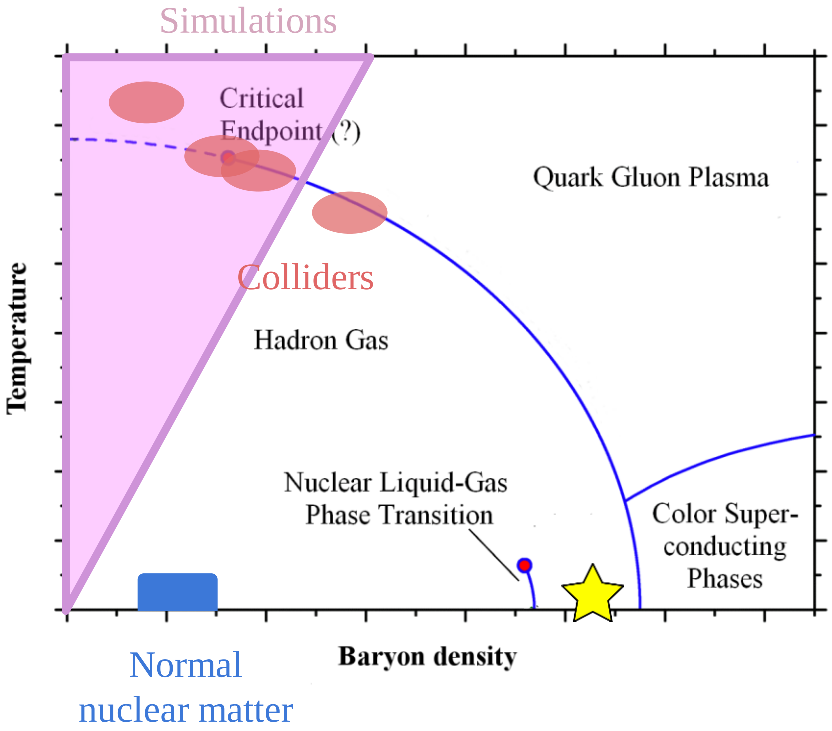
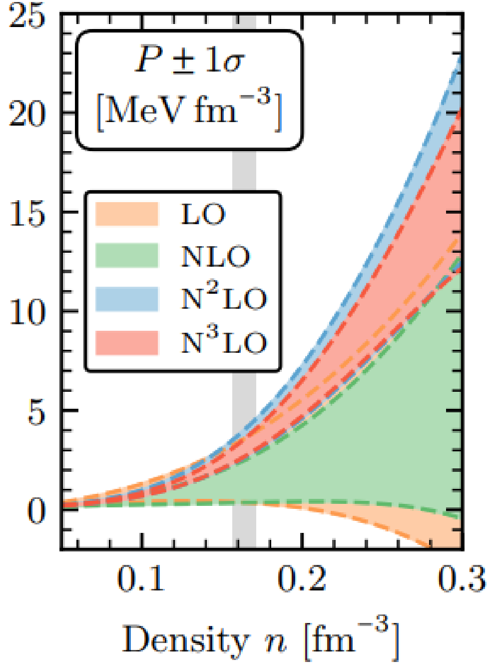
What neutron stars tell about phase transitions in QCD

János Takátsy

QCD Master Class, 2023.06.08.

Why study neutron stars?





Basic properties of neutron stars

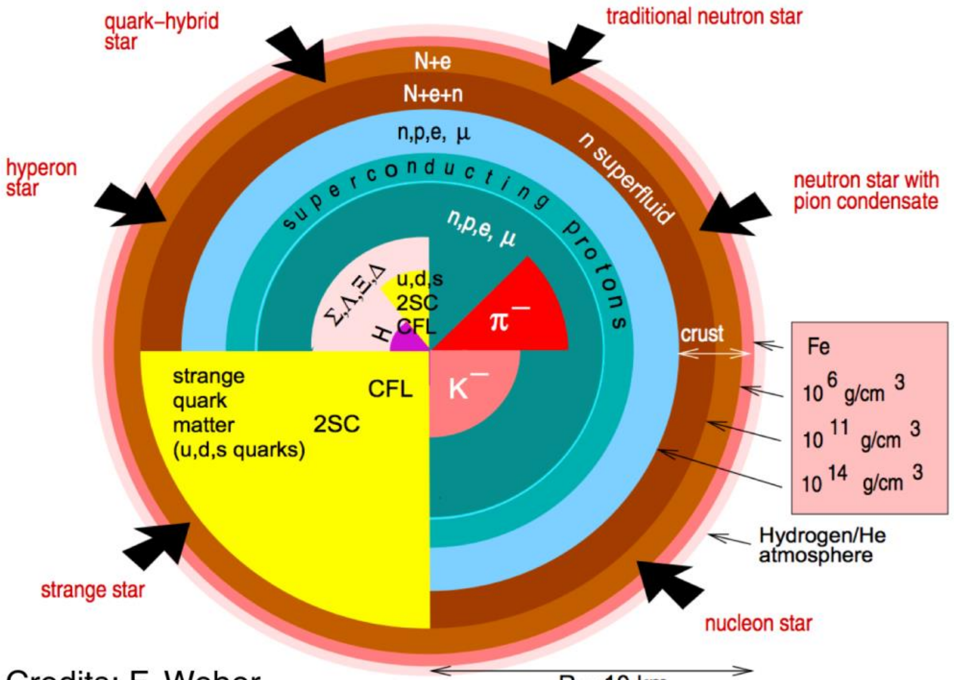
Size: R ~ 10 km

Mass: M = 1.2 $M_\odot$ - 2.3 $M_\odot$

$\longrightarrow \rho \cong 5 \cdot 10^{17}$ kg/m^3

Strong magnetic field: 10^4 - 10^{11} T (16 T in laboratory)

Fast rotation (rotational period can be as low as several ms)



Credits: F. Weber

Tolman-Oppenheimer-Volkoff equation

———

Spherically symmetric metric: $$\mathrm{d}s^2 = e^{2\nu(r)}\mathrm{d}t^2 - e^{2\lambda(r)}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

Einstein's equations (ideal fluid): $$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = (p+\varepsilon)u_\mu u_\nu - pg_{\mu\nu}$$

After fiddling with the equations one can get:

$$\boxed{\frac{\mathrm{d}p}{\mathrm{d}r} = -[\varepsilon(r)+p(r)]\,\frac{M(r)+4\pi r^3 p(r)}{r^2-2M(r)r}}$$

$$e^{-2\lambda} = 1 + \frac{2M(r)}{r} \equiv 1 + \frac{2}{r}\int_0^r 4\pi r^2\varepsilon(r)\mathrm{d}r$$

The mass-radius relation

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -[\varepsilon(r) + p(r)]\frac{M(r)+4\pi r^3 p(r)}{r^2-2M(r)r}$$

How to get a mass radius relation:

→ get an equation of state p(ε)

→ start with a specific central density: ε_c, p_c, M(0) = 0

→ integrate the TOV equations until p(R) = 0 → R is the radius of the NS

→ M(R) is the mass of the NS

→ change ε_c and repeat → M-R relation

The mass-radius relation

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -[\varepsilon(r) + p(r)]\frac{M(r)+4\pi r^3 p(r)}{r^2 - 2M(r)r}$$

How to get a mass radius relation:

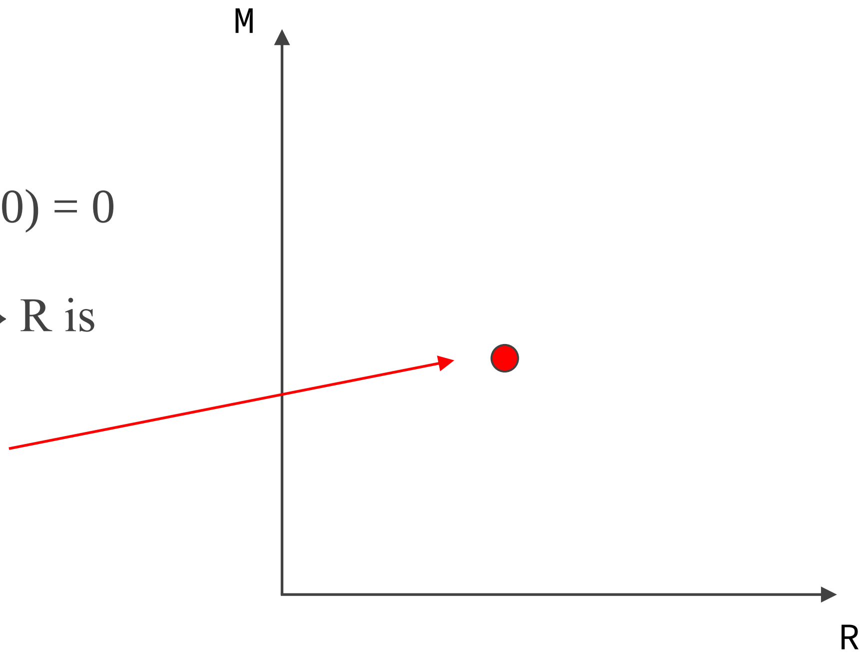
→ get an equation of state p(ε)

→ start with a specific central density: ε_c, p_c, M(0) = 0

→ integrate the TOV equations until p(R) = 0 → R is the radius of the NS

→ M(R) is the mass of the NS

→ change ε_c and repeat → M-R relation



The mass-radius relation

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -[\varepsilon(r) + p(r)]\,\frac{M(r)+4\pi r^3 p(r)}{r^2-2M(r)r}$$

How to get a mass radius relation:

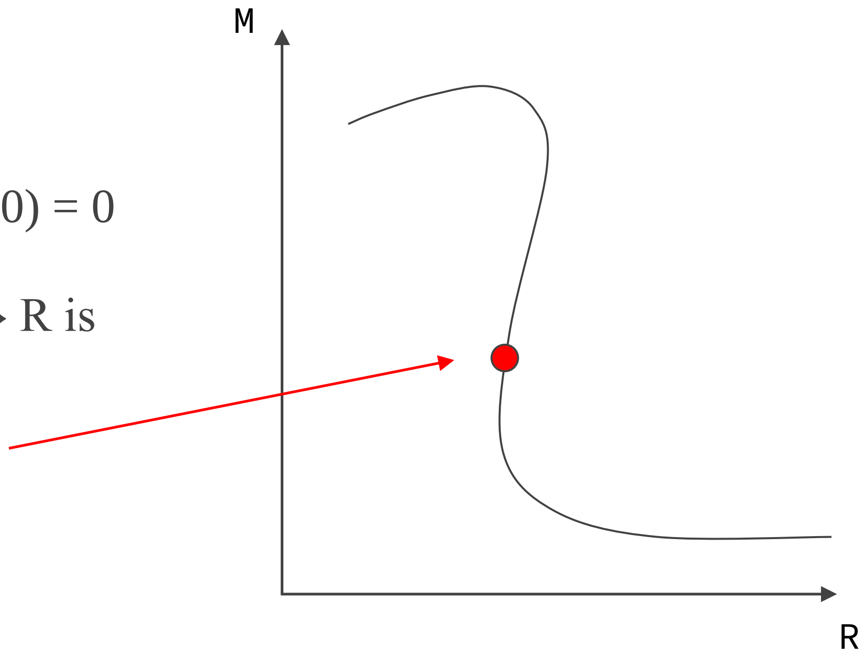
→ get an equation of state p(ε)

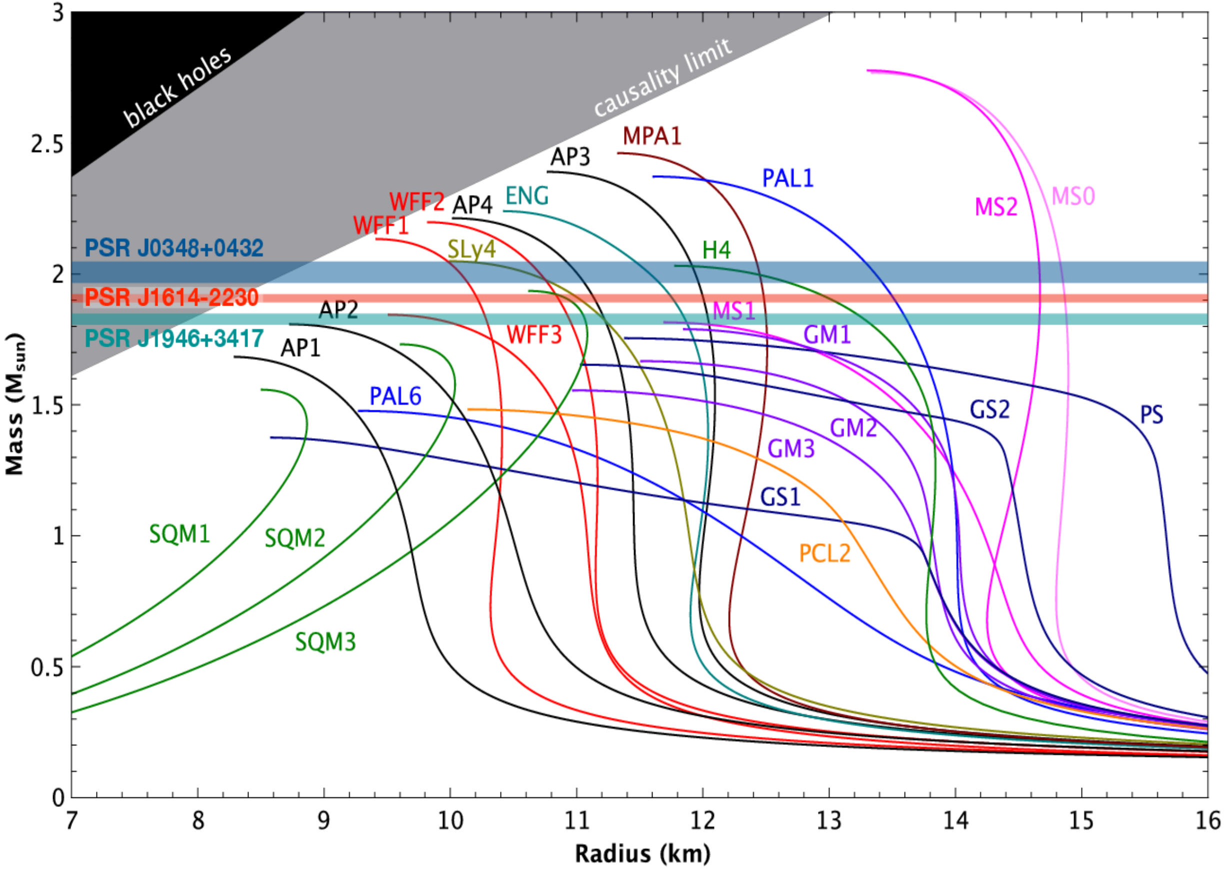
→ start with a specific central density: ε_c, p_c, M(0) = 0

→ integrate the TOV equations until p(R) = 0 → R is the radius of the NS

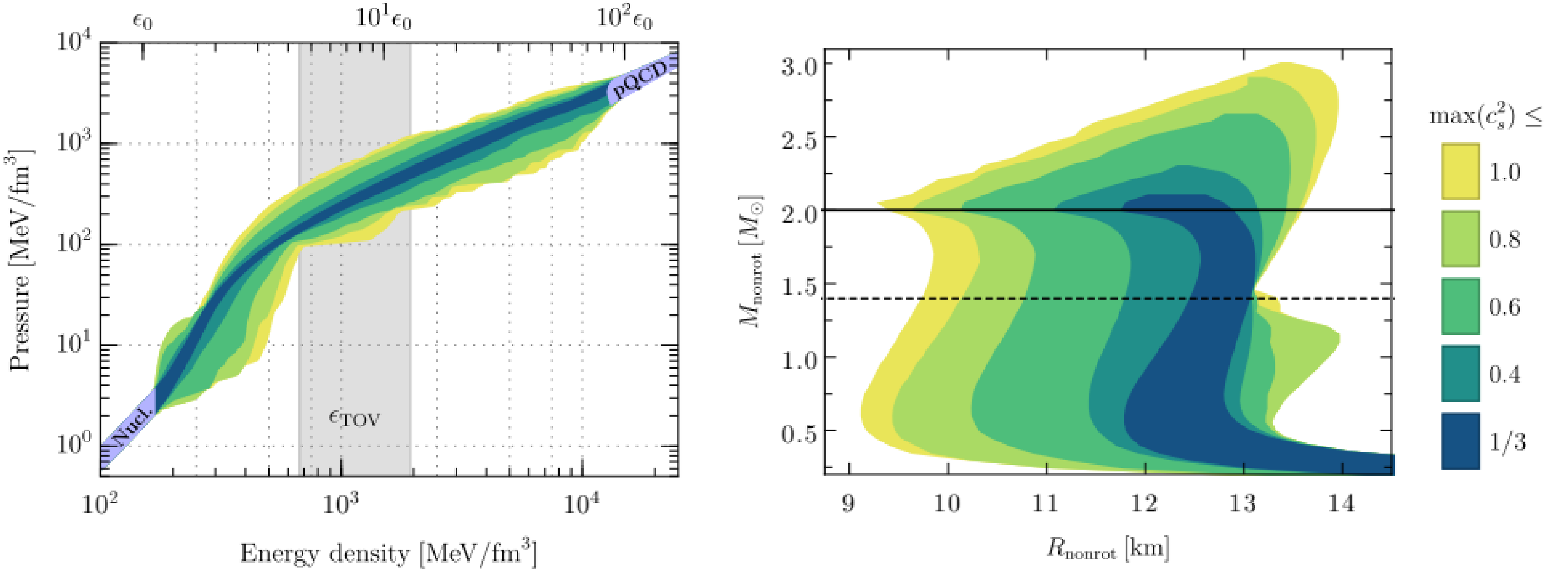
→ M(R) is the mass of the NS

→ change ε_c and repeat → M-R relation



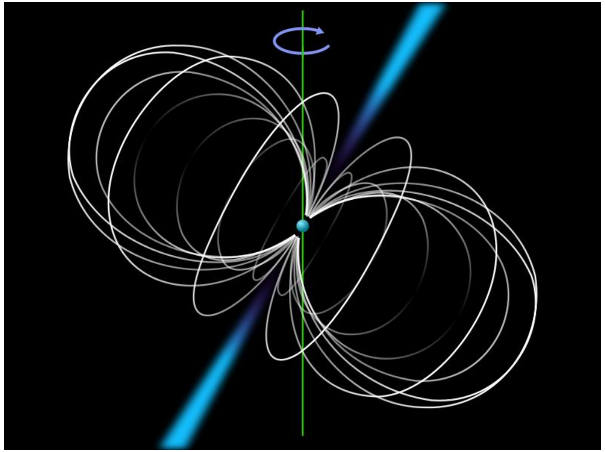

black holes
causality limit
MPA1
AP3
PAL1
MS2
MS0
WFF2
AP4
ENG
WFF1
PSR J0348+0432
SLy4
H4
PSR J1614-2230
MS1
AP2
WFF3
GM1
PSR J1946+3417
AP1
PAL6
GS2
PS
GM2
GM3
GS1
SQM1
SQM2
PCL2
SQM3
Mass (M_{sun})
Radius (km)
3
2.5
2
1.5
1
0.5
0
7
8
9
10
11
12
13
14
15
16

Neutron star EoS constraints



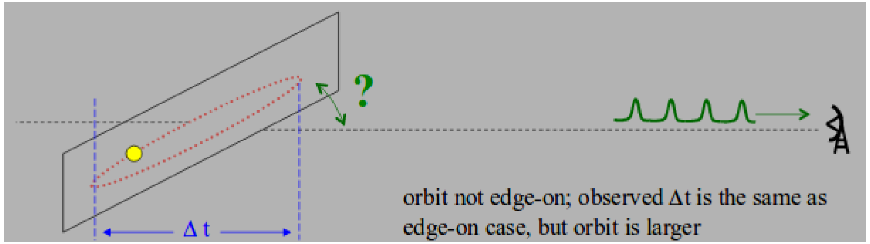
Source: E. Annala, et al., Phys.Rev.X 12 (2022) 1, 011058

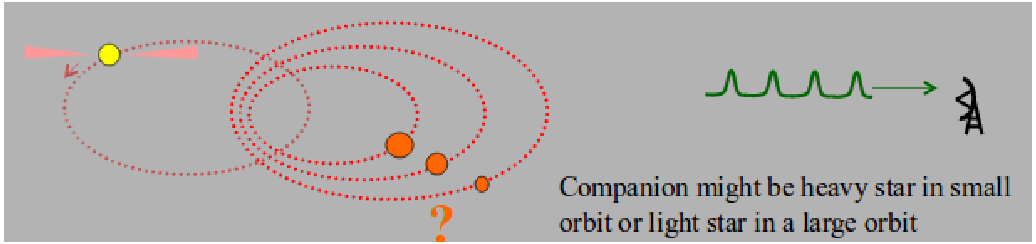
Mass measurement



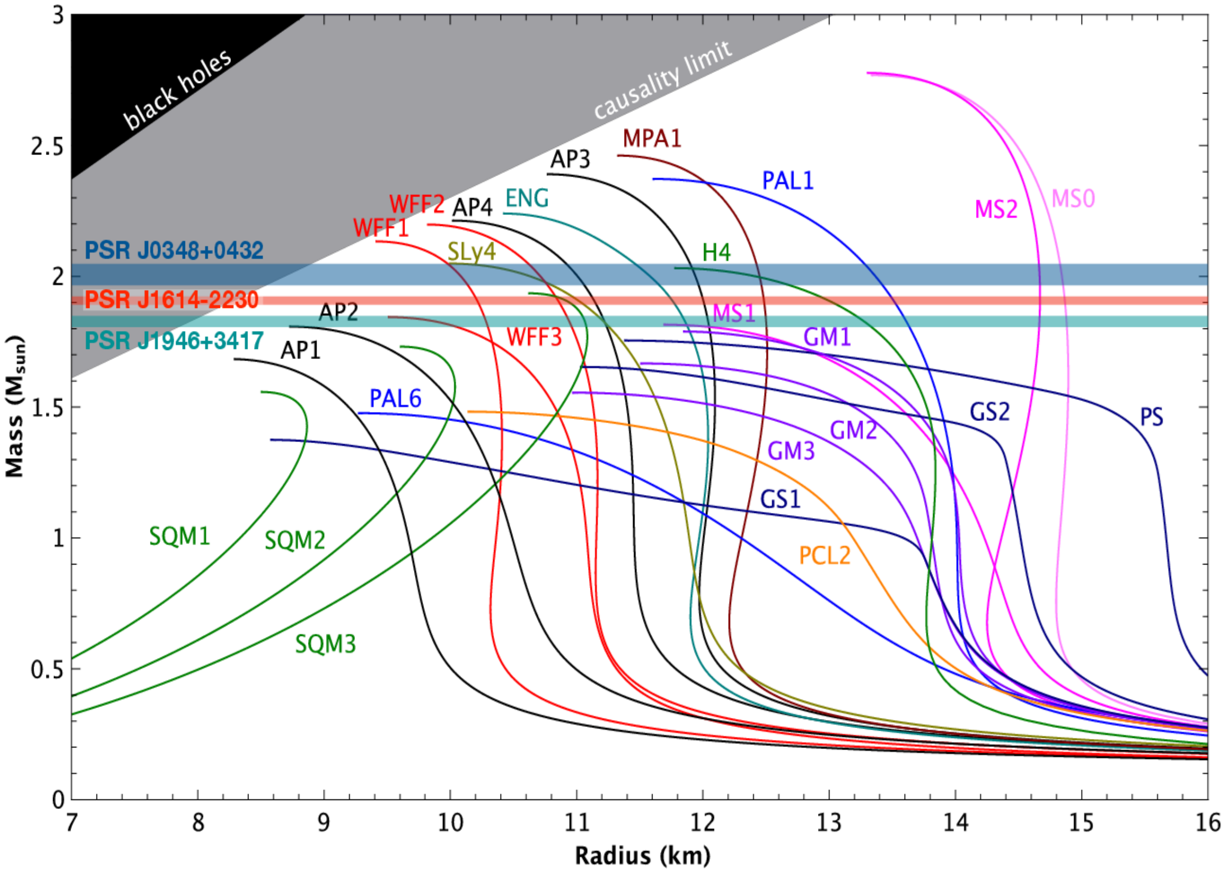
→ Pulsars in binary systems + Doppler shift

→ Degeneracies → only projected semi-major axis

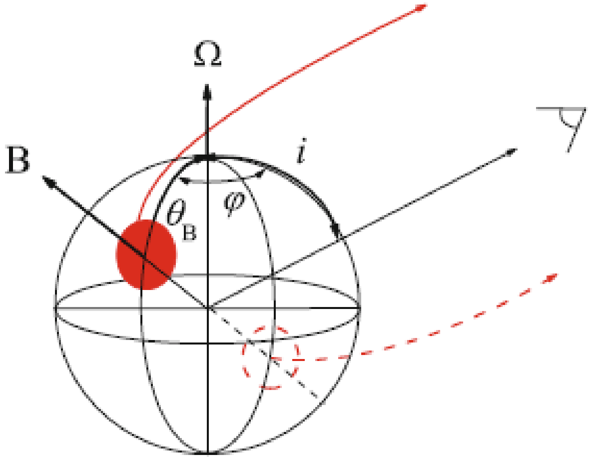


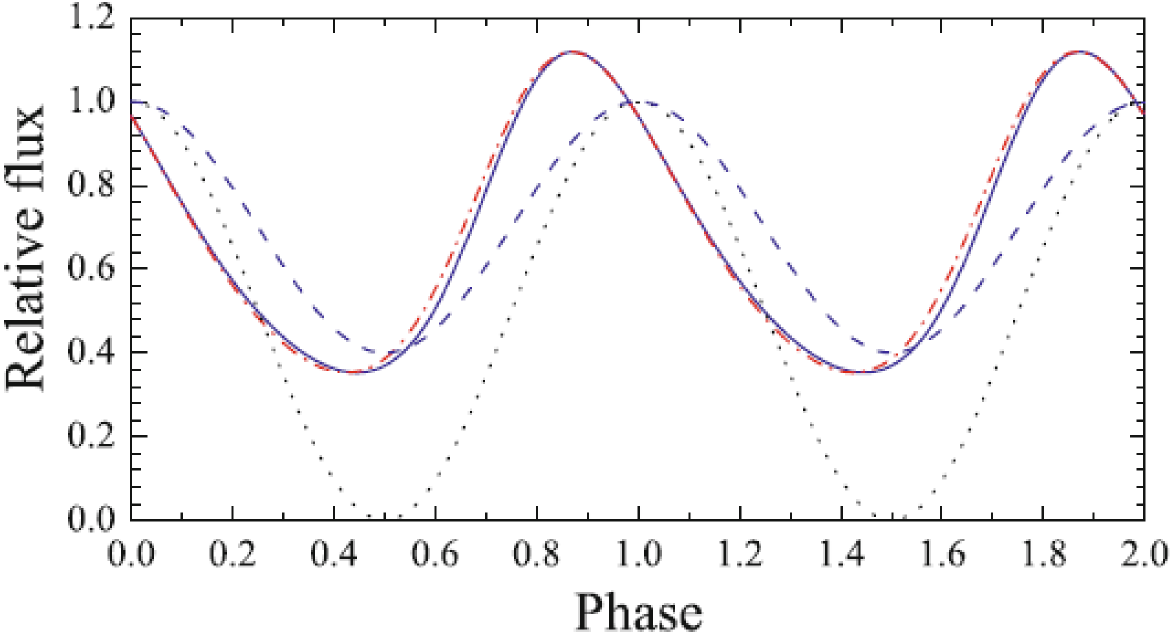


→ Observation of the companion object: another pulsar, white dwarf (+X-ray binaries)

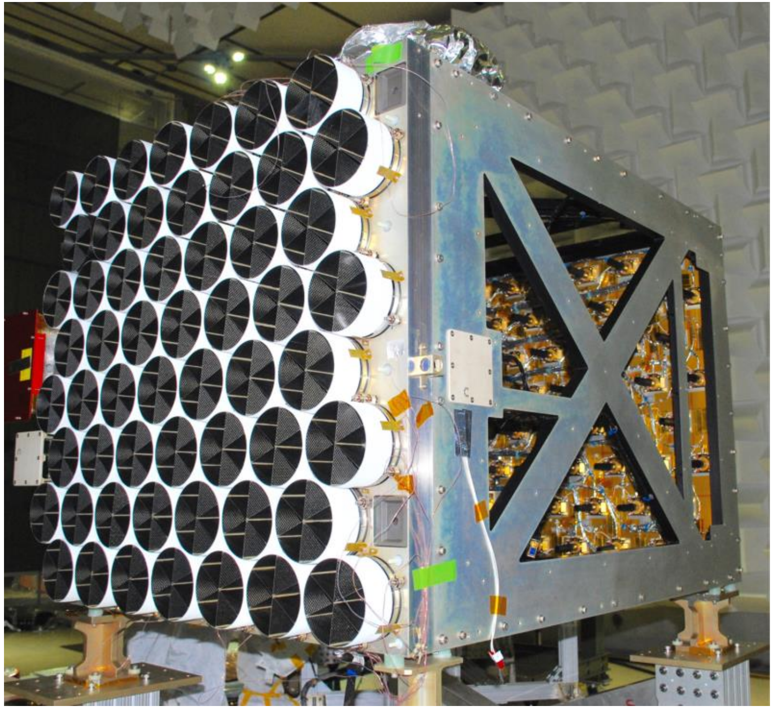

black holes
causality limit
MPA1
AP3
PAL1
MS2
MS0
WFF2
AP4
ENG
WFF1
PSR J0348+0432
SLy4
H4
PSR J1614-2230
AP2
MS1
PSR J1946+3417
WFF3
GM1
AP1
PAL6
GS2
PS
GM2
GM3
GS1
SQM1
SQM2
PCL2
SQM3
Mass (M_{sun})
Radius (km)

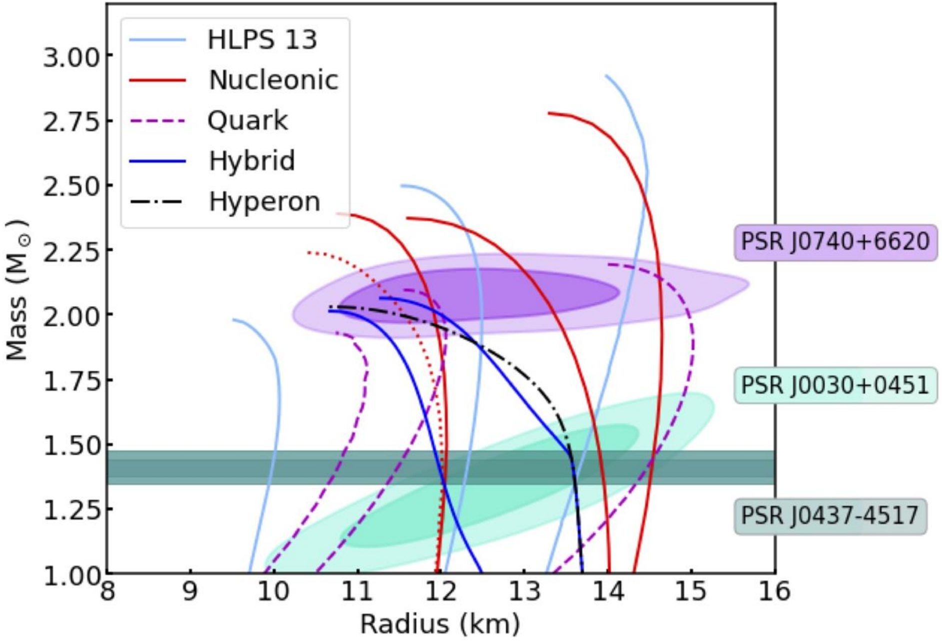
Radius measurement: pulse profile modeling



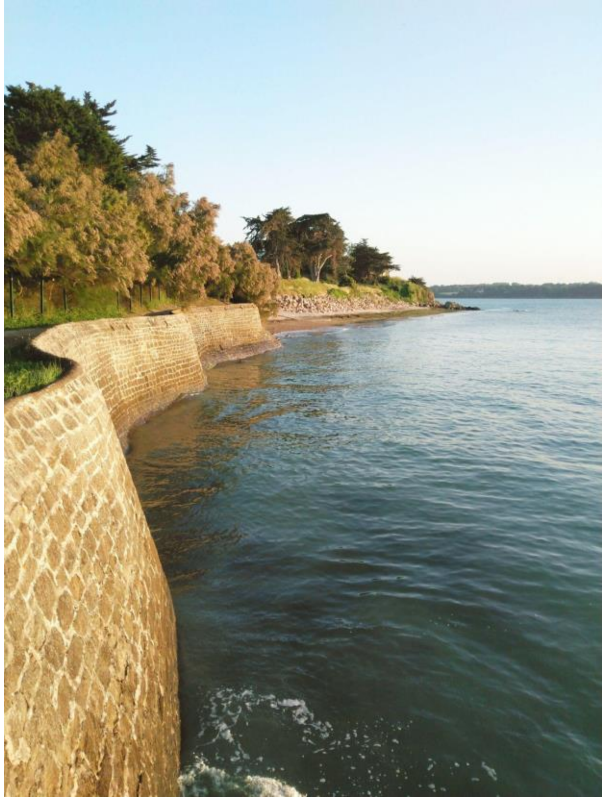


NICER measurements





Tidal deformability



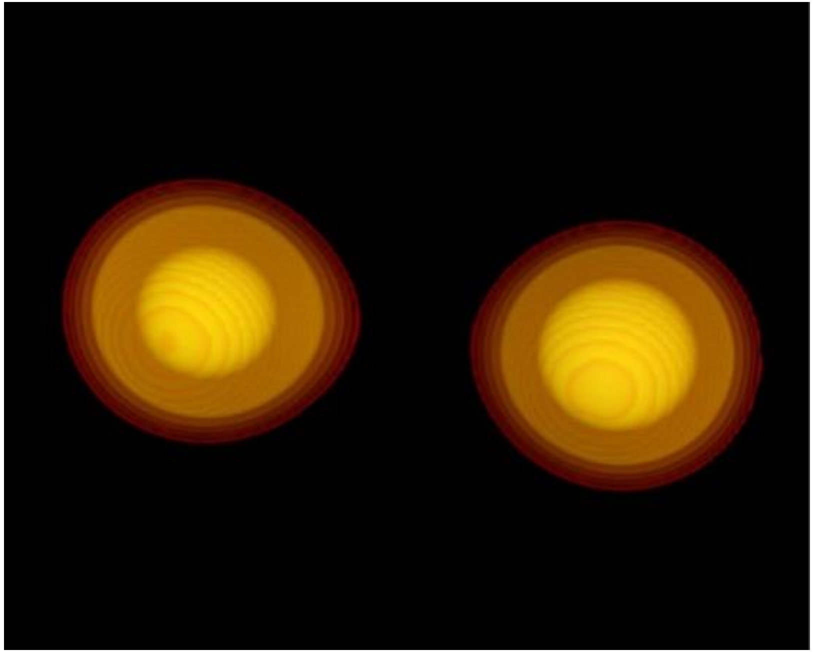
Tidal deformability

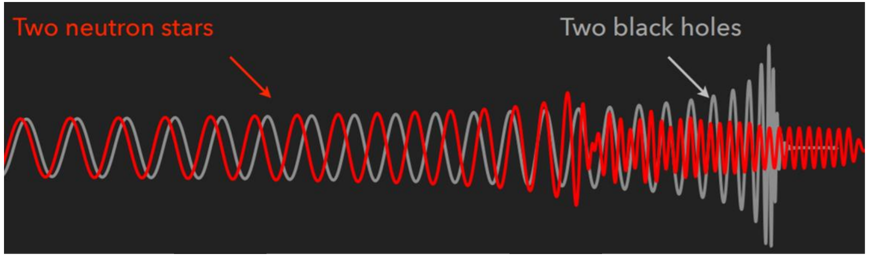
→ neutron stars have physical extension

→ they can be deformed by external tidal fields

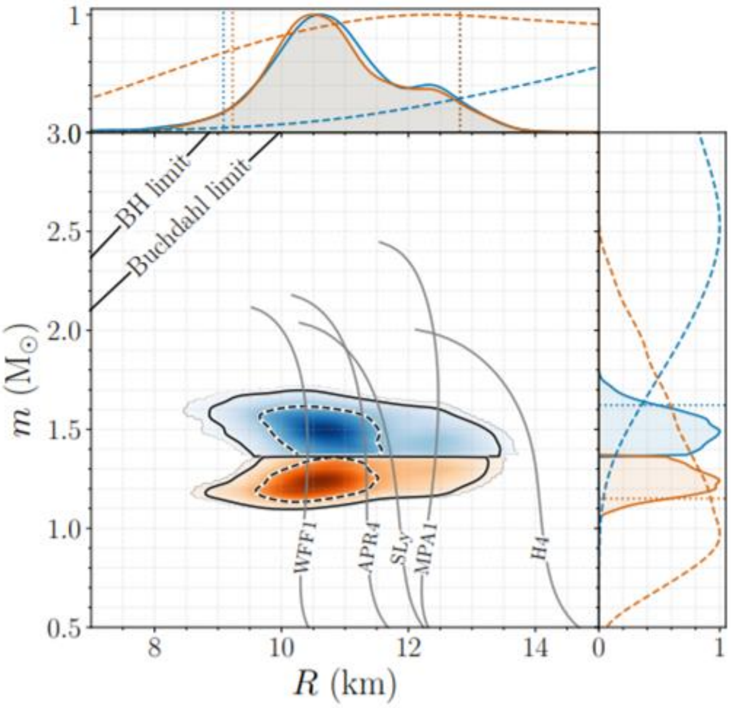
In the inspiral phase, far from the merger tidal effects cause a phase shift in the gravitational wave signal:

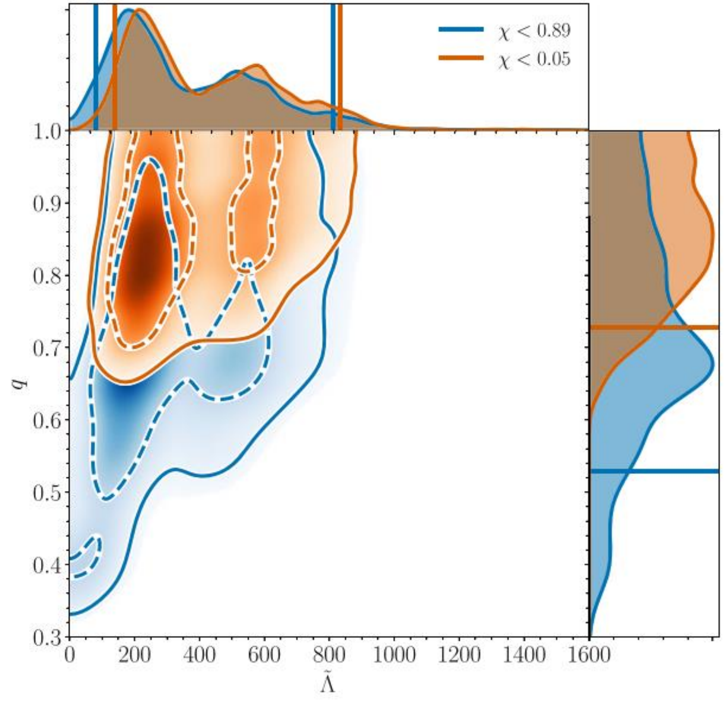
$$\delta\Psi \propto \tilde{\Lambda} f^{5/3}$$





GW170817


BH limit
Buchdahl limit
WFF1
APR4
SLy
MPA1
H4
m ($M_\odot$)
R (km)
3.0
2.5
2.0
1.5
1.0
0.5
8
10
12
14
0
1


$\chi < 0.89$
$\chi < 0.05$
q
$\tilde{\Lambda}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
200
400
600
800
1000
1200
1400
1600

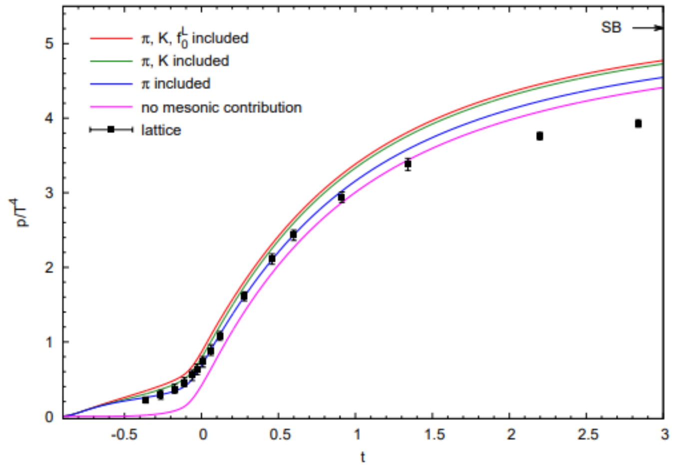
The constituent quark model

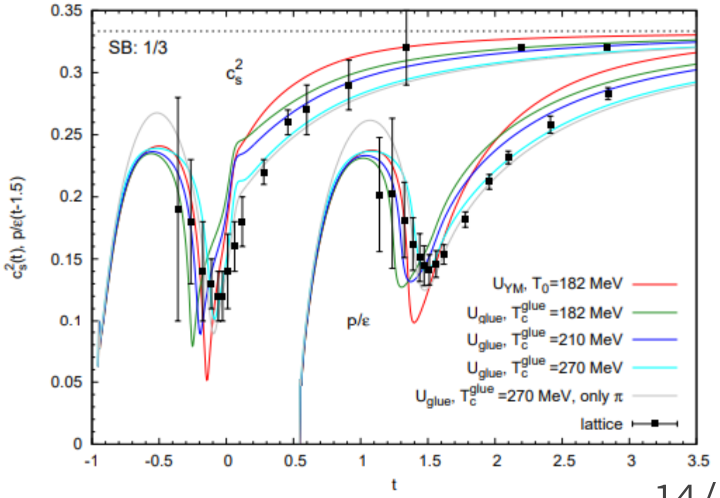
We use the (axial)vector meson extended linear sigma model

↪ SU(3) constituent quark-meson model with the complete (pseudo)scalar and (axial)vector meson nonets

↪ parameterized with meson vacuum masses and decay widths

↪ agrees well with lattice results at finite temperature



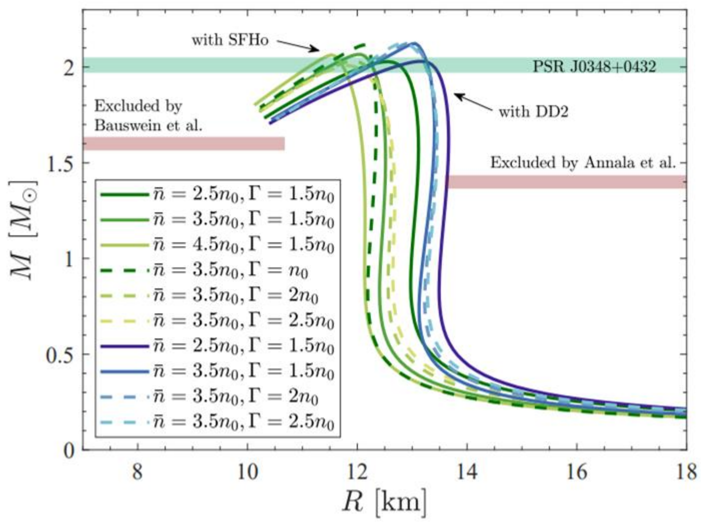


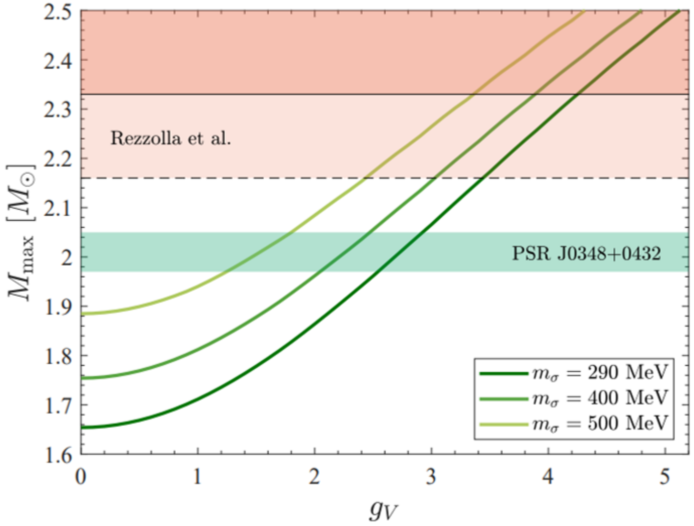
Hybrid equation of state

Hybrid stars also have a hadronic crust and outer core:

- at low densities we use hadronic EoSs:
  ↪ the SFHo EoS to represent soft hadronic EoSs
  ↪ the DD2 as a stiff EoS
- we apply a smooth connection between the two phases:
  ↪ $\varepsilon(n_B)$ interpolation with polynomial
- we have 4 tunable parameters:
  ↪ 2 from the constituent quark model: m_σ, g_V
  ↪ 2 describing the concatenation: $\tilde{n}$, Γ

Constraint from maximum mass of neutron stars



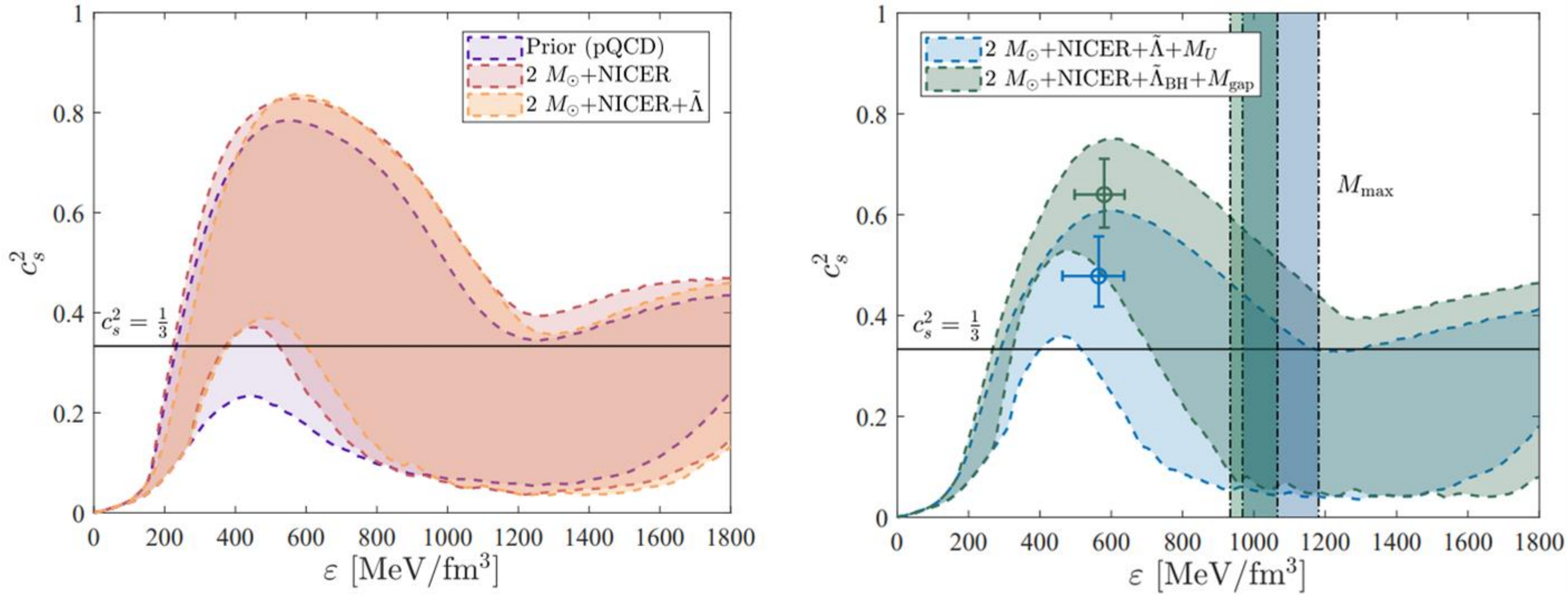


↪ maximum mass mostly depend on quark model parameters

↪ with m_σ=290 MeV g_V is constrained to $2.5 < g_V < 4.3$

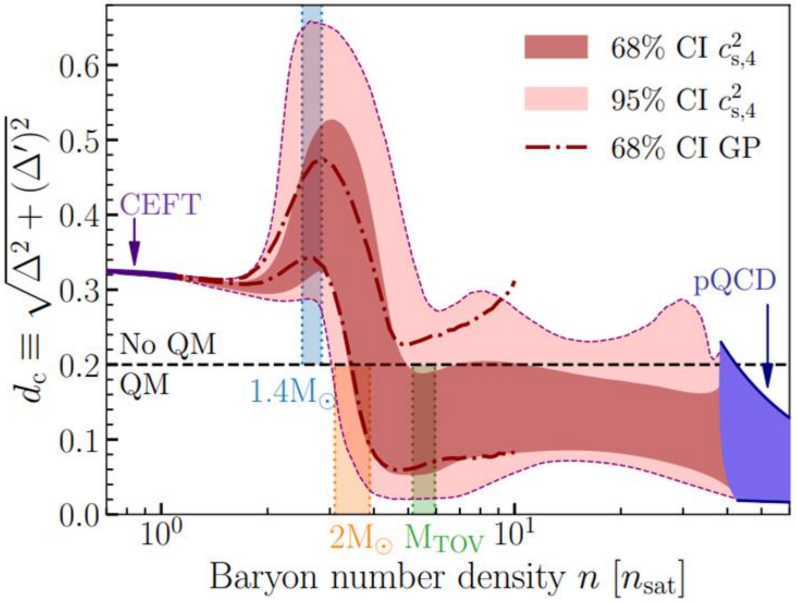
Speed of sound peak

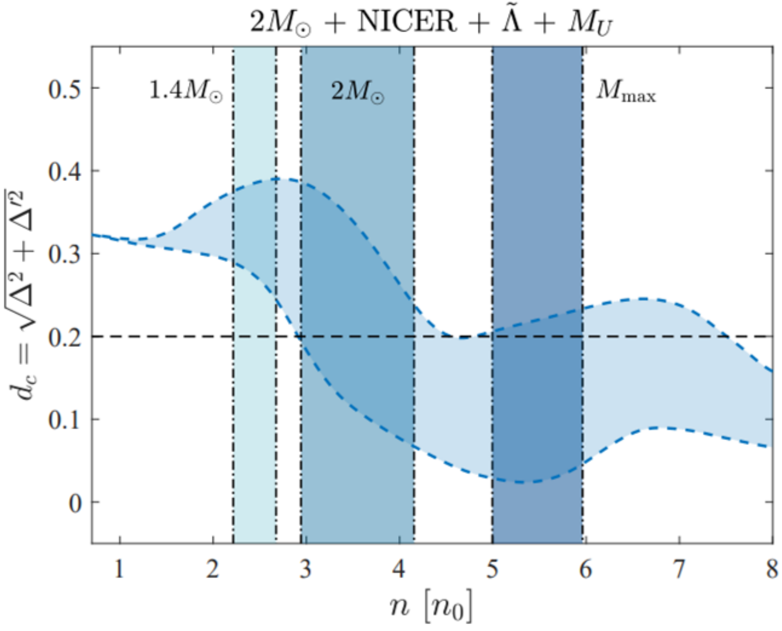
$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\varepsilon}$$



Conformality

$$\Delta = \frac{1}{3} - \frac{p}{\varepsilon} \qquad \Delta' = \frac{d\Delta}{d\,\ln\varepsilon} = c_s^2\left(\frac{1}{\gamma} - 1\right)$$





Source: E. Annala, et al., arXiv:2303.11356

Thank you for your attention!

Some references

[1] T. Hinderer, The Astrophysical Journal 677, 1216 (2008)

[2] S. Postnikov, M. Prakash, Phys. Rev. D 82, 024016 (2010)

[3] K. Yagi, N. Yunes, Physics Reports 681, 1 (2017)

[4] T. Hinderer, et al., Phys. Rev. D 81, 123016 (2010)

[5] B. P. Abbott, et al., Phys. Rev. Lett. 121, 161101 (2018)

[6] B. P. Abbott, et al., Astrophysical Journal Letters 892, L3 (2020)

Backup slides

Mass measurement

$$f_{ns} = \left(\frac{2\pi}{P_{orb}}\right)^2 \frac{(a_{ns}\sin i)^3}{G} = \frac{(M_c \sin i)^3}{M_T^2}$$

$$q = \frac{M}{M_c} = \frac{(a_c \sin i)}{(a_{ns}\sin i)}$$

Two other measurements:

1) Projected semi-major axis of the companion star → mass ratio

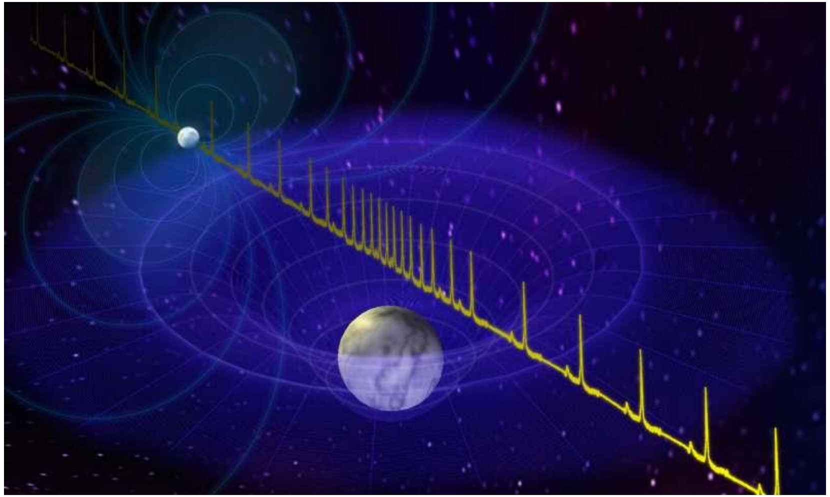
a) White dwarf → optical measurements

b) Another pulsar

2.1) Eclipses → observed edge-on

2.2) Independent mass measurement of the companion: Shapiro-delay

2.3) Relativistic effects: precession, gravitational radiation



Vector condensates and asymptotic behaviour

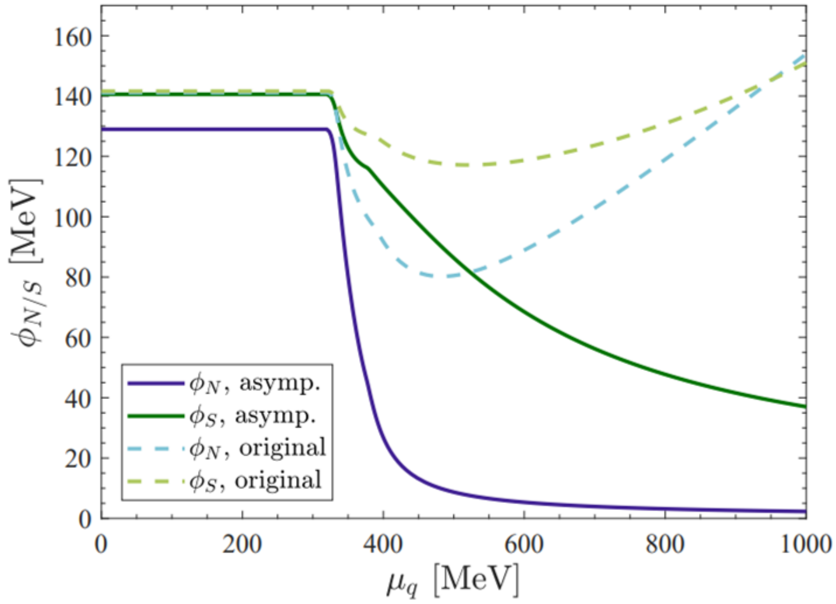
———

For arbitrary parametrization **chiral symmetry is not restored** at high density.

↪ we need to include this extra requirement

↪ we get an **extra constraint for the parameters**

Increasing the **vector coupling**, the phase transition **turns into a crossover** (g_V>3.1)



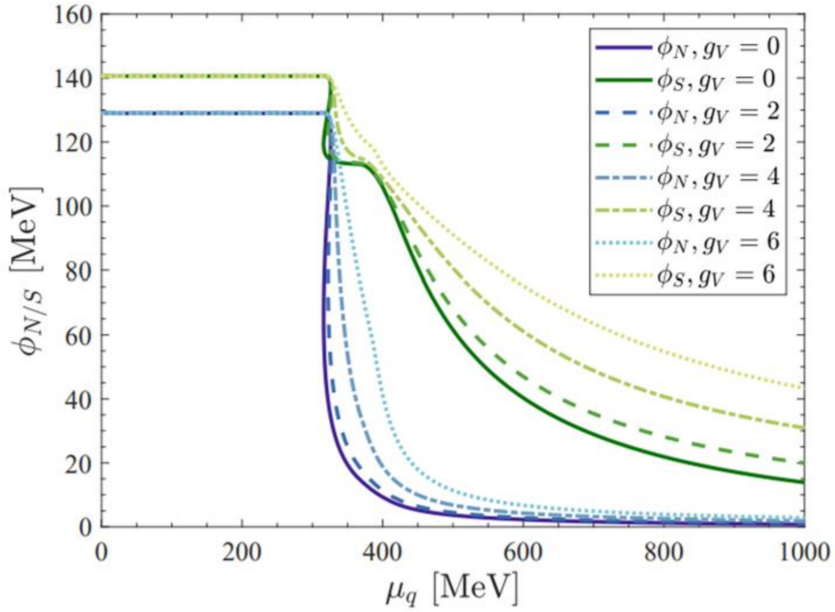
Vector condensates and asymptotic behaviour

For arbitrary parametrization **chiral symmetry is not restored** at high density.

↪ we need to include this extra requirement

↪ we get an **extra constraint for the parameters**

Increasing the **vector coupling**, the phase transition **turns into a crossover** (g_V>3.1)



Bayesian analysis

Bayes' theorem:

$$p(\vartheta|\text{data}) = \frac{p(\text{data}|\vartheta)p(\vartheta)}{p(\text{data})}$$

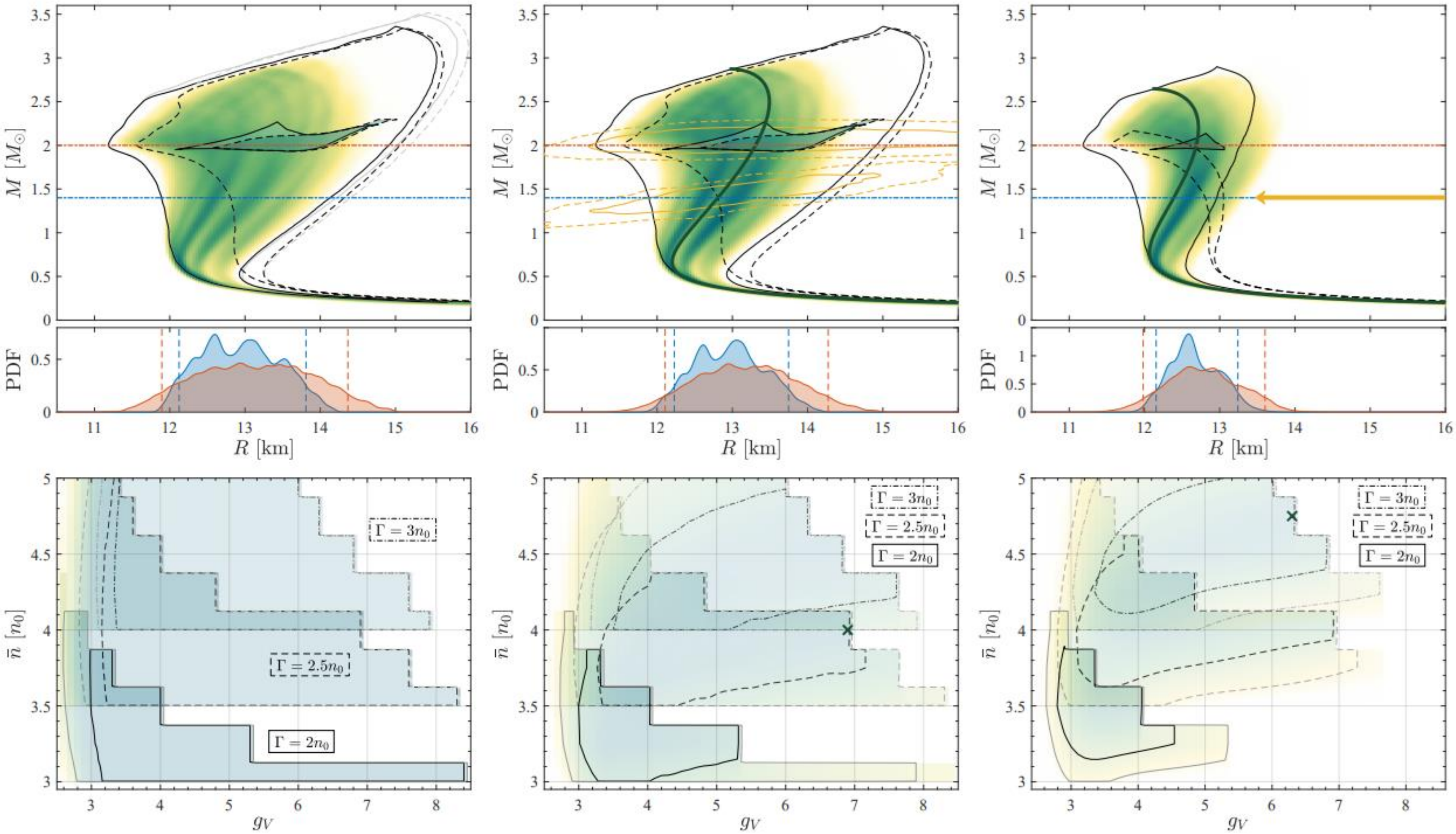
$$p(\text{data}|\vartheta) = p(M_{\text{max}}|\vartheta)p(\text{NICER}|\vartheta)p(\tilde{\Lambda}|\vartheta)$$

↪ Lower limit on maximum mass from $2M_{\odot}$ NS observations

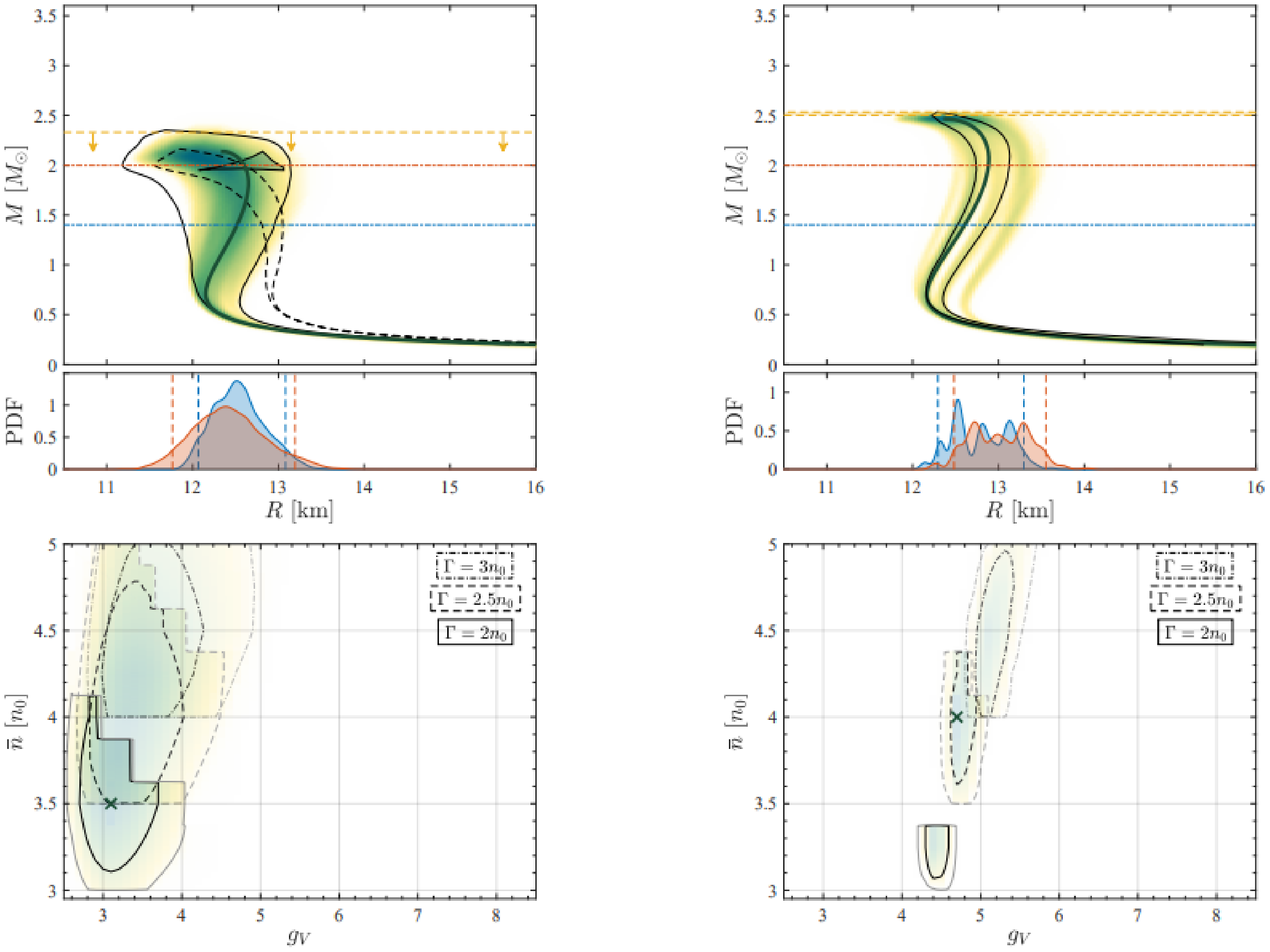
↪ Mass-radius probability densities from observations of PSR J0030+0451 and PSR J0740+6620 with NICER

↪ Tidal deformability data from LVC for GW170817 + constraint from no prompt collapse to BH

↪ Upper mass constraint from hypermassive NS hypothesis

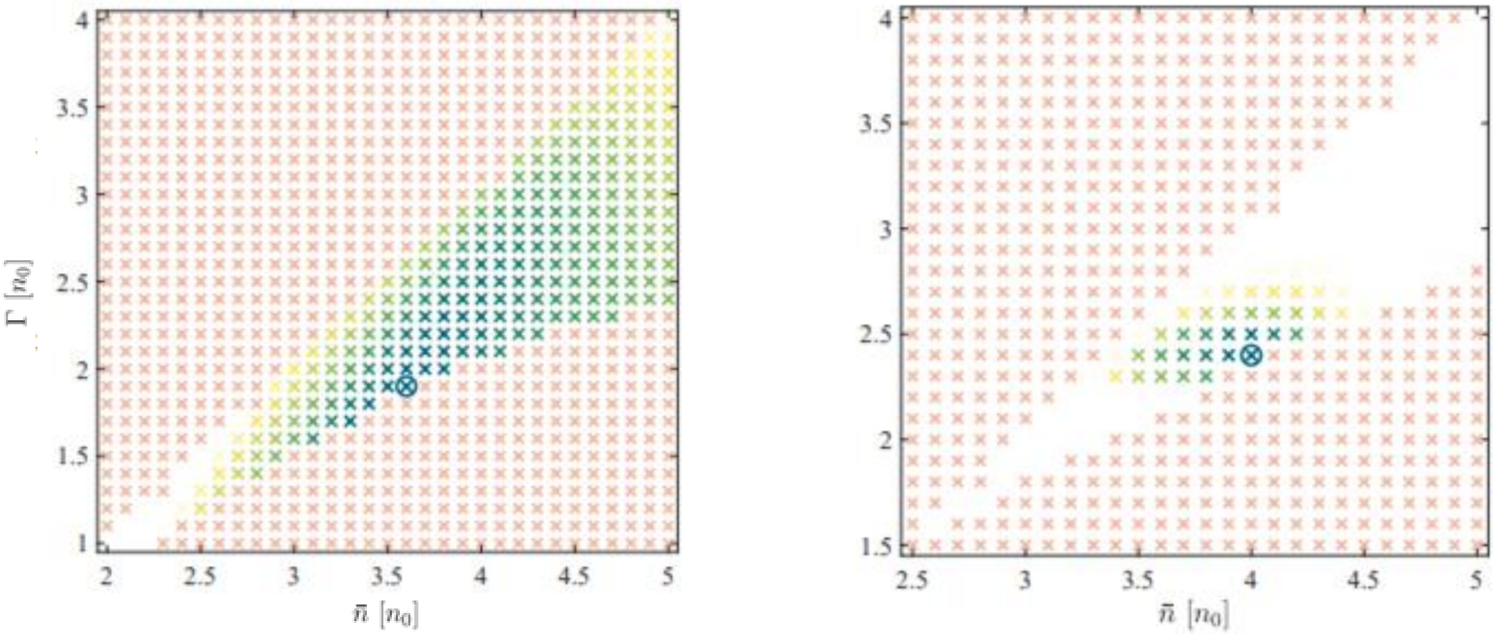


$M\ [M_\odot]$
PDF
R [km]
$\bar{n}\ [n_0]$
g_V
$\Gamma = 3n_0$
$\Gamma = 2.5n_0$
$\Gamma = 2n_0$



M $[M_\odot]$
PDF
R [km]
$\bar{n}$ $[n_0]$
g_V
$\Gamma = 3n_0$
$\Gamma = 2.5n_0$
$\Gamma = 2n_0$

Constraints on concatenation parameters



Speed of sound peak

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\varepsilon}$$

$$\Delta = \frac{1}{3} - \frac{p}{\varepsilon}$$