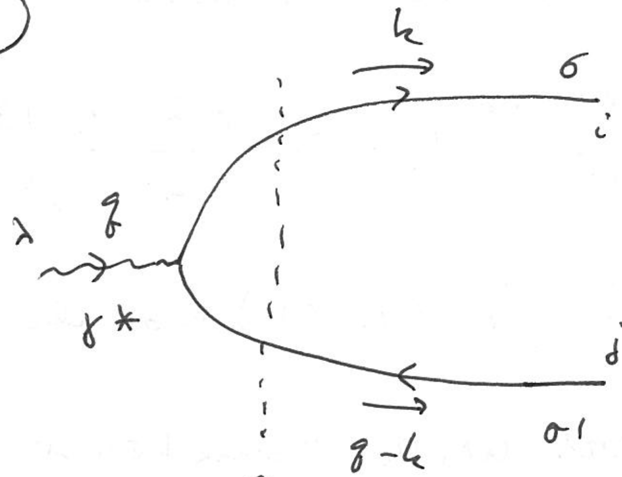
Using the rules for calculating wave functions derived in class, we write



intermediate state

$$\Psi_{L,T}^{\gamma^* \to \bar{q}q} = \frac{1}{q^+}\,\theta(k^+)\,\theta(q^+-k^+)\,\frac{1}{q^- - k^- - (q-k)^-}\, e Z_f$$

$$\cdot\ \bar{u}_\sigma(k)\,\not\epsilon^{\lambda}_{L,T}\,v_{\sigma'}(q-k)\,\delta_{ij}$$

← color indices.

Defining $z = k^+/q^+$ we simplify a part of the wave function as follows:

$$\frac{1}{q^+}\,\theta(k^+)\,\theta(q^+-k^+)\,\frac{1}{q^- - k^- - (q-k)^-} = \frac{1}{q^+}\,\theta(z)\,\theta(1-z)$$

$$\cdot\ \frac{1}{\underbrace{-\frac{Q^2}{q^+}}_{q^-} - \frac{\underline{k}^2+m_f^2}{k^+} - \frac{(\underline{q}-\underline{k})^2+m_f^2}{q^+-k^+}} = -\,\theta(z)\,\theta(1-z)$$

$$\cdot\ \frac{z(1-z)}{Q^2 z(1-z) + (1-z)(\underline{k}^2+m_f^2) + z((\underset{\overset{\|}{0}}{\underline{q}}-\underline{k})^2+m_f^2)} = \frac{-\theta(z)\,\theta(1-z)\,z(1-z)}{Q^2 z(1-z) + \underline{k}^2 + m_f^2}$$

(see eq. (4.1) in KL, $q^\mu = (q^+, -\frac{Q^2}{q^+}, \underline{0})$)

$\Rightarrow$ the wave function becomes

$$\Psi_{L,T}^{\gamma^* \to q\bar{q}}(\underline{k}, z) = - \frac{e Z_f \, \theta(z)\theta(1-z)\, z(1-z)}{\underline{k}^2 + m_f^2 + Q^2 z(1-z)} \, \bar{u}_\sigma(k) \, \not{\epsilon}_{L,T}^{\lambda} \, v_{\sigma'}(q-k) \, \delta_{ij}$$

(cf. Eq. (4.13), modulo sign)

Start with transverse polarizations: $\epsilon_T^\mu = (\overset{+}{0}, \overset{-}{0}, \overset{\perp}{\underline{\epsilon}_\lambda})$

$\underline{\epsilon}_\lambda = -\frac{1}{\sqrt{2}}(\lambda, i)$

$$\Rightarrow \bar{u}_\sigma(k) \, \not{\epsilon}_T^\lambda \, v_{\sigma'}(q-k) = - \underline{\epsilon}_\lambda \cdot \bar{u}_\sigma(k) \underline{\gamma} \, v_{\sigma'}(q-k) =$$

$$= - \epsilon_\lambda^i \, \bar{u}_\sigma(k) \gamma^i v_{\sigma'}(q-k) = - \epsilon_\lambda^i \left[\bar{v}_{\sigma'}(q-k) \gamma^i u_\sigma(k) \right]^* = \Big| \text{using table A.2}$$

$$\underset{\underline{q}=0}{\uparrow} = - \epsilon_\lambda^i \Bigg[\delta_{\sigma,-\sigma'} \left(- \frac{k_\perp^i + i\sigma \epsilon^{ij} k_\perp^j}{q^+ - k^+} + \frac{k_\perp^i - i\sigma \epsilon^{ij} k_\perp^j}{k^+} \right) - \delta_{\sigma\sigma'} \, \sigma \, m_f \cdot$$

$\underline{a} \times \underline{b} \equiv \epsilon^{ij} a^i b^j = a^1 b^2 - a^2 b^1$

$$\cdot \frac{q^+}{(q^+ - k^+) k^+} \left(\delta^{i1} - i\sigma \delta^{i2} \right) \Bigg] \sqrt{(q^+ - k^+) k^+} = + \delta_{\sigma,-\sigma'} \Bigg[\frac{1}{1-z} \Big(\underline{\epsilon}_\lambda \cdot \underline{k}$$

$$+ i\sigma \, \underline{\epsilon}_\lambda \times \underline{k} \Big) - \frac{1}{z} \left(\underline{\epsilon}_\lambda \cdot \underline{k} - i\sigma \, \underline{\epsilon}_\lambda \times \underline{k} \right) \Bigg] \sqrt{z(1-z)} + \delta_{\sigma\sigma'} \, \sigma \, m_f \frac{1}{\sqrt{z(1-z)}} \cdot$$

$$\cdot \left(-\frac{1}{\sqrt{2}} \right) \underbrace{(\lambda - i\sigma i)}_{\lambda + \sigma} = \Big| \begin{array}{l} \text{using} \\ \underline{\epsilon}_\lambda \times \underline{k} = -i\lambda \, \underline{\epsilon}_\lambda \cdot \underline{k} \end{array} = \frac{\delta_{\sigma,-\sigma'}}{\sqrt{z(1-z)}} \Big[\underline{\epsilon}_\lambda \cdot \underline{k} \, (2z - 1)$$

$$+ \sigma\lambda \, \underline{\epsilon}_\lambda \cdot \underline{k} \Big] - \frac{1}{\sqrt{2}} \delta_{\sigma\sigma'} \, m_f \, (1 + \sigma\lambda) \frac{1}{\sqrt{z(1-z)}} = - \frac{1}{\sqrt{z(1-z)}} \cdot$$

$$\cdot \left\{ (1 - \delta_{\sigma,\sigma'}) \, \underline{\epsilon}_\lambda \cdot \underline{k} \, (1 - 2z - \sigma\lambda) + \frac{1}{\sqrt{2}} \delta_{\sigma\sigma'} \, m_f \, (1 + \sigma\lambda) \right\}$$

Therefore, assuming $0 \le z \le 1$, we write

$$\boxed{\psi_T^{\gamma^* \to q\bar{q}}(\underline{k}, z) = \frac{e Z_f \sqrt{z(1-z)}}{\underline{k}^2 + m_f^2 + Q^2 z(1-z)} \Big[(1-\delta_{\sigma,\sigma'}) \, \underline{\varepsilon}_\lambda \cdot \underline{k} \, (1-2z-\sigma\lambda) + \frac{1}{\sqrt{2}} \delta_{\sigma\sigma'} m_f (1+\sigma\lambda) \Big] \delta_{ij}}$$

(cf. 4.14)

Fourier transformed wave function

$$\psi_{T,L}^{\gamma^* \to q\bar{q}}(\underline{x}, z) = \int \frac{d^2 k}{(2\pi)^2} e^{i \underline{k} \cdot \underline{x}} \, \psi_{T,L}^{\gamma^* \to q\bar{q}}(\underline{k}, z)$$

is obtained using Eq. (A.11) in KL. Defining $a_f^2 = m_f^2 + Q^2 z(1-z)$,

we get
$$\int \frac{d^2 k}{(2\pi)^2} \frac{e^{i \underline{k} \cdot \underline{x}}}{\underline{k}^2 + a_f^2} = \frac{1}{2\pi} K_0(x_\perp a_f)$$

$$\int \frac{d^2 k}{(2\pi)^2} e^{i \underline{k} \cdot \underline{x}} \frac{\underline{\varepsilon}_\lambda \cdot \underline{k}}{\underline{k}^2 + a_f^2} = -i \underline{\varepsilon}_\lambda \cdot \underline{\nabla}_x \int \frac{d^2 k}{(2\pi)^2} \frac{e^{i \underline{k} \cdot \underline{x}}}{\underline{k}^2 + a_f^2} = \frac{-i \underline{\varepsilon}_\lambda \cdot \underline{\nabla}_x K_0(x_\perp a_f)}{2\pi}$$

$$= -\frac{i}{2\pi} \left(- K_1(x_\perp a_f) \right) a_f \frac{\underline{\varepsilon}_\lambda \cdot \underline{x}}{|\underline{x}|}$$ $\Rightarrow$ the final result for the transverse wave function is

$$\boxed{\psi_T^{\gamma^* \to q\bar{q}}(\underline{k}, z) = \frac{e Z_f}{2\pi} \sqrt{z(1-z)} \, \delta_{ij} \Big[(1-\delta_{\sigma\sigma'})(1-2z-\sigma\lambda) \, i a_f \frac{\underline{\varepsilon}_\lambda \cdot \underline{x}}{x_\perp} \cdot K_1(x_\perp a_f) + \delta_{\sigma\sigma'} \frac{m_f}{\sqrt{2}} (1+\sigma\lambda) K_0(x_\perp a_f) \Big]}$$

cf. (4.16)

Similarly, for the longitudinal wave function we use $\varepsilon_L^\mu = \left(\overset{+}{\frac{q^+}{Q}}, \overset{-}{\frac{Q}{q^+}}, \overset{\perp}{\underline{0}}\right)$ to write

$$\bar{u}_\sigma(k) \not{\varepsilon}_L v_{\sigma'}(q-k) = \frac{q^+}{Q}\left[\bar{v}_{\sigma'}(q-k)\gamma^- u_\sigma(k)\right]^*$$

$$+ \frac{Q}{q^+}\left[\bar{v}_{\sigma'}(q-k)\gamma^+ u_\sigma(k)\right]^* = \frac{1}{2}\frac{q^+}{Q}\left[\delta_{\sigma,-\sigma'}\frac{2}{(q^+-k^+)k^+}\cdot(-\underline{k}^2-m_f^2)\right.$$

$$\left. - \delta_{\sigma\sigma'}\,\sigma\,\frac{2m_f}{k^+(q^+-k^+)}\cdot\emptyset\right]\sqrt{(q^+-k^+)k^+} + \frac{1}{2}\frac{Q}{q^+}\,2\,\delta_{\sigma,-\sigma'}\sqrt{k^+(q^+-k^+)} =$$

$$= \frac{1}{2}\sqrt{z(1-z)}\,\delta_{\sigma,-\sigma'}\left[-\frac{2}{Q}\frac{k^2+m_f^2}{z(1-z)} + 2Q\right] = -\frac{1}{Q}\frac{\delta_{\sigma,-\sigma'}}{\sqrt{z(1-z)}}\cdot$$

$$\cdot\left[\underline{k}^2+m_f^2 - z(1-z)Q^2\right] = -\frac{1}{Q}\frac{\delta_{\sigma,-\sigma'}}{\sqrt{z(1-z)}}\left[\underbrace{\underline{k}^2+m_f^2+z(1-z)Q^2}_{\text{gives } \sim\delta^2(\underline{x}) \text{ in coordinate space} \Rightarrow \text{drop}}\right.$$

$$\left. -2z(1-z)Q^2\right] = 2Q\sqrt{z(1-z)}\,\delta_{\sigma,-\sigma'} = 2Q\sqrt{z(1-z)}\,(1-\delta_{\sigma,\sigma'})$$

$\Rightarrow$

$$\boxed{\Psi_L^{\gamma^*\to q\bar{q}}(\underline{k},z) = -\frac{e\,Z_f\,[z(1-z)]^{3/2}(1-\delta_{\sigma\sigma'})\,2Q}{\underline{k}^2+m_f^2+Q^2\,z(1-z)}}$$

(cf. (4.19), modulo sign)

Fourier-transforming we arrive at

$$\Psi_L^{\gamma^*\to q\bar{q}}(\underline{x},z) = -\frac{e\,Z_f}{2\pi}\cdot 2Q[z(1-z)]^{3/2}(1-\delta_{\sigma\sigma'})\,K_0(x_\perp a_f)$$

cf. (4.20), modulo sign.