}$ above $\sqrt{s_-}$ (below $\sqrt{s_+}$)
  - The energy spread tends to decrease the difference $A_{FB}(s_+)$ - $A_{FB}(s_-)$ by about 10^{-3} (relative)
    - With a similar relative effect on $\alpha(m_Z)$
  - This effect is two orders of magnitude larger than the statistical uncertainty
    - It must be corrected for, e.g., by a measurement of the centre-of-mass energy spread
      - ➔ Typically with a precision better than 1% to reduce the uncertainty on $\alpha(m_Z)$ to 10^{-5} NEW!

Experimental uncertainties

- **In-situ measurement of the centre-of-mass energy spread with the same events !** NEW!
  - Energy spread = relative longitudinal boost $x_\gamma = p_z^{miss} / \sqrt{s}$
  - Full spectrum obtained from μ directions and E,p conservation

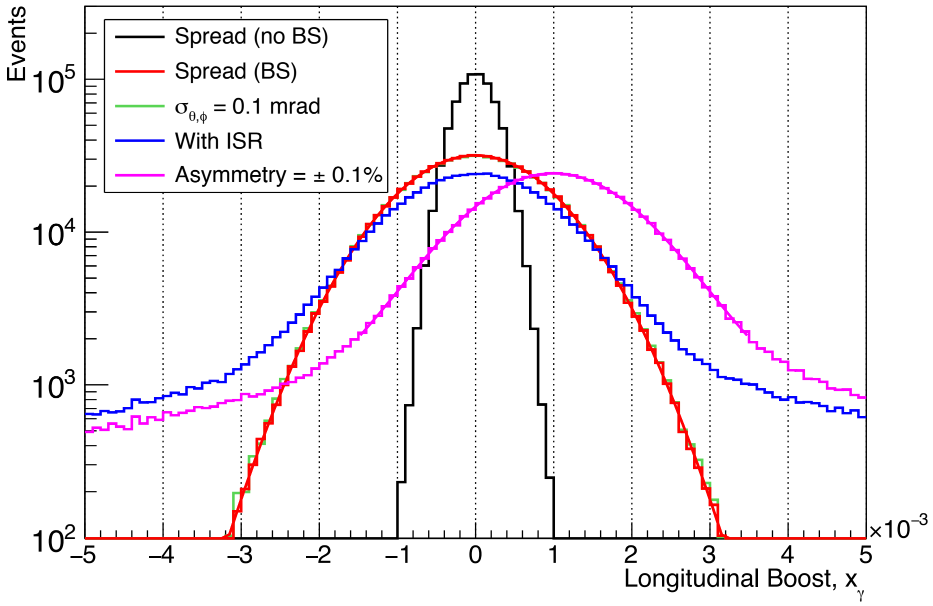
$$x_\gamma = -\frac{x_+ \cos\theta^+ + x_- \cos\theta^-}{\cos(\alpha/2) + |x_+ \cos\theta^+ + x_- \cos\theta^-|}$$

$$x_\pm = \frac{\mp \sin\theta^\mp \sin\varphi^\mp}{\sin\theta^+ \sin\varphi^+ - \sin\theta^- \sin\varphi^-} \qquad \alpha = 2\arcsin\left[\frac{\sin(\varphi^- - \varphi^+)\sin\theta^+\sin\theta^-}{\sin\varphi^-\sin\theta^- - \sin\varphi^+\sin\theta^+}\right]$$

- Method also provides absolute directions wrt the beams
- Requires ~0.1 mrad angular resolution or better
- Good ISR description needed: to be checked

1% precision every 10-20 seconds at $\sqrt{s_+}$ and $\sqrt{s_-}$

One million dimuon events



- Even better: it is an event-by-event measurement of $\sqrt{s}$
  - Automatic self-calibrated energy scan
    - ➔ Gives the possibility of measuring $A_{FB}(s)$ around s_+ and s_- without $\sqrt{s}$-spread-related uncertainty

Experimental uncertainties

- **Selection efficiency and acceptance : getting rid of it experimentally**
  - **At tree level, the angular distribution of $\mu^\pm$ reads**

$$\frac{dN^\pm}{dc} \propto \left\{ \frac{3}{8}(1+c^2) \pm A(s)c \right\} \times \varepsilon(c)$$

- c = cosθ in the μμ cm frame
- A(s) = asymmetry parameter (=A_{FB})
- Ɛ(c) = event selection efficiency

  - **The selection efficiency Ɛ(c) is eliminated in the charge asymmetry $A_Q(c)$**

$$A_Q(c) = \frac{N^+(c) - N^-(c)}{N^+(c) + N^-(c)} \implies A(s) = \kappa(c) A_Q(c), \text{ with } \kappa(c) = \frac{3}{8}\frac{1+c^2}{c}$$

- **Average over all c values returns the optimal statistical precision on A(s)**
  - ➔ Singularity at c = 0 can be avoided by rejecting these events, which carry no information on A(s)
- **Electric-charge-dependent efficiency can be measured in situ with T&P method (e.g. at the Z)**

- **Other experimental uncertainties are found to be negligible**
  - **Charge inversion, tau background, angular resolution (at tree level), etc.**

Theoretical uncertainties

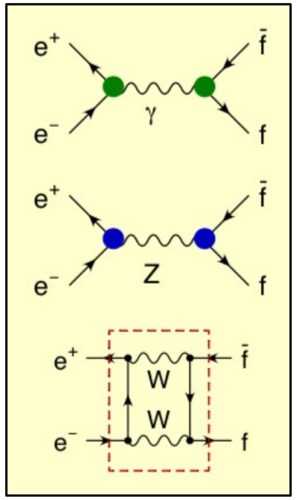
- Impact of missing higher orders on A_{FB} prediction from A. Freitas (2016)

Current status: full 1-loop for g_γ^ℓ, $g_Z^{\ell\prime}$, S_{box}

Bardin et al. '99 (ZFITTER 6.21)

Relative impact on $A_{FB}^{\mu\mu}(s_2) - A_{FB}^{\mu\mu}(s_1)$ for $\sqrt{s_1} = 88$ GeV, $\sqrt{s_2} = 95$ GeV:

g_γ^ℓ 1-loop:	2×10^{-4}
g_Z^ℓ 1-loop:	1×10^{-3}
S_{box} 1-loop:	1×10^{-3}



Expected impact of future corrections (order of magnitude):

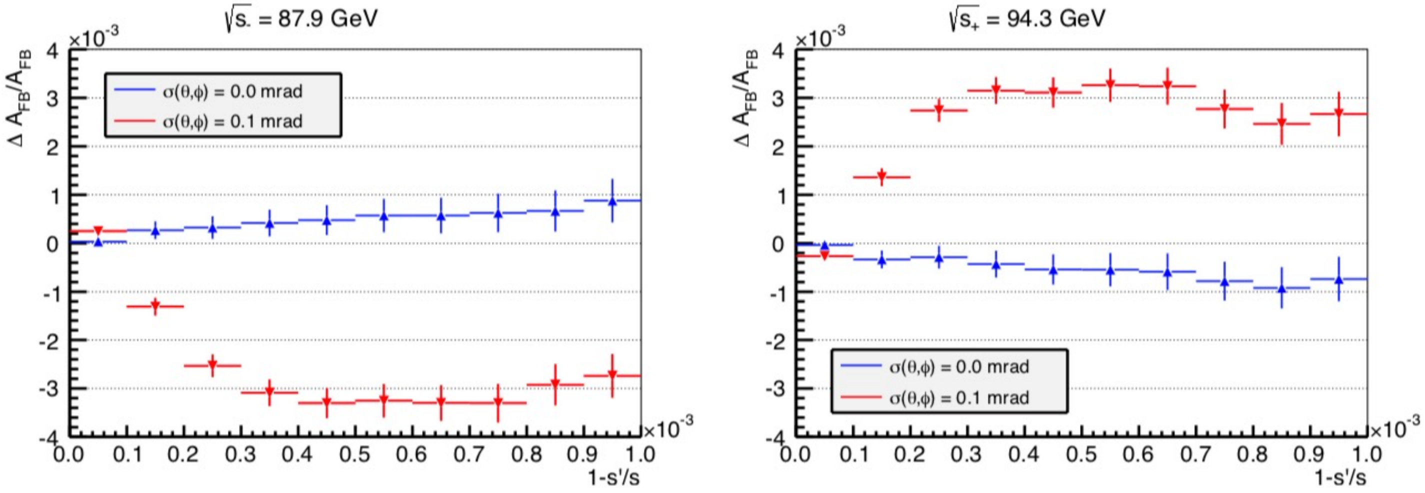
$\mathcal{O}(\alpha)$	$\sim 2 \times 10^{-3}$	(known)
$\mathcal{O}(\alpha\alpha_s)$	$\sim 7 \times 10^{-4}$	(current techniques)
$\mathcal{O}(\alpha\alpha_s^2)$	$\sim 2 \times 10^{-4}$	(current techniques)
$\mathcal{O}(N_f\alpha^2)$	$\sim 2 \times 10^{-4}$	(current techniques)
$\mathcal{O}(\alpha_{\text{bos}}^2)$	$\lesssim 10^{-4}$	(new methods)
$\mathcal{O}(\alpha^2\alpha_s)$	$\sim 7 \times 10^{-5}$	(new methods)
$\mathcal{O}(N_f^2\alpha^3)$	$\sim 2 \times 10^{-5}$	(new methods)
$\mathcal{O}(N_f\alpha^3)$	$< 10^{-5}$	(speculative)
$\mathcal{O}(N_f\alpha^2\alpha_s^2)$	$\sim 10^{-5}$	(speculative)

- Two- and three-loop calculations needed to match missing orders with statistics
  - These are estimates only – need to perform the actual calculation to know for sure
    - ➔ In particular to evaluate the level of cancellations in $A(s_+) - A(s_-)$

Theoretical uncertainties

- **Initial state radiation : several consequences on A_{FB} measurement**
  - Smear the centre-of-mass energy spread distribution
    - Will need to check if the current knowledge of ISR suffices for this purpose
  - Reduces the centre-of-mass energy to $s^* = s\,(1\!-\!2x_\gamma)$
    - Modifies in turn A(s) to A(s*) ; Solution: measure A as a function of s* (self-calibrated scan)
  - Modifies the angular distributions from the two muons with longitudinal boost x_γ
    - Solution: Boost back to the centre-of-mass energy frame (with the knowledge of x_γ)
  - Result : [A_{FB}-A_{FB}(SM)] / A_{FB} as a function of $1\text{-}s^*/s$ exhibits large measurement biases
    - Biases originate from
      - → ISR angular distribution
      - → Several ISR photons
      - → Muon angular resolution
      - → Beam energy spread

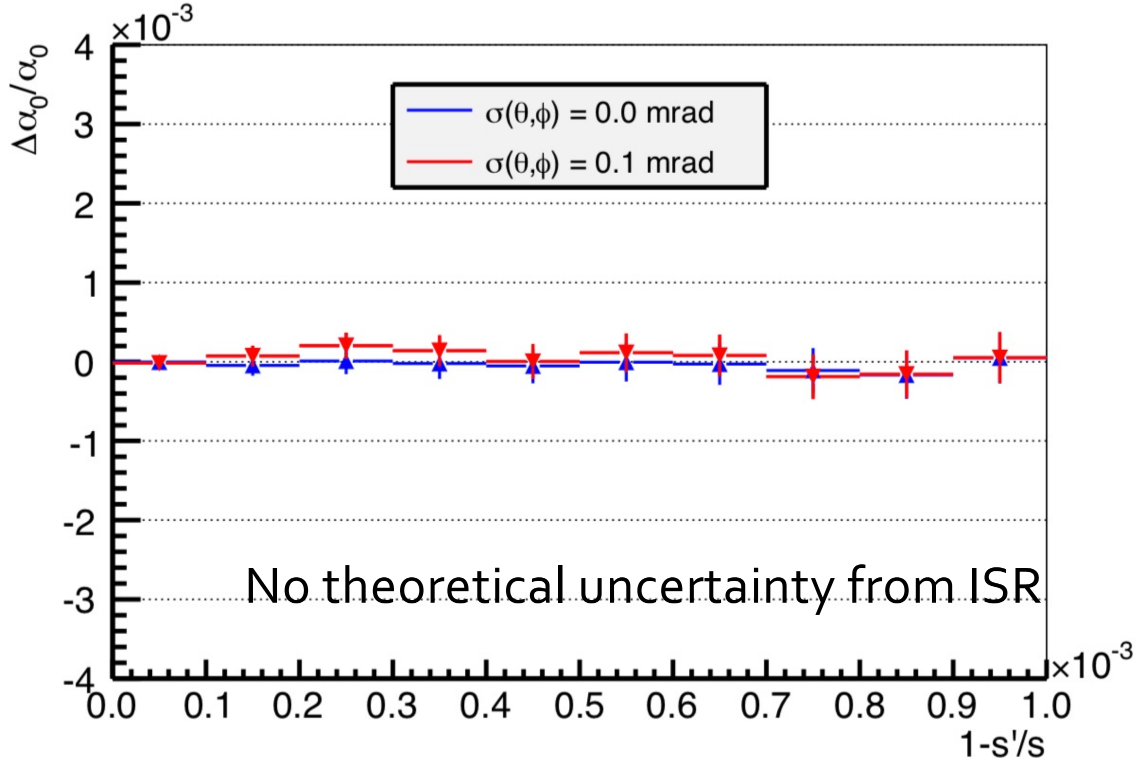
        (not shown here)
    - Effects common to s_+ and s_-



Theoretical uncertainties

- Initial state radiation (ISR): consequences mitigated for $\alpha(m_Z)$
  - Biases on A_{FB} are two orders of magnitude larger than statistical target
    - They appear to be universal at both centre-of-mass energies
      - ➔ With perfect cancelation in the difference $A(s_+) - A(s_-)$

<u>IFF muon angular resolution better than 0.1 mrad</u>



Theoretical uncertainties

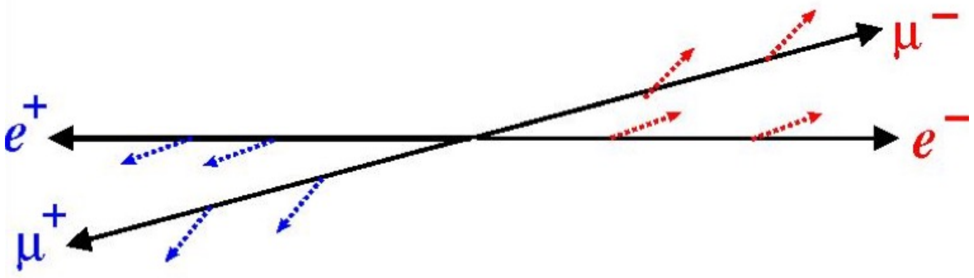
- **Final state radiation (FSR)**
  - Mostly collinear
    - Effect on the muon direction (and the determination of s*) very much suppressed
  - Symmetric around the muon directions at all orders in α
    - Effect on A_{FB} expected to be unmeasurably small on average
  - Residual tiny biases on the asymmetry parameter independent of $\sqrt{s}$
    - Expect exact cancellation anyway in the difference $\Delta A = A(s_+) - A(s_-)$

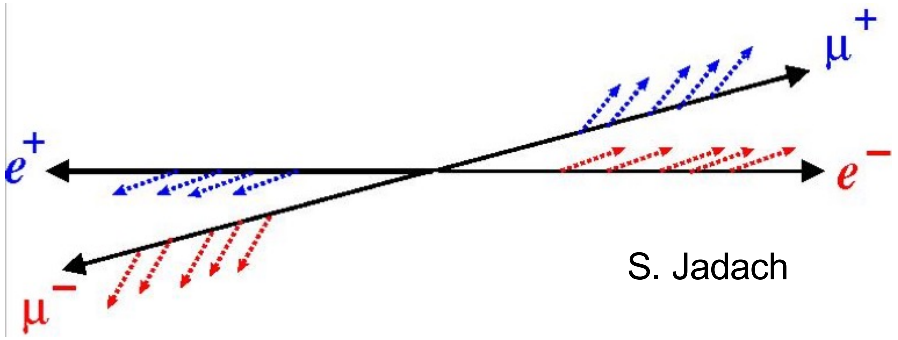
Theoretical uncertainties

- **Initial-final state radiation interference (IFI)**

NEW!

  - Angular distribution modified with another totally asymmetric function





IFI ≈ $Q_{\mu-} \times Q_{e-}$ or $Q_{\mu+} \times Q_{e+} > 0$

IFI ≈ $Q_{\mu+} \times Q_{e-}$ or $Q_{\mu-} \times Q_{e+} < 0$

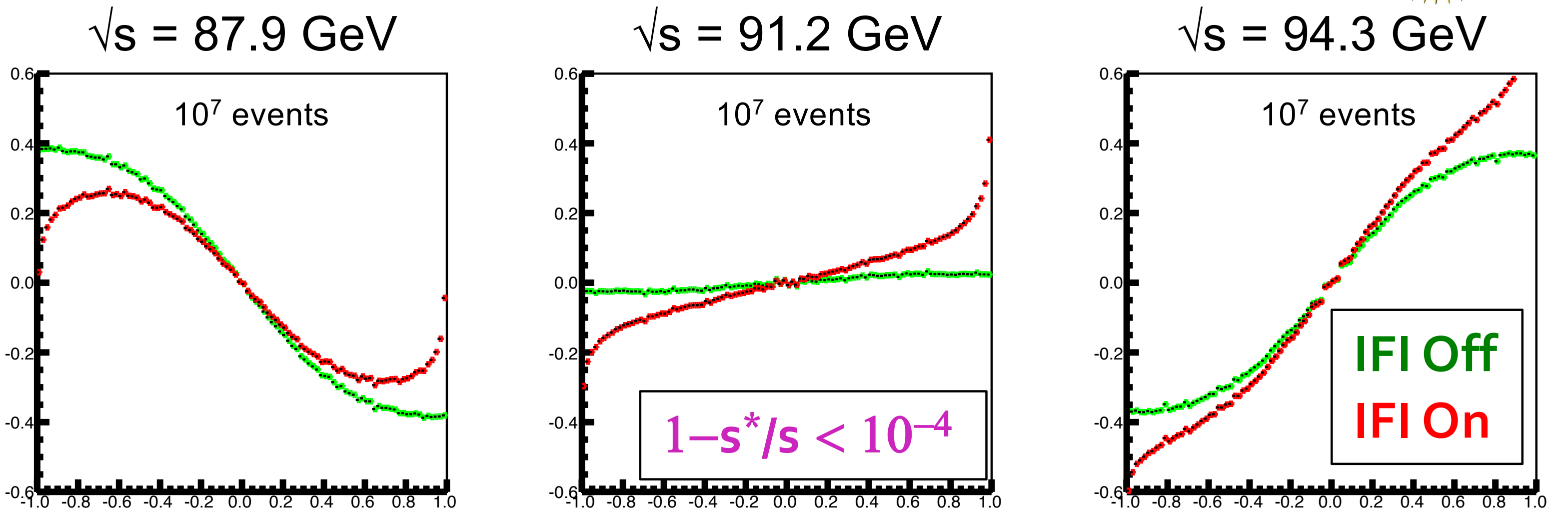
$$\frac{dN^{\pm}}{ds^* dc^*}(s, s^*, c^*) \propto \left\{ \frac{3}{8}\left(1 + c^{*2}\right) \pm A(s^*)c^* \right\} \times \left[1 \pm f(s, s^*, c^*)\right] \times \varepsilon(c^*)$$

- Here, f is supposed to be an odd function of c*, without any loss of generality
  - ➔ The even part, if any, can safely be absorbed in the normalization factor, which disappears in the ratio

Theoretical uncertainties

- **Angular distributions and charge asymmetry distributions (here for $1-s^*/s < 10^{-4}$)** NEW!

Angular distributions

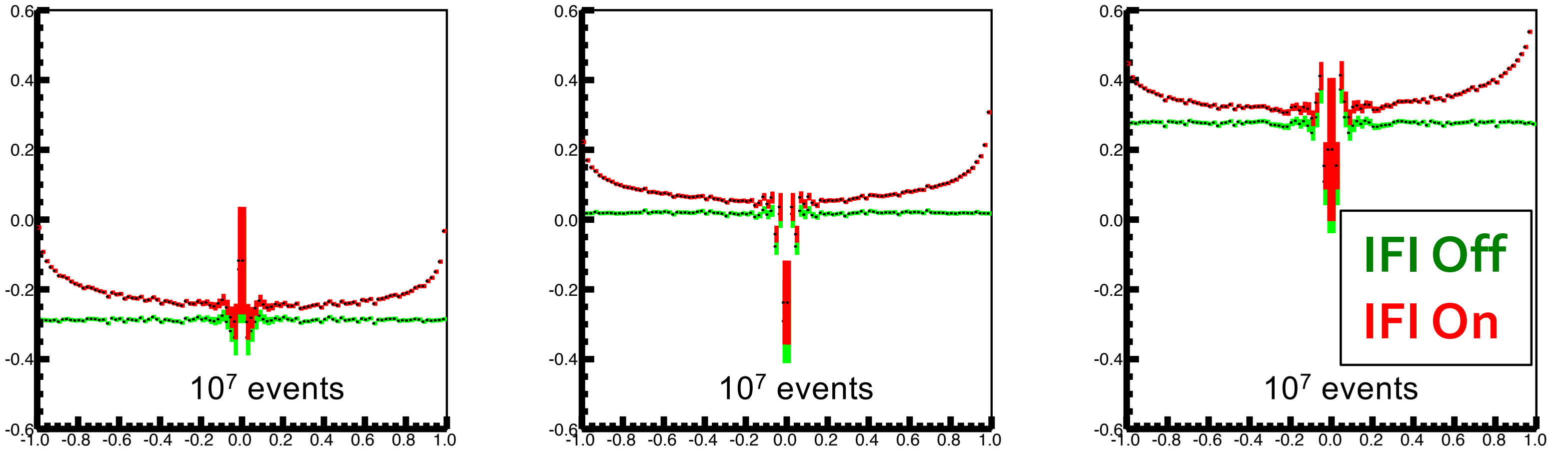


Charge asymmetries

$$A_Q = \frac{N^+ - N^-}{N^+ + N^-} = \frac{\kappa^* f + A}{\kappa^* + A f}, \text{ with } \kappa^* = \frac{3}{8}\frac{1+c^{*2}}{c^*}$$

10^7 events produced with KKMC by Scott Yost and Staszek Jadach

$\kappa^* A_Q^{on} \neq A$ and $\kappa^* A_Q^{off} = A$



$$A_Q^{on} = \frac{\kappa^* f_{on} + A}{\kappa^* + A f_{on}} \text{ and } A_Q^{off} = \frac{\kappa^* f_{off} + A}{\kappa^* + A f_{off}}$$

Theoretical uncertainties

- **Is it possible to fit the IFI function f away from the data ?**

From MC only

$$A_Q^{\rm on} = \frac{\kappa^* f_{\rm on} + A}{\kappa^* + A f_{\rm on}} \quad \text{and} \quad A_Q^{\rm off} = \frac{\kappa^* f_{\rm off} + A}{\kappa^* + A f_{\rm off}} \Longrightarrow \frac{f_{\rm on} - f_{\rm off}}{1 - f_{\rm on} f_{\rm off}} \; (\equiv f_{\rm on}) = \frac{A_Q^{\rm on} - A_Q^{\rm off}}{1 - A_Q^{\rm on} A_Q^{\rm off}}$$

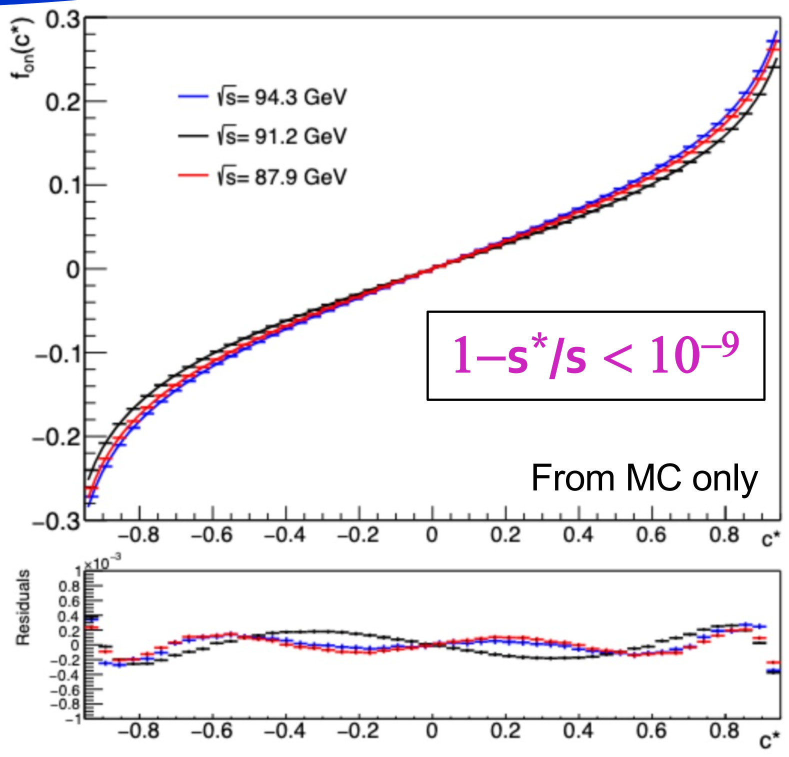
$$f_{\rm on} = \frac{A - \kappa^* A_Q^{\rm on}}{A A_Q^{\rm on} - \kappa^*}$$

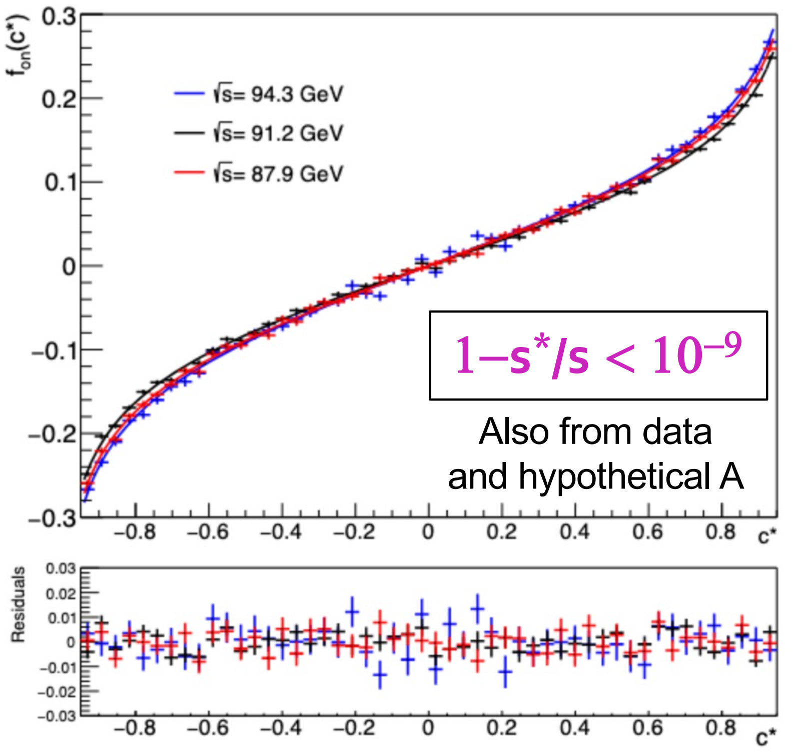
Also from data and hypothetical A

- **Fit to the function**

$$\left(a + b c^{*2}\right) \times \log\left[(1 + c^*)/(1 - c^*)\right]$$

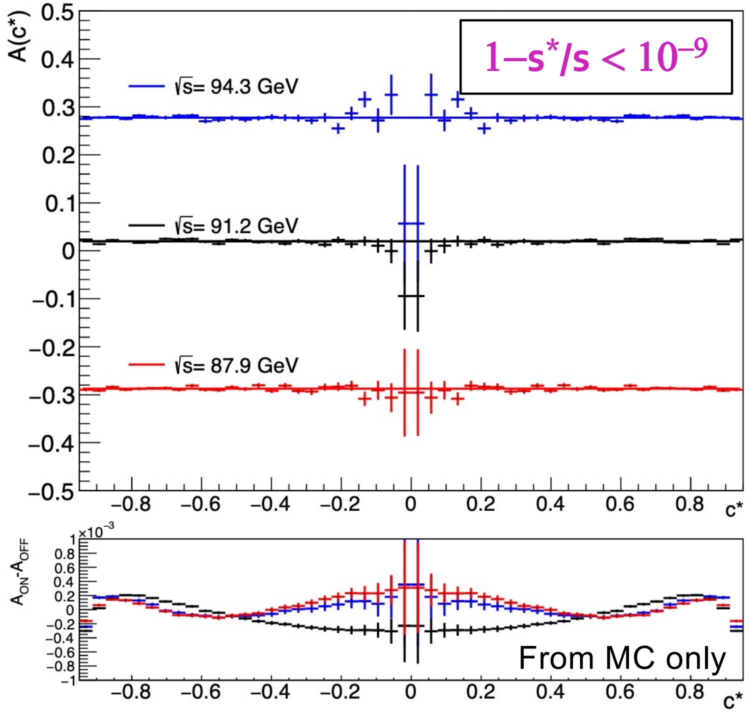
(with non universal a and b)

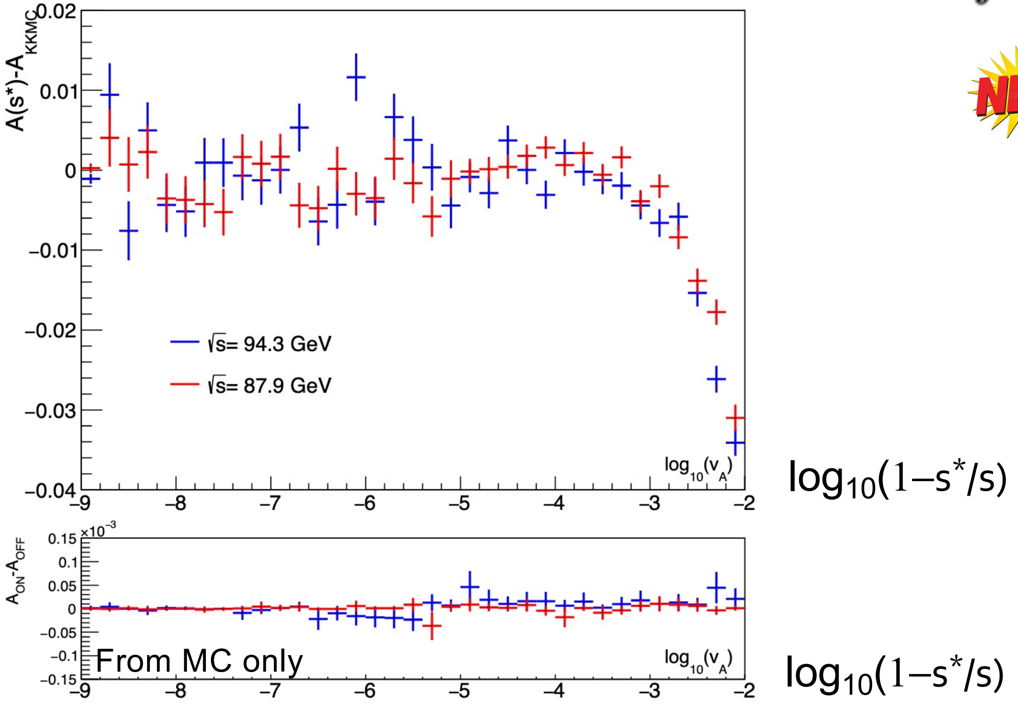




Theoretical uncertainties

- Re-inject the function f as obtained from the hypothetical A value
  - Determine (iteratively) a new value of the asymmetry parameter A with $A = \kappa^* \frac{A_Q - f}{1 - f A_Q}$



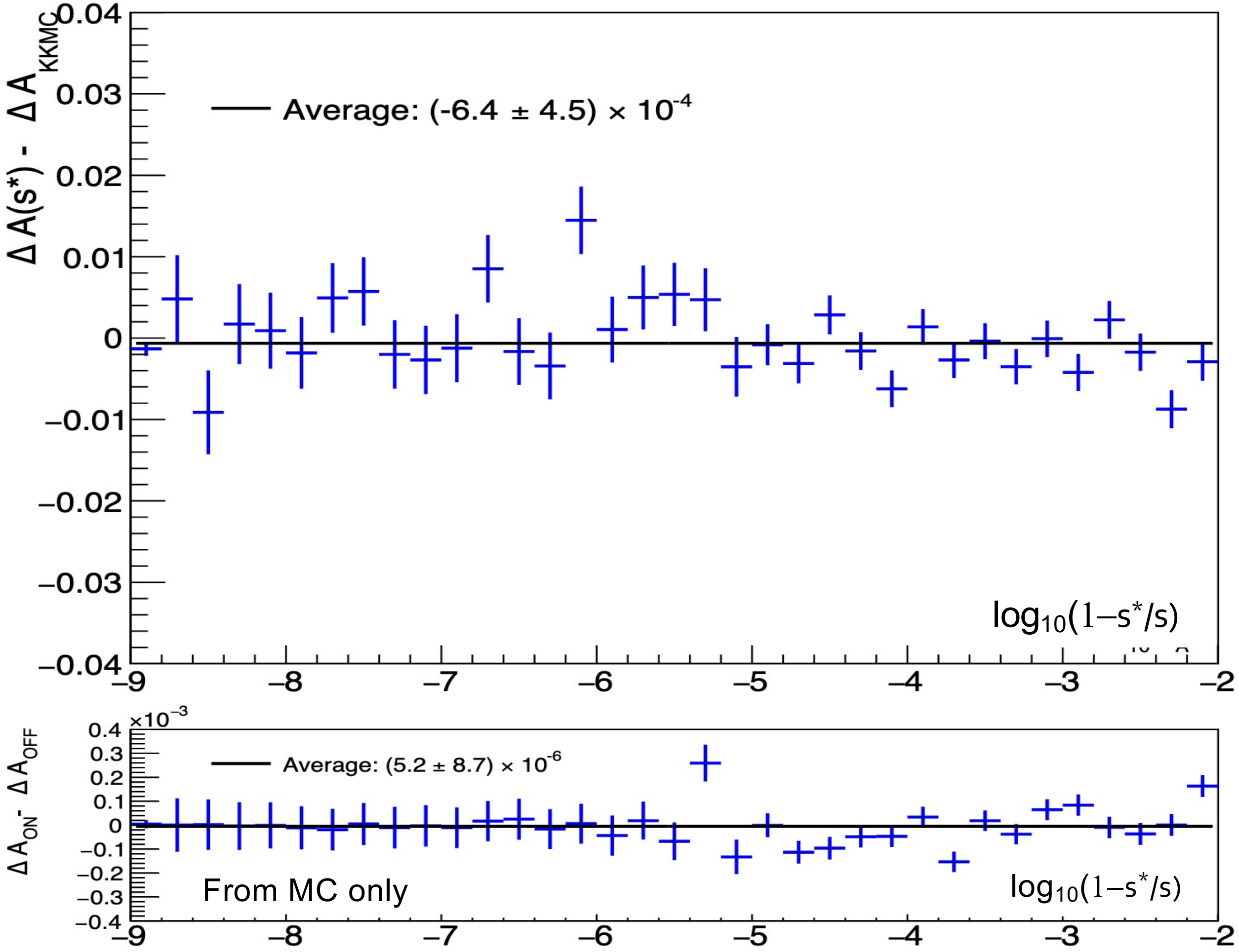


- Biases due to the imperfections of the functional form for f(c*)
  - ➔ Cancellation in $A(s_+)-A(s_-)$ at the level of 10^{-5} or less

Theoretical uncertainties

- **Absorbing biases in the difference $\Delta A = A(s_+) - A(s_-)$**

NEW!



$$\overline{\Delta A} - \Delta A_{\mathrm{KKMC}} = (-6.4 \pm 4.5) \times 10^{-4}$$

ΔA compatible with input value
(would need more statistics)

$$\Delta A_{ON} - \Delta A_{OFF} = (5.2 \pm 8.7)\times 10^{-6}$$

ΔA with IFI on = ΔA with IFI off
= ΔA with ISR off
(within statistical target)

Theoretical uncertainties

- **Does it mean that no theoretical effort is needed for IFI prediction ?**

NOT AT ALL !

- The previous plots about IFI (slides 16 – 19) were produced at generator level
  - No $\sqrt{s}$ spread
  - No muon angular resolution

- These two experimental realities affect the calculation of s^* and mix with IFI (also ISR)
  - Which in turn will create more biases on the determination of $f(s^*,c^*)$ and $A(s^*)$
    - ➔ Though we might still expect cancellations in the difference $\Delta A = A(s_+)-A(s_-)$

      TO BE CHECKED THOROUGHLY !

- The determination of $\sin^2\theta_{W,eff}$ relies only on the asymmetry parameter at $\sqrt{s}$ = mZ
  - No difference is at play in that case to cancel the ISR and IFI biases
    - ➔ ISR/IFI need to be predicted with a precision suited to match the 1.5×10^{-6} statistical target on $\sin^2\theta_{W,eff}$

Summary after this first feasibility study

- A direct measurement of $\alpha_{QED}(m_Z)$ with dimuon events at FCC-ee is statistics limited
  - Negligible exp'tal errors
  - Negligible parametric errors
  - ISR and IFI seem under control
    - To be checked in full detail
  - Bottleneck today is A_{SM} prediction
    - Full NNNLO needed ?

Type	Source	Uncertainty
Experimental (UPDATED)	E_{beam} calibration	6×10^{-6}
	E_{beam} spread	$< 10^{-5}$
	Acceptance and efficiency	negl.
	Charge inversion	negl.
	Backgrounds	negl.
Parametric (UPDATED)	m_Z and Γ_Z	5×10^{-7}
	$\sin^2 \theta_W$	1×10^{-6}
	G_F	5×10^{-7}
Theoretical (UPDATED)	QED (ISR, FSR, IFI)	$< 10^{-6}$
	Missing EW higher orders	few 10^{-4}
	New physics in the running	0.0
Total (except missing EW higher orders)	Systematics	$< 10^{-5}$
	Statistics (1 year, 4 IP)	3×10^{-5}

- A full analysis is now needed with all effects (and correlations) studied together
  - Until now, effects have been studied either in isolation or in pairs

Outlook – Shopping list for newcomers

- Implement a full analysis beyond this feasibility study
  - And publish a paper !
- Check with full simulation / reconstruction
- Go to the full statistics to unveil other issues
  - 10^{11} dimuon events !
- Measure asymmetry parameter off-peak with more channels
  - Repeat with di-tau events
    - τ direction from the decay vertex position ? (check τ angular resolution)
  - What can be done with di-electron events ?
- Study indirect measurements with low-angle Bhabha events (Lumical)
  - Measure $\Delta\alpha_{had}(t)$ See, e.g., arXiv:1504.0228, arXiv:hep-ex/0505072 (OPAL), arXiv:hep-ex/0002035 (L3)