Heavy quarkonia production in ultra-pheripheral heavy-ion collisions

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Introduction

- heavy quarkonia = bound states of quark and antiquark pairs ($\bar{c}c$, $\bar{b}b$)
- the most studied are S-wave $J/\psi(\bar{c}c), \psi'$ and $\Upsilon(\bar{b}b)$
- heavy quarkonia are important tool for studying dense matter created in heavy-ion collisions and QCD dynamics
- the study of production mechanism deepens our knowledge of QCD and help us to test various production models as well as models of $q\bar{q}$ interaction.
- despite a large progress in the theoretical description over the past years there are still remaining theoretical uncertainties which need to be addressed.

Introduction

- ultra-pheripheral collisions (UPC) are dominant source of heavy quarkonia and provide unique acces to photon-nucleus interactions
- description of coherent quarkonium photoproduction $\gamma A \to VA$ within the light-cone color dipole approach

  B.Z. Kopeliovich et al., PRD 107 (2023) 054005

  M. Krelina, J. Nemchik, PRD 102 (2020) 114033

Light-cone color dipole approach

- eigenstates of interaction are color dipoles
- suitable for description of nuclear collisions
- formulated in the target rest frame
- possible expansion of a projectile into the Fock states, e.g. photon

$$|\gamma\rangle = |\bar{Q}Q\rangle + |\bar{Q}Qg\rangle + |\bar{Q}Q2g\rangle + \ldots$$

- **coherence lenght** defined for each Fock state
  $l_c^{\bar{Q}Q} = \frac{2k}{M_{\bar{Q}Q}^2 + Q^2} \gg l_c^{\bar{Q}Qg} = \frac{2k}{M_{\bar{Q}Qg}^2 + Q^2} \gg l_c^{\bar{Q}Q2g} \gg \ldots$
- **shadowing phenomena** $\Rightarrow$ destructive interference of amplitudes related to interactions on different bound nucleons
- $l_c^{\bar{Q}Q} \Rightarrow$ **quark shadowing**, $l_c^{\bar{Q}Qg} \Rightarrow$ **gluon shadowing**

Light-cone color dipole approach

- in the LC dipole approach, the amplitude of a diffractive process is treated as elastic scattering of a $\bar{q}q$ fluctuation of the incident particle
- the amplitude of vector meson photo- or electroproduction $\gamma^* N \to VN$ is in the form

$$\mathcal{M}_{\gamma^* N \to VN}(x, Q^2) = \langle V | \sigma_{\bar{q}q}^N(r, x) | \gamma^* \rangle$$
$$= \int_0^1 d\alpha \int d^2 r \Psi_V^*(\vec{r}, \alpha) \sigma_{\bar{q}q}^N(r, x) \Psi_{\gamma^*}(\vec{r}, \alpha, Q^2)$$

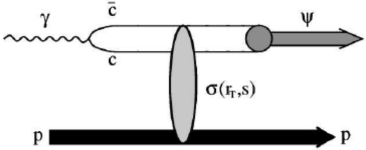

Fig.1: Schematic representation of the ampltidute $\mathcal{M}_{\gamma^* N \to VN}(s, Q^2)$.

Dipole cross section $\sigma_{\bar{q}q}(r, x)$

- interaction of nucleon with a dipole ($\bar{q}q$) described by universal phenomenological dipole cross section $\sigma_{\bar{q}q}(r, x)$
- $\sigma_{\bar{q}q}$ cannot be determind from first principles
- several parametrizations of $\sigma_{\bar{q}q}(r, x)$, parameters fitted to the HERA DIS data
- popular parametrizations - GBW *(K. Golec-Biernat, M. Wüsthoff, PRD 59 (1999) 014017)*, KST *(B.Z. Kopeliovich, A. Schäfer, A.V. Tarasov, PRD 62 (2000) 054022)*

$$\sigma_{\bar{q}q}(r, x) = \sigma_0 \left[1 - \exp\left(-\frac{r^2}{R_0^2(x)}\right)\right] \ ,$$

where $R_0^2(x) = \frac{4}{Q^2}(\frac{x}{x_0})^\lambda$

LC wave function for photon

- the perturbative LC photon wave function is given by

$$\Psi^{(\mu,\bar{\mu})}_{\gamma^*_{T,L}}(\vec{r},\alpha,Q^2) = \frac{\sqrt{N_c\alpha_{\rm em}}}{2\pi} Z_q \chi^{\mu\dagger}_Q \hat{\mathcal{O}}_{T,L} \tilde{\chi}^{\mu}_{\bar{Q}} K_0(\epsilon r) \ ,$$

  where $\epsilon^2 = \alpha(1-\alpha)Q^2 + m_q^2$, Z_q = electric charge of the quark,
  χ^μ_Q = two-component spinors in the infinite momentum frame,
  $\hat{\mathcal{O}}_{T,L}$ = operators for transverse and longitudinal photon polarization

- $\vec{r}$ is transverse separation of $\bar{Q}Q$ pair, $\alpha = \frac{p_Q^+}{p_\gamma^+}$ - fraction of photon momentum carried by quark

LC wave function for S-wave vector meson

- we assume factorization of spatial and spin-depentend part of the VM wave function and non-photon-like structure of VM vertex
- radial part found by solving the Schrödinger equation with heavy quark interaction potential $V_{\bar{q}q}$ in the rest frame
- boost to the LC frame by Terent'ev prescription *(M. V. Terent'ev, SJNP 24 (1976) 106)* :

$\Rightarrow$ Fourier transformation of the spatial part from coordinate space to momentum space $\psi(p)$

$\Rightarrow$ boost to the LC frame $\psi_V(p_T, \alpha) = \left(\frac{p_T^2 + m_q^2}{16(\alpha(1-\alpha)^3)}\right)^{1/4} \psi(p)$

$\Rightarrow$ the LF wave function in the coordinate space give by

$$\Psi_V(\vec{r}, \alpha) = \int_0^\infty dp_T p_T J_0(p_T r) \psi(p_T, \alpha)$$

$\Rightarrow$ Melosh spin-rotation = boost of the spin-dependent part to the LC frame *(H.J. Melosh, PRD 9 (1974) 1095)*

Quarkonium photoproduction cross-section in UPC

- cross section for photoproduction of a vector meson in the rest frame of the target nucsleus A is of the form:

$$k\frac{d\sigma}{dk} = \int d^2 b_A \int d^2 b\, n_\gamma(k, \vec{b} - \vec{b}_A, y)\frac{d^2\sigma_A(b,s)}{d^2 b} + \{y \to -y\},$$

  $\vec{b}_A > 2R_A \Rightarrow$ relative impact parameter of the collision
  $\vec{b} \Rightarrow$ impact parameter of photon-nucleon collision relative to the center of one of the nuclei

- photon flux $n(k, \vec{b})$ induced by the projectile nucleus within the one-photon-exchange

$$n_\gamma(k, \vec{b}) = \frac{\alpha_{em} Z^2 k^2}{\pi^2\gamma^2}\left[K_1^2\left(\frac{bk}{\gamma}\right) + \frac{1}{\gamma^2}K_0^2\left(\frac{bk}{\gamma}\right)\right],$$

  the Lorenz factor gamma $\gamma = 2\gamma_{\rm col}^2 - 1$, where $\gamma_{\rm col} = \frac{\sqrt{s_N}}{2M_N}$

Cross section for coherent quarkonia production

J. Nemchik et al., PRD 107 (2023) 054005

- In UPC at LHC the photon energy is sufficiently high $\Rightarrow l_c^{\bar{Q}Q} \gg R_A$
- coherent (elastic) cross section for quarkonia production $\gamma A \to VA$ within the LF color dipole approach

$$\frac{d^2\sigma_A(b,s)}{d^2 b}\Big|_{l_c^{\bar{Q}Q} \gg R_A} = \Big| \int d^2 r \int_0^1 d\alpha \Psi_V^*(\vec{r},\alpha) \left(1 - \exp\left[-\frac{1}{2}\sigma_{\bar{q}q}(r,s)\, T_A(b)\right]\right) \Psi_\gamma(\vec{r},\alpha)\Big|^2$$

where $T_A(b) = \int_{-\infty}^{\infty} dz \rho_A(b,z)$ is the nuclear thickness function and c.m energy $s = M_V \sqrt{s_N} \exp(y)$

- "frozen" approximation - transverse separation of the $|\bar{Q}Q\rangle$ Fock state does not change during propagation through the medium
- VM wave function calculated for several models of $\bar{q} - q$ interaction: Büchmüller-Tye (BT) or Power-like (POW).

Correction for finite coherence length $l_c^{\bar{Q}Q}$

J. Nemchik et al., PRD 107 (2023) 054005

- eikonal approximation ("frozen") for cross section valid for long-living $\bar{Q}Q$ photon fluctuations at $y = 0 \Rightarrow$ long coherence length
- forward or backward rapidities at LHC or RHIC $\Rightarrow$ coherence length becomes too short ($l_c^{\bar{Q}Q} \lesssim R_A$) at least for one of the colliding nuclei
- correction for finite coherence length using form factors

$$\frac{d^2\sigma_A(b,s)}{d^2b} = \frac{d^2\sigma_A(b,s)}{d^2b}\Big|_{l_c^{\bar{Q}Q} \gg R_A} \cdot F^{coh}(s, l_c(s))$$

- form factor calculated using Green function technique $\Rightarrow$ harmonic-oscillator VM wave function + analytic form of Green function for HO
- $F^{coh}(s, l_c(s)) < 1$ for energies $s \lesssim 10^3$ GeV^2 $\Rightarrow$ suppression of the cross section

Gluon shadowing

J. Nemchik et al., PRD 107 (2023) 054005

- gluon shadowing (GS) = another source of nuclear suppression, leading twist effect
- at sufficiently high energies $l_c^{\bar{Q}Qg} \gg 2$ fm and gluon radiation does not resolve multiple interaction and acts as one accumulated kick $\Rightarrow$ intensity of gluon radiation reduced compared to Bethe-Heitler regime $\Rightarrow$ gluon shadowing
- GS is a part of Gribov corrections and corresponds to higher Fock components of the photon $|\bar{Q}Qg\rangle, |\bar{Q}Q2g\rangle, \ldots$
- transverse size of $\bar{Q}Q - g$ dipole fluctuates during propagation throught the nucleus $\Rightarrow$ GS calculated using the Green function formalism

Gluon shadowing

M.Krelina et al., PRD 105 (2022) 054023

- GS calculated for $|\bar{Q}Qg\rangle$ Fock compoment as the ratio of gluon densities in nuclei and nucleon
$$R_G(x, b_A) = 1 - \frac{\triangle\, \sigma_{tot}^{\gamma^* A}(b_A)}{T_A(b_A)\ \sigma_{tot}^{\gamma^* N}}$$
- GS correction is included in the calculations via the substitution
$$\sigma_{\bar{q}q}(r, s) \Rightarrow \sigma_{\bar{q}q}(r, s) \cdot R_G(x, b_A)$$

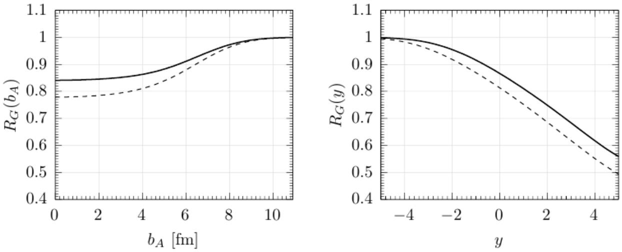

Fig.2: $R_G(b_A)$ for photoproduction of J/ψ on lead as a function of impact parameter b_A (left) and rapidity y (right) for b_A integrated cross section at $\sqrt{s_N} = 5.02$ TeV (solid) and 13 TeV (dashed) *(M.K. et al., PRD 105 (2022) 054023)*.

Coherent J/ψ photoproduction in UPC

J. Nemchik et al., PRD 107 (2023) 054005

gluon shadowing

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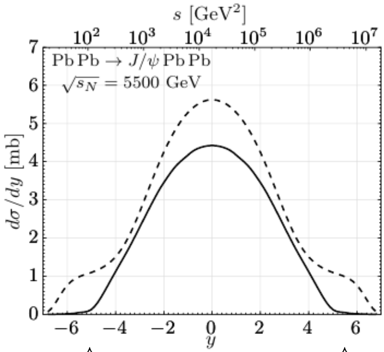

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reduced coherence length $l_c^{\bar{Q}Q}$ effect

Fig.3: Coherent cross section for photoproduction of J/ψ in UPC at $\sqrt{s_N} = 5.5$ TeV. Comparison of cross section calculated with eikonal formula (dashed line) and with corrections for finite CL and GS (solid line).

J. Nemchik et al., PRD 107 (2023) 054005

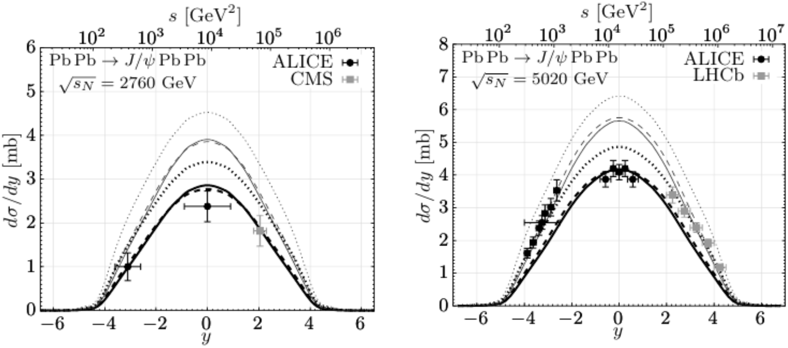


Fig.4: Rapidity distribution of coherent cross section for photoproduction of J/ψ in UPC at $\sqrt{s_N} = 2.76$ TeV (left) and $\sqrt{s_N} = 5.02$ TeV (right). Charmonium WF generated by the POW (thin lines) and BT (thick lines) potentials with GBW (solid), KST (dashed), and BGBK (dotted) models for the dipole cross section.

Quarkonia production using the Green function approach

- we want to calculate coherent cross section for quarkonia production using the Green function approach
- Green function naturally includes the effect of finite coherence length
- proper treatment of coherence length is important at forward(backward) rapidities at current LHC and RHIC energies
- it will be important for future experiments at EIC collider

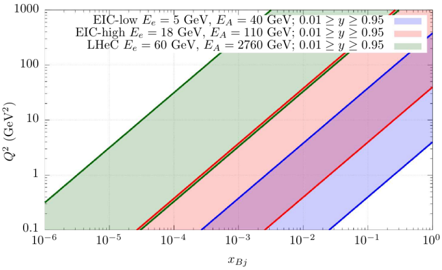


Fig.5: Kinematic regions covered by future EIC and LHeC experiments. *M.Krelina, J. Nemchik, EPJP 135 (2020) 444*.

Quarkonia production using the Green function approach

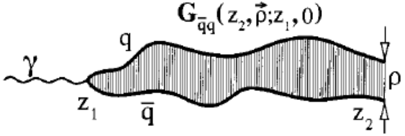
- coherent cross section for quarkonia production $\gamma A \to VA$

$$\frac{d^2\sigma_A(b,s)}{d^2b} = |\int_{-\infty}^{\infty} dz_1 \rho_A(b,z_1) H_1(s,b,z_1)|^2,$$

where

$$H_1(s,b,z_1) = \int_0^1 d\alpha \int d^2 r_1 \int d^2 r_2 \psi_V^*(\vec{r}_2,\alpha) G_{q\bar{q}}(z_2,\vec{r}_2;z_1,\vec{r}_1) \tilde{\sigma}_{q\bar{q}}(\vec{r}_1,s) \psi_\gamma(\vec{r}_1,\alpha)|_{z_2\to\infty}.$$

- $G_{q\bar{q}}(z_2,\vec{r}_2;z_1,\vec{r}_1)$ - Green function describes the propagation of $\bar{q}q$ fluctuation throught the medium



Quarkonia production using the Green function approach

- evolution equation for the Green function on LC reads

$$i\frac{d}{dz_1}G_{q\bar{q}}(z_2,\vec{r}_2;z_1,\vec{r}_1) = \left[\frac{\epsilon^2 - \triangle_{r_2}}{2k\alpha(1-\alpha)} + V_{\bar{q}q}(z_2;\vec{r}_2,\alpha)\right]G_{q\bar{q}}(z_2,\vec{r}_2;z_1,\vec{r}_1) \quad (1)$$

  with the boundary condition $G_{q\bar{q}}(z_2,\vec{r}_2;z_1,\vec{r}_1)|_{z_1=z_2} = \delta^2(\vec{r}_1-\vec{r}_2)$

- $V_{\bar{q}q}(z_2;\vec{r}_2,\alpha)$ - complex potential on the light-cone
  - $\mathrm{Im}\,V_{\bar{q}q}(z_2;\vec{r}_2,\alpha) = -i\frac{\sigma_{\bar{q}q}}{2}\rho_A(b,z)$
  - $\mathrm{Re}\,V_{\bar{q}q}(z_2;\vec{r}_2,\alpha) = \frac{a^4(\alpha)\vec{r}^2}{2k\alpha(1-\alpha)}$ for HO potential
- analytic solution possible for HO potential
- other potential models (BT, POW, ...) $\Rightarrow$ numerical solution of Eq.(1) + boost of the $V_{\bar{q}q}(r)$ to the light-cone

Numerical solution of GF evolution equation

- using substitution
  $g_1(\vec{r}_2, z_2; z_1) = \int d^2 r_1 K_0(\epsilon \vec{r}_1) G_{q\bar{q}}(z_2, \vec{r}_2; z_1, \vec{r}_1) \sigma_{q\bar{q}}(\vec{r}_1, s)$
- the function g_1 satisfies the following Schrödinger equation

$$i \frac{d}{dz_2} g_1(\vec{r}_2, z_2; z_1) = \left[\frac{\epsilon^2 - \triangle_{r_2}}{2k\alpha(1-\alpha)} + V_{\bar{q}q}(z_2; \vec{r}_2, \alpha) \right] g_1(\vec{r}_2, z_2; z_1) \quad (2)$$

  with boundary condition $g_1(\vec{r}_2, z_2; z_1)|_{z_2 = z_1} = K_0(\epsilon \vec{r}_2) \sigma_{q\bar{q}}(\vec{r}_2, s)$
- soulution of Eq.(2) using the Crank-Nicholson method
  J. Nemchik, PRC 68 (2003) 035206

Quarkonia production using the Green function approach

- calculations of coherent cross section using the Green function are in progress!
- LC form of $V_{\bar{q}q}$ for realistic potential models under discussion

Conclusions

- study of coherent photoproduction of heavy quarkonia within LC color dipole approach
- important effects of quantum coherence included
  - reduction of coherence length for $\bar{Q}Q$ fluctuation included via form fators
  - gluon shadowing for $|\bar{Q}Qg\rangle$ state calculated using the Green function technique
- calculations of coherent cross section for quarkonia production using the Green function formalism in progress!

Thank you for your attention!

Back-up slides

$V_{\bar{q}q}$ on the light-cone

- we know the VM wave function on the light-cone $\Psi_V(\vec{r},\alpha)$ for any rest frame $\bar{q}q$ potential
- possible derivation of LC $V_{\bar{q}q}(\vec{r},\alpha)$ from LC Schrödinger equation

$$\left(\frac{A^2-\Delta}{2k\alpha(1-\alpha)}+V_{\bar{q}q}(\vec{r},\alpha)\right)\Psi_V(\vec{r},\alpha)=E_{LC}\Psi_V(\vec{r},\alpha)$$

$$\Downarrow$$

$$V^*_{\bar{q}q}(r)=V_{\bar{q}q}(r,\alpha)+\frac{A^2}{2k\alpha(1-\alpha)}=\frac{E_{LC}\Psi_V(r,\alpha)+\frac{\Delta\Psi_V(r,\alpha)}{2k\alpha(1-\alpha)}}{\Psi_V(r,\alpha)}$$

- we assume that $E_{LC}=E_{rest}\frac{m_q}{2k\alpha(1-\alpha)}$
- shift A^2 determined from the non-relativistic limit