# Measuring the Dark Matter environments of black hole binaries with gravitational waves

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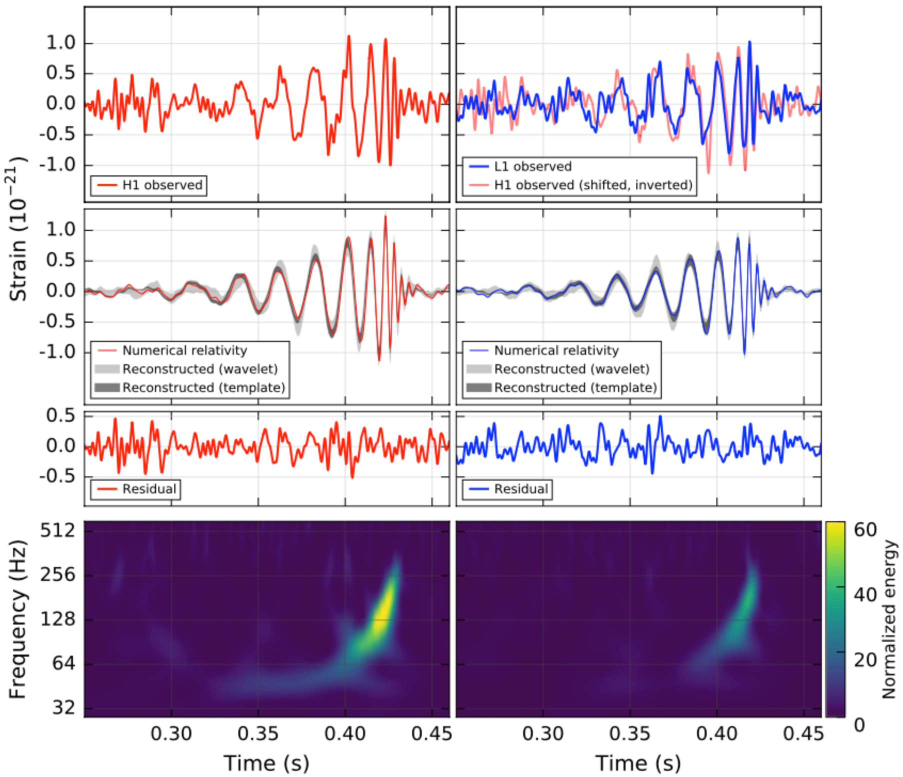
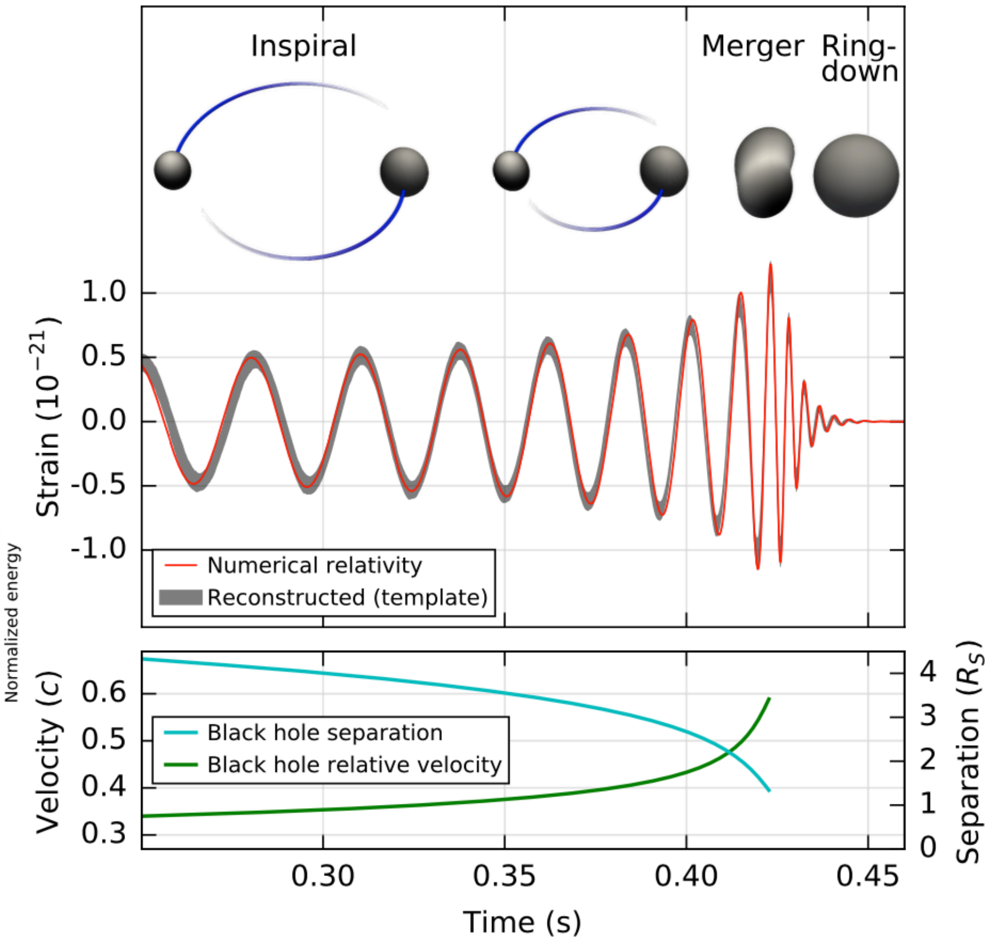
# First LIGO detection during O1: GW150914

(Abbott et al. PRL 116 (2016) 061102)



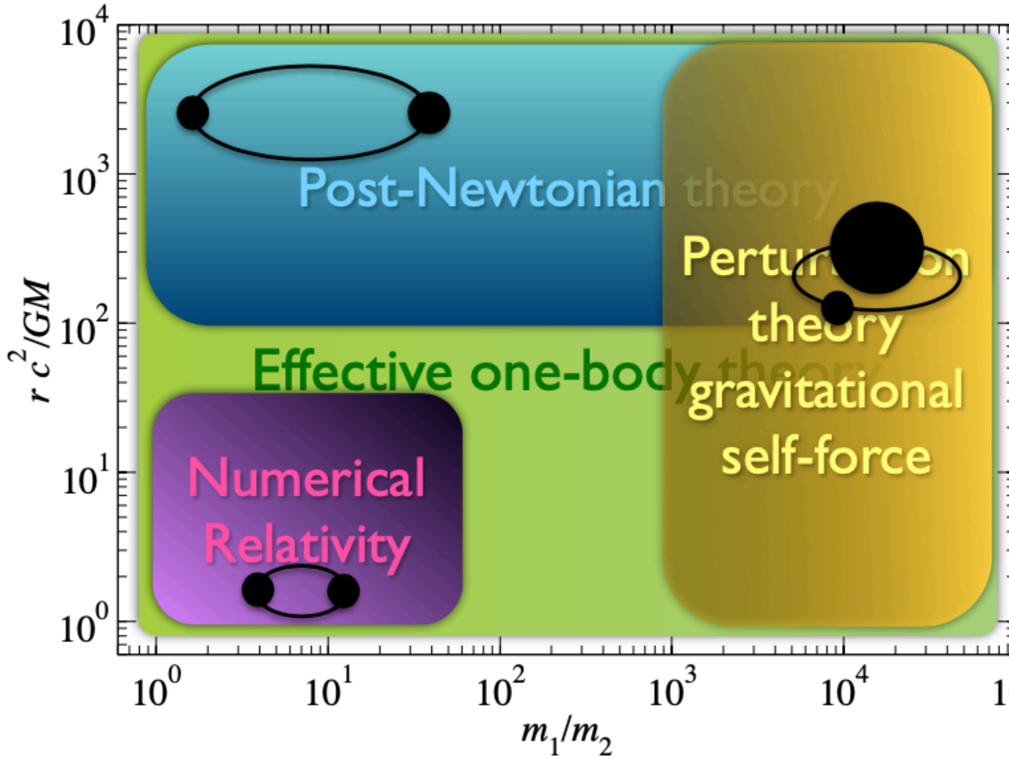

- **Black holes** of radius of 90 km at separation of 350 km are **making 75 orbits** per second **before merging**.
- **Black holes** collide at (almost) speed of light, like **fundamental particles.**
- **Gravitational waves** carry **fingerprints** of **source**.

# Solving two-body problem in General Relativity (including radiation)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

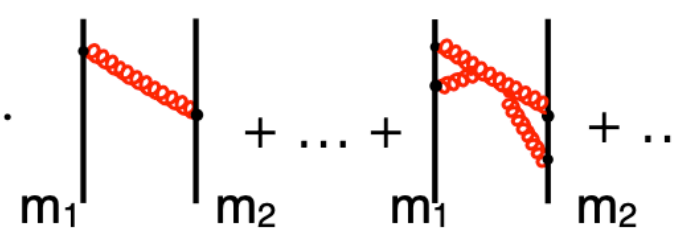
- **GR** is **non-linear theory**. Complexity similar to QCD.
- Einstein's field equations can be solved:
  - **approximately**, but **analytically** (**fast** way)
  - **exactly**, but **numerically** on supercomputers (**slow** way)

**$v^2/c^2 \sim GM/rc^2$**




(AB & Sathyaprakash 14)

- **Analytical methods**: post-Newtonian/post-Minkowskian/post-Test-Body expansions effective-one-body theory
  - **effective field-theory**, **dimensional regularization**, etc.
  - **diagrammatic** approach to organize expansions



## Tidal effects in the gravitational wave signal emitted in NS-NS binary coalescence

$$h(f) = \mathcal{A}(f) e^{i\psi(f)} \qquad \psi(f) = \psi_{PP} + \psi_{\bar{Q}} + \psi_{\bar{\lambda}}$$

$x = (m\pi f)^{5/3}$ PN expansion parameter

point-particle contribution

$$\psi_{PP}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{1 + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)x - (16\pi - 4\beta)x^{3/2} + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2 - 10\sigma\right)x^2 + \mathcal{O}(x^{5/2})\right\}$$

$\sigma$ contains spin-spin and spin-orbit terms. Note that it appears in the 2-PN term ($x^2$)

Quadrupole contribution:

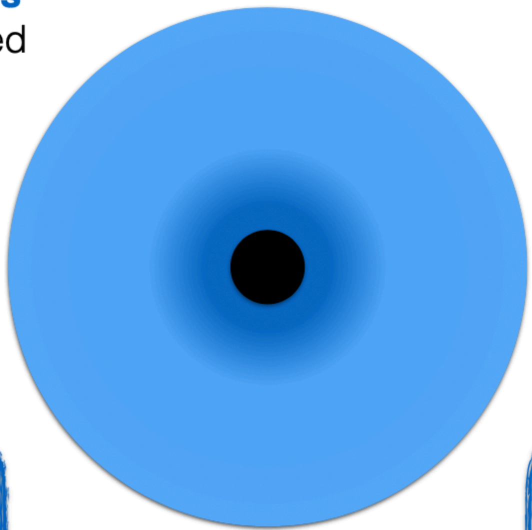
$$\psi_Q = \frac{3}{128}(\mathcal{M}\pi f)^{-5/3}\left\{-50\left[\left(\frac{m_1^2}{m^2}\chi_1^2 + \frac{m_2^2}{m^2}\chi_2^2\right)(Q_S - 1) + \left(\frac{m_1^2}{m^2}\chi_1^2 - \frac{m_2^2}{m^2}\chi_2^2\right)Q_a\right]x^2\right\}$$

$$Q_S = \frac{\bar{Q}_1 + \bar{Q}_2}{2}, \quad Q_a = \frac{\bar{Q}_1 - \bar{Q}_2}{2}$$

both the quadrupole moments and the spin terms appear at the 2-PN order and cannot be measured independently : in this sense we say that there is complete degeracy

# Dark Matter Spikes

Consider now a cold **DM** '**spike**' or '**dress**' around the central BH (not to be confused with ultralight boson clouds).



Study the following benchmarks:

$$m_1 = 1 M_\odot$$
$$m_2 = 10^{-2} - 10^{-4} M_\odot$$
$$\rho_{\rm DM} = \rho_6 \left(\frac{10^{-6}\,\rm pc}{r}\right)^{\gamma_{\rm sp}}$$

**Astrophysical scenario**

$$\gamma_{\rm sp} = 7/3 \approx 2.3333\ldots$$
$$\rho_6 \approx 5.45 \times 10^{15}\, M_\odot\, \rm pc^{-3}$$

...depending on a number of environmental factors...

[astro-ph/9906391, astro-ph/0509565, 1305.2619, ...]

**PBH scenario**

$$\gamma_{\rm sp} = 9/4 \approx 2.25$$
$$\rho_6 \approx 5.35 \times 10^{15}\, M_\odot\, \rm pc^{-3}$$

[Bertschinger (1985), astro-ph/0608642, 1901.08528, ...]


Gondolo, P. & Silk, J. 1999, Phys. Rev. Lett., 83, 1719.

Bertone, G. & Merritt, D. 2005, Phys. Rev. D, 72, 103502.

Ullio, P., Zhao, H., & Kamionkowski, M. 2001, Phys. Rev. D, 64, 043504.

Feng, W.-X., Parisi, A., Chen, C.-S., et al. 2021, arXiv:2112.05160

Eroshenko, Y. N. 2016, Astronomy Letters, 42, 347.

Boucenna, S. M., Kühnel, F., Ohlsson, T., et al. 2018, J. Cosmology Astropart. Phys., 2018, 003.

# Phase space distribution

Follow semi-analytically the phase space distribution of DM:

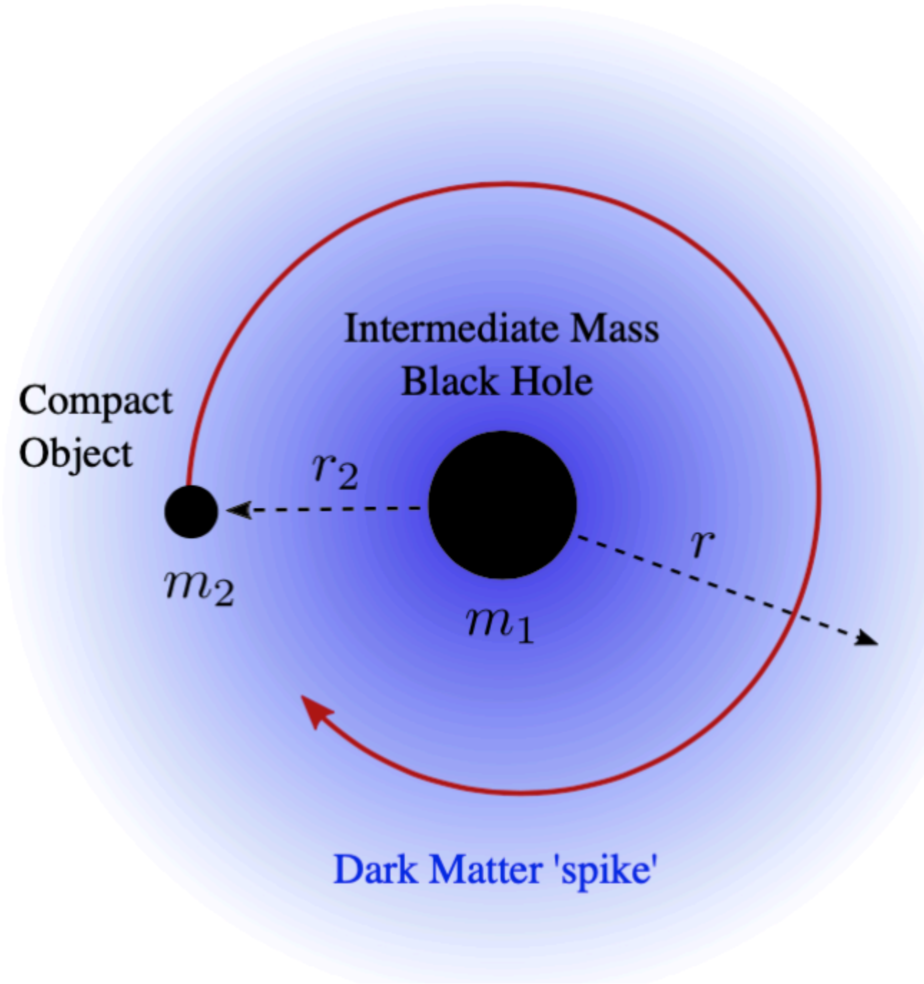
$$f = \frac{\mathrm{d}N}{\mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{v}} \equiv f(\mathcal{E})$$

$$\mathcal{E} = \Psi(r) - \frac{1}{2}v^2$$

Each particle receives a 'kick'

$$\mathcal{E} \to \mathcal{E} + \Delta\mathcal{E}$$

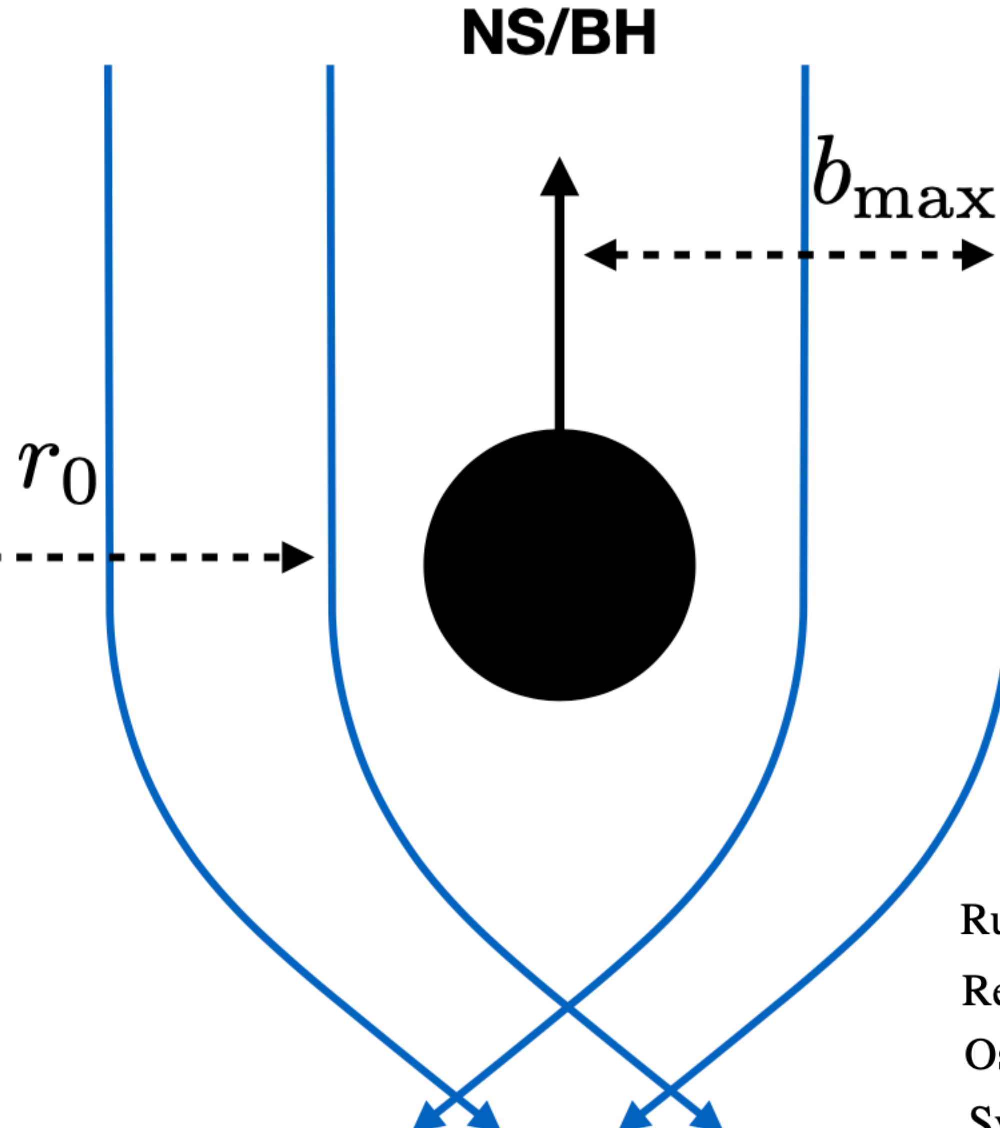
through gravitational scattering




Reconstruct density from distribution function:

$$\rho(r) = \int \mathrm{d}^3\mathbf{v}\, f(\mathcal{E})$$

# Dynamical Friction




$$\frac{dE_{DF}}{dt} = 4\pi(Gm_2)^2\rho_{DM}(r_2)\xi(v)v^{-1}\log\Lambda$$

$$\Lambda = \sqrt{\frac{b_{max}^2 + b_{90}^2}{b_{min}^2 + b_{90}^2}},$$


Chandrasekhar, S. 1943, ApJ, 97, 255.

Lee, E. P. 1969, ApJ, 155, 687.

Ruderman, M. A. & Spiegel, E. A. 1971, ApJ, 165, 1.

Rephaeli, Y. & Salpeter, E. E. 1980, ApJ, 240, 20.

Ostriker, E. C. 1999, ApJ, 513, 252.

Syer, D. 1994, MNRAS, 270, 205.

Barausse, E. 2007, MNRAS, 382, 826.

# Gravitational Wave

$$\frac{dE_{\rm orb}}{dt} = -\frac{dE_{\rm GW}}{dt} - \frac{dE_{\rm DF}}{dt}.$$

$$\frac{dE_{\rm GW}}{dt} = \frac{32G^4M(m_1m_2)^2}{5(cr_2)^5}.$$

$$\frac{dE_{\rm DF}}{dt} = 4\pi(Gm_2)^2\rho_{\rm DM}(r_2)\xi(v)v^{-1}\log\Lambda.$$

$$\dot{r}_2 = -\frac{64G^3Mm_1m_2}{5c^5(r_2)^3} - \frac{8\pi G^{1/2}m_2\rho_{\rm sp}\xi\log\Lambda r_{\rm sp}^{\gamma_{\rm sp}}}{\sqrt{M}m_1r_2^{\gamma_{\rm sp}-5/2}}$$

$$h_+(t) = \frac{4G_N\mu}{c^4D_L}\frac{1+\cos^2\iota}{2}(\omega r_2)^2\cos[2\Phi_{\rm orb}(t)+2\phi],$$

$$h_\times(t) = \frac{4G_N\mu}{c^4D_L}\cos\iota(\omega r_2)^2\sin[2\Phi_{\rm orb}(t)+2\phi],$$

$$E(v) = -\frac{1}{2}\eta M v^2\left(1+\#(\eta)v^2+\#(\eta)v^4+\ldots\right)$$

$$P(v) \equiv -\frac{dE}{dt} = \frac{32}{5G_N}v^{10}\left(1+\#(\eta)v^2+\#(\eta)v^3+\ldots\right)$$

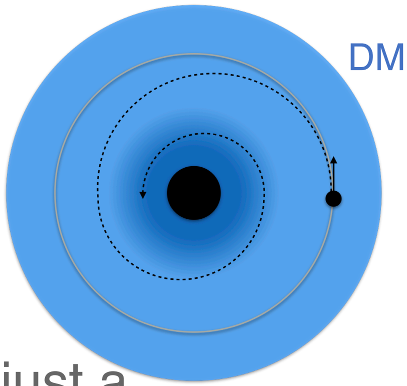
E(v)(P(v)) known up to 3(3.5)PN

$$\frac{1}{2\pi}\phi(T) = \frac{1}{2\pi}\int^T\omega(t)dt = -\int^{v(T)}\frac{\omega(v)dE/dv}{P(v)}dv$$

$$\sim \int\left(1+\#(\eta)v^2+\ldots+\#(\eta)v^6+\ldots\right)\frac{dv}{v^6}$$

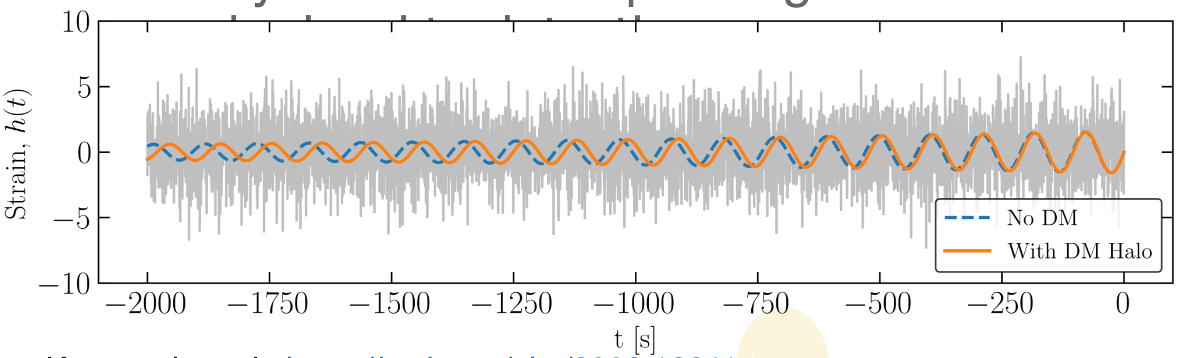
# Detecting DM with Einstein Telescope

- Presence of DM ‘spikes’ around BHs can alter inspiral dynamics




- GW waveform gradually goes out of phase with the corresponding
  vacuum-only waveform

- Possibility to detect and constrain dense DM ‘spikes’ with just a few cycles of GW ‘dephasing’ → but these subtle differences




Ideal case for
**Machine learning!**

Kavanagh et al., https://arxiv.org/abs/2002.12811
Coogan et al., https://arxiv.org/abs/2002.12811

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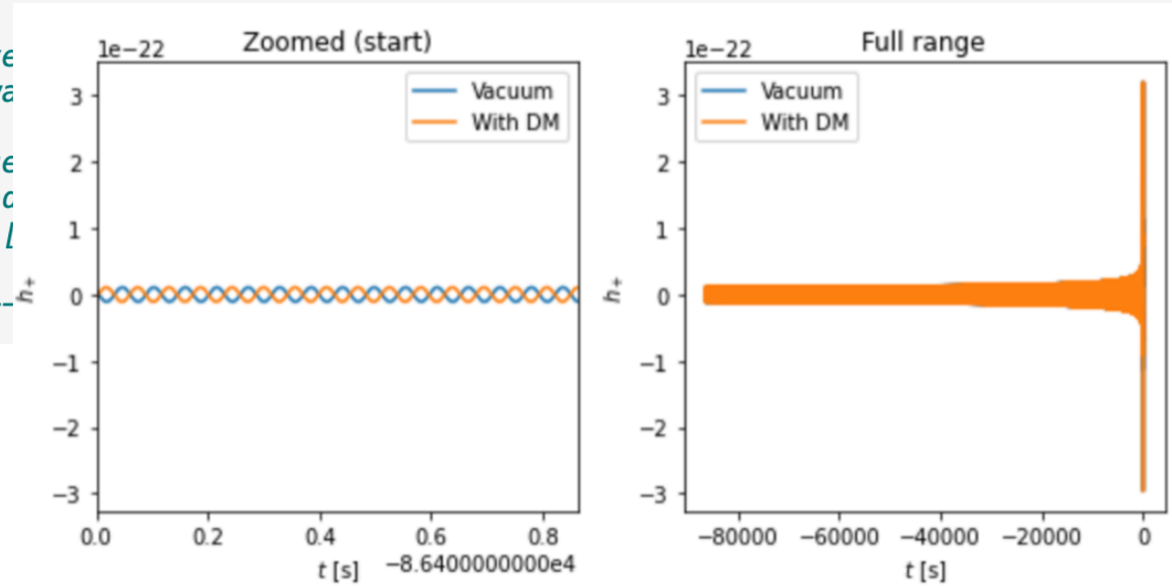
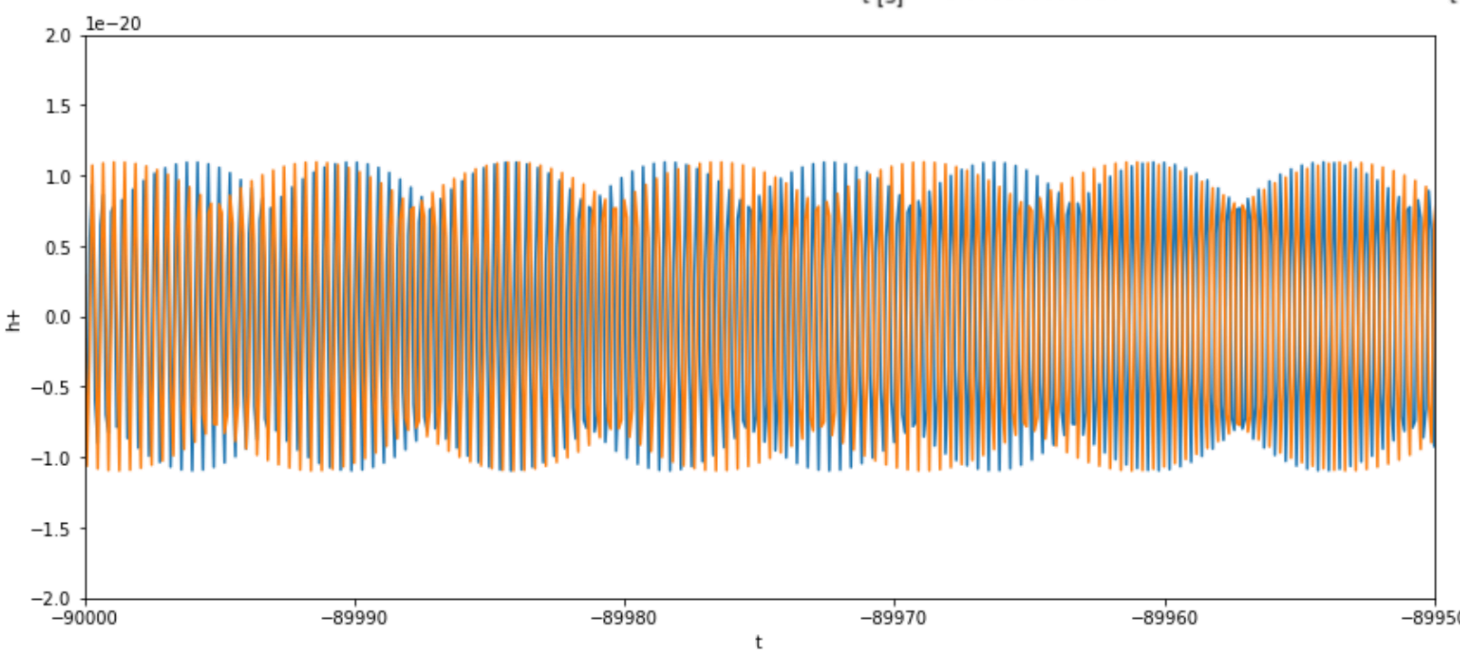
```
#EDIT WAVEFORM PARAMETERS BELOW:
#-----------------------------------------------------
m_1      = 1.0     # [MSun]
m_2      = 0.001   # [MSun]
t_obs    = 1*DAY   # Duration of the wave
t_end    = -0.0*YR   # End time of the wa
f_samp   = 200 # Sampling frequency [Hz]
d_l      = 100000   # Luminosity distance
iota     = 0.0   # Inclination angle [rad
phi_c    = 0.0   # Phase at coalescence [

#-----------------------------------------
```

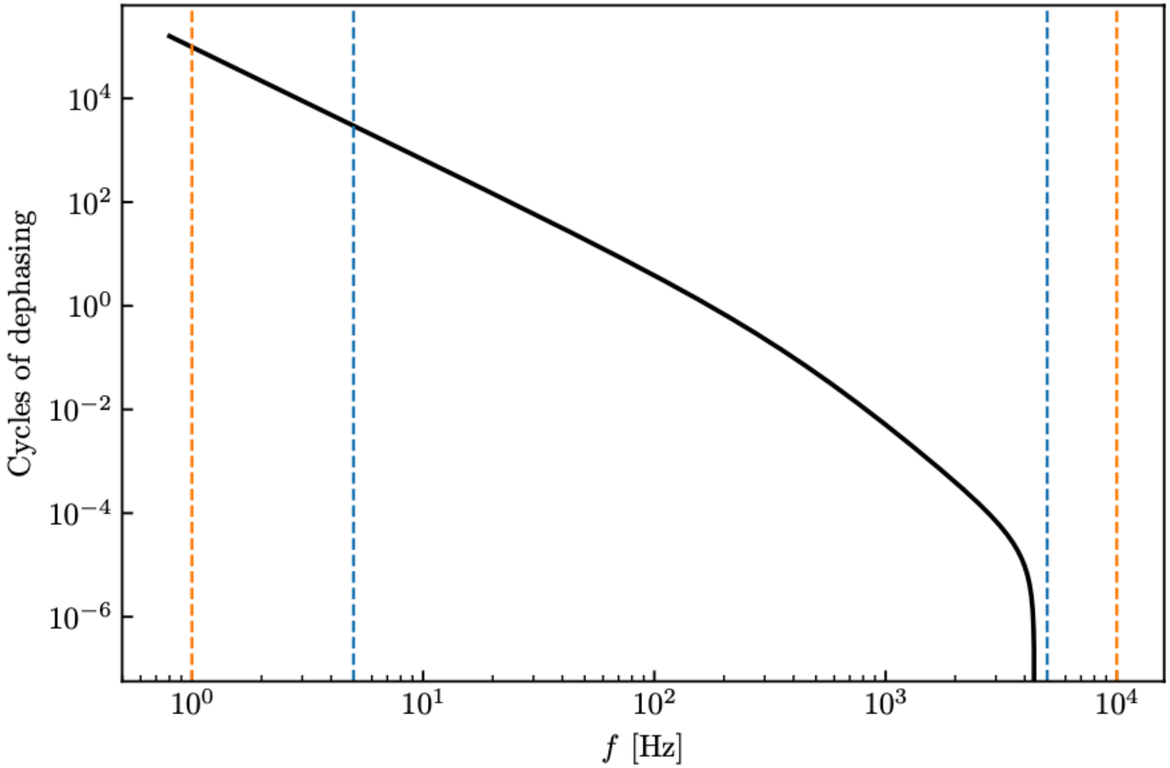


# Dephasing

$$N_{\text{cycles}}(t_{\max}, t_{\min}) = \int_{t_{\min}}^{t_{\max}} f_{\text{gw}}(t)dt = \int_{f_{\min}}^{f_{\max}} df_{\text{gw}} \frac{f_{\text{gw}}}{\dot{f}_{\text{gw}}}$$

$$\Delta N_{\text{cycles}} = N_{\text{cycles}}^{\text{vac}}(f_{\max}, f_{\min}) - N_{\text{cycles}}^{\text{DM}}(f_{\max}, f_{\min})$$




$m_1 = 1M_\odot$

$m_2 = 10^{-2} - 10^{-4}M_\odot$

$\rho_{\text{sp}} = 835M_\odot/\text{pc}^3$

$\gamma_{\text{sp}} = 9/4.$

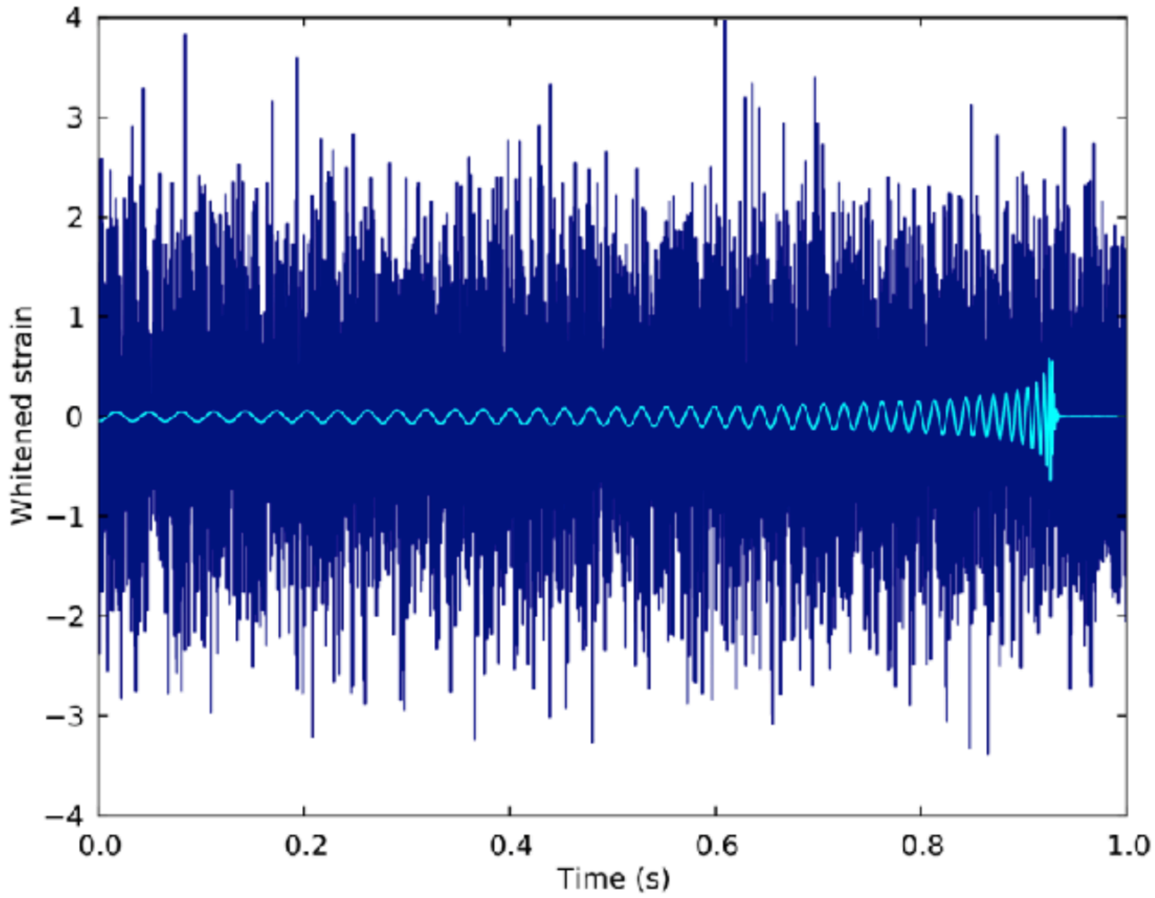
# Matched Filtering

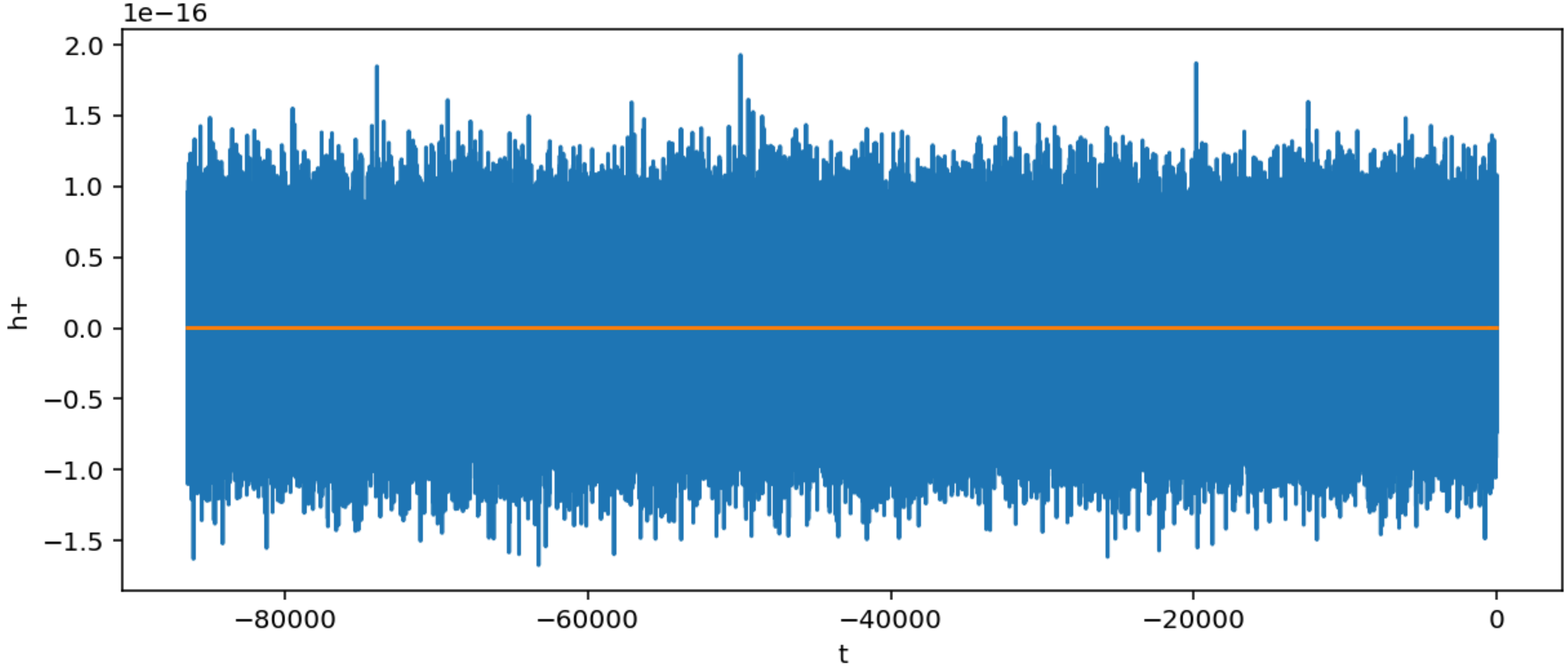
Naively, one might think that we can only make confident detections when $|h(t)| > |n(t)|$

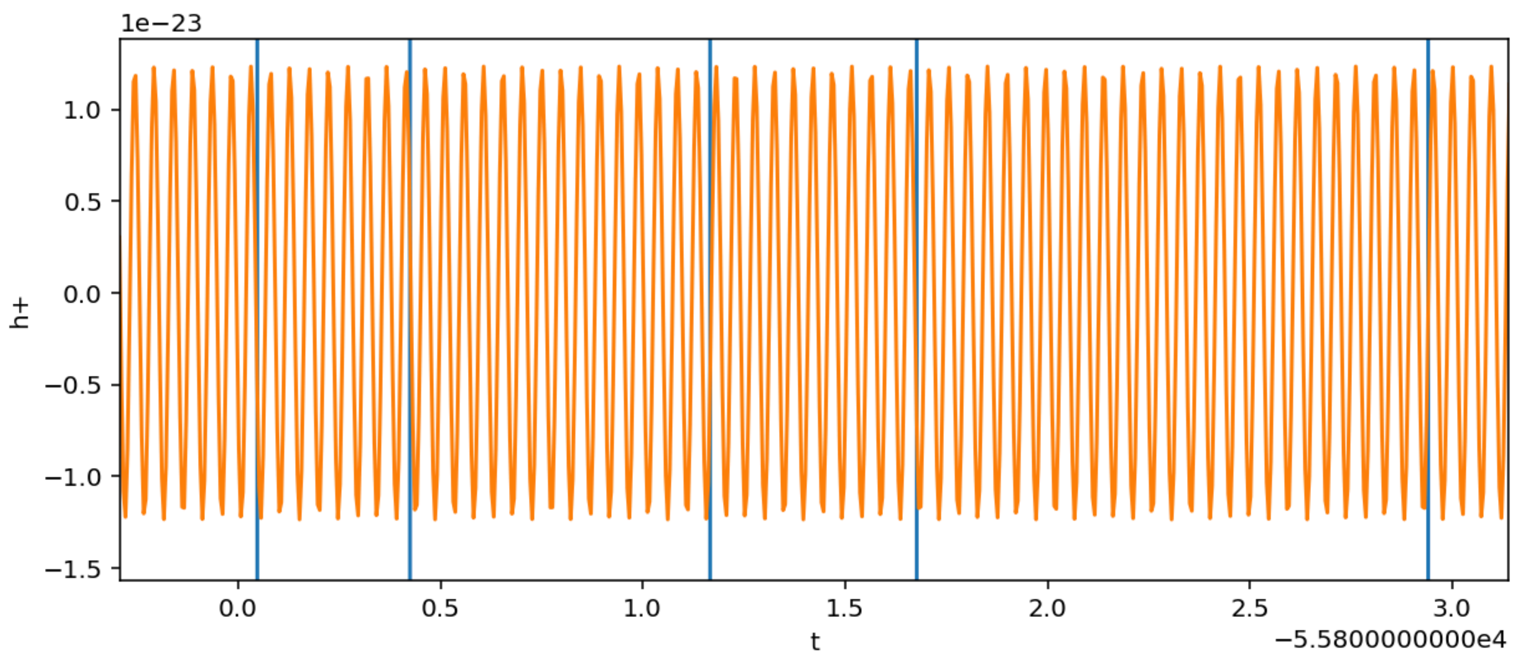
However, the **majority of signals are expected to be $|h(t)| \ll |n(t)|$**

Therefore, we need a method to detect signals from noise-dominated data

If we know the possible forms of h(t), we can "filter" out things that are non-signal-like




1e−16
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
h+
−80000
−60000
−40000
−20000
0
t


1e−23
1.0
0.5
0.0
−0.5
−1.0
−1.5
h+
0.0
0.5
1.0
1.5
2.0
2.5
3.0
t
−5.5800000000e4

# Detector Antenna Sensitivity

- **Antenna patterns**

$$F = \begin{bmatrix} \cos(2\psi) & \sin(2\psi) \\ -\sin(2\psi) & \cos(2\psi) \end{bmatrix} \begin{bmatrix} F_+[\theta,\phi] \\ F_\times[\theta,\phi] \end{bmatrix}$$

$$F_+[\theta,\phi] = \tfrac{1}{2}(1+\cos^2\theta)\cos$$

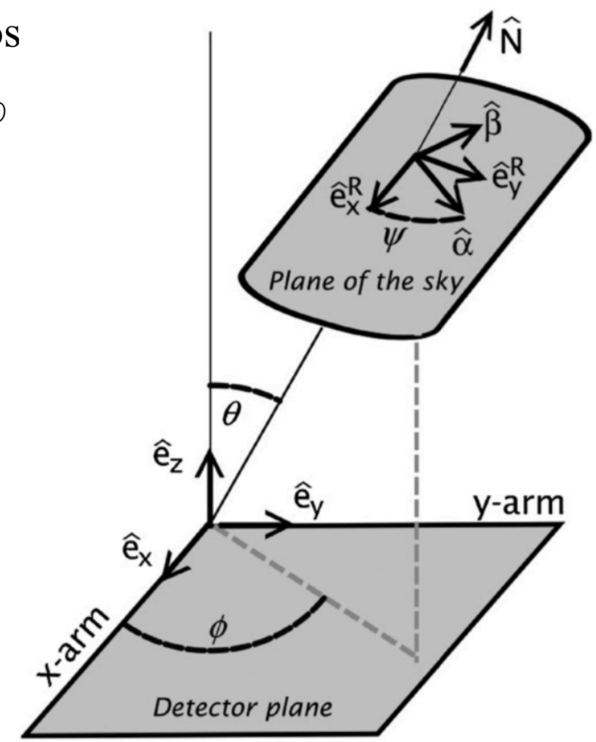
$$F_\times[\theta,\phi] = \cos\theta \sin 2\varphi$$

- **Sampled GW signal**

$$h[i] = \begin{bmatrix} \cos(2\psi) & \sin(2\psi) \\ -\sin(2\psi) & \cos(2\psi) \end{bmatrix} \begin{bmatrix} h_+[i] \\ h_\times[i] \end{bmatrix}$$

- **Sampled detector response**

$$\xi[i] = F_+ h_+[i] + F_\times h_\times[i] = F^T \cdot h[i]$$




- **Direction to the source $\theta,\varphi$ and polarization angle $\Psi$ define relative orientation of the detector and wave frames.**
- **Rotation of the wave frame $R_z(2\Psi)$ induces transformations both for F and h, but $\xi$ is INVARIANT**

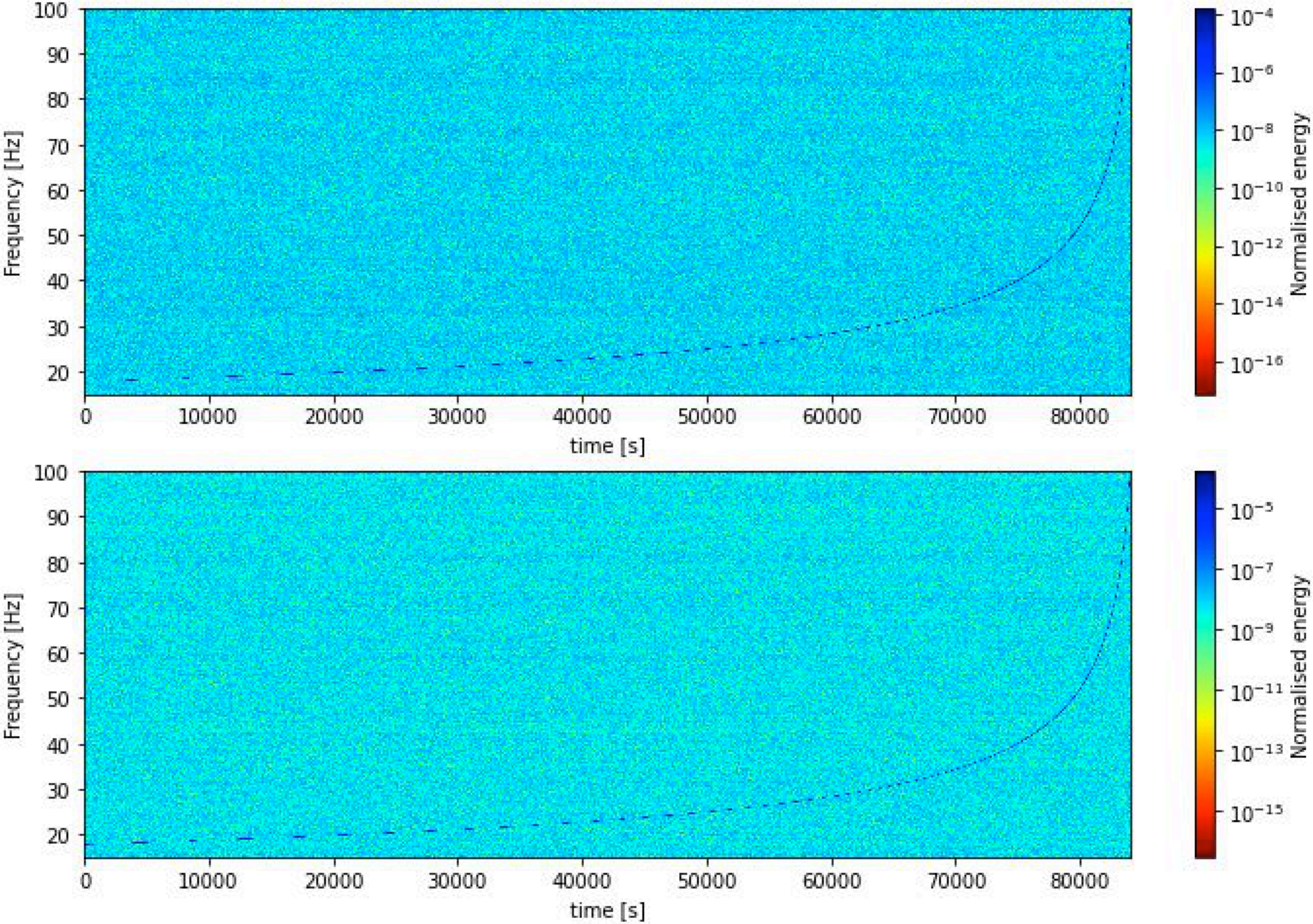
# Waveform Dataset

- Develop a catalog of waveforms for different luminosity distances and masses

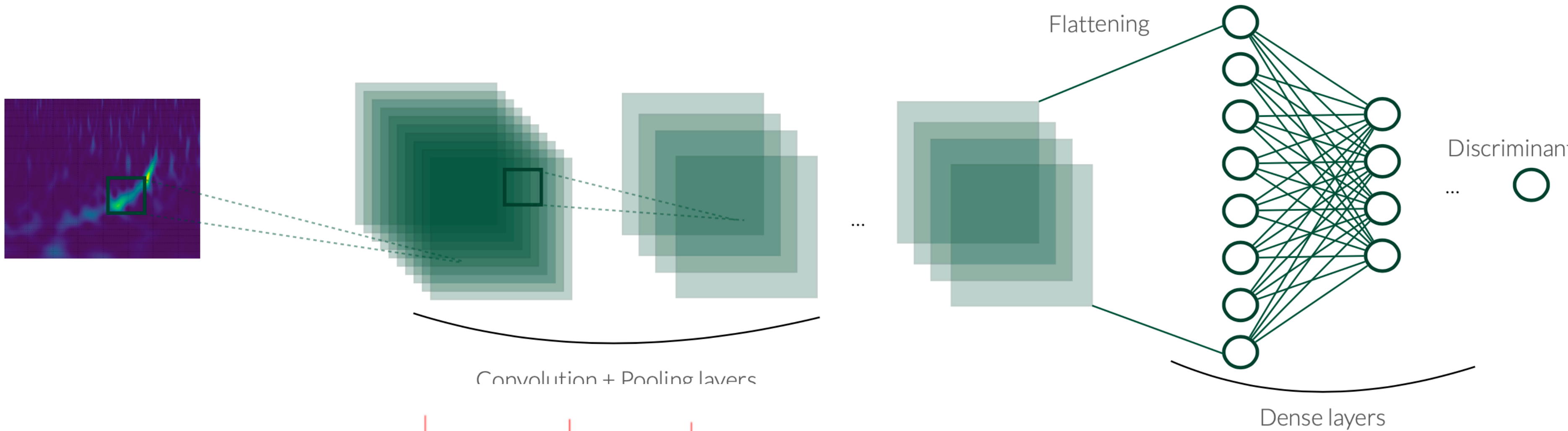
- **Luminosity distance d=10kpc, 100kpc, 1Mpc, 10Mpc, 100Mpc**
- **Mass** $m_1 = 1M_\odot$ $m_2 = 10^{-2} - 10^{-4} M_\odot$ $\Delta m_2 = 0.001 M_\odot$
- **Antenna Sensitivity 100 different directions**

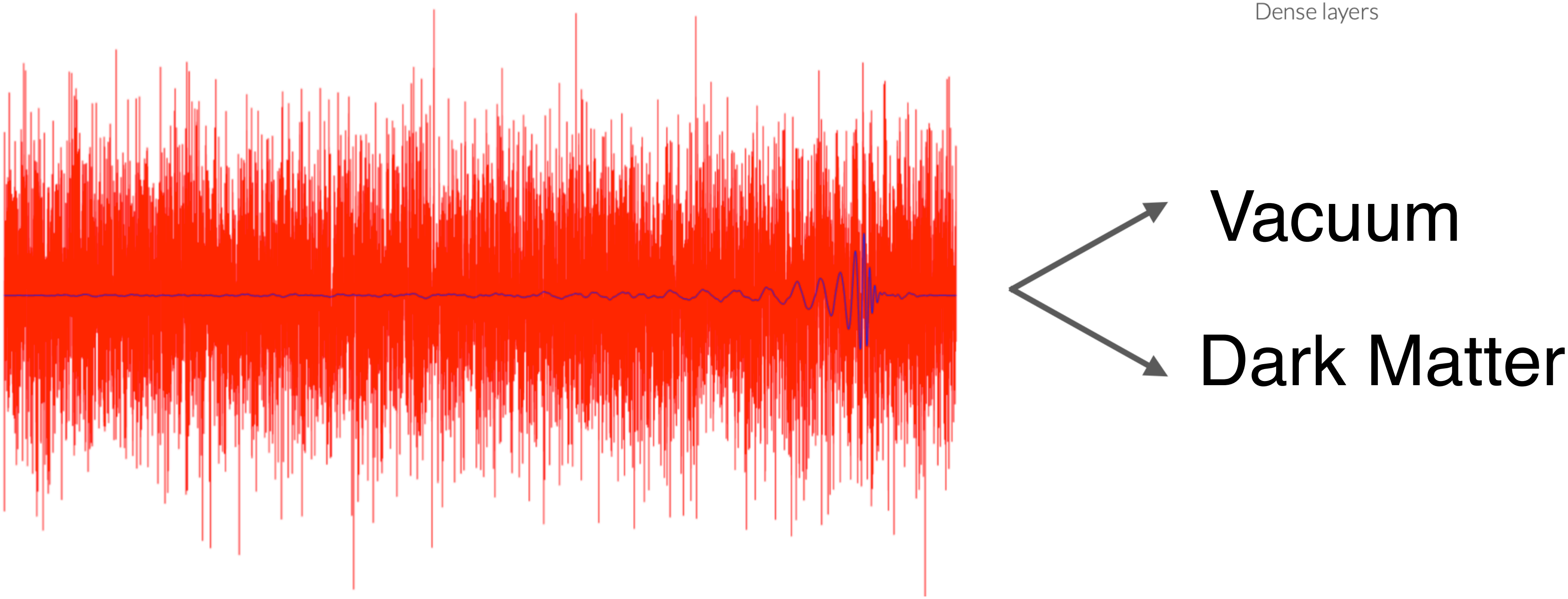
We have 9500 GW for the vacuum and 9500 GW with dark matter

Total: 19000 waveform


Frequency [Hz]
time [s]
Normalised energy
100
90
80
70
60
50
40
30
20
0
10000
20000
30000
40000
50000
60000
70000
80000
$10^{-4}$
$10^{-6}$
$10^{-8}$
$10^{-10}$
$10^{-12}$
$10^{-14}$
$10^{-16}$
Frequency [Hz]
time [s]
Normalised energy
$10^{-5}$
$10^{-7}$
$10^{-9}$
$10^{-11}$
$10^{-13}$
$10^{-15}$

# Machine Learning for GW Classification



# Pipeline Structure

## Input GW data

- **Basic GW wavedorm**
- **Add a noise**
- **Antenna Sensitivity 100 different directions**
- **Whitened strain**

## Classification

- **Basic GW Image creation from time frequency (spectrograms)**
- **Tested various networks, including a 4-block layers**

# High Performance Computing Center

Scuola Normale Superiore

# Conclusions

- We can measure the properties of dark matter spike around binaries with Einstein Telescope
- We can distinguish between vacuum and dark matter for distance up to 100kpc

## Futuro work:

- Eccentric waveforms
- Post-Newtonian corrections

Thank you for your attention