

Transverse Momentum Distributions of Heavy Hadrons and Polarized Heavy Quarks

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arXiv: 2305.15461



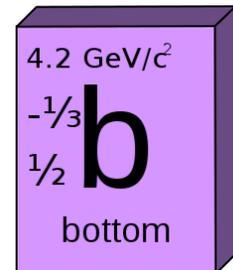
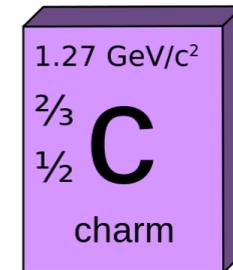
Massachusetts
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QCD MASTER CLASS
SAINT-JACUT-DE-LA-MER, FRANCE

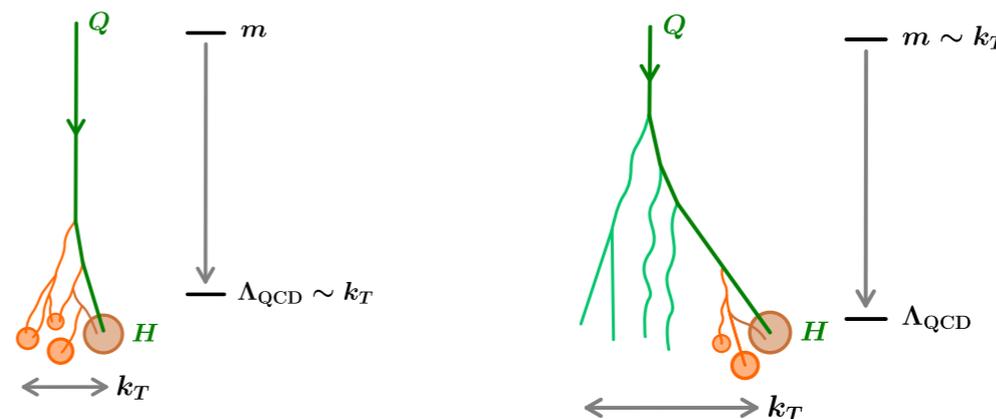
June 15th, QCD Masterclass 2023, France

Outline

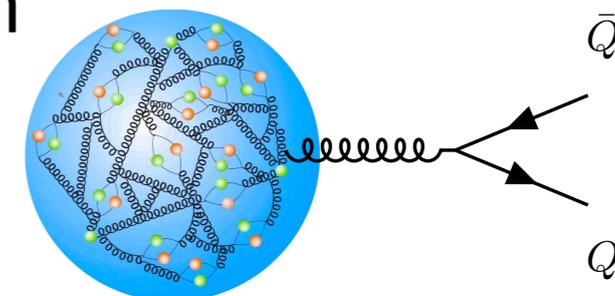
- Motivation: heavy quarks as probes for hadronization
- Heavy quark TMD fragmentation dynamics in two regimes



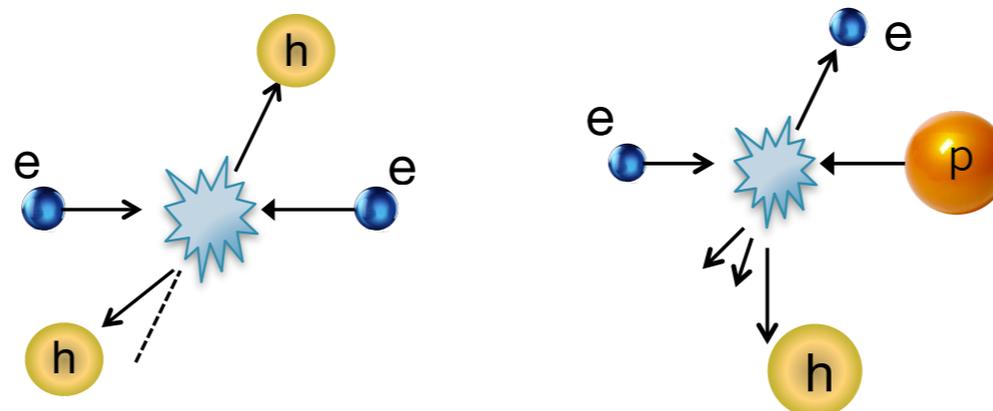
- $\Lambda_{\text{QCD}} \lesssim k_T \ll m$
- $\Lambda_{\text{QCD}} \ll m \lesssim k_T$



- Polarized heavy quark TMD PDFs within nucleon
- Phenomenology at colliders



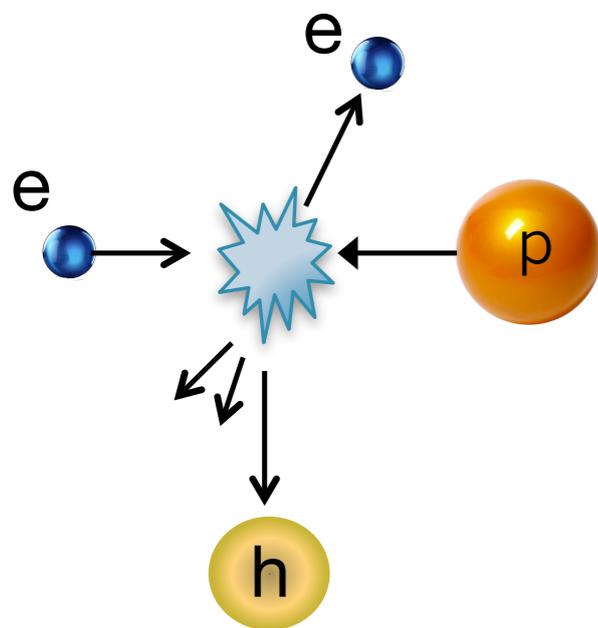
- e^+e^- colliders
- EIC



TMD Factorization in one slide

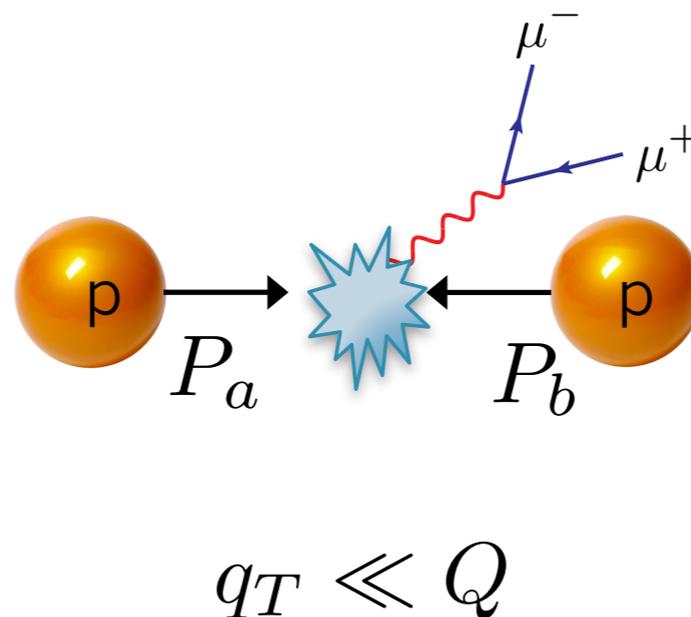
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(z, k_T)$$



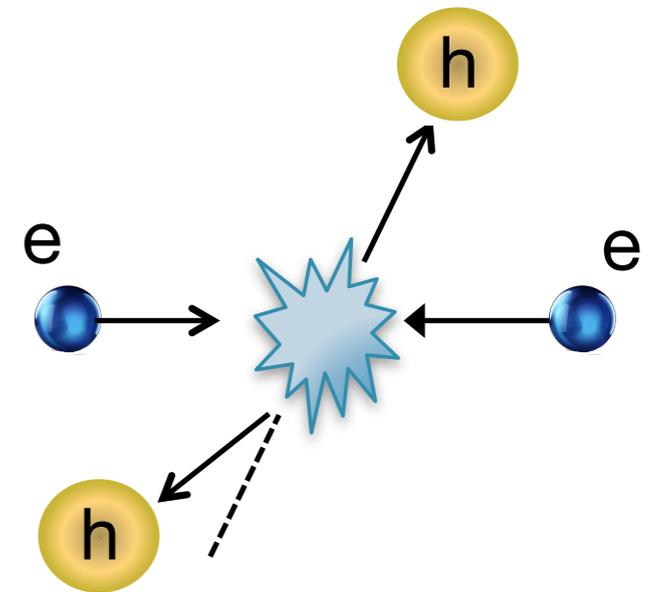
Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



Dihadron in e+e-

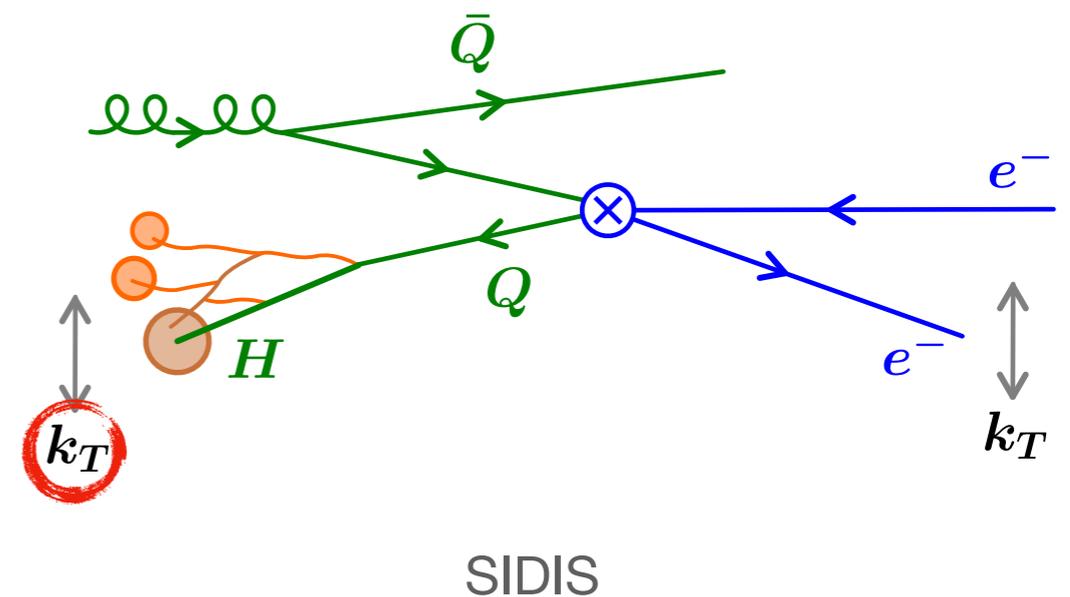
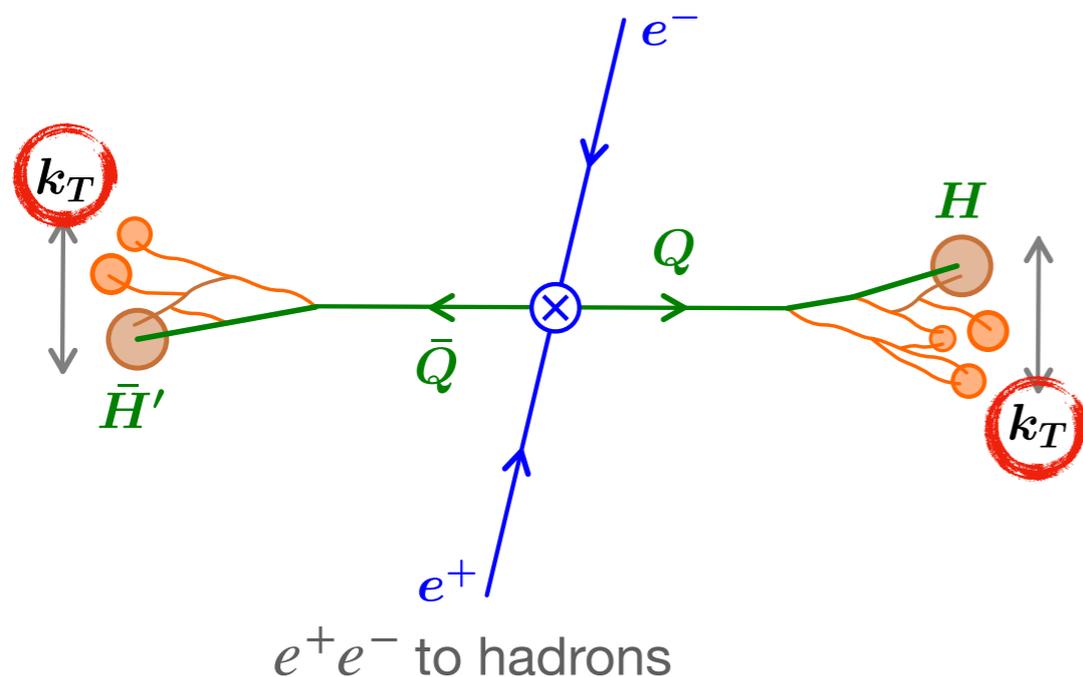
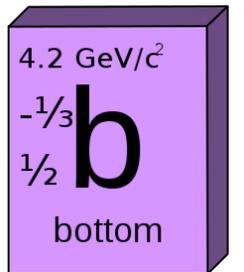
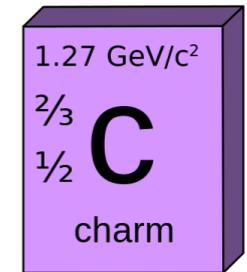
$$\sigma \sim D_{h/q}(z, k_T) D_{h/q}(z, k_T)$$



- Transverse momentum distributions (TMDs) are universal across processes
- Two scales q_T, Q allows natural power counting

Motivation

- Heavy quark (b, c) mass provides perturbative scale and static color source in hadronization process
- TMD factorization is a rigorous framework to study fragmentation
- TMDs enter other observables, e.g. EEC for (heavy) quarks
- Together with TMD PDFs, can explore EIC pheno



What this talk is not about

- Heavy quarks will be used as hard probes of gluon TMDs
→ already-planned heavy-flavor program at the EIC

[1309.0780] R. Zhu, P. Sun, F. Yuan

[1709.08970] G-P. Zhang

[2008.07531] R. del Castillo, M. Echevarria, Y. Makris, I. Scimemi

- Also considered as probe for gluon nuclear PDF

[2002.05880] I. Vitev et. al.

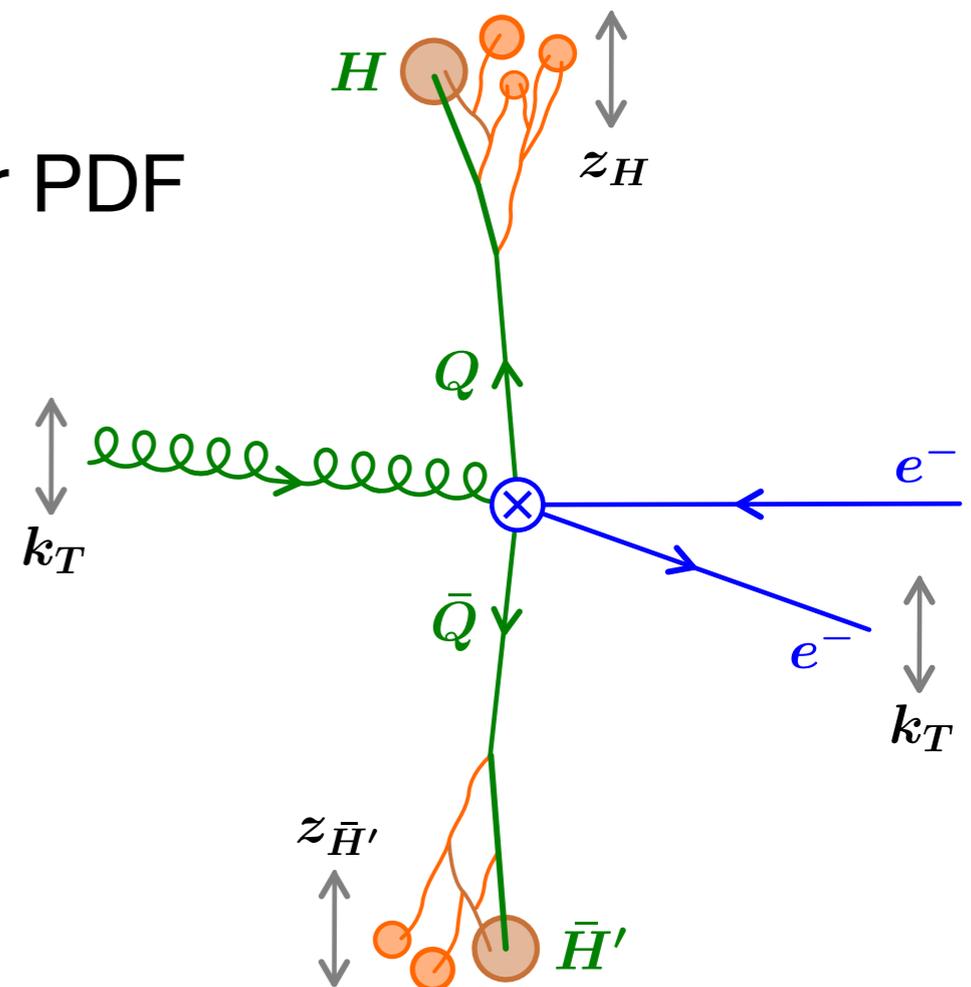
[2007.10994] I. Vitev, H-T. Li, Z-L. Liu

- Only needs the well-understood collinear (z_H) distribution

[0706.2136] M. Neubert

[1606.07737] M. Fickinger, S. Fleming, C. Kim, E. Mereghetti

- Instead, we are interested in the heavy hadron **transverse** momentum away from endpoint, $1 - z_H \sim 1$



heavy quark pair production in eN collisions

Heavy Quark TMD Fragmentation

Heavy Quark + TMD FF basics

- Heavy quark effective theory (HQET) Lagrangian: tree-level matching

$$\mathcal{L} = \bar{h}_v(i v \cdot D) h_v + \mathcal{L}_{\text{light}} + \mathcal{O}\left(\frac{1}{m}\right)$$

$$\psi_Q(x) = e^{-imv \cdot x} h_v(x) \left[1 + \mathcal{O}\left(\frac{1}{m}\right)\right]$$

- Sterile quark decoupling:

$$h_v(x) = Y_v(x) h_v^{(0)}(x), \quad Y_v(x) = P \left[\exp\left(ig \int_0^\infty ds v \cdot A(x + vs)\right) \right]$$

separate color and spin

$$u(v, h_Q) = u(mv, h_Q) / \sqrt{m}$$

HQET spinor QCD Dirac spinor

$$h_v(x) |s_Q, h_Q; s_\ell, h_\ell, f_\ell; X\rangle = u(v, h_Q) Y_v(x) |s_\ell, h_\ell, f_\ell; X\rangle$$

- TMD fragmentation function correlator:

$$\Delta_{H/Q}^{\beta\beta'}(z_H, b_\perp) = \frac{1}{2z_H N_c} \int \frac{db^+}{4\pi} e^{ib^+(P_H^-/z_H)/2}$$

$$\sum_X |HX\rangle \langle HX| \equiv \sum_X \sum_{h_H} |H, h_H; X\rangle \langle H, h_H; X|$$

sum over hadron helicity

$$\times \text{Tr} \sum_X \langle 0 | W^\dagger(b) \psi_Q^\beta(b) |HX\rangle \langle HX| \bar{\psi}_Q^{\beta'}(0) W(0) |0\rangle$$

constrained sum over states

- Unpolarized TMD FF:

$$D_{1H/Q}(z_H, b_T) = \text{tr} \left[\frac{\not{n}}{2} \Delta_{H/Q}(z_H, b_\perp) \right]$$

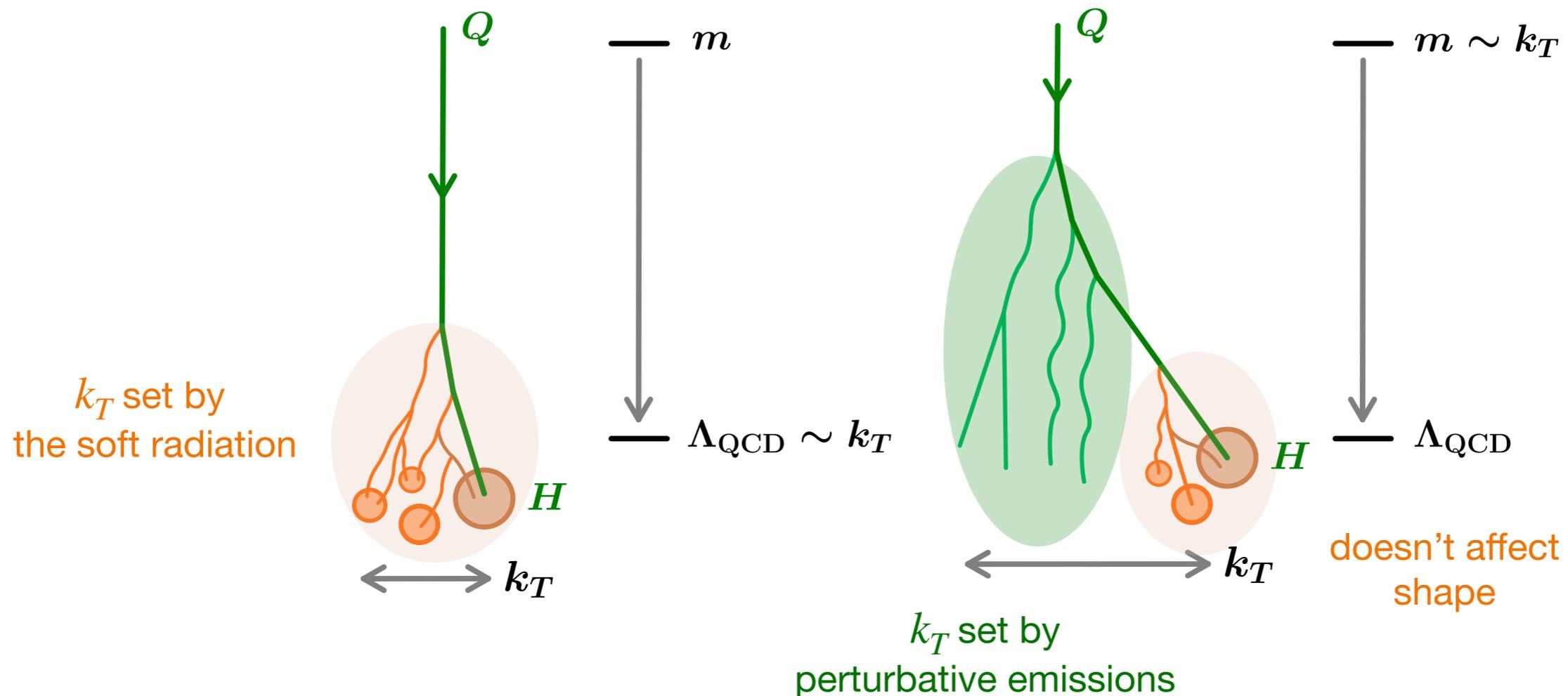
- Collins FF:

$$H_{1H/Q}^{\perp(1)}(z_H, b_T) = \text{tr} \left[\frac{\not{n}}{2} \frac{\not{b}_\perp}{b_T} \Delta_{H/Q}(z_H, b_\perp) \right]$$

Two Regimes

- m_c, m_b are perturbative scales
- Multi-scale problem: two hierarchies to consider

1.27 GeV/c ² 2/3 1/2 c charm	4.2 GeV/c ² -1/3 1/2 b bottom
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- In both regimes, TMD factorization still applies: $m_Q, k_T \ll \text{hard scale}$

Regime 1: $\Lambda_{\text{QCD}} \lesssim k_T \ll m$

- Strategy: match TMD FF correlator onto bHQET

$$\Delta_{H/Q}^{\beta\beta'}(z_H, b_\perp) = \frac{1}{2zN_c} \int \frac{db^+}{4\pi} e^{ib^+ P_H^- / (2z_H)} \text{Tr} \not{x} \langle 0 | W^\dagger(b) \psi_Q^\beta(b) |HX\rangle \langle HX | \bar{\psi}_Q^{\beta'}(0) W(0) |0\rangle$$

↓ matching to HQET at tree level

$$\Delta_{H/Q}(z_H, b_\perp) = \frac{\delta(1-z_H)}{\bar{n} \cdot v} \frac{1}{2N_c} \text{Tr} \not{x} \langle 0 | W^\dagger(b_\perp) h_v(b_\perp) |H_v X\rangle \langle H_v X | \bar{h}_v(0) W(0) |0\rangle$$

bHQET matrix element $\equiv F_H(b_\perp)$

- Unpolarized and Collins can then be written as

$$D_{1H/Q}(z_H, b_T, \mu, \zeta) = \delta(1-z_H) C_m\left(m, \mu, \frac{\zeta}{m^2}\right) \chi_{1,H}(b_T, \mu, \zeta) + \mathcal{O}\left(\frac{1}{m}\right)$$

$$b_T M_H H_{1H/Q}^{\perp(1)}(z_H, b_T, \mu, \zeta) = \delta(1-z_H) C_m\left(m, \mu, \frac{\zeta}{m^2}\right) \chi_{1,H}^\perp(b_T, \mu, \zeta) + \mathcal{O}\left(\frac{1}{m}\right)$$

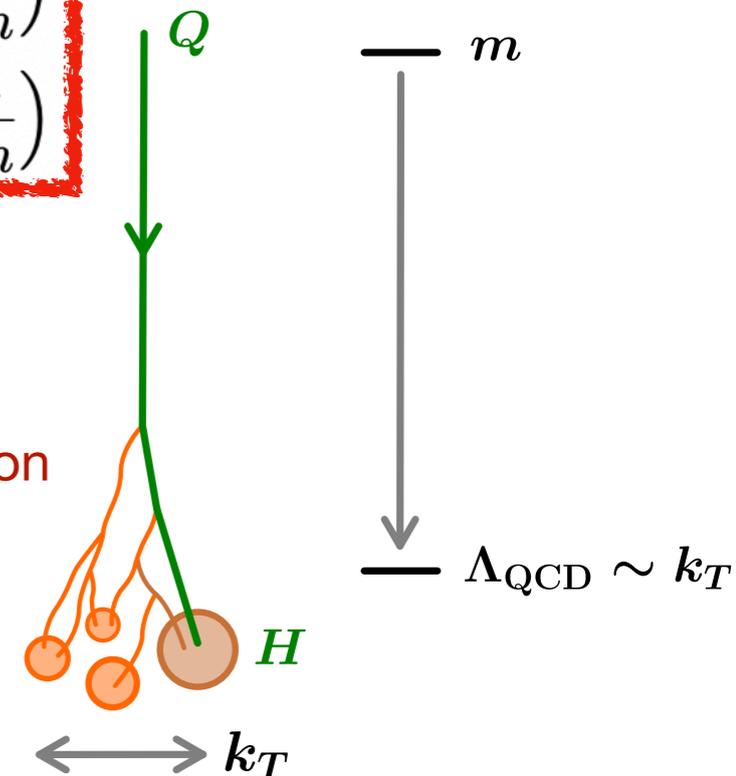
[1508.04137] A. Hoang, A. Pathak, P. Pietrulewicz, I. Stewart

where the the two scalar functions are explicitly:

parameterize Wilson line direction

$$\chi_{1,H}(b_T) = \frac{1}{2} \text{tr} F_H(b_\perp), \quad \chi_{1,H}^\perp(b_T) = \frac{1}{2} \text{tr} \left[\frac{b_\perp}{b_T} \not{z} F_H(b_\perp) \right]$$

Nonperturbative heavy quark TMD fragmentation factors



Regime 1: $\Lambda_{\text{QCD}} \lesssim k_T \ll m$

- Now decouple the sterile heavy quark:

$$F_H(b_\perp) = \frac{1}{2} \sum_{h_H} \sum_{h_Q, h'_Q} \sum_{h_\ell, h'_\ell} u(v, h_Q) \bar{u}(v, h'_Q) \langle s_Q, h_Q; s_\ell, h_\ell | s_H, h_H \rangle \langle s_H, h_H | s_Q, h'_Q; s_\ell, h'_\ell \rangle$$

$$\times \frac{1}{N_c} \text{Tr} \int_X \langle 0 | W^\dagger(b_\perp) Y_v(b_\perp) | s_\ell, h_\ell, f_\ell; X \rangle \langle s_\ell, h'_\ell, f_\ell; X | Y_v^\dagger(0) W(0) | 0 \rangle \equiv \rho_{\ell, h_\ell h'_\ell}(b_\perp)$$

spin-density matrix for light dof

- Unpolarized:

Clebsch-Gordan coefficients

$$\chi_{1,H}(b_T) = \frac{1}{2} \sum_{h_H} \sum_{h_Q} \sum_{h_\ell} |\langle s_Q, h_Q; s_\ell, h_\ell | s_H, h_H \rangle|^2 \rho_{\ell, h_\ell h_\ell}(b_\perp)$$

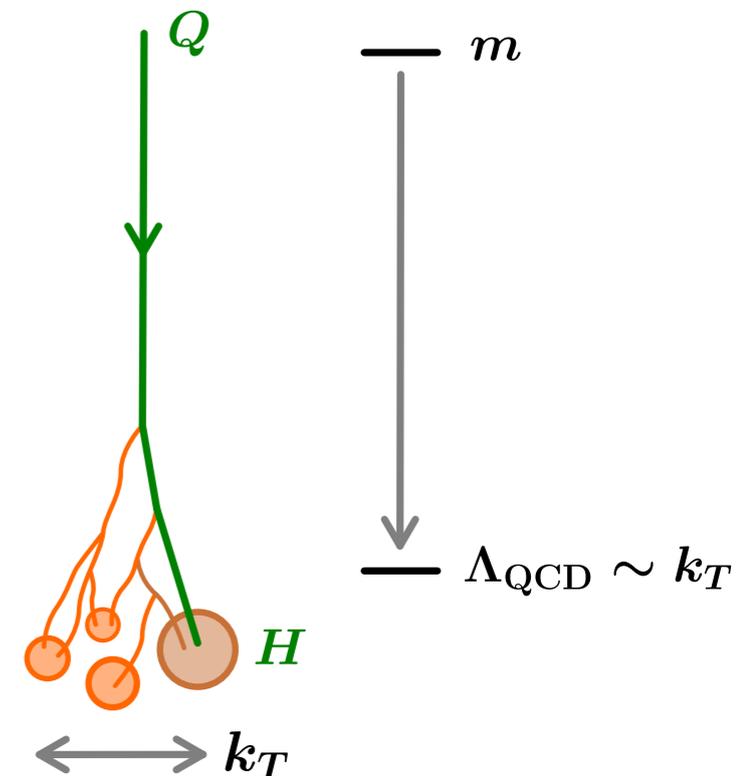
e.g. $s_\ell = 1/2, s_H = 0$: $\chi_{1,H}(b_T) = \frac{1}{4} [\rho_{\ell, ++}(b_\perp) + \rho_{\ell, --}(b_\perp)]$

- Further define: $\chi_{1,\ell}(b_T) \equiv \sum_{H \in M_\ell} \chi_{1,H}(b_T) = \sum_{h_\ell} \rho_{\ell, h_\ell h_\ell}(b_\perp)$
sum over hadrons with same light flavor and spin

$$s_\ell = \frac{1}{2} : \quad \chi_{1,H}(b_T) = \frac{1}{4} \chi_{1,\ell}(b_T), \quad \chi_{1,H^*}(b_T) = \frac{3}{4} \chi_{1,\ell}(b_T)$$

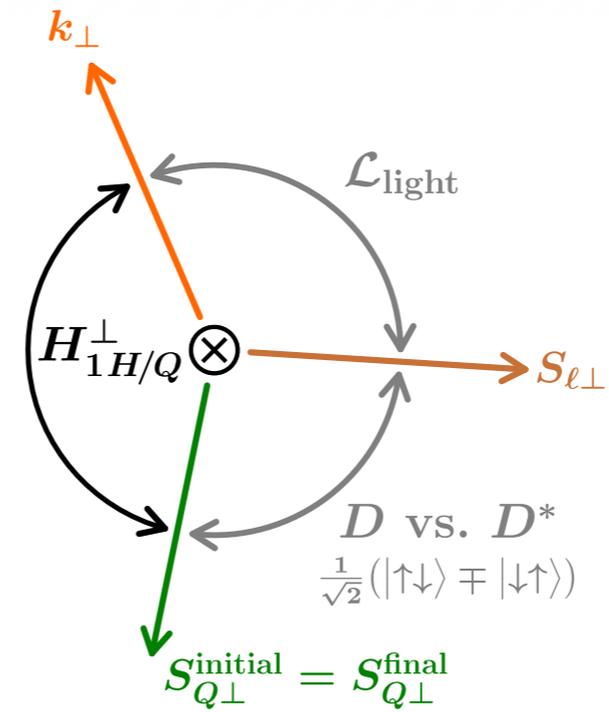
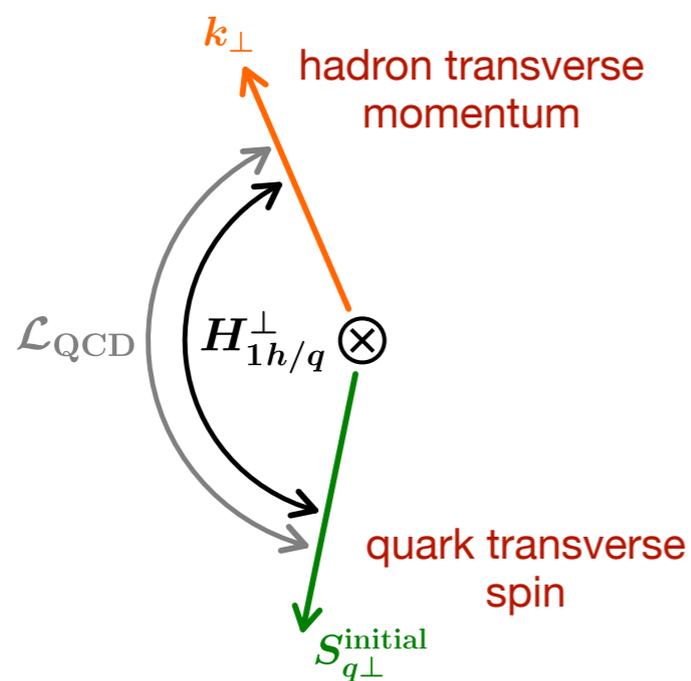
e.g. D, D^*

Result of heavy quark spin symmetry



Regime 1: $\Lambda_{\text{QCD}} \lesssim k_T \ll m$

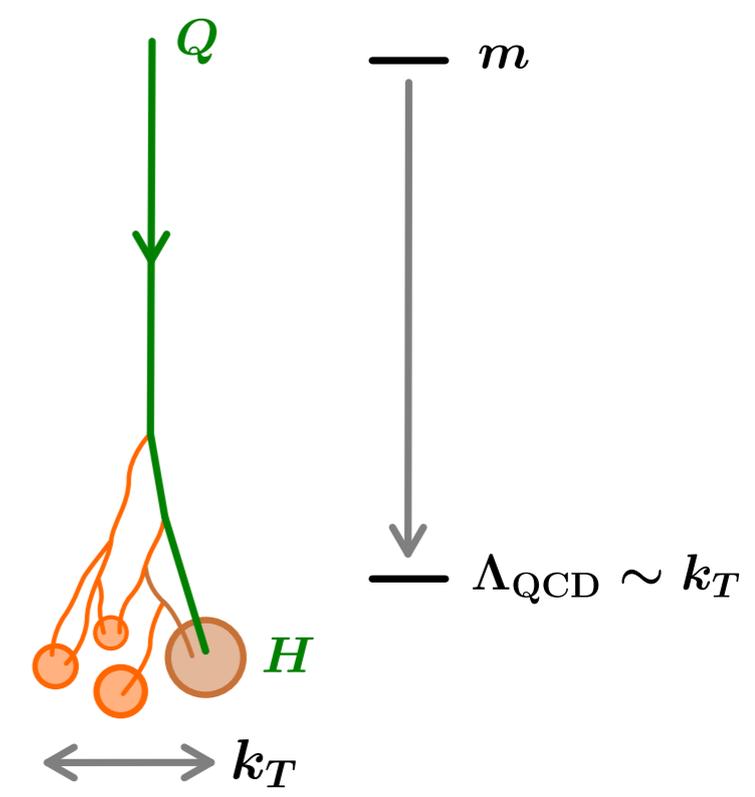
- For Collins, can similarly compute: $\chi_{1,H}^\perp(b_T) = \frac{1}{2} \text{tr} \left[\frac{\not{b}_\perp}{b_T} \not{F}_H(b_\perp) \right]$
 e.g. $s_\ell = 1/2, s_H = 0$: $\chi_{1,H}^\perp(b_T) = \frac{1}{4} [\rho_{\ell,-+}(b_\perp) - \rho_{\ell,+-}(b_\perp)]$



- Similarly, arrive at sum rule: $\sum_{H \in M_\ell} \chi_{1,H}^\perp(b_T) = 0$
 sum over hadrons with same light flavor and spin

$s_\ell = \frac{1}{2}$: $\chi_{1,H}^\perp(b_T) + \chi_{1,H^*}^\perp(b_T) = 0$

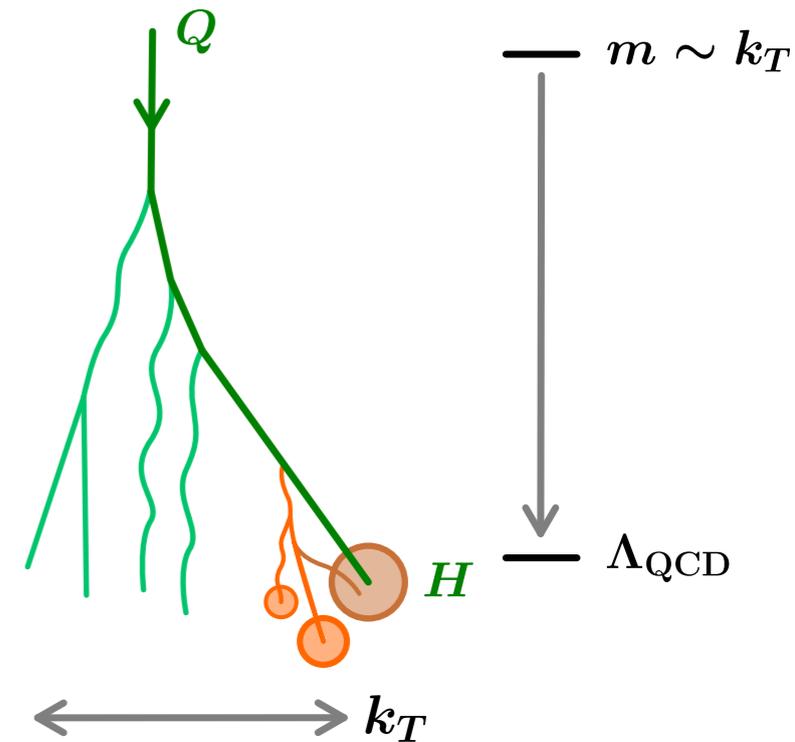
e.g. D, D^* Result of heavy quark spin symmetry



Regime 2: $\Lambda_{\text{QCD}} \ll m \sim k_T$

- Unpolarized TMD FF:
new perturbative matching coefficient

$$D_{1H/Q}(z_H, b_T, \mu, \zeta) = d_{1Q/Q}(z_H, b_T, \mu, \zeta) \chi_H + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m}\right) + \mathcal{O}(\Lambda_{\text{QCD}} b_T)$$



partonic heavy-quark TMD FF

$$d_{1Q/Q}(z, b_T) = \text{tr} \left[\frac{\not{b}_T}{2} \Delta_{Q/Q}(z, b_\perp) \right] = \delta(1-z) + \mathcal{O}(\alpha_s)$$

Refactorizes \downarrow in the limit $m \ll k_T$

$$\sum_i \int \frac{dz}{z} \mathcal{J}_{i/Q}(z, b_T, \mu, \zeta) d_{Q/i}\left(\frac{z_H}{z}, \mu\right) + \mathcal{O}(m^2 b_T^2)$$

total probability for the quark to fragment into H

$$\frac{1}{4N_c} \text{Tr} \text{tr} \sum_X \langle 0 | W^\dagger(0) h_v(0) | H_v X \rangle \langle H_v x | \bar{h}_v(0) W(0) | 0 \rangle$$

[hep-ph/9306201] R. Jaffe, L. Randall
[hep-ph/9308241] A. Falk, M. Peskin
[0706.2136] M. Neubert

- From HQ spin symmetry we still have: $D_{1H/Q} = \frac{1}{3} D_{1H^*/Q}$

Regime 2: $\Lambda_{\text{QCD}} \ll m \sim k_T$

- Strategy for Collins: use the well-known light quark matching onto collinear FFs, then match onto bHQET

known result for light quark

$$b_T M_h H_{1h/q}^{\perp(1)}(z_h, b_T) = b_T \hat{H}_{h/q}(z_h) + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\text{QCD}}^2 b_T^2)$$

Generalize to heavy quark

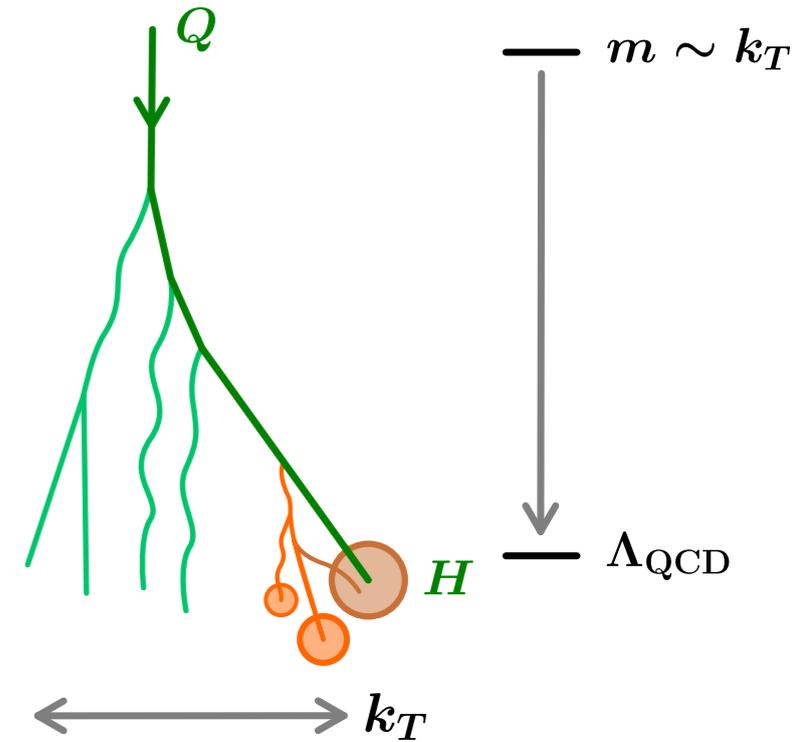
$$\hat{H}_{H/Q}(z_H) = \delta(1 - z_H) \chi_{H,G} + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{1}{m}\right)$$

$$\chi_{H,G} \equiv \frac{1}{2N_c} \text{Tr tr} \sum_X \left\{ \langle 0 | W^\dagger \sigma_{\beta\alpha} z^\beta [iD_\perp^\alpha + g\mathcal{G}_\perp^\alpha] h_v | H_v X \rangle \langle H_v X | \bar{h}_v W | 0 \rangle + \text{h.c.} \right\}$$

bHQET matrix element from matching

$$b_T M_H H_{1H/Q}^{\perp(1)}(z_H, b_T) = \delta(1 - z_H) b_T \chi_{H,G} + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\text{QCD}}^2)$$

- From HQ spin symmetry we still have: $\sum_{H \in M_\ell} \chi_{H,G} = 0$



Polarized Heavy Quark TMD PDF

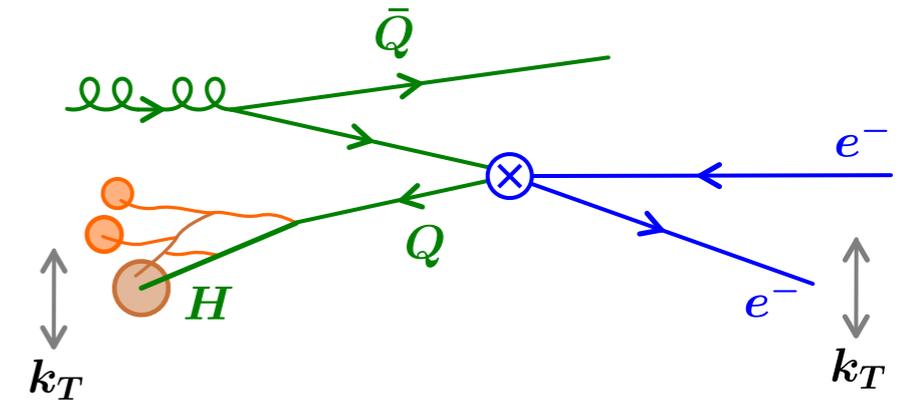
Heavy TMD PDF in nucleon

- Focus on leading gluon contribution
- Collinear gluon PDF decomposition:

$$\Phi_{g/N}^{\mu\nu}(x) = -\frac{g_{\perp}^{\mu\nu}}{2} f_{g/N}(x) + \frac{i\epsilon_{\perp}^{\mu\nu}}{2} g_{g/N}(x) S_L$$

- TMD PDF decomposition:

$$\Phi_{Q/N}(x, k_{\perp}) = \left\{ f_{1Q/N} + g_{1LQ/N} S_L \gamma_5 + h_{1LQ/N}^{\perp} S_L \gamma_5 \frac{\not{k}_{\perp}}{M_N} + i h_{1Q/N}^{\perp} \frac{\not{k}_{\perp}}{M_N} + (\text{terms } \propto S_{\perp}) \right\} \frac{\not{n}}{4}$$



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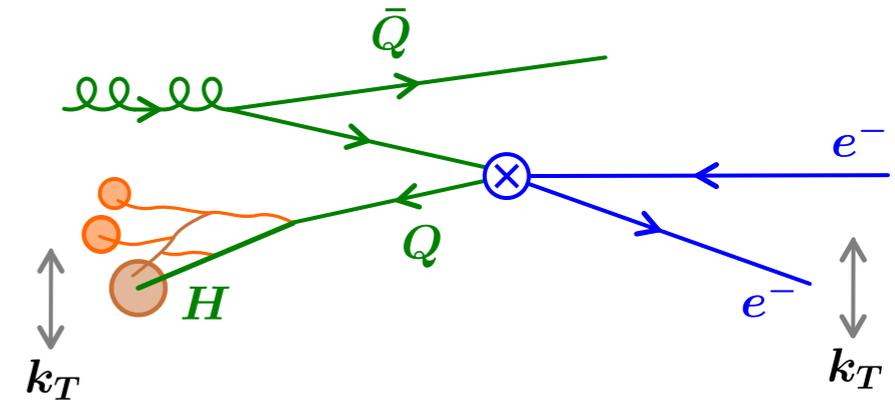
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- Allowed matching by polarization :

- Unpolarized f_1 and Boer-Mulders h_1^{\perp} onto unpolarized f_g
- Helicity g_{1L} and worm-gear $L h_{1L}^{\perp}$ onto helicity g_g

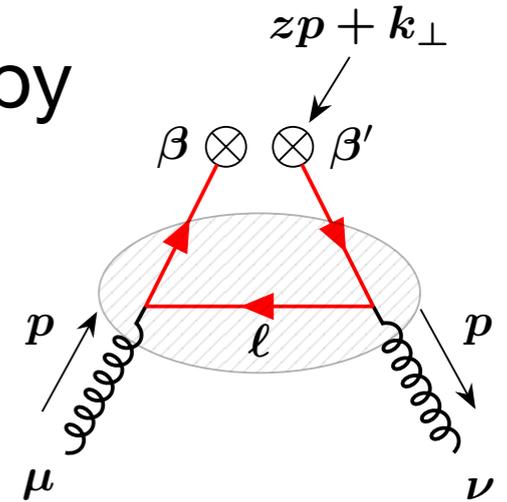
- Actually: all terms proportional to S_{\perp} vanish to all orders in α_s !



TMD PDF matching coefficients

- Momentum space matching coefficient $C_{Q/g}$ defined by

$$\Phi_{Q/N}^{\beta\beta'}(x, k_{\perp}) = \int \frac{dp^{-}}{p^{-}} C_{Q/g, \mu\nu}^{\beta\beta'}(xP_N^{-}, p^{-}, k_{\perp}) \Phi_{g/N}^{\mu\nu}\left(\frac{p^{-}}{P_N^{-}}\right)$$



- At $\mathcal{O}(\alpha_s)$ with heavy quark (gluon) polarization λ (λ'):

$$C_{Q/g}^{(1)}(z, k_T, m) = T_F \Theta(z)\Theta(1-z) \frac{2}{\pi} \frac{m^2 + k_T^2(1-2z+2z^2)}{(k_T^2 + m^2)^2} \quad \text{unpolarized from unpolarized}$$

[hep-ph/0210082] P. Nadolsky, N. Kidonakis, F. Olness, C-P. Yuan
 [1703.09702] P. Pietrullewicz, D. Samitz, A. Spiering, F. Tackmann



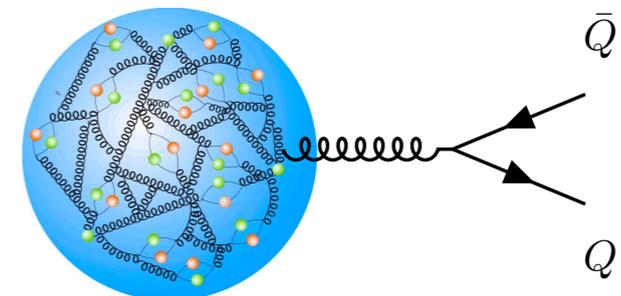
$$C_{Q_{\parallel}/g_{\parallel}}^{(1)}(z, k_T, m) = T_F \Theta(z)\Theta(1-z) \frac{2}{\pi} \frac{k_T^2(1-2z) - m^2}{(k_T^2 + m^2)^2} \quad \text{linearly polarized from linearly polarized}$$

NEW

$$C_{Q_{\perp}/g_{\parallel}}^{(1)}(z, k_T, m) = T_F \Theta(z)\Theta(1-z) \frac{4}{\pi} \frac{mk_T(1-z)}{(k_T^2 + m^2)^2} \quad \text{transversely polarized from linearly polarized}$$

NEW

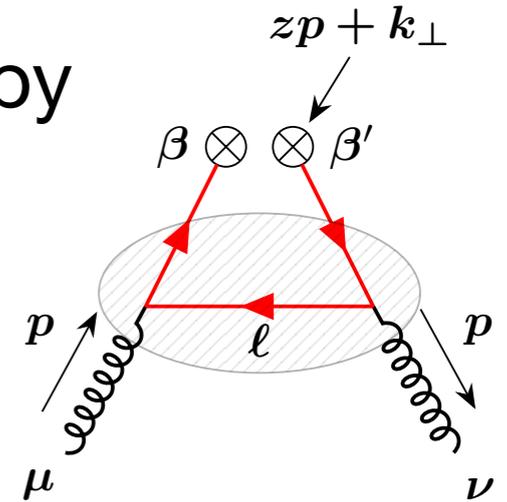
Note that Boer-Mulders function cannot map onto twist-2 gluon PDFs due to time reversal invariance



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[hep-ph/0210082] P. Nadolsky, N. Kidonakis, F. Olness, C-P. Yuan
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$$C_{Q_{\parallel}/g_{\parallel}}^{(1)}(z, k_T, m) = T_F \Theta(z)\Theta(1-z) \frac{2}{\pi} \frac{k_T^2(1-2z) - m^2}{(k_T^2 + m^2)^2}$$

linearly polarized from linearly polarized

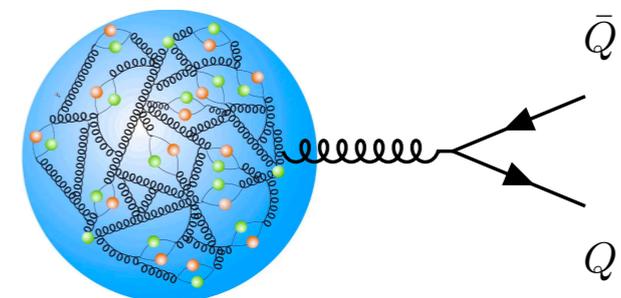
NEW

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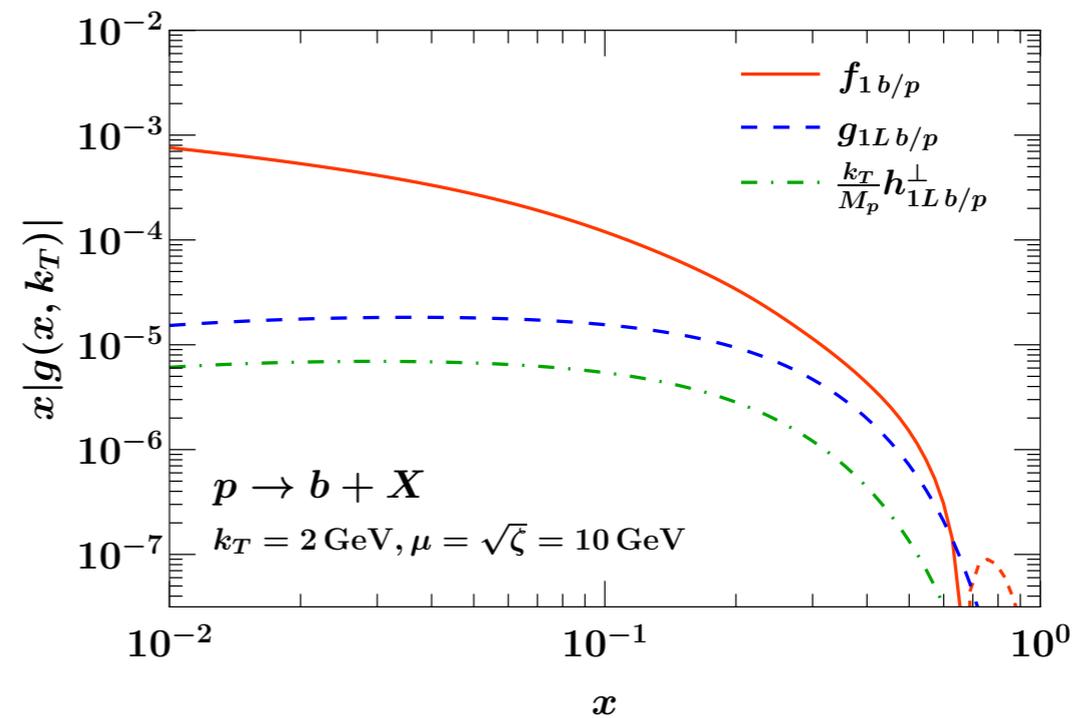
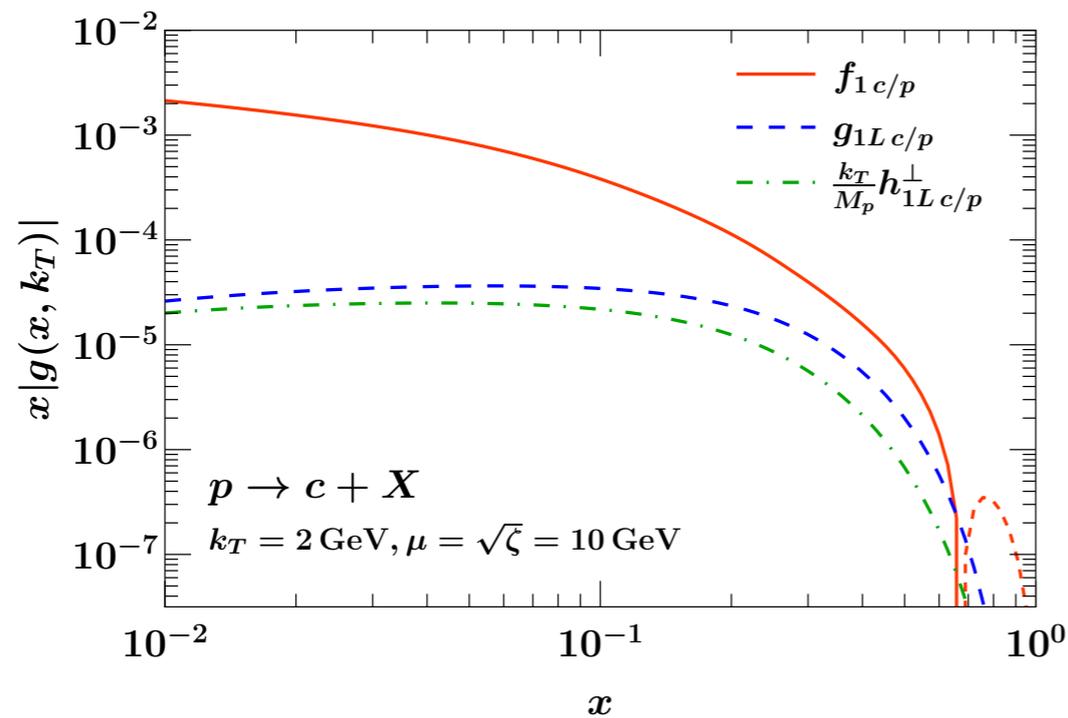
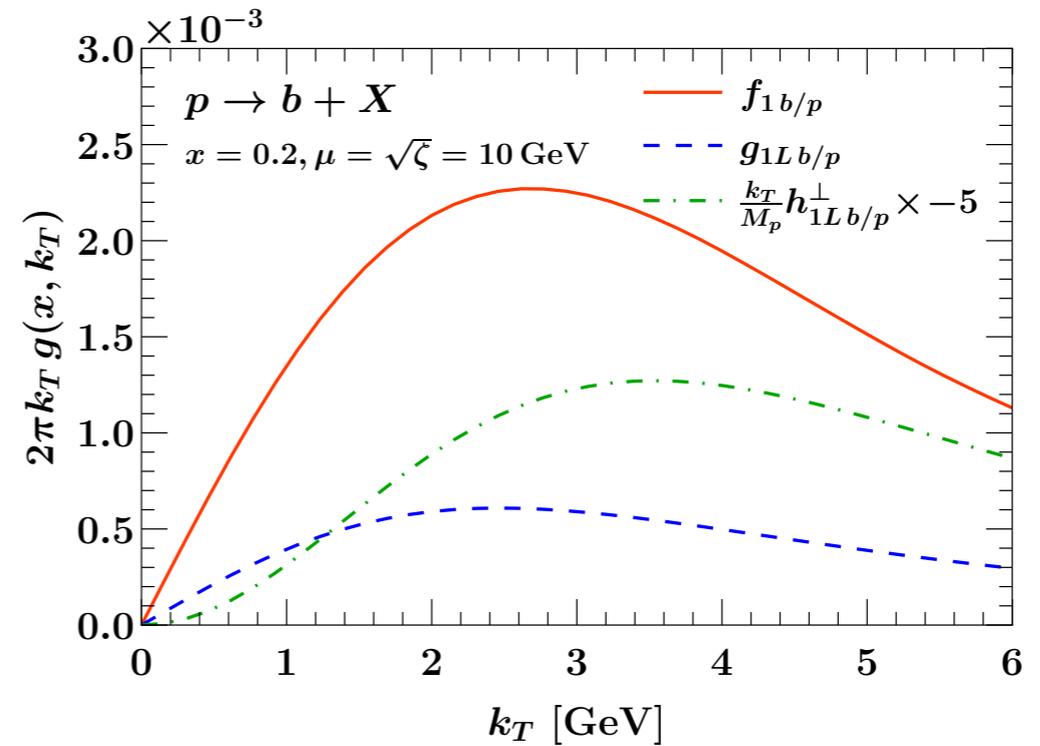
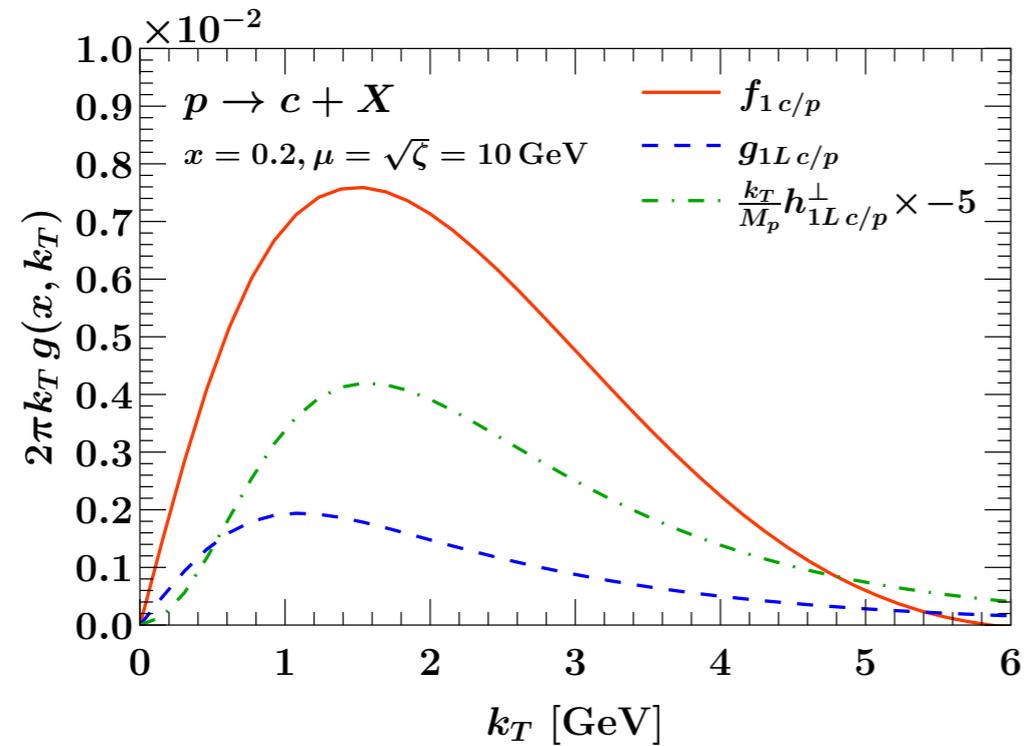
transversely polarized from linearly polarized

NEW

Note that Boer-Mulders function cannot map onto twist-2 gluon PDFs due to time reversal invariance

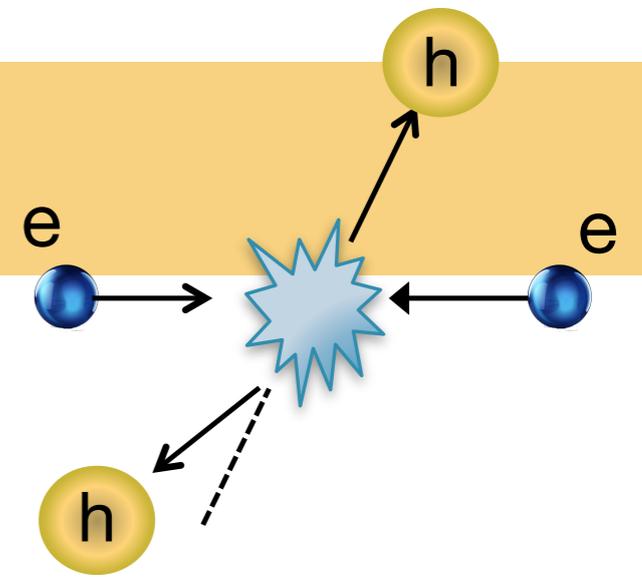


Produce polarized heavy quarks



Towards Phenomenology

e^+e^- colliders



- Cross section can still be factorized since $Q \gg q_T, m$

$$\frac{d\sigma_{e^+e^- \rightarrow H_a H_b X}}{d \cos \theta d\phi dz_a dz_b d^2 \vec{P}_{a,T}} = \frac{3\alpha_{\text{em}}^2}{Q^2} \left[\left(\frac{1}{2} - y + y^2 \right) W_{\text{incl}}(Q^2, z_a, z_b, P_{a,T}/z_a) \right. \\ \left. + y(1-y) \cos(2\phi_0) W_{\cos(2\phi_0)}(Q^2, z_a, z_b, P_{a,T}/z_a) \right] \\ + (\text{odd under } y \leftrightarrow 1-y) \quad y = (1 + \cos \theta)/2$$

$\uparrow \quad \uparrow$
 Spherical coords of H_b
 in CM frame relative to beam

unpolarized D_1

Collins H_1^\perp

ϕ_0 : azimuthal angle of hadron with respect to beam plane

- Gaussian model for nonperturbativity + leading log evolution for heavy quark TMD FFs to predict cross section

$$\int_{z_{\text{cut}}} dz_H D_{1H/Q}(z_H, b_T, \mu, \zeta) = \chi_H \exp\left(-\kappa_H^2 b_T^2\right) U_q(\mu_0, \zeta_0, \mu, \zeta) \quad \kappa_H, \kappa_{H,\perp} \sim \Lambda_{\text{QCD}} \\ \lambda_{H,\perp} = \chi_{H,G}/\chi_H \sim \Lambda_{\text{QCD}}$$

$$b_T M_H \int_{z_{\text{cut}}} dz_H H_{1H/Q}^{\perp(1)}(z_H, b_T, \mu, \zeta) = \chi_H \lambda_{H\perp} b_T \exp\left(-\kappa_{H\perp}^2 b_T^2\right) U_q(\mu_0, \zeta_0, \mu, \zeta)$$

e^+e^- colliders

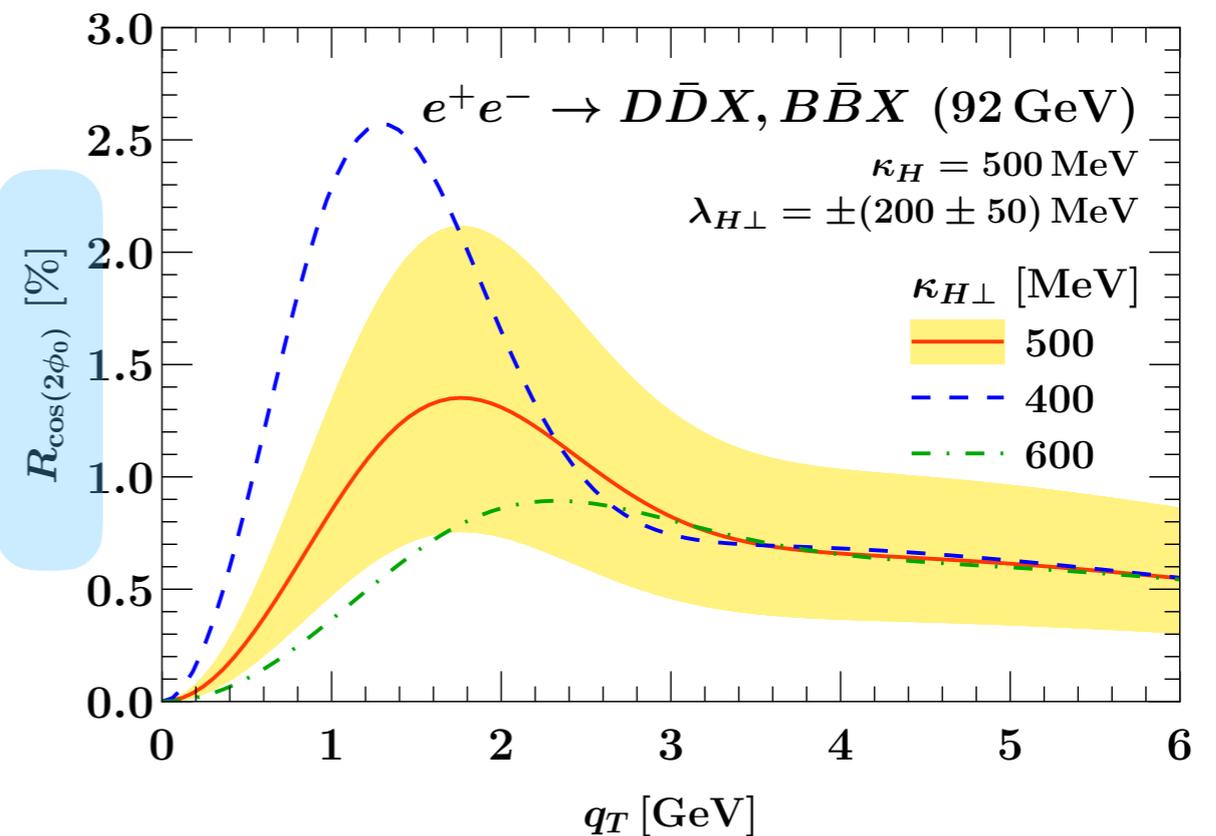
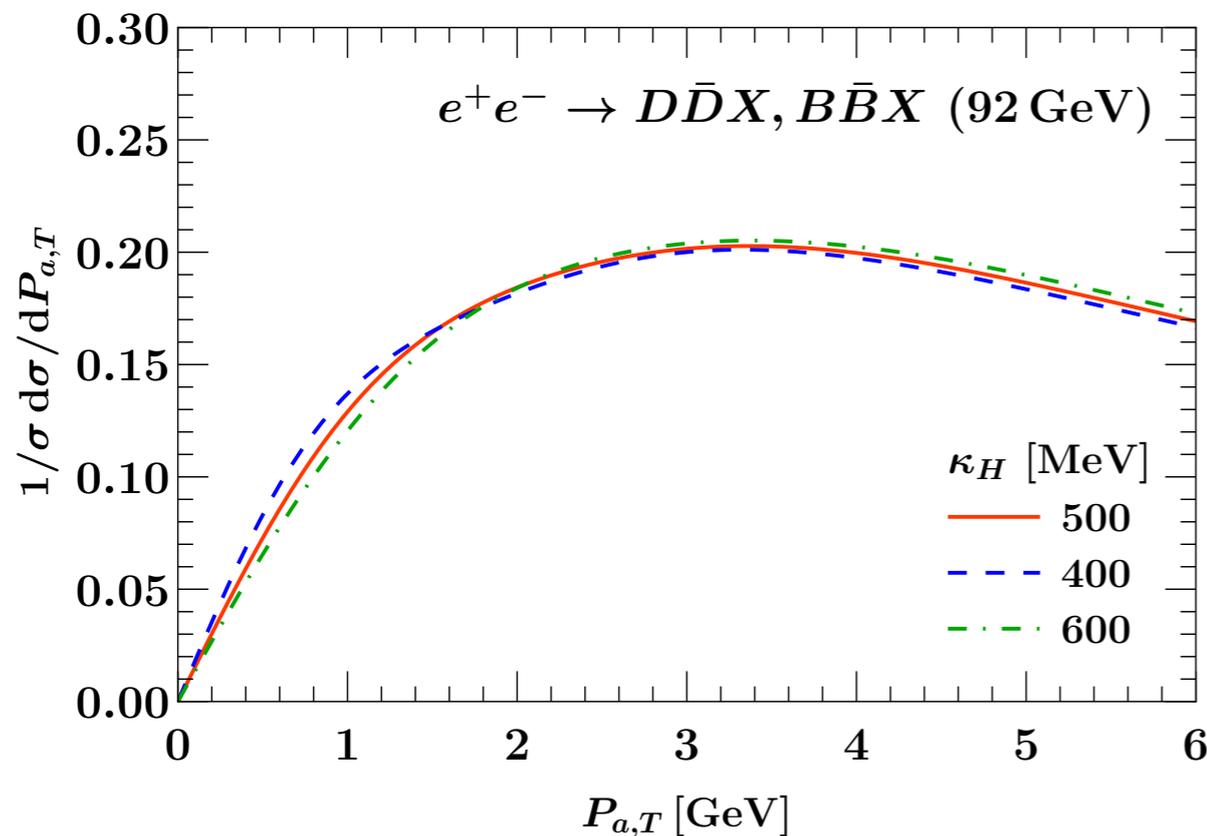
- e^+e^- cross section Collins effect

$$R_{\cos(2\phi_0)} \propto \frac{H_1^\perp \otimes H_1^\perp}{D_1 \otimes D_1}$$

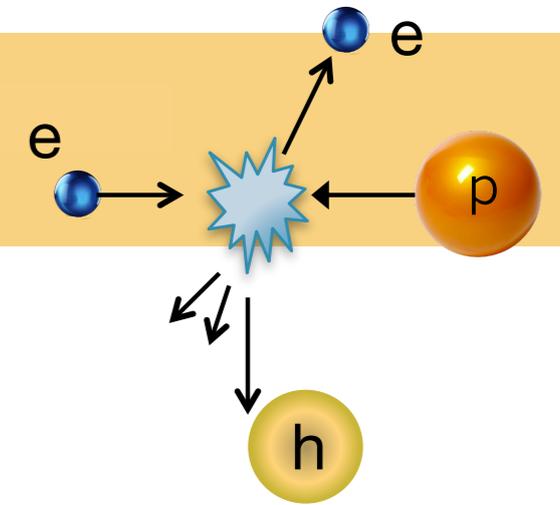
- HQ spin symmetry predict

$$R_{\cos(2\phi_0)}^{D\bar{D}} = -\frac{1}{3}R_{\cos(2\phi_0)}^{D^*\bar{D}} = -\frac{1}{3}R_{\cos(2\phi_0)}^{D\bar{D}^*} = +\frac{1}{9}R_{\cos(2\phi_0)}^{D^*\bar{D}^*}$$

can be tested in experiments!



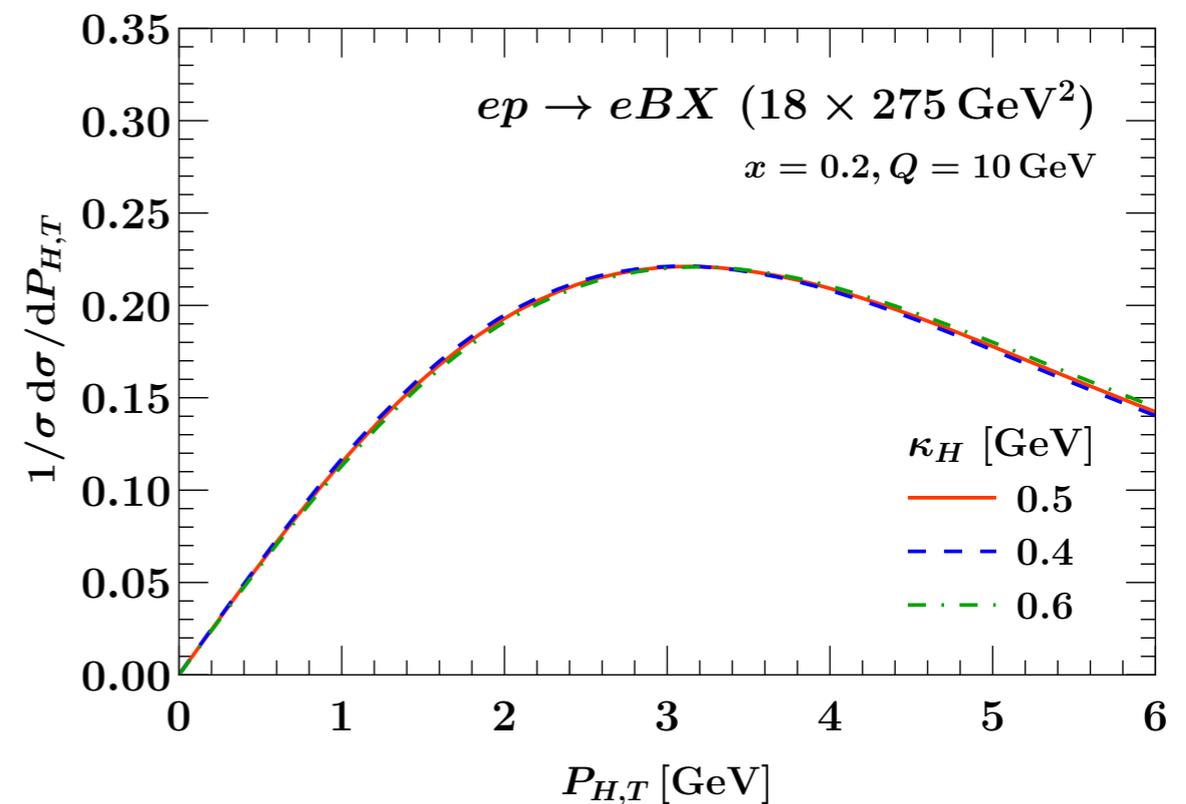
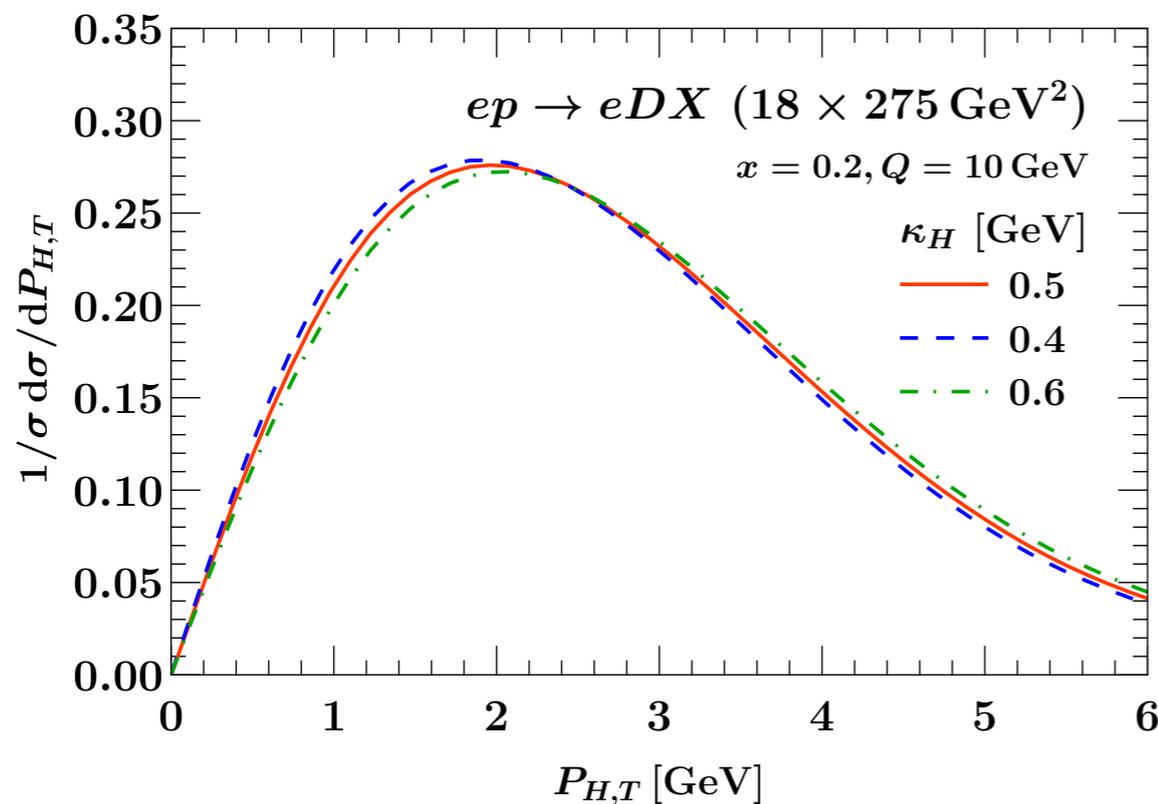
Future EIC



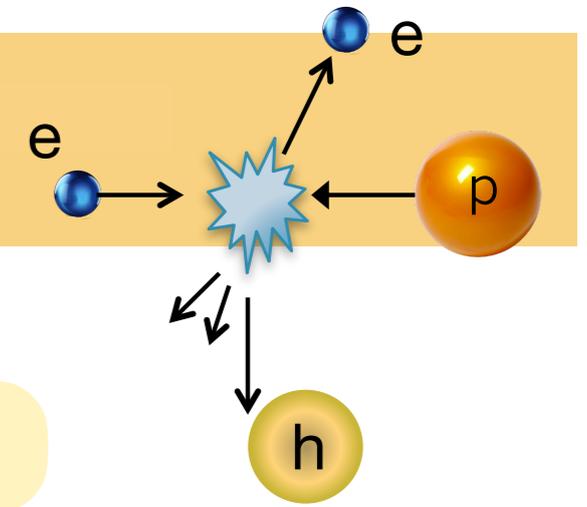
- SIDIS cross section also factorizes:

$$\frac{d\sigma_{eN \rightarrow eHX}}{dx dy dz_H d^2\vec{P}_{H,T}} = \sigma_0 \left\{ \begin{aligned} &W_{UU,T}(Q^2, x, z_H, \vec{P}_{H,T}/z_H) \quad \text{unpolarized } f_1 D_1 \\ &+ \lambda_e S_L \sqrt{1 - \epsilon^2} W_{LL}(Q^2, x, z_H, \vec{P}_{H,T}/z_H) \quad \text{helicity + unpolarized } g_{1L} D_1 \\ &+ S_L \epsilon \sin(2\phi_H) W_{UL}^{\sin(2\phi_H)}(Q^2, x, z_H, \vec{P}_{H,T}/z_H) \end{aligned} \right\} \text{worm-gear } L + \text{Collins } h_{1L}^\perp H_1^\perp$$

ϕ_H : azimuthal angle of the hadron transverse momentum in photon frame



Future EIC



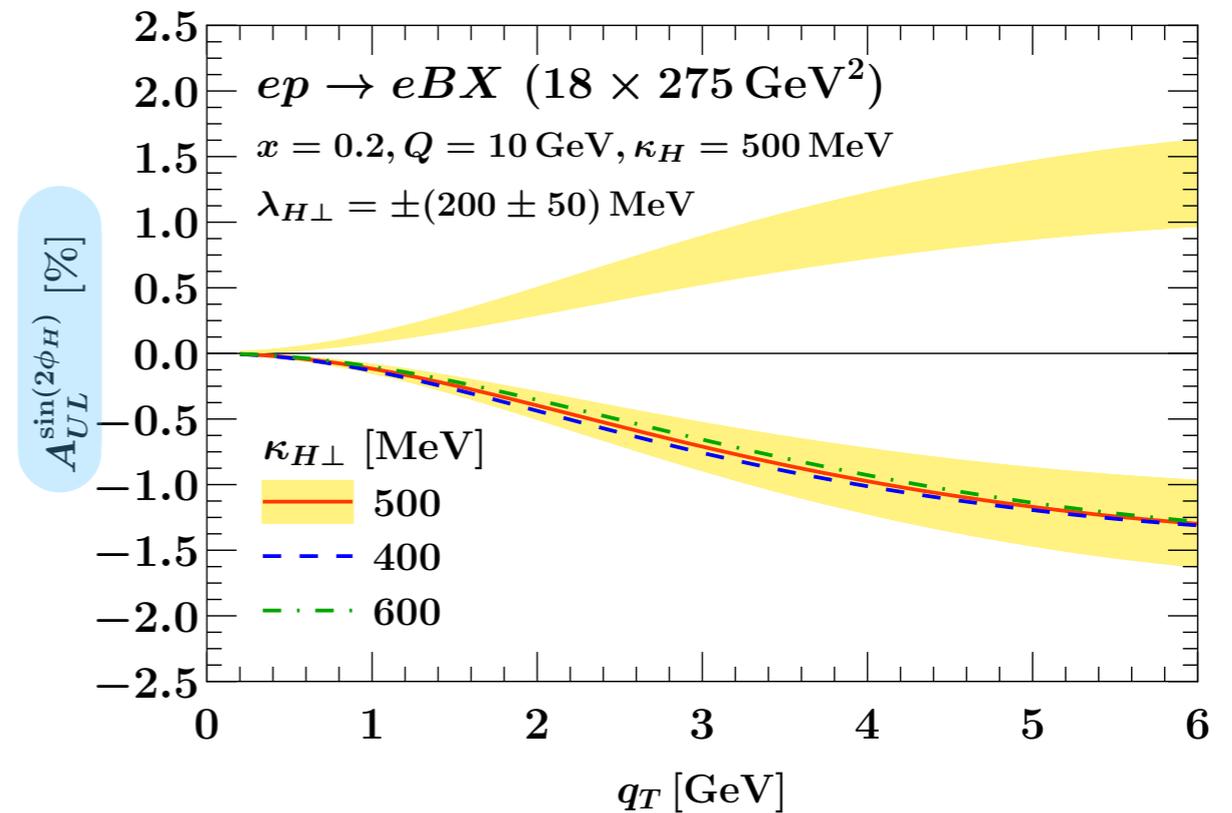
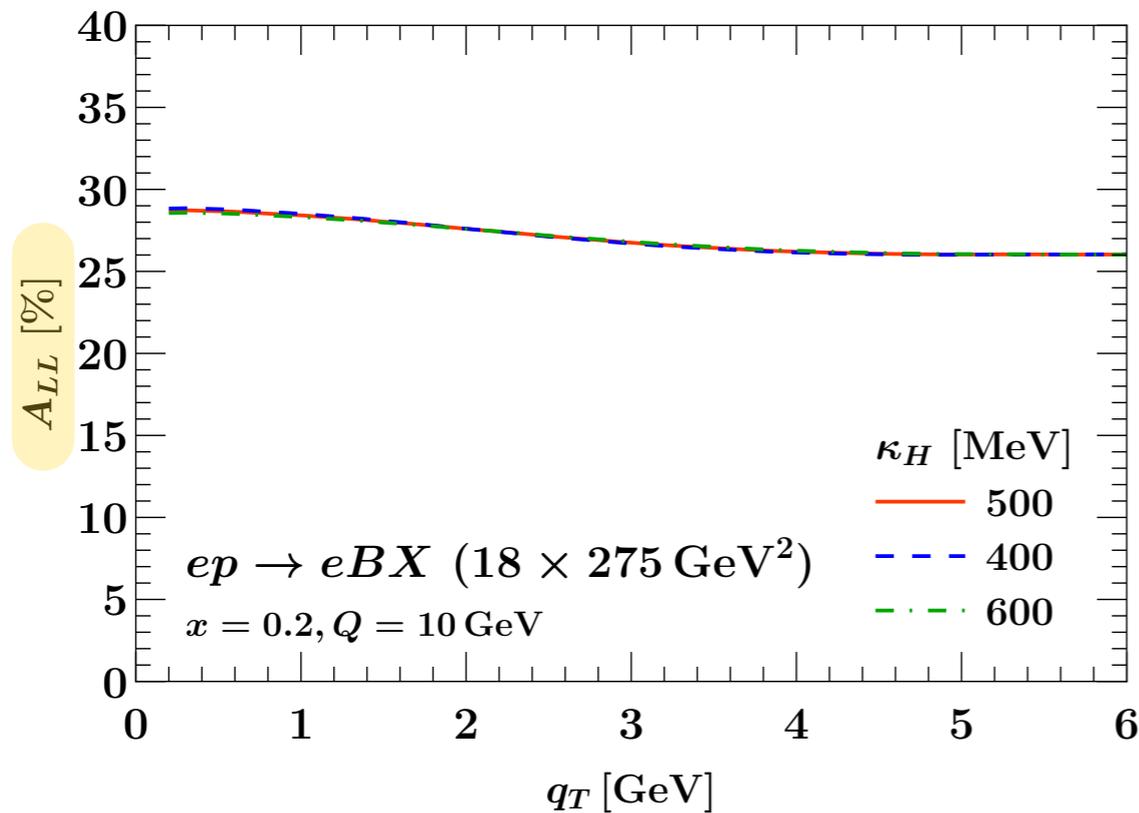
- Two nonzero spin asymmetries:

$$A_{LL} \propto \frac{g_{1L} \otimes D_1}{f_1 \otimes D_1}$$

$$A_{UL}^{\sin(2\phi_H)} \propto \frac{h_{1L}^\perp \otimes H_1^\perp}{f_1 \otimes D_1}$$

Can extract sign!

- EIC provides a clean probe of the heavy quark Collins FF



Conclusions

- Initiated the study of heavy quark TMD FFs and identified new bHQET nonperturbative matrix elements
 - ➔ Understand hadronization with rich hierarchies of scales
 - ➔ Experimentally testable results from HQ spin symmetry
- Studied perturbative matching of polarized heavy quark TMD PDFs onto twist-2 collinear gluon PDFs
 - ➔ Transversely polarized heavy quarks can be produced by linearly polarized gluons
 - ➔ Get access to a single Collins function with its sign at the EIC

That's all, thank you