In-medium potentials from their vacuum counterpart

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Introduction

Our objective is to study the interaction of particles bound by the strong force inside a QCD hot and dense medium. This is an ideal setup for the open quantum systems (OQS) formalism.

In order to use the OQS formalism we need to know how the potential between quarks is affected by the presence of a medium.

Our goal is then to obtain a general formula that relates the in-medium potential with the vacuum potential.

We are going to apply the result to the $X(3872)$ inside quark-gluon plasma (QGP).

We are going to be able to get a rough estimation of the decay rate and of the survival probability of the $X(3872)$.

Quark potential

The force between quarks can be modeled with potentials, such as the popular Cornell potential: the addition of a Coulomb contribution ($\sim -1/r$) and a confinement term ($\sim r$):

$$V_{\text{Cornell}}(r) = -\frac{\tilde{\alpha}_s}{r} + \sigma r,$$

with $\tilde{\alpha}_s = C_F g^2/(4\pi)$. We will look for different heavy quark model potentials using potential non-relativistic QCD (pNRQCD).

The Born-Oppenheimer (BO) approximation allows to decouple the dynamics of the light components (light quarks or gluonic excitations) from the heavy quark dynamics.

As the potential can be obtained with Wilson loops, results can be derived from both hard-thermal loop (HTL) perturbation theory and lattice QCD.

Permittivity

In momentum space, using HTL perturbation theory, the inverse of the permittivity is:

$$\varepsilon^{-1}(p, m_D) = \frac{p^2}{p^2 + m_D^2} - i\pi T \frac{p m_D^2}{(p^2 + m_D^2)^2},$$

which in position space is:

$$\varepsilon^{-1}(r, m_D) = \frac{\delta(r)}{4\pi r^2} - \frac{m_D^2 e^{-m_D r}}{4\pi r} - i\frac{m_D T}{4\sqrt{\pi} r} G_{1,3}^{2,1}\left(\begin{matrix} -\frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{4} m_D^2 r^2 \right).$$

It is important to notice that the permittivity is valid in the vacuum case $m_D = T = 0$, since $\varepsilon^{-1}(r, 0) = \delta(r)/(4\pi r^2)$.

Debye screening

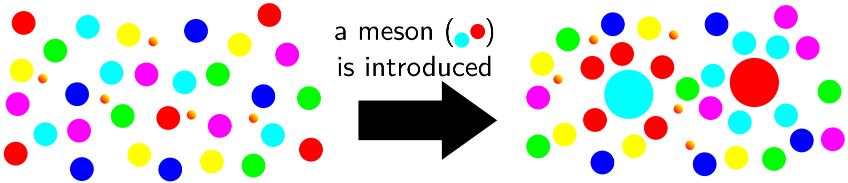

Figure: The medium size circles represent quarks and antiquarks, while the small double-coloured circles are meant to be gluons; the big circles on the right are the quark and antiquark that conformed the meson of interest.

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The general formula I

Starting with a vacuum potential between two static (anti)quarks $V_{\text{vac}}(\mathbf{r})$, we have that, in momentum space, the in-medium potential is:

$$V(\mathbf{p}) = \frac{V_{\text{vac}}(\mathbf{p})}{\varepsilon(\mathbf{p}, m_D)},$$

with $\varepsilon(\mathbf{p}, m_D)$ the permittivity, being m_D the Debye or screening mass. In coordinate space using the convolution theorem, we have:

$$V(\mathbf{r}, m_D) = (V_{\text{vac}} * \varepsilon^{-1})(\mathbf{r}, m_D).$$

Instead of performing the convolution, we will come with a variation of the Gauss's (or Poisson's) law and we will solve the corresponding equation [D. Lafferty, and A. Rothkopf (2020)].

The general formula II

We can start from the Gauss's law for a unit charge, which states the identity:

$$\nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2} \right) = 4\pi\delta(\mathbf{r}),$$

where $\hat{\mathbf{r}}/r^2$ plays the role of a chromoelectric field. A general radial field $\mathbf{E}(r) = E(r)\hat{\mathbf{r}}$ verifies trivially:

$$\nabla \cdot \left(\frac{\mathbf{E}(r)}{r^2 E(r)} \right) = 4\pi\delta(\mathbf{r}).$$

In terms of the potential, $\mathbf{E}(r) = -\nabla V_{\text{vac}}(r) = -\,\partial V_{\text{vac}}(r)/\partial r\, \hat{\mathbf{r}}$, so:

$$\nabla \cdot \left(\frac{-\nabla V_{\text{vac}}(r)}{r^2 E(r)} \right) = 4\pi\delta(\mathbf{r}),$$

with $E(r) = -\,\partial V_{\text{vac}}(r)/\partial r$.

The general formula III

In components, the equation is:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{-1}{E(r)}\frac{\partial V_{\rm vac}(r)}{\partial r}\right) = 4\pi\delta(\mathbf{r}),$$

which can be written as:

$$\left(\frac{1}{r^2 E(r)^2}\frac{\partial E(r)}{\partial r}\frac{\partial}{\partial r} - \frac{1}{r^2 E(r)}\frac{\partial^2}{\partial r^2}\right) V_{\rm vac}(r) = \mathcal{G}(r) V_{\rm vac}(r) = 4\pi\delta(\mathbf{r}).$$

Where this linear operator can be applied to the in-medium potential $V(r, m_D) = (V_{\rm vac} * \varepsilon^{-1})(r, m_D)$:

$$\mathcal{G}(r)V(r) = [(\mathcal{G}V_{\rm vac}) * \varepsilon^{-1}](r, m_D) = 4\pi\varepsilon^{-1}(r, m_D).$$

The general formula IV

So we arrive to the general equation:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{-1}{E(r)}\frac{\partial V(r,m_D)}{\partial r}\right) = 4\pi\varepsilon^{-1}(r,m_D),$$

which formally solves as:

$$V(r,m_D) = C - b\int^r \mathrm{d}r' E(r') - 4\pi\int^r \mathrm{d}r' E(r')\int^{r'} \mathrm{d}r'' r''^2 \varepsilon^{-1}(r'',m_D),$$

where we can substitute $E(r) = -\,\partial V_{\mathrm{vac}}(r)/\partial r$ so:

$$V(r,m_D) = C + bV_{\mathrm{vac}}(r) + 4\pi\int^r \mathrm{d}r' \frac{\partial V_{\mathrm{vac}}(r')}{\partial r'}\int^{r'} \mathrm{d}r'' r''^2 \varepsilon^{-1}(r'',m_D),$$

where we considered $\int^r \mathrm{d}r'\,\partial V_{\mathrm{vac}}(r')/\partial r' = V_{\mathrm{vac}}(r)$.

The general formula V

We need to determine C and b. We are going to use the fact that $V_{\text{vac}}(r) = V(r,0)$. Being $\varepsilon^{-1}(r,0) = \delta(r)/(4\pi r^2)$, we have:

$$V(r,0) = C + bV_{\text{vac}}(r) + \int^r \mathrm{d}r' \frac{\partial V_{\text{vac}}(r')}{\partial r'} = C + (b+1)V_{\text{vac}}(r),$$

so $b = C = 0$ for $m_D = 0$. Moreover, by dimensional analysis we can set $b = 0$ for all m_D. We also enforce a smooth transition for $m_D \to 0$. Then, the general formula can then be written as [V. López-Pardo, N. Armesto, M. A. Escobedo and E. G. Ferreiro (in preparation)]:

$$V(r,m_D) = C(m_D) + 4\pi \int^r \mathrm{d}r' \frac{\partial V(r',0)}{\partial r'} \int^{r'} \mathrm{d}r'' r''^2 \varepsilon^{-1}(r'', m_D),$$

with the prescription that $C(m_D)$ must ensure that $V(r,m_D) = V_{\text{vac}}(r)$ up to $\mathcal{O}(m_D)$.

The general formula VI

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Using the permittivity we can specify further the in-medium potential:

$$\mathrm{Re}[V(r,m_D)] = \mathrm{Re}[C(m_D)] + \int^r \mathrm{d}r' \frac{\partial V(r',0)}{\partial r'} e^{-m_D r'}(m_D r' + 1),$$

$$\mathrm{Im}[V(r,m_D)] = \mathrm{Im}[C(m_D)] - \int^r \mathrm{d}r' \frac{\partial V(r',0)}{\partial r'} \frac{\sqrt{\pi} m_D T}{2} r'^2 \times$$

$$\times\, G^{2,1}_{1,3}\left(\begin{matrix} \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, -1 \end{matrix} \,\middle|\, \frac{1}{4} m_D^2 r'^2 \right).$$

$C(m_D)$ is real since the integral on the second line is, at least, $\mathcal{O}(m_D^2)$, so:

$$\mathrm{Re}[V(r,m_D)] = C(m_D) + \int^r \mathrm{d}r' \frac{\partial V(r',0)}{\partial r'} e^{-m_D r'}(m_D r' + 1),$$

$$\mathrm{Im}[V(r,m_D)] = -\int^r \mathrm{d}r' \frac{\partial V(r',0)}{\partial r'} \frac{\sqrt{\pi} m_D T}{2} r'^2 G^{2,1}_{1,3}\left(\begin{matrix} \frac{1}{2} \\ \frac{1}{2}, \frac{1}{2}, -1 \end{matrix} \,\middle|\, \frac{1}{4} m_D^2 r'^2 \right).$$

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Aplication to the $X(3872)$

Let's apply the method to the $X(3872)$ ($m_{X(3872)} = 3871.65(06)$ MeV, $I^G(J^{PC}) = 0^+(1^{++})$)

We have to postulate a vacuum potential for the $X(3872)$. Once we have the vacuum potential, we can compute its in-medium counterpart and obtain some results.

We present here a model for the vacuum potential, but any other sufficiently well-reasoned model is equally valid.

We will rely on lattice QCD results for hybrids to find a suitable model.

The vacuum potential

Similarities between (heavy) hybrids and tetraquarks:

- Contain a heavy quark-antiquark pair.
- The potential within the pair is modified by colour-charged light particles.

Colour charge differs between gluons (8 representation) and quarks (3 representation), but we can assume that a light quark-antiquark pair behaves in a similar way to a set of gluons:

$$3 \otimes \bar{3} = 8 \oplus 1, \qquad 8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1.$$

Hybrid Π_u^- ($c\bar{c}$, $m_{\Pi_u^-} = 4184$ MeV y $J^{PC} = 1^{++}$), potential [S. Capitani, O. Philipsen, C. Reisinger, C. Riehl, and M. Wagner (2019)]:

$$V_{\text{vac}}(r) = \frac{A_{-1}}{r} + A_0 + A_2 r^2,$$

with A_{-1}, A_0 y A_2 constants.

The in-medium potential I

By simply substituting the previous vacuum potential into our general formula we get that the real part is:

$$\mathrm{Re}[V(r,m_D)] = C(m_D) + A_{-1}\frac{e^{-m_D r}}{r} - A_2\left(2r^2 + \frac{6r}{m_D} + \frac{6}{m_D^2}\right)e^{-m_D r},$$

where we only must determine the value of $C(m_D)$. The imaginary part is:

$$\mathrm{Im}[V(r,m_D)] = \frac{\sqrt{\pi}T}{m_D}\frac{A_{-1}}{r}G^{2,1}_{1,3}\left(\begin{matrix}\frac{3}{2}\\ \frac{3}{2},\frac{3}{2},0\end{matrix}\,\middle|\,\frac{1}{4}m_D^2r^2\right) + \\ + 4\frac{\sqrt{\pi}T}{m_D^3}A_2G^{2,1}_{1,3}\left(\begin{matrix}\frac{3}{2}\\ \frac{5}{2},\frac{5}{2},0\end{matrix}\,\middle|\,\frac{1}{4}m_D^2r^2\right)$$

but we will not use it due to the appearence of divergencies.

The in-medium potential II

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The fixing of the value of $C(m_D)$ can be done thanks to the prescription $\lim_{m_D \to 0} \mathrm{Re}[V(r, m_D)] - V(r, 0) = 0$, this is:

$$\lim_{m_D \to 0} \mathrm{Re}[V(r, m_D)] - V(r,0) = C(m_D \to 0) - A_{-1} m_D - \frac{6A_2}{m_D^2} - A_0 = 0,$$

so:

$$C(m_D \to 0) = C(m_D) = A_{-1} m_D + \frac{6A_2}{m_D^2} + A_0,$$

and then:

$$\mathrm{Re}[V(r, m_D)] = A_{-1}\left(m_D + \frac{e^{-m_D r}}{r}\right) + A_0 + \\ + A_2\left[\frac{6}{m_D^2}(1 - e^{-m_D r}) - \left(2r^2 + \frac{6r}{m_D}\right) e^{-m_D r}\right].$$

In-medium potential for different Debye masses

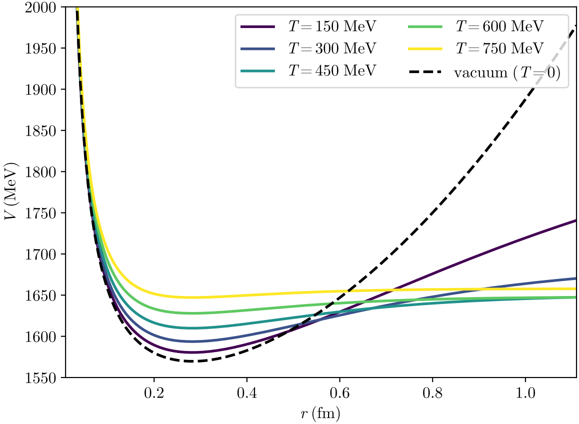

Figure:
$A_{-1} = 17.778$ MeV $\cdot$ fm,
$A_0 = 1475.23$ MeV and
$A_2 = 394.742$ MeV $\cdot$ fm^{-2}.

The octet imaginary part I

It can be shown that the imaginary part we derived corresponds to the decay rate of a singlet state ($\Gamma^s(r, m_D)$), so we need to obtain the octet counterpart ($\Gamma^o(r, m_D)$). In order to do so we can use the relationship:

$$\Gamma^o(r, m_D) = \Gamma^s(\infty, m_D) + \frac{1}{N_c^2 - 1}(\Gamma^s(\infty, m_D) - \Gamma^s(r, m_D)),$$

where we noted $\Gamma^s(\infty, m_D) = \lim_{r\to\infty} \Gamma^s(r, m_D)$. In the $N_c \to \infty$ limit we arrive to:

$$\Gamma^o(m_D) = \lim_{r\to\infty} \Gamma^s(r, m_D) = \lim_{r\to\infty} \text{Im}[V(r, m_D)],$$

where we dropped the radial dependency because the decay rate only depends on the temperature (or Debye mass) in this approximation.

The octet imaginary part II

Since the imaginary part diverges as $r \to \infty$, in order to use the previous formula, we need to introduce a regularization (which corresponds to take into account the string breaking effect). Once we do so we arrive to:

$$\Gamma^{o}(T) = A_{-1}T + \frac{A_2 T}{m_D^3}\frac{6\pi}{\Delta},$$

where we know that $m_D = m_D(T)$, and Δ is a regularization constant to be determined.

Moreover, if we consider $m_D \propto T$, we can model:

$$\Gamma(T) \approx A_{-1}T + \frac{b_2}{T^2},$$

where b_2 can be determined through experimental data.

Decay rate of the $X(3872)$

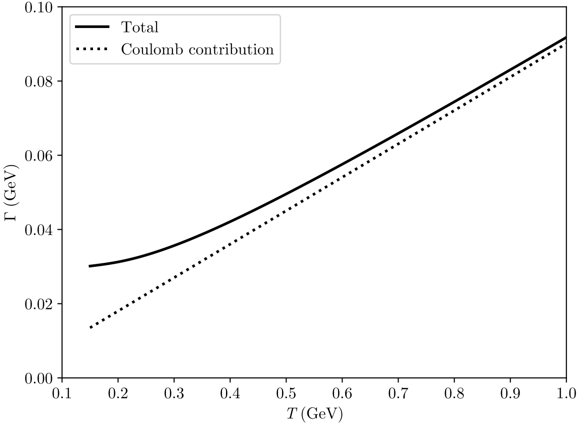

Figure: $\Gamma(T)$ with $\Delta \approx 6\pi$. It is lower than expected, similar to the decay rate of the $\Upsilon(1S)$.

The survival probability

We can compute the survival probability:

$$S(t) = \exp\left[-\int_{t_0}^{t} \mathrm{d}\tau \Gamma(T(\tau))\right].$$

We are going to use the approximation $\Gamma(T) = A_{-1}T + b_2/T^2$. Using Bjorken evolution ($tT^3 =$ const.) we can arrive to an expression of the survival probability:

$$S(t) = \exp\left\{-\frac{3t_0 A_{-1}T(t_0)}{2}\left[\left(\frac{t}{t_0}\right)^{2/3} - 1\right]\right\}\times$$
$$\times \exp\left\{-\frac{3t_0 b_2}{5T^2(t_0)}\left[\left(\frac{t}{t_0}\right)^{5/3} - 1\right]\right\}.$$

The $X(3872)$ survival probability

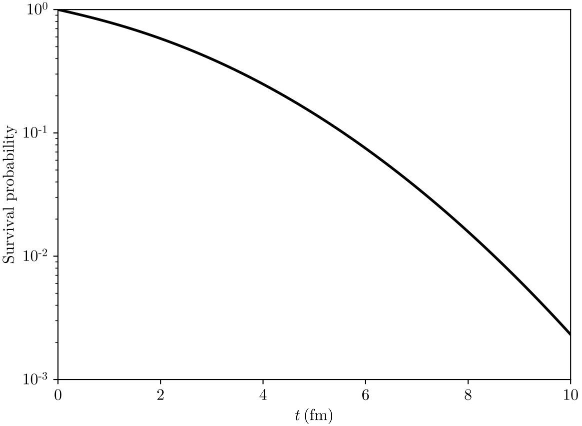

Figure: The time evolution of the survival probability. The time is in fm ($c = 1$), and 10 fm is around 3 ys.

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Conclusions

- The general method developed here can be applied to a wide range of particles.
- Useful to predict melting temperatures, lifetimes – when dealing with the imaginary contribution – or resonances, among others.
- We need a solid argument to justify the form of the vacuum potential for a hadron.
- We have powerful tool to extrapolate the results to the case of these particles in a medium.
- Preliminary results for the $X(3872)$ are promising.

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Merci beaucoup!

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Mesonic molecules vs. tetraquarks

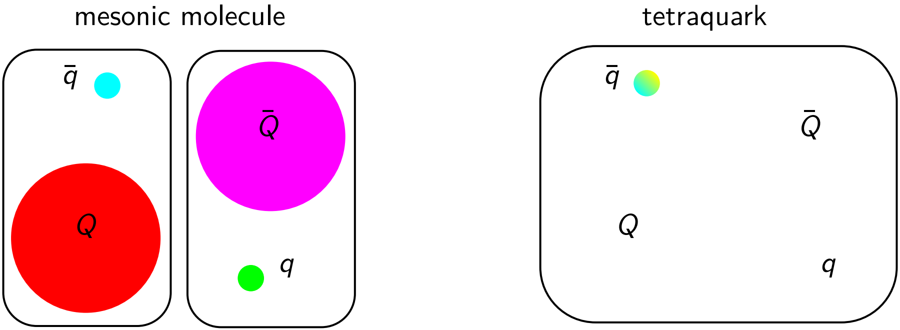

Figure: On this schematic representation the colour neutrality is represented by the black lines surrounding the groups of (anti)quarks and the lower case q and capital Q are reserved for light and heavy quarks, respectively.

The $X(3872)$ meson

The $X(3872)$ (also $\chi_{c1}(3872)$) is an exotic meson candidate first observed by Belle. Since it does not fit inside the quark model it is proposed as a tetraquark. Relevant information:

- Mass: $m_{X(3872)} = 3871.65(06)$ MeV.
- Composition: $c\bar{c}$ + two light quarks, probably $c\bar{c}u\bar{u}$.
- Decay modes: $X(3872) \to D^0\bar{D}^0$, $X(3872) \to \bar{D}^{*0}D^0$, $X(3872) \to \pi^+\pi^- J/\psi$...
- Quantum numbers: $I^G(J^{PC}) = 0^+(1^{++})$

Evidence of $X(3872)$ production inside QGP in January, 2022.

Debye mass versus T

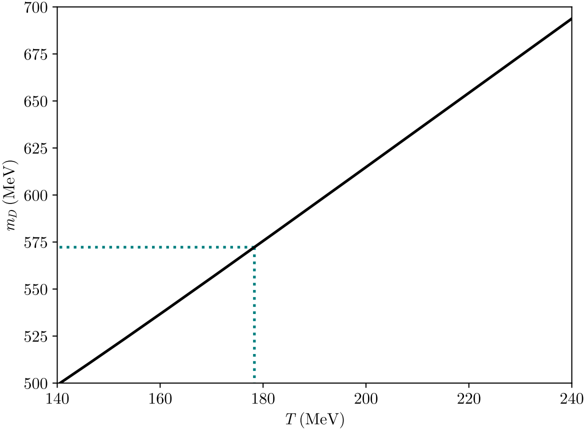

Figure:

$$m_D^2 \simeq \frac{4\pi^2 T^2}{3\ln\left(7.547\,T/\Lambda_{\overline{\mathrm{MS}}}\right)}$$

with $\Lambda_{\overline{\mathrm{MS}}} \approx 375$ MeV.

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