

## I. QCD SUMMARY

The  $SU(N_c)$  QCD Lagrangian without gauge fixing

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}G_{\mu\nu}^A G^{\mu\nu A}, & G_{\mu\nu}^A &= \partial_\mu A_\nu^A - \partial_\nu A_\mu^A - gf^{ABC}A_\mu^B A_\nu^C \\ D_\mu &= \partial_\mu + igA_\mu^A T^A, & [D_\mu, D_\nu] &= igG_{\mu\nu}^A T^A.\end{aligned}\quad (1)$$

The equations of motion and Bianchi

$$(i\not{D} - m)\psi = 0, \quad \partial^\mu G_{\mu\nu}^A = gf^{ABC}A^{B\mu}G_{\mu\nu}^C + g\bar{\psi}\gamma_\nu T^A\psi, \quad \epsilon^{\mu\nu\lambda\sigma}(D_\nu G_{\lambda\sigma})^A = 0. \quad (2)$$

Color identities

$$\begin{aligned}[T^A, T^B] &= if^{ABC}T^C, & \text{Tr}[T^A T^B] &= T_F \delta^{AB}, & \bar{T}^A &= -T^{A*} = -(T^A)^T, \\ T^A T^A &= C_F \mathbf{1}, & f^{ACD} f^{BCD} &= C_A \delta^{AB}, & f^{ABC} T^B T^C &= \frac{i}{2} C_A T^A, \\ T^A T^B T^A &= \left(C_F - \frac{C_A}{2}\right) T^B, & d^{ABC} d^{ABC} &= \frac{40}{3}, & d^{ABC} d^{A'BC} &= \frac{5}{3} \delta^{AA'},\end{aligned}\quad (3)$$

where  $C_F = (N_c^2 - 1)/(2N_c)$ ,  $C_A = N_c$ ,  $T_F = 1/2$ , and  $C_F - C_A/2 = -1/(2N_c)$ . The color reduction formula and Fierz formula are

$$T^A T^B = \frac{\delta^{AB}}{2N_c} \mathbf{1} + \frac{1}{2} d^{ABC} T^C + \frac{i}{2} f^{ABC} T^C, \quad (T^A)_{ij} (T^A)_{kl} = \frac{1}{2} \delta_{il} \delta_{kj} - \frac{1}{2N_c} \delta_{ij} \delta_{kl}. \quad (4)$$

Feynman gauge rules, fermion, gluon, ghost propagators, and Fermion-gluon vertex

$$\frac{i(\not{p} + m)}{p^2 - m^2 + i0}, \quad \frac{-ig^{\mu\nu} \delta^{AB}}{k^2 + i0}, \quad \frac{i}{k^2 + i0}, \quad -ig\gamma^\mu T^A. \quad (5)$$

Triple gluon and Ghost Feynman rules in covariant gauge for  $\{A_\mu^A(k), A_\nu^B(p), A_\rho^C(q)\}$  all with incoming momenta, and  $\bar{c}^A(p)A_\mu^B c^C$  with outgoing momenta  $p$ :

$$-gf^{ABC} [g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu], \quad gf^{ABC} p^\mu. \quad (6)$$

Triple gluon Feynman rule in bkgnd Field covariant gauge  $\mathcal{L}_{gf} = -(D_\mu^A Q_\mu^A)^2/(2\xi)$  for  $\{A_\mu^A(k), Q_\nu^B(p), Q_\rho^C(q)\}$  with  $A_\mu^A$  a bkgnd field:

$$-gf^{ABC} \left[ g^{\mu\nu} \left( k - p - \frac{q}{\xi} \right)^\rho + g^{\nu\rho} (p - q)^\mu + g^{\rho\mu} \left( q - k + \frac{p}{\xi} \right)^\nu \right]. \quad (7)$$

Lorentz gauge:

$$\mathcal{L} = -\frac{(\partial_\mu A^\mu)^2}{2\xi}, \quad D^{\mu\nu}(k) = \frac{-i}{k^2 + i0} \left( g^{\mu\nu} - (1 - \xi) \frac{k^\mu k^\nu}{k^2} \right), \quad (8)$$

where Landau gauge is  $\xi \rightarrow 0$ . Coulomb gauge:

$$\begin{aligned}\vec{\nabla} \cdot \vec{A} &= 0, & D^{\mu\nu}(k) &= \frac{-i}{k^2 + i0} \left( g^{\mu\nu} - \frac{[g^{\nu 0} k^0 k^\mu + g^{\mu 0} k^0 k^\nu - k^\mu k^\nu]}{\vec{k}^2} \right), \\ D^{00}(k) &= \frac{i}{\vec{k}^2 - i0}, & D^{ij}(k) &= \frac{i}{k^2 + i0} \left( \delta^{ij} - \frac{k^i k^j}{\vec{k}^2} \right).\end{aligned}\quad (9)$$

Running coupling with  $\beta_0 = 11C_A/3 - 4T_F n_f/3 = 11 - 2n_f/3$ :

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{\beta_0}{2\pi} \alpha_s(\mu_0) \ln \frac{\mu}{\mu_0}} = \frac{2\pi}{\beta_0 \ln \frac{\mu}{\Lambda_{\text{QCD}}}}, \quad \frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(\mu_0)} + \frac{\beta_0}{2\pi} \ln \frac{\mu}{\mu_0}. \quad (10)$$