

## Soft-Collinear Effective Theory

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@ QCD Masterclass 2023

- EFT treatment of Soft & Collinear IR physics for hard collisions in QCD (or decays with large  $E$  released)
- $\Rightarrow$  jets, energetic hadrons, soft partons/hadrons

eg. $e^+e^- \rightarrow 2\text{-jets}$ $\quad\quad\quad \rightarrow 3\text{-jets}$ $\quad\quad\quad \rightarrow h_1 h_2 X$ $\quad\quad\quad \vdots$ $\quad\quad\quad \vdots$	$pp \rightarrow \mu^+\mu^- X (\text{DY})$ $\quad\quad\quad \rightarrow H X$ $\quad\quad\quad \rightarrow H + 1\text{-jet}$ $\quad\quad\quad \rightarrow n\text{-jets}$ $\quad\quad\quad \vdots$	$e^- p \rightarrow e^- X (\text{DIS})$ $\quad\quad\quad \rightarrow e^- \pi X (\text{SIDIS})$ $\quad\quad\quad \rightarrow e^- \text{jet} X$ $\quad\quad\quad \vdots$
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**LHC**      **EIC**

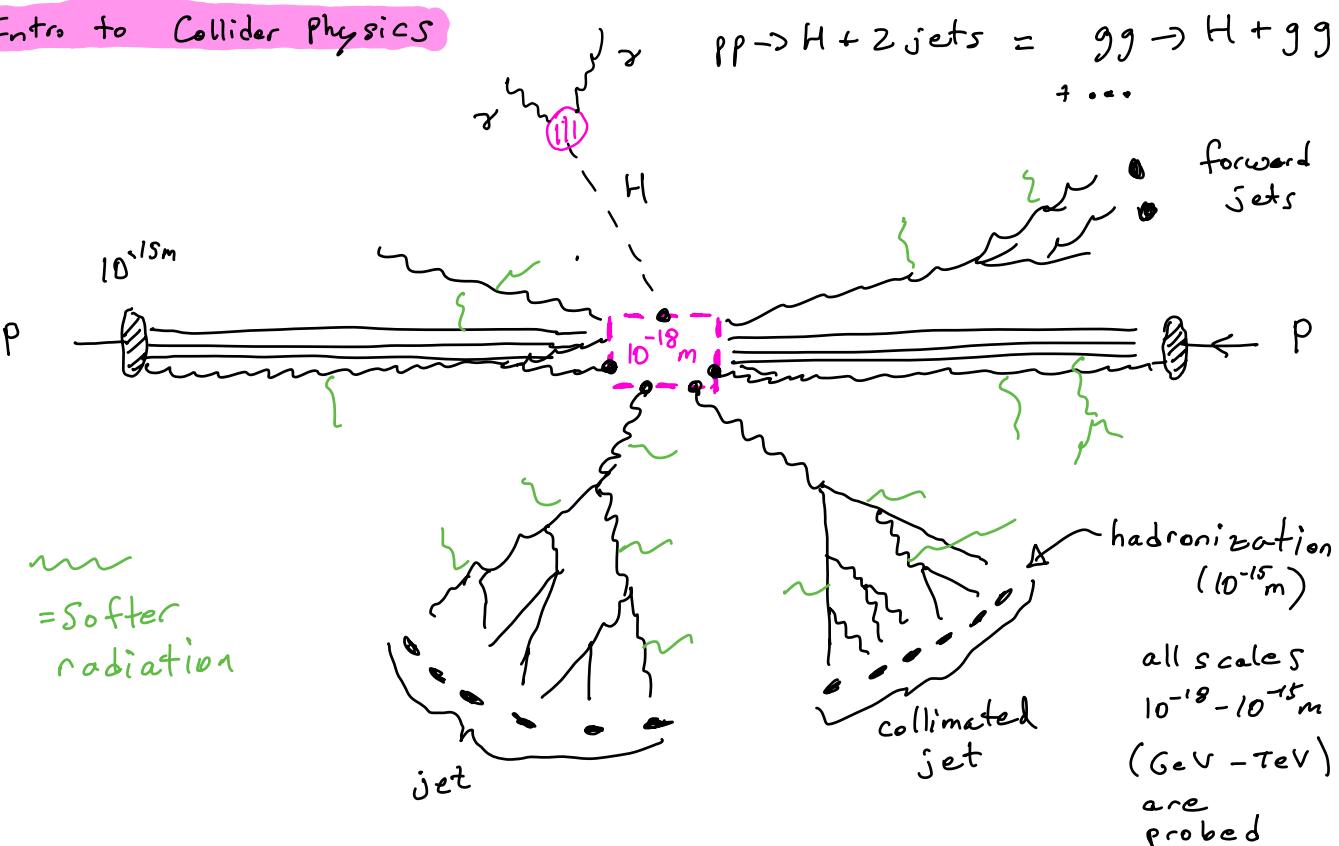
$\gamma^* \gamma \rightarrow \pi^0$ $\gamma^* \pi^+ \rightarrow \pi^+$ $\vdots$	$B \rightarrow \pi \pi$ $\quad\quad\quad \rightarrow \pi l \bar{\nu}$ $\quad\quad\quad \rightarrow X_S \gamma$ $\quad\quad\quad \vdots$	$e^+e^- \rightarrow J/4 X + \text{jet substructure}$ $T \rightarrow X \gamma$ $\vdots$	$+ \text{Regge / High Energy}$ $+ \text{Heavy-Ion}$ $+ \text{electro-weak logs}$ $+ \text{DM cross sections}$ $+ \dots$
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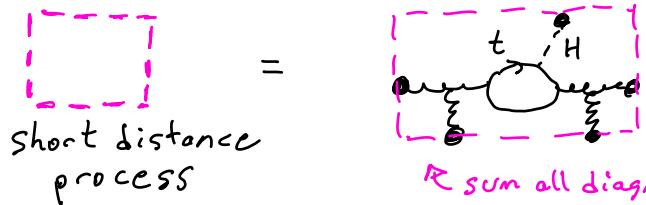
**(SCET + HQET)**      **(SCET + NRQCD)**

see SCET in QCD c50 [2212.11107 pg 223]

\* Discuss Refs Posted \*

## Intro to Collider Physics





- Why? Collinear & Soft dominate
- Complicated!?  $\rightarrow$  Incoherent: physics of proton, jets, Higgs production mostly decouple

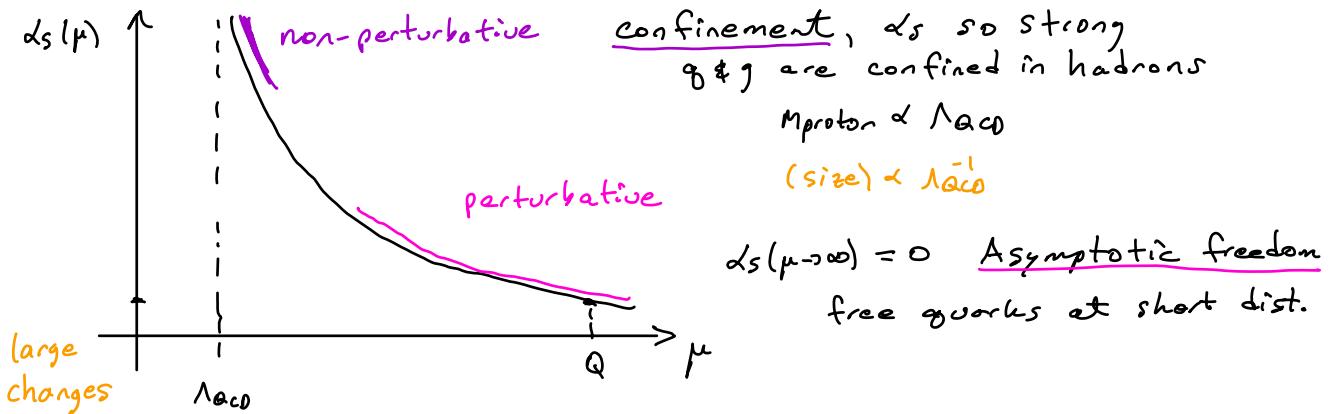
### Running Coupling

In QCD resolution scale  $\mu$  of a process is very important

$$\alpha_s = \frac{g^2}{4\pi} = \alpha_s(\mu) \quad \text{parameters in QFT defined by}$$

$\overline{\text{MS}}$  renormalization scheme here

scheme parameter  $\mu$  ( $\alpha_s^{\text{bare}} = 2d \mu^{2\epsilon} l^\epsilon \alpha_s(\mu)$ ,  $l \in \frac{\pi}{4\pi}$  poles)



$$\beta\text{-function} \quad \mu \frac{d}{d\mu} \alpha_s(\mu) = \beta[\alpha_s] = -\frac{\beta_0 \alpha_s(\mu)^2}{2\pi} + \dots$$

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{\beta_0}{2\pi} \alpha_s(\mu_0) \ln \frac{\mu}{\mu_0}} = \frac{2\pi}{\beta_0 \ln \left( \frac{\mu}{\lambda_{QCD}} \right)} \quad ; \quad \lambda_{QCD} \sim 250 \text{ MeV}$$

dimensional transmutation

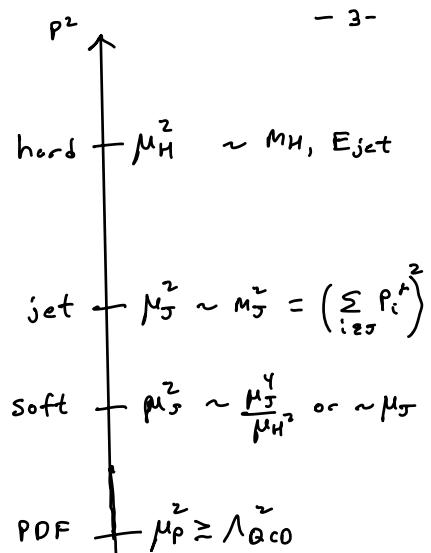
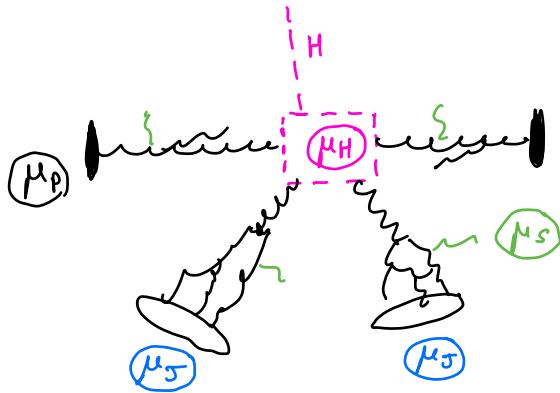
$$\beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f$$

$$C_A = 3$$

$$n_f = \# \text{ light quarks at } \mu$$

- processes with physical scale  $s$  will involve  $\alpha_s(\mu) \ln \frac{\mu}{s}$  terms, so we pick  $\mu \approx s$  to avoid large logs
- $\Rightarrow$  more than one scale  $s_i \Rightarrow$  more than one relevant  $\alpha_s(\mu_i)$

## Collider Scales



Note: (ask) distinguishing 2 jets also requires angular information

## Factorization

key tool to calculate cross sections is the ability to independently consider different parts of the process

$$d\sigma \sim \left( \begin{array}{c} \text{Prob. for} \\ \text{gluons taken} \\ \text{from protons} \end{array} \right) \left( \begin{array}{c} \hat{\sigma}(gg \rightarrow H), \\ \hat{\sigma}(gg \rightarrow Hg), \\ \dots \end{array} \right) \left( \begin{array}{c} \text{Prob. for gluons} \\ \text{to produce} \\ \text{jets} \end{array} \right)$$

Another key idea is to exploit inclusive observables

$e+e^- \rightarrow X$  (any hadrons)

$e-p \rightarrow e^- X$  DIS

the less we measure the simpler the physics

e.g. Higgs Production via gluon fusion

$p p \rightarrow H + X_{\text{g}} \quad (\text{any hadrons or } 0+1+2+\dots \text{ jets})$

$$d\sigma \sim \int d\zeta_a d\zeta_b f_g(\zeta_a, \mu) f_g(\zeta_b, \mu) \underbrace{[\hat{\sigma}_{gg \rightarrow H+X}(\zeta_a, \zeta_b, m_H, \dots, \mu)]}_{\text{universal parton dist'n function (PDF)}} * (1) \quad (1)$$

$f_g = \text{Prob. of finding } g \text{ in proton}$  = Probability Density  
with momentum fraction  $\zeta_a$  (proton snapshot)

$(1) = \sum_i \text{Prob}(i)$  sum over everything that can happen to final state quarks & gluons, so we are not sensitive to this dynamics (jets etc)

Practical limits on  $\sum_i$  → restrict final state

→ cuts on jets to control background or enhance signals ( $\geq N$  jets SUSY)

→ need more exclusive events to determine expt. efficiencies etc.

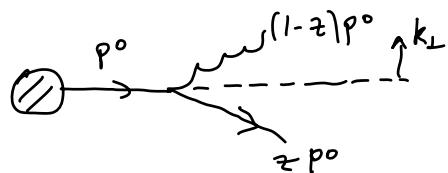
Still sum over dynamics inside the jet & characterize

it by a few variables : jet momentum  $P_J^{\mu} = \sum_{i \in J} p_i^{\mu}$

angular size   $R$

## Jets Why does QCD produce Jets?

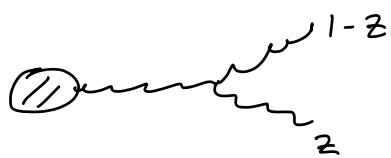
log enhancement from collinear singularities,  $k_T \rightarrow 0$



$$\frac{|A_{N+1}(z)|^2}{|A_N(z)|^2} \propto \frac{ds C_F}{\pi} \frac{dk_L}{k_L} dz P_{gg}(z)$$

splitting fn.

$$\hat{\tau} \frac{1+z^2}{1-z}$$



$$\propto \frac{ds C_A}{\pi} \frac{dk_L}{k_L} dz P_{gg}(z)$$

$$P_{gg}(z) = \frac{z}{1-z} + \frac{1-z}{z} + z(1-z)$$

$$\text{Collinear limit } k_L \rightarrow 0 \text{ enhanced : prefer to split in collimated manner}$$

$$\hat{\tau} \frac{1+(1-z)^2}{z}$$

$$\text{Soft limit : } z \xrightarrow{z \rightarrow 0} \frac{1}{2} P_{gg}(z)$$

$$\hat{\tau} z^2 + (1-z)^2$$

Collinear limit  $k_L \rightarrow 0$  enhanced : prefer to split in collimated manner

Soft limit :  $z \rightarrow 0, 1$  enhanced too. (In shower producing jet the soft gluons are preferentially emitted within cone of collinear emissions)

(angular ordering).)

### Parton Shower $\rightarrow$ Jet

- Leading contribution is strongly ordered

$$k_{1\perp} \gg k_{2\perp} \gg k_{3\perp} \dots \gg k_{n\perp} \sim \Lambda_{QCD}$$

If  $d_s \ln \left( \frac{k_{i\perp}}{k_{i+1\perp}} \right) \approx 1$  no perturbative suppression

- If we measure jet mass but don't probe physics inside the jet then we are only sensitive to emissions above this "IR cutoff"

$$Q \gg k_{1\perp} \gg \dots \gg k_{n\perp} \sim M_J, \text{ each } \int \frac{dk_{i\perp}}{k_{i\perp}} \sim \ln \frac{Q}{M_J} \equiv L$$

Also acts as energy cut off  $\int_{M_J^2/Q^2}^1 \frac{dz}{z} \sim \ln \frac{Q^2}{M_J^2} = 2L$

$\rightarrow$  double logarithmic enhancement

$$\int_0^{M_J^2} \frac{dM_J^2}{dM_J^2} \frac{d\sigma}{dM_J^2} \sim \exp \left( L \lesssim (d_s L)^1 + \sum_i (d_s L)^i + d_s \sum_i (d_s L)^i + \dots \right)$$

$\uparrow$

LL                    NLL                    NNLL  
= leading log      = next-to-LC

$$S(M_J^2) \Rightarrow 1 \sim 1 + d_s L^1 + d_s^2 L^4 + \dots \quad \text{"Sudakov Double Logs"}$$

+ ...

### Concepts to Explore with SCET

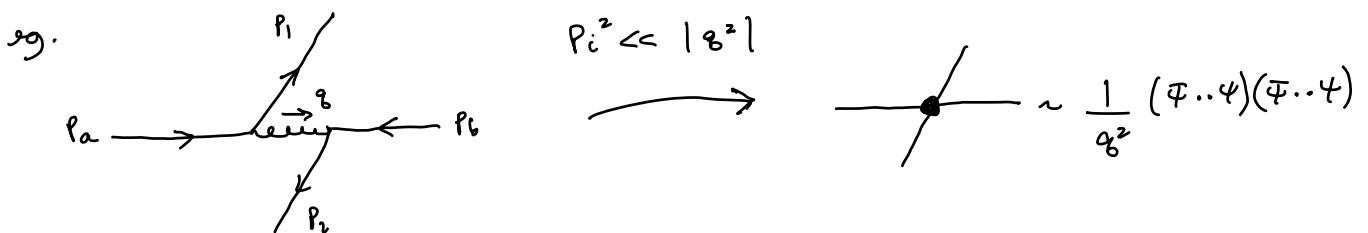
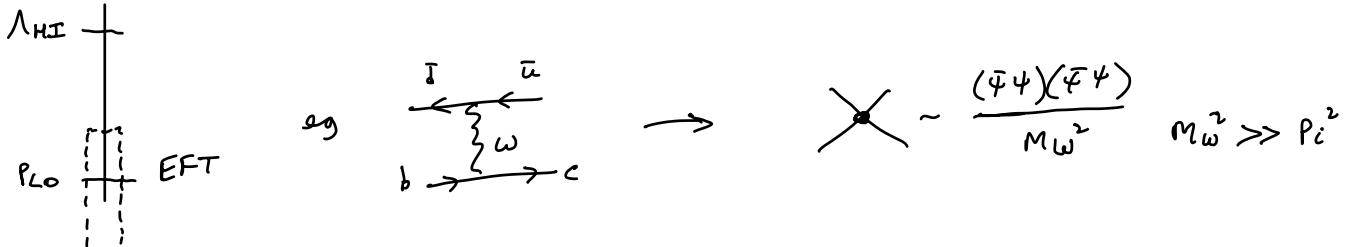
new?

- Factorization (Wilson Lines, ...)
- Power Expansions  $\rightarrow \frac{k_T}{Q} \ll 1, \frac{\Lambda_{QCD}}{Q_0} \ll 1, \dots, \frac{\mu_1}{\mu_2} \ll 1$ 
  - $\rightarrow$  Study Power Corrections
  - $\rightarrow$  interface with other EFTs (HQET, NRQCD)
- Sum Double Logs with RGE, N<sup>K</sup>LL
  - precision
  - exploit universal objects
  - interface w/ fixed order
- Hadronization parameters/functions w/ Field Theory
  - beyond parton distn functions & fragmentation functions

## First, Review key EFT Concepts

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Decoupling Effects from heavy or offshell particles are suppressed / decouple  $p_{lo} \ll \Lambda_{HI}$



$$\text{say } p_i^2 = 0 \text{ on-shell}, \quad q = p_a - p_i = n_a E_a - n_i E_i$$

$$n_a = (1, \hat{z})$$

$$\bar{n}_a = (1, -\hat{z})$$

$$q^2 = -2 E_a E_i \cdot n_a \cdot n_i$$

$$n_i = (1, \hat{n})$$

$$\bar{n}_i = (1, -\hat{n})$$

$$= -2 E_a E_i (1 - \hat{z} \cdot \hat{n})$$

large if energies big &  
deflection angles large

$q^2 \sim Q^2$  "hard"

### Construct $\mathcal{L}_{eff}$

- degrees of freedom? low energy / nearly onshell modes  
→ what fields
- symmetries → constrain interactions / operators  
[Lorentz, Gauge theory, Global, ...]
- expansions, leading order description  
→ power counting

$$\mathcal{L}_{EFT} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$

- often expand in mass dimension of operators, but not in SCET]

- $\infty$  # operators, but only specific subset needed at given order

### Matching

$$\mathcal{L}_{\text{EFT}}^{(k)} = \sum_i c_i(\mu) \mathcal{O}_i^{(k)}(\mu)$$

short. dist.  
 (offshell)      long dist.  
 ( $\sim$  on-shell)

- $\mathcal{L}_{\text{HI}}$  &  $\mathcal{L}_{\text{EFT}}$  have same IR, differ in UV
- $c_i(\mu)$  does not depend on IR scales (masses in EFT,  $\Lambda_{\text{QCD}}$ , IR regulators, ...)

splits HI from LO

"top-down EFT"  
 if we know  $\mathcal{L}_{\text{HI}}(\Lambda_{\text{HI}}, p_0)$  we can (perturbatively) construct  $\mathcal{L}_{\text{EFT}}$ .

Calculate  $C$ , construct  $\mathcal{O}$

[Hweak, HQET  
NRQCD, SCET, ...]

**Ask**

Do this by demanding equality of S-matrix elts.  
in  $\mathcal{L}_{\text{HI}}$  &  $\mathcal{L}_{\text{EFT}}$

"bottom-up EFT"  
 form  $\sum_i c_i \mathcal{O}_i$  complete basis  
 exploit symmetries

**L\_EFT**

[e.g. SM as EFT, chiral Lagrangians ...]

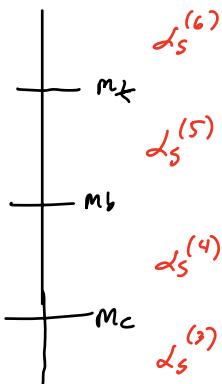
### Renormalization

- parameters  $g$ ,  $C$  in QFT must be defined by a renormalization scheme, also  $\mathcal{O}$  ( $\bar{m}_s$ , Wilsonian cutoff, ...)
- schemes depend on cutoff / renormalization scale "μ"  $g(\mu)$ ,  $C(\mu)$

## Renormalization Group

e.g.  $\alpha_s^{(nf)}(\mu)$  in QCD

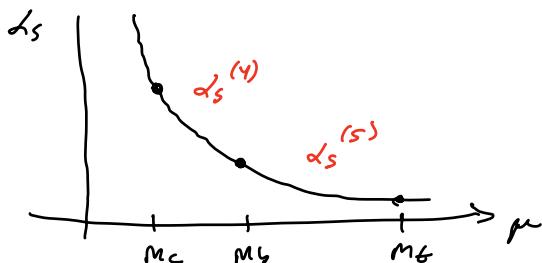
$$\mu \frac{d}{d\mu} \alpha_s^{(nf)}(\mu) = -\frac{\beta_0}{2\pi} [\alpha_s^{(nf)}(\mu)]^2 + \dots$$



$$\beta_0^{(nf)} = 11 - \frac{2}{3} n_f$$

• sums logs between mass scales  
 $\alpha_s^{(k)} \ln^{(k)}(\mu/m_t)$

Homework #1  
for anyone  
unfamiliar  
with this



Ask: Is slope of  $\alpha_s(\mu)$  continuous?

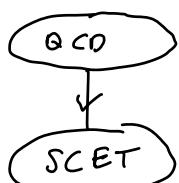
Is  $\alpha_s(\mu)$  continuous?

e.g.  $\alpha_s^{(4)}(\mu_m) = \alpha_s^{(5)}(\mu_m) \left[ 1 + \frac{\alpha_s^{(5)}}{\pi} \left( -\frac{1}{6} \ln \frac{\mu_m^2}{m_b^2} \right) + \left( \frac{\alpha_s^{(5)}}{\pi} \right)^2 \left( \frac{11}{72} - \frac{1}{24} \ln \frac{\mu_m^2}{m_b^2} + \frac{1}{36} \ln^2 \frac{\mu_m^2}{m_b^2} \right) + \dots \right]$

e.g.  $\mu \frac{d}{d\mu} f_{i/p}(x, \mu) = \dots$  (later)

- Power counting handles powers  $\frac{P_{LO}}{\Lambda_{HI}} \ll 1$  [Typically]
- Renormalization group handles logs  $\ln\left(\frac{P_{LO}}{\Lambda_{HI}}\right)$   
which may be large as  $\ln(\dots) \sim 1$

SCET



$$\Lambda_{QCD}^2, P_{IR}^2 \ll p^2 \sim Q^2$$

hard collision

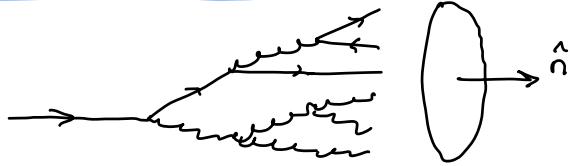
$\frac{\uparrow L1}{\downarrow L2}$

degrees of freedom

consider  $e^+e^- \rightarrow 2 \text{ jets}$

$$e^+ \rightarrow g^* \rightarrow q \bar{q} \quad Q^2 = g^2$$

## Jets/collinear



due to collinear (soft) enhancement -q-  
in QCD

- collimated radiation in  $\hat{n}$
- $E_{\text{jet}} \sim Q$

Let  $n^\mu = (1, \hat{n})$  - encode direction of jet/radiation  
 $\bar{n}^\mu = (1, -\hat{n})$  - auxiliary vector for decomposition  
 $n^2 = \bar{n}^2 = 0, n \cdot \bar{n} = 2$

$$p^\mu = \underbrace{\bar{n} \cdot p}_{P^-} \frac{n^\mu}{2} + \underbrace{n \cdot p}_{P^+} \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$

$$p^2 = n \cdot p \bar{n} \cdot p + \underbrace{p_\perp^2}_{-P_\perp^2}$$

## Collinear?

1 massless particle :  $p^\mu = \bar{n} \cdot p \frac{n^\mu}{2}$

2 massless :  $\rightarrow \bar{n}^\mu$   $p_i^\mu = \bar{n} \cdot p_i \frac{n^\mu}{2} + p_{i\perp}^\mu + n \cdot p_i \frac{\bar{n}^\mu}{2}$   
 $i = 1, 2$

$$\bar{n} \cdot p_i \sim Q$$

$$p_{i\perp}^\mu \ll Q \quad \text{collimated}$$

say  $p_{i\perp} \sim \lambda Q$

$$\lambda \ll 1$$

dimensionless power counting parameter

on-shell  $n \cdot p_i = +\frac{p_i^2}{n \cdot p_i} \Rightarrow n \cdot p_i \sim \lambda^2 Q$  nearly on-shell

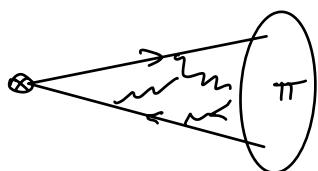
n particles : same (ignore legs for now)

n-Collinear :  $p^\mu \sim Q (\lambda^2, 1, \lambda)$

Mention not simply mass dimension

SCET n-Collinear Fields : quark  $\xi_n$   
 gluon  $A_n^\mu$

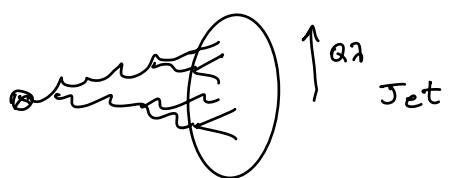
energetic hadron :

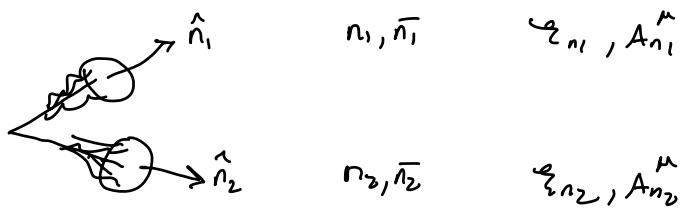


$$\uparrow p_\perp \sim \Lambda_{QCD} \Rightarrow \lambda \sim \frac{\Lambda_{QCD}}{Q}$$

energetic quarks & gluons confine into single hadron

jet of hadrons :  $1 \gg \lambda \gg \frac{\Lambda_{QCD}}{Q}$



Z-jets

Often simplify using back-to-back frame :

$$\begin{aligned}
 n_2 = \bar{n} &= (1, -\hat{n}) & \leftarrow \text{circle with } \hat{n} \rightarrow & n_1 = n = (1, \hat{n}) \\
 \bar{n}_2 = n & & & \bar{n}_1 = \bar{n} \\
 & & \frac{(+, -, \perp)}{(\lambda^2, 1, \lambda)} & (\text{not possible for } \geq 3 \text{ jets}) \\
 & & q_n, A_n & (1, \lambda^2, \lambda) \\
 & & q_{\bar{n}}, A_{\bar{n}} & (1, \lambda^2, \lambda)
 \end{aligned}$$

**Soft**

$$\underline{p_s^\mu \sim Q \lambda^\alpha}$$

all components small  
& homogeneous

soft + soft = soft

soft + hard = hard

collinear + hard = hard

$n_1$ -collinear +  $n_2$ -collinear = hard  $\hookrightarrow$  hard interaction produces jets

collinear + soft ?

$$\begin{array}{c}
 \sum_i p_{in}^\mu \\
 \hline
 p_n^\mu
 \end{array}
 \quad (p_n + p_s)^2 = 2 p_n \cdot p_s = \bar{n} \cdot p_n \bar{n} \cdot p_s + \dots \sim Q^2 \lambda^\alpha$$

*f suppressed*

Value of  $\alpha$  depends on what we measure

eg 1 Mass in (large enough) region  $a$ ,  $M_a^2 = \left( \sum_{i \in a} p_i^\mu \right)^2$   
 [mass of  $R=1$  jet, hemisphere mass, ...]

demand  $M_a^2 \sim Q^2 \lambda^2 \ll Q^2$  [collimated jet has  $E_J \gg M_J$ ]

$$\begin{array}{l}
 \text{collinear + collinear} \quad (p_n + p_n')^2 = 2 p_n \cdot p_n' \sim Q^2 \lambda^2 \\
 \hline
 \begin{matrix} + & - \\ - & + \\ \perp & \perp \end{matrix}
 \end{array}$$

collinear + soft  $(p_n + p_s)^2 \sim Q^2 \lambda^\alpha$

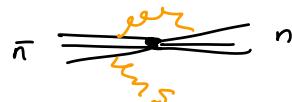
$\therefore \alpha = 2$  to contribute "ultrasoft"

eg 2 Transverse Momenta  $\rightarrow$  broadening  $B_\perp = \sum_{i \neq n} |\vec{p}_{i\perp}| \ll Q$   
 $\sum_{i \neq n}$  collinear ✓

soft  $p_{s\perp} \sim \lambda^\alpha \Rightarrow \alpha = 1$  "soft"

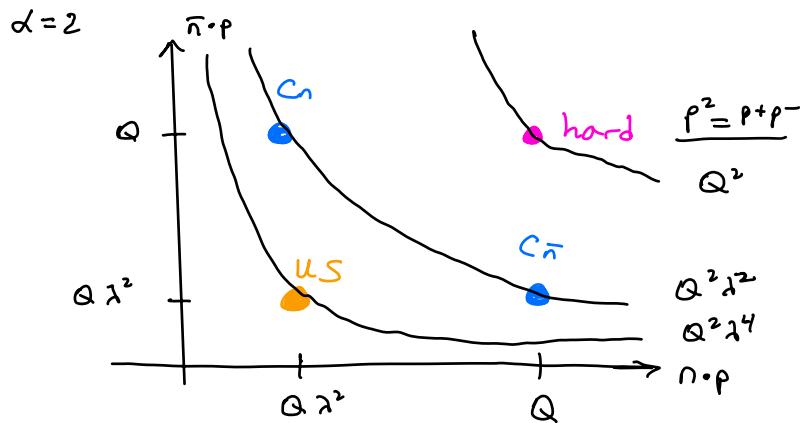
### DOF Picture

$e^+e^- \rightarrow 2$  jets  
(cm frame)



[virtual too]

#### ① SCET<sub>I</sub>



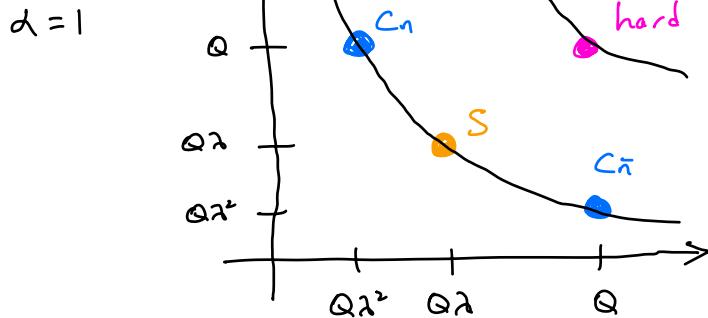
- power counting requires multiple fields for same particle

- relative scaling of modes is important  
[boost invariant, unlike absolute scaling]

- modes not classified by  $p^2$  alone  
rapidity  $\sim$  polar angle

$$e^{-2Y} = \frac{p^+}{p^-} \approx \tan^2 \frac{\Theta}{2}$$

#### ② SCET<sub>II</sub>



- modes cover regions of momentum space, extend into IR

Study SCET<sub>I</sub>, come back to SCET<sub>II</sub>

## Field Power Counting

use free kinetic term

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$\xi_n$  propagator

$$p^2 = n \cdot p \bar{n} \cdot p + p_\perp^2$$

$$\lambda^2 * \lambda^0 + (\lambda)^2 \text{ same size}$$

$$\frac{i\cancel{x}}{p^2+i0} = \frac{i\cancel{x}}{2} \frac{\bar{n} \cdot p}{p^2+i0} + \dots = \frac{i\cancel{x}}{2} \frac{1}{n \cdot p + \frac{p_\perp^2}{\bar{n} \cdot p} + i0 \operatorname{sign}(\bar{n} \cdot p)} + \dots$$

must have

$$\int d^4x \underbrace{e^{-ip \cdot x}}_{\lambda^4} \langle 0 | T \xi_n(x) \bar{\xi}_n(0) | 0 \rangle = \frac{i\cancel{x}}{2} \frac{\bar{n} \cdot p}{p^2+i0} \quad \textcircled{*}$$

$\lambda^0$

$(d^4p \sim \lambda^4)$  thus  $\xi_n \sim \lambda$  [differs from  $\frac{3}{2}$  mass dimension]

Note:  $\textcircled{*}$  implies  $\cancel{x} \xi_n = 0$  since  $\cancel{x}^2 = n^2 = 0$

take  $\xi_n = \frac{\cancel{x} \cancel{\not{p}}}{4} \psi$  for spin  
 projection op.  $\not{1} = \frac{\cancel{x} \cancel{\not{p}}}{4} + \frac{\cancel{\not{p}} \cancel{x}}{4}$

spinors  $u_n = \frac{\cancel{x} \cancel{\not{p}}}{4} u(p)$

$$\sum_s u_n^s \bar{u}_n^s = \frac{\cancel{x} \cancel{\not{p}}}{4} \sum_s u^s \bar{u}^s \frac{\cancel{\not{p}} \cancel{x}}{4} = \frac{\cancel{x}}{2} \bar{n} \cdot p \quad \checkmark$$

$$u_+(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} \cancel{\not{p}}^- \\ \cancel{\not{p}}^+ e^{i\phi_p} \\ \cancel{\not{p}}^- \\ \cancel{\not{p}}^+ e^{i\phi_p} \end{pmatrix} \rightarrow \frac{\cancel{x} \cancel{\not{p}}}{4} u(p) \text{ kills small terms}$$

Dirac Rep:  $\frac{\cancel{\not{p}}}{4} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$   
 cf. Dixon hep-ph/9601359

similar for  $u_-(p)$  & antiquarks  $u_+(p), u_-(p)$

$A_n^\mu$  some propagator as QCD w/ gauge fixing

$$p^\mu \sim (\lambda^2, 1, \lambda) \sim i \partial_n^\mu$$

$$i D_n^\mu = i \partial_n^\mu + g A_n^\mu$$

want  $i \partial_n^\mu \sim A_n^\mu$  so

$$A_n^\mu \sim (\lambda^2, 1, \lambda) \quad \text{true in any gauge}$$

[or derive from free propagator]

Soft

Similar analysis

$$p_s \sim \lambda^\alpha$$

$$\text{eg. } \int d^4x \bar{\psi}_s i\partial^\mu \psi_s \sim 1$$

$$\lambda^{-4\alpha} \quad \lambda^\alpha$$

$$A_s^\mu \sim p_s^\mu \sim \lambda^\alpha$$

$$4s \sim \lambda^{3\alpha/2}$$

$\alpha = 2$  for SCET<sub>I</sub> ("ultrasoft"  $\alpha = 2$  vs. "soft"  $\alpha = 1$ )

### Structure of SCET $\mathcal{L}$

interactions with  $\geq 2$  collinear sectors  
(or soft-collinear) = hard scattering

interactions with 1-collinear sector  
or ultrasoft-collinear = dynamics

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_{\text{hard}} + \mathcal{L}_{\text{dyn}}$$

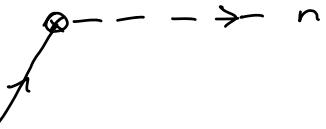
$$= \underbrace{\left( \mathcal{L}_{\text{hard}}^{(0)} + \sum_{i \geq 1} \mathcal{L}_{\text{hard}}^{(i)} \right)}_{\text{leading}} + \underbrace{\left( \mathcal{L}_{\text{dyn}}^{(0)} + \mathcal{L}_G^{(0)} + \sum_{i \geq 1} \mathcal{L}_{\text{dyn}}^{(i)} \right)}_{\text{leading}} + \underbrace{\text{subleading}}$$

potential fact.  
violating terms  
(more later on)

Start by studying  $\mathcal{L}_{\text{hard}}$

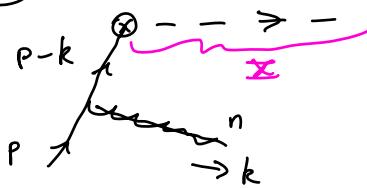
### Collinear Wilson Lines

- $\vec{n} \cdot \vec{A}_n \sim \lambda^0$  ? no suppression for building operators

  
not  $n$   
( $n'$  or  
massive etc.)

①   
nearly on-shell  
part of  
 $\mathcal{L}_{\text{dyn}}^{(0)}$

(2)



$$(p-k)^2 = \cancel{p^2 - m^2 + k^2} - 2p \cdot k = -\bar{n} \cdot k \bar{n} \cdot p + \dots$$

 $\therefore$  offshell

$\cancel{\gamma^0}$   $\cancel{\gamma^0}$  since  
not  $n$ -collinear

integrate it out

$$= \sum \frac{i(\gamma - k + m)}{(p-k)^2 - m^2 + i\alpha} (\gamma^\mu \gamma^\nu \epsilon_n^\alpha) u(p)$$

$$\uparrow \frac{i}{2} \bar{n} \cdot \epsilon_n^\alpha + \dots$$

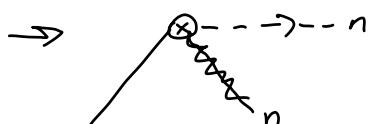
expand

Homework #2

$$\text{since } A^\mu = \bar{n} \cdot A \frac{\gamma^\mu}{2} + \dots$$

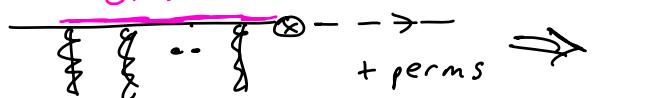
$$= \sum \frac{(-g) \bar{n} \cdot A_n^\alpha}{-\bar{n} \cdot k + i\alpha} T^\alpha u(p)$$

- universal,  
independent of  $p, m, \dots$



keep going

offshell



More Homework

Gives Wilson line

$$\langle W_n \rangle = \sum_m (-g)^m \sum_{\substack{\text{perms} \\ \{i_1, \dots, i_m\}}} \frac{\bar{n}^{\mu_m} T^{A_m} \dots \bar{n}^{\mu_1} T^{A_1}}{[\bar{n} \cdot g_{i_1}] [\bar{n} \cdot (g_{i_1} + g_{i_2})] \dots [\bar{n} \cdot \sum_{i=1}^m g_i]}$$

in position space:

$$W_n(y, -\infty) = \rho \exp \left( ig \int_{-\infty}^y ds \bar{n} \cdot A_n(s \bar{n} + y) \right)$$

W<sub>n</sub> ~  $\lambda^0$

SCET operator  $(\overline{\xi}_n \omega_n)$  (r+)

generic, operator "building block"

quark  $\chi_n = \omega_n^+ \xi_n$

gluon  $\mathcal{O}_{Bn\perp}^\mu = \frac{1}{g} [\omega_n^+ iD_{n\perp}^\mu \omega_n] = \left[ \frac{1}{g \bar{n} \cdot \bar{n}} \omega_n^+ [i\bar{n} \cdot D_n, iD_\perp^\mu] \omega_n \right]$   
 $= A_{n\perp}^\mu(k) - \frac{k_\perp^\mu}{\bar{n} \cdot k} \bar{n} \cdot A_n(k) + \dots$  [vanishes if  $A^\mu \rightarrow k^\mu$ , g.-inv.]

Gauge Symmetry symmetry transfm. must leave or within the EFT

$$U(x) = e^{i\alpha^A(x) T^A} \quad i\partial^\mu U(x) \sim P_n^\mu U(x) \quad \text{collinear}$$

$$i\partial^\mu U_{us}(x) \sim P_{us}^\mu U_{us}(x) \quad \text{ultrasoft}$$

•  $\xi_n \rightarrow U_n \xi_n \quad iD_n^\mu \rightarrow U_n iD_n^\mu U_n^+ \quad \text{for } A_n$

$g_{us} \rightarrow g_{us}$  [else not ultrasoft],  $\omega_n \rightarrow U_n \omega_n$

•  $g_{us} \rightarrow U_{us} g_{us} \quad iD_{us} \rightarrow \dots$   
 $\xi_n \rightarrow U_{us} \xi_n \quad A_n^\mu \rightarrow U_{us} A_n^\mu U_{us}^+, \quad \omega_n \rightarrow U_{us} \omega_n U_{us}^+$

$\Rightarrow \chi_n = \omega_n^+ \xi_n \rightarrow \omega_n^+ U_n^+ \chi_n \xi_n$  protected by g.-inv.  
 e.g. stays together when we add loop corrections

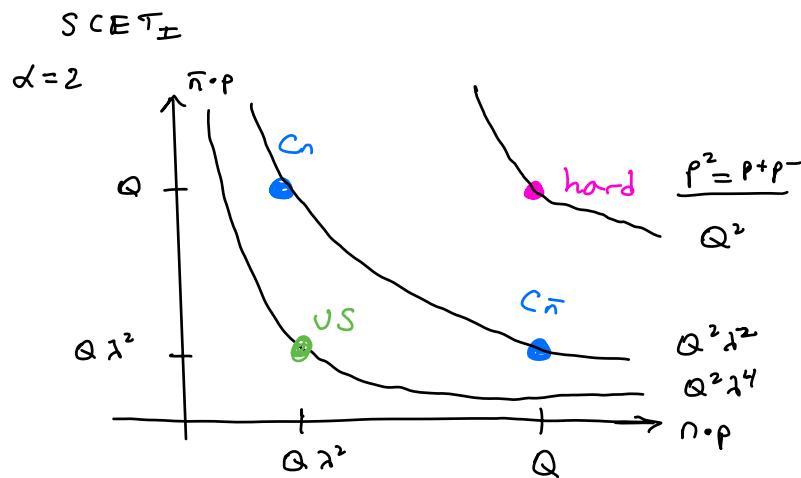
build operators out of n-collinear gauge invariant building blocks  $\chi_n, \mathcal{O}_{Bn\perp}^\mu$

Wilson lines needed to ensure gauge invariance in presence of operators where gluons only couple in on-shell manner to single colored field.

~~one-gluon~~  $\rightarrow$  ~~one-gluon~~  $\omega_n$  + vs. vs.

Review L2

-15.5-



$(p^+, p^-, p_\perp)$

$C_n : (\lambda^2, 1, \lambda)$

fields

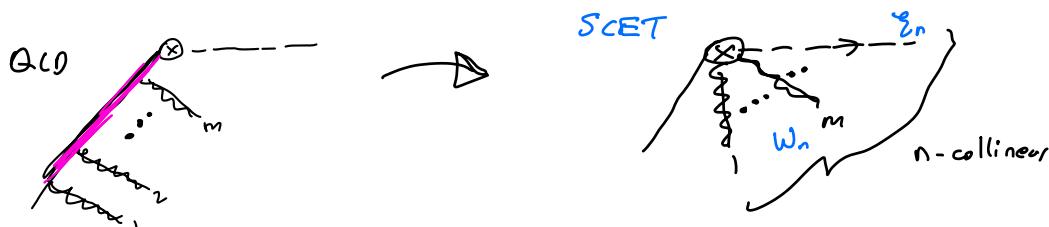
collinear quark  $\xi_n \sim \lambda$

collinear gluon  $A_n^\mu \sim (\lambda^2, 1, \lambda)$

ultrasoft:  $(\lambda^2, \lambda^2, \lambda^2)$

$\psi_{us} \sim \lambda^3$ ,  $A_{us}^\mu \sim \lambda^2$

Integrate out hard  $p^2 \sim Q^2$  modes:



$$\bar{\chi}_n \equiv \overline{\xi_n} w_n$$

$$\chi_n \equiv w_n^\dagger \xi_n$$

"quark parton field", "jet field"

wilson line  $w_n(y, -\infty) = P \exp \left( ig \int_{-\infty}^y ds \bar{n} \cdot A(\bar{n} s + y) \right)$

Hard-Collinear Factorization:

$$C(i\bar{n} \cdot \partial_n) \chi_n = \int d\omega C(\omega) \delta(\omega - i\bar{n} \cdot \partial_n) \chi_n = \int d\omega C(\omega) \chi_{n,\omega}$$

Mention Homework

Mention Conventions

traces  $\bar{n} \cdot A_n \rightarrow w_n$

-16-

$$w_n^+ w_n = \mathbb{1} = w_n w_n^+$$

$$[\bar{n} \cdot D_n w_n] = 0$$

$$\therefore i\bar{n} \cdot D_n w_n \cancel{\underline{}} = w_n i\bar{n} \cdot D_n \cancel{\underline{}}$$

$$w_n^+ i\bar{n} \cdot D_n w_n = i\bar{n} \cdot D_n \text{ as operator}$$

$$i\bar{n} \cdot D_n = w_n i\bar{n} \cdot D_n w_n^+$$

collinear gauge singlet

### Hard - Collinear Factorization

$$\mathcal{L}_{\text{hard}} = C \otimes \mathcal{O}$$

What do Wilson Coefficients depend on?

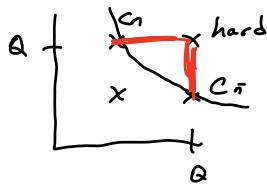
$$i\bar{n} \cdot D_n \sim \lambda^0$$

$$\text{Allows } C(i\bar{n} \cdot D_n) X_n = \underbrace{\int d\omega C(\omega)}_{\text{gauge inv.}} \underbrace{\delta(\omega - i\bar{n} \cdot D_n)}_{\text{operator}} X_n$$

Hard & collinear modes communicate through  $\sim \lambda^0$  momenta

constrained by gauge inv.

& momentum conservation



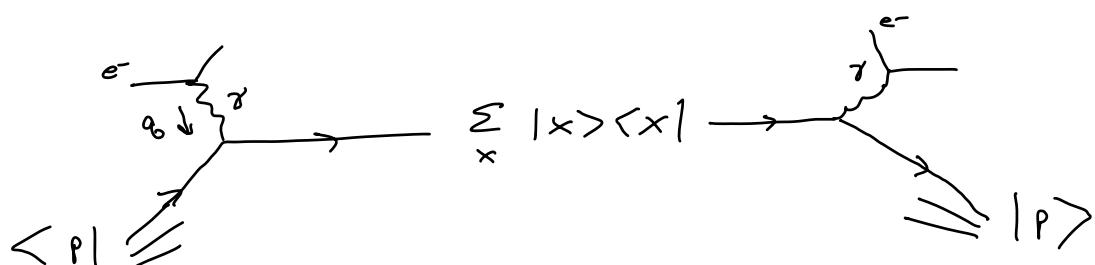
$\uparrow L_2$   
 $\downarrow L_3$

DIS

$$e^- p \rightarrow e^- X$$

Inclusive Factorization

[ full analysis requires more knowledge, eg  $L$ , cover few key parts ]



Take  $q = (0, 0, 0, Q) = \frac{Q}{2} (\bar{n} - n)$     $q^2 = -Q^2$  spacelike

$$\text{Bjorken } x = \frac{Q^2}{2 p \cdot q}$$

Breit frame, where  
proton is n-collinear

proton  $P_p^\mu = \frac{\pi^\mu}{2} \bar{n} \cdot p_p + \frac{\pi^\mu}{2} \frac{M_p^2}{\bar{n} \cdot p_p}$ , big  $\bar{n} \cdot p_p = \frac{Q}{x} \sim 2^\circ$  -17-

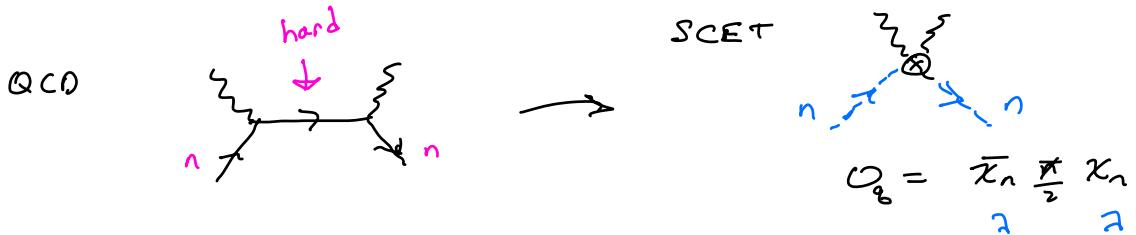
small

$$P_x = P_p + q, \quad P_x^2 = Q^2 \left( \frac{1}{x} - 1 \right) + M_p^2$$

regions:	$\frac{P_x^2}{Q^2}$	$\left( \frac{1}{x} - 1 \right)$	study of this
	$\sim Q^2$	$\sim 1$	inclusive
	$\sim Q/\lambda$	$\sim N_Q$	endpoint $x \rightarrow 1$ (jet)
	$\sim \lambda^2$	$\sim \lambda^2/Q$	resonance $(e-p \rightarrow e-p')$
	$\gg Q^2$	$\gg 1$	small $x$

$P_x = P + q = \text{hard}$

$$\lambda = \frac{\Lambda_{QCD}}{Q} \ll 1$$



Add arbitrary pert.

$\alpha_s^K$  corrections:

actually  $C_i = 1, 2, \dots, 100$   
structures for  $W^{KK}$

$$\sigma \sim \lambda^2 \text{ twist-2}$$

also gluon  $O_g = \bar{B}_{n\perp} \frac{\lambda}{2} B_{n\perp \mu}$

$$\mathcal{L}_{\text{hard}} = \int d\omega d\omega' C(\omega, \omega'; Q) \bar{X}_n \frac{\lambda}{2} \delta(\omega + i\pi \cdot \partial_n) \delta(\omega - i\pi \cdot \partial_n) X_n$$

forward  $\langle p | \dots | p \rangle$  matrix element fixes  $\omega = \omega'$

$$\sigma \sim \int d\omega \text{ Im } C(\omega, Q) \langle p | \bar{X}_n \frac{\lambda}{2} \delta(\omega - i\pi \cdot \partial_n) X_n | p \rangle$$

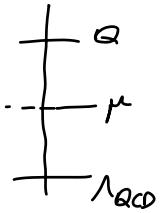
dimensionless  $\frac{\omega}{Q}$  momentum of quark in proton

$$\sim \int \frac{d\zeta}{\zeta} H\left(\frac{\zeta}{Q}, \frac{Q}{\mu}, \alpha_s(\mu)\right) f_{q/p}\left(\frac{\zeta}{\mu}, \frac{\mu}{\Lambda_{QCD}}\right)$$

$$\frac{Q}{\omega} = \frac{Q}{\zeta \bar{n} \cdot \ell} = \frac{x}{\zeta}$$

$$\zeta = \frac{\omega}{\bar{n} \cdot \ell}$$

Hard Collinear PDF



- all orders in  $\alpha_s$  (no use of pert. theory),  $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$  - 18-  
corrections
- universal  $f_{g/p}$
- $H$  dimensionless  $\rightarrow$   $\alpha_s \ln \mu/Q$  dependence on  $Q$ , Bjorken scaling
- $f_{g/p}(\xi, \mu)$  encodes a set of "twist-2" operators

In position space

$$f_{g/p}(\xi) = \int \frac{db^+}{4\pi} e^{-i(\xi p^-)b^+} \langle p | \bar{\xi}_n(b^+) W_n(b^+, 0) \frac{i}{2} \xi_n(0) | p \rangle$$

Same in  
QCD & SCET

### Comments on Renormalization

**Ask:** Operators with same Quantum numbers mix under ren.  
 $\Rightarrow$  Loops can change  $\xi$ ,  $f_g(\xi)$  mixes with  $f_g(\xi')$ . Also  
mix parton types  $i = g, q$ .

$$f_i^{\text{bare}}(\xi) = \sum_j \int d\xi' \gamma_{ij}(\xi, \xi') f_j(\xi', \mu)$$

$\tau_{\mu \text{ indep.}}$        $\tau_{\text{renormalized}}$

$$\Rightarrow \mu \frac{d}{d\mu} f_i(\xi, \mu) = \sum_j \underbrace{\int d\xi' \gamma_{ij}(\xi, \xi') f_j(\xi', \mu)}_{\propto P_{ij}(\xi, \xi')} \text{ splitting functions}$$

$$\text{with } \gamma_{ij} = - \sum_k \int d\xi'' \bar{\xi}_{ik}(\xi, \xi'') \mu \frac{d}{d\mu} \xi_{kj}(\xi'', \xi')$$

### More Hard Operators

power counting, symmetry & matching calc imply  $\mathcal{O}$   
are built from

[Note: true at any order  
other collinear ops  
eliminated by operator identities  
& eqns. of motion.]

$\chi_n$   
 $\partial B_{n\perp}^\mu$

Discuss  $n=2$   
 $n=3$

$P_\perp^\mu$   
& Soft Fields

often suppressed

Example $e^+e^- \rightarrow 2 \text{ jets}$ Operators

$\bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}}$



Amplitude

 $gg \rightarrow H$ 

$\bar{B}_{n_1}^\mu \bar{B}_{\bar{n}_1 \perp \mu} H$



Ampl.

[ quark PDF

$\bar{\chi}_n \frac{\not{p}}{2} \delta(\omega - i\bar{n} \cdot \not{p}) \chi_n$

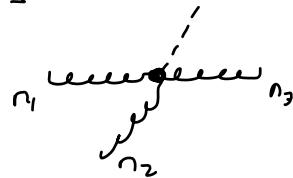
Ampl.<sup>2</sup> ]

gluon PDF

$+ r [\bar{B}_{n_1}^\mu \delta(\omega - i\bar{n}_1 \cdot \not{p}) \bar{B}_{n_1 \perp \mu}]$

Ampl.<sup>2</sup> $p p \rightarrow H + 1\text{-jet}$ 

(remove top)



Ampl

no  $\delta^{a_1 a_2 a_3}$  by charge conj.

$\bullet H \bar{B}_{n_1 \perp}^{a_1 \mu_1} \bar{B}_{n_2 \perp}^{a_2 \mu_2} \bar{B}_{n_3 \perp}^{a_3 \mu_3} \quad (\text{if } \delta^{a_1 a_2 a_3})$

spin

$$\begin{cases} T_{\mu_1 \mu_2 \mu_3} \\ \bar{T}_{\mu_1}^{\alpha \bar{\rho}} \end{cases}$$

Ask

$\bullet H \bar{B}_{n_1 \perp}^{a_1 \mu_1} \bar{\chi}_{n_2}^{\alpha} T^{\beta} \bar{\chi}_{n_3}^{\rho}$

how many operators?

Helicity basis: natural in SCET since we have direction to use  $\hat{n}$ 

$\bar{B}_{n \pm}^a = - \epsilon_{\mp}^r(n, \bar{n}) \bar{B}_{n \perp \mu}^{\perp}, \quad \epsilon_{\mp} = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$

$\bar{\chi}_{n_1 n_2 \pm} \propto \epsilon_{\mp}^r(n_1, n_2) \bar{\chi}_{n_1 \pm} \not{p} \underbrace{\bar{\chi}_{n_2 \pm}}_{[(\frac{1 \pm i}{2}) \chi]} \not{p}$

Allowed  $H \bar{B} \bar{B} \bar{B} \bar{B}$ 

$+ \quad + \quad +$

$+ \quad + \quad -$

$- \quad - \quad +$

$- \quad - \quad -$

} Wilson Coeff  
fixed by Parity

 $H \bar{B} \bar{J}$ 

$+ \quad +$

$- \quad +$

$+ \quad -$

$- \quad -$

} fixed by  
charge Conj.

4 non-trivial coefficients

[ note: no even-odd operators in leading power SCET due to helicity conservation ]

Easy to exploit modern spinor-helicity results.

[ see 1508.02397 for more on helicity operators in SCET. ]

$\text{SCET} \overset{(o)}{\mathcal{L}_{\text{dyn}}}$

$\text{SCET}_I (\alpha=2)$

-20-

For interactions that are isolated and purely  $n$ -collinear or purely ultrasoft we just have full QCD  $\mathcal{L}$  for each sector.

ultrasoft: nothing to expand

$n$ -collinear: boost  $(\gamma^2, 1, \gamma) \rightarrow (\gamma, \gamma, \gamma)$   
everything some

Key thing SCET describes is interactions between sectors

For  $\mathcal{L}^{(0)}$

- $(\gamma^2, 1, \gamma) \xrightarrow[\sum_{\text{US}}]{\sum_{\text{UR}}} (\gamma^2, 1, \gamma)$  usoft leave collinear non-shell
- hard interactions produce collinear quarks with  $\not{x} \cdot \not{\xi}_n = 0$   
[hard int. breaks boost argument]

$$\not{A} \not{\psi} = \left( \frac{\alpha \not{x}}{4} + \frac{\not{x} \alpha}{4} \right) \not{\psi} = \not{\xi}_n + \not{\gamma}_n$$

Good & Bad components

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i i \not{D} \psi_i = \bar{\xi}_n \not{\partial} \text{in}\cdot D \not{\xi}_n + \bar{\gamma}_n \not{\partial} \text{in}\cdot D \not{\gamma}_n + \bar{\xi}_n i \not{\partial}_\perp \not{\gamma}_n + \bar{\gamma}_n i \not{\partial}_\perp \not{\xi}_n$$

$$\text{e.o.m. } \frac{\delta}{\delta \bar{\psi}_n} \Rightarrow \not{\gamma}_n = \frac{1}{i \bar{n} \cdot 0} i \not{\partial}_\perp \frac{\not{\partial}}{2} \not{\xi}_n \quad \begin{matrix} \text{smaller} \\ \text{than} \\ \not{\xi}_n \end{matrix} \quad \begin{matrix} \text{for hard} \\ \text{production} \end{matrix}$$

$$\mathcal{L}_{\text{QCD}} = \bar{\xi}_n \left( \text{in}\cdot D + i \not{\partial}_\perp \frac{1}{i \bar{n} \cdot 0} i \not{\partial}_\perp \right) \frac{\not{x}}{2} \not{\xi}_n \quad \text{still QCD}$$

Expand

- couple only to  $\not{\xi}_n$  in path integral  $\int \mathcal{D} \not{\xi}_n$

$$\text{in}\cdot D = \text{in}\cdot \not{\partial} + g n \cdot A_n + g \bar{n} \cdot A_{\bar{n}}$$

multipole expansion

label comment

$$i \not{\partial}_\perp = i \not{\partial}_{n\perp} + g A_{n\perp} + \dots$$

$$A_{ns}^\perp \ll A_{n\perp}$$

$$i \bar{n} \cdot D = i \bar{n} \cdot \not{\partial}_{\bar{n}} + g \bar{n} \cdot A_{\bar{n}} + \dots$$

$$i \not{\partial}_{\bar{n}s}^\perp \ll i \not{\partial}_{\bar{n}}^\perp$$

$$g_{\bar{n}} A_{\bar{n}s} \ll \bar{n} \cdot A_{\bar{n}}$$

$$i \bar{n} \cdot \not{\partial}_{\bar{n}s} \ll i \bar{n} \cdot \not{\partial}_{\bar{n}}$$

$$\mathcal{L}_{nq}^{(0)} = \bar{\xi}_n \left( i n \cdot D + i \partial_{n\perp} \frac{1}{i \bar{n} \cdot D_n} i \partial_{n\perp} \right) \frac{i \not{k}}{2} \not{\xi}_n \sim \mathcal{O}(x^4) \quad -21-$$

$\int d^4x \mathcal{L}^{(0)} \sim 1$

gluons

$$\mathcal{L}_{nq}^{(0)} = \mathcal{L}_{nq}^{(0)} [n \cdot D, D_{n\perp}, \bar{n} \cdot D_n] \text{ too}$$

(+ gauge fixing & ghosts)

$\not{k}$  gives  $\frac{(k_\perp)^2}{n \cdot p + \frac{p_\perp^2}{\bar{n} \cdot p} + i \alpha \text{sign}(\bar{n} \cdot p)}$  ✓

$\not{k}$  bit more work  
for particle vs. antiparticle  
see EFTX

If we drop  $n \cdot A_{05}$  these are QCD Lagrangians

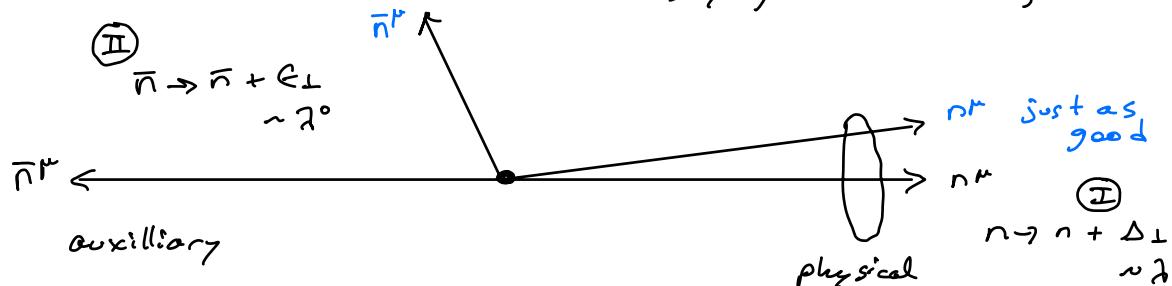
Higher Orders e.g.  $\mathcal{L}^{(1)} = (\bar{\xi}_n \omega_n) \frac{i \not{\partial}_{n\perp}^{\text{QCD}}}{2} \left( \omega_n^\dagger \frac{1}{i \bar{n} \cdot D_n} i \not{\partial}_{n\perp} \not{\xi}_n \right) + \text{h.c.} = 2^5$

e.g.  $\mathcal{L}^{(1)} = (\bar{\xi}_n \omega_n) \frac{i \not{\partial}_{n\perp}}{2} \frac{g_{05}}{2} g_{05} + \text{h.c.} = 2^5$

Gauge Inv ✓

Reparameterization Inv (RPI) freedom to choose  $n \neq \bar{n}$

satisfying  $n^2 = \bar{n}^2 = 0, n \cdot \bar{n} = 2$



III  $n \rightarrow K n$      $\bar{n} \rightarrow \frac{\bar{n}}{K} \Rightarrow$  Numerator (<# n's - # n̄'s>)  
= Denominator (<# n's - # n̄'s>)

Each collinear sector has its own RPI symmetry

[protects  $\mathcal{L}^{(k)}$  coeff from loop corrections, relates operator coeffs.]

$$\mathcal{L}_{SCET_I}^{(0)} = \mathcal{L}_{05}^{(0)} + \sum_n (\mathcal{L}_{nq}^{(0)} + \mathcal{L}_{ng}^{(0)}) + \mathcal{L}_G^{(0)}$$

↗ 1<sup>22</sup>  
↙ L3      Just full QCD  
             $q_{05}, A_{05}$

↗ sum over distinct RPI equivalence classes  
             $n_1 \cdot n_2 \gg \lambda^2$

$\not{k}$  only factorization violating term (more later)

## RG Evolution & Matching

UV renormalization  
in SCET  
[now]

compare renormalized QCD  
to " "  
SCET  
& extract  $C_S$  [later]

$e^+e^- \rightarrow 2 \text{ jets}$

$$\bar{\chi}_n \gamma^\mu \chi_{\bar{n}} = (\bar{\epsilon}_n w_n) \gamma^\mu (w_{\bar{n}}^+ \epsilon_{\bar{n}})$$

(use Feyn. Gauge, offshell IR regulator  $p^2, \bar{p}^2 \neq 0$ )

$$= \frac{\alpha_s C_F}{4\pi} \left[ -\frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln\left(\frac{(-p^2)(-\bar{p}^2)}{\mu^2 (-Q^2)}\right) + \dots \right] \quad \downarrow \text{finite terms}$$

$\int \frac{d^4 k \ n \cdot \bar{n}}{(n \cdot k + \frac{P^2}{Q})(\bar{n} \cdot k + \frac{\bar{P}^2}{Q}) k^2}$

$$= \frac{\alpha_s C_F}{4\pi} \left[ \frac{2}{\epsilon^2} + \frac{2}{\epsilon} - \frac{2}{\epsilon} \ln\left(\frac{(-p^2)}{\mu^2}\right) + \dots \right]$$

$\int d^4 k \left[ \frac{\bar{n} \cdot (k+p)}{\bar{n} \cdot k (k+p)^2 k^2} - \frac{\bar{n} \cdot p}{\bar{n} \cdot k (\bar{n} \cdot p n \cdot k + p^2) k^2} \right]$

naive collinear integrand      0-bin subtraction

**0-bin:** collinear modes in  $SCET_I$  have 0-bin subtractions  
from region  $k^2 \sim Q^2$  to avoid double counting  
IR region described by soft mode.  
[part of proper multipole expansion]

$$= \frac{\alpha_s C_F}{4\pi} \left[ \frac{2}{\epsilon^2} + \frac{2}{\epsilon} - \frac{2}{\epsilon} \ln\left(\frac{(-\bar{p}^2)}{\mu^2}\right) + \dots \right]$$

$$= - \frac{\alpha_s C_F}{4\pi} \left[ \frac{1}{\epsilon} + \dots \right]$$

in sum  $\frac{\ln(-\epsilon^2)}{\epsilon} \approx \frac{\ln(-\bar{\epsilon}^2)}{\epsilon}$  cancel [mixed UV\* IR]  
 [crossed out above]

-23-

$$\text{sum} = \frac{\alpha_s C_F}{4\pi} \left[ \frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{\mu^2}{-\Omega^2 - i0} + \frac{3}{\epsilon} + \dots \right]$$

$$C^{\text{bare}} = z_c C$$

$\overline{\text{MS}}$  counter term

$$(z_{c-1}) \otimes \left[ \dots \right] = -\frac{\alpha_s C_F}{4\pi} \left[ \frac{2}{\epsilon^2} + \frac{2}{\epsilon} \ln \frac{\mu^2}{-\Omega^2 - i0} + \frac{3}{\epsilon} \right]$$

$$0 = \mu \frac{d}{d\mu} C^{\text{bare}} = \mu \frac{d}{d\mu} [z_c(\mu, \epsilon) C(\mu)] \\ = [\mu \frac{d}{d\mu} z_c] C + z_c [\mu \frac{d}{d\mu} C]$$

$$\mu \frac{d}{d\mu} C(\mu) = \underbrace{[-z_c^{-1} \mu \frac{d}{d\mu} z_c]}_{\gamma_c} C(\mu)$$

$$\underline{\alpha_s} \quad z_c^{-1} \rightarrow 1 \quad \mu \frac{d}{d\mu} \alpha_s = -2\epsilon \alpha_s + \mathcal{O}(\beta_0 \alpha_s^2)$$

[recall  
 $\alpha_s^{\text{bare}} = \mu^{\epsilon} \alpha_s(\mu) / z_c$   
 implies this]

$$\mu \frac{d}{d\mu} z_c = \frac{C_F}{4\pi} \alpha_s (-2\epsilon) \left( -\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{\mu^2}{-\Omega^2} - \frac{3}{\epsilon} \right)$$

$$+ \frac{C_F \alpha_s}{4\pi} \left( -\frac{4}{\epsilon} \right) \quad \text{from } \mu \frac{d}{d\mu} \ln \mu^2 = 2$$

$$\gamma_c = -\frac{\alpha_s(\mu)}{4\pi} \left[ 4 C_F \ln \frac{\mu^2}{-\Omega^2} + 6 C_F \right] \quad \text{finite}$$

cusp anomalous dimension

when we square the amplitude we get  
 hard function  $H = |C(Q, \mu)|^2$

$\frac{\uparrow L^3}{\downarrow L^4}$

$$\mu \frac{d}{d\mu} H(Q, \mu) = (H + H^*) H = -\frac{\alpha_s(\mu)}{2\pi} \left[ 8 C_F \ln \frac{\mu}{Q} + 6 C_F \right] H(Q, \mu)$$



leading dble logs  
 $\alpha_s \ln \sim 1$

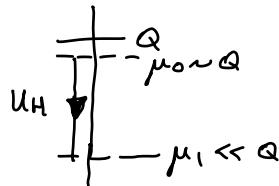
Part of NLL, -24-  
 also need  
 2-loop cusp  $\alpha_s^2 \ln \frac{\mu}{Q}$

$$H(Q, \mu) = H(Q, \mu_0) U_H(\theta, \mu_0, \mu_1)$$

boundary condition

$$= H(Q, \mu_0) \exp \left[ - \# \alpha_s \ln^2 \left( \frac{\mu_1}{Q} \right) + \dots \right] \quad \text{Frozen coupling result}$$

$$= H(Q, \mu_0) \exp \left[ - \frac{\#}{\alpha_s(\mu_0)} f \left( \frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)} \right) \right] \quad \text{Running coupling result}$$



Details on  
Hawking #3

Sudakov Form Factor

no emission until  $\mu_1$

Ask about scales

$\bar{x}_n \Gamma \chi_{\pi}$  SCET operator restricts radiation  
(collinear & soft emissions below  $\mu_1$ )

- To discuss the order we're working look at series in

$$\ln C(\omega, \mu) \sim \alpha_s^K \ln^{K+1} + \alpha_s^K \ln^K + \alpha_s^K \ln^{K-1} + \dots$$

**LL**                    **NLL**                    **NNLL**

- What do we need to compute?

	tree-level	1-loop	2-loop	3-loop
LL	matching	$\gamma_{\epsilon^2}$	-	-
NLL	matching	$\gamma_{\epsilon}$	$\gamma_{\epsilon^2}$	-
NNLL		matching	$\gamma_{\epsilon}$	$\gamma_{\epsilon^2}$

$\frac{1}{\epsilon^2} \rightarrow$  cusp.  
 anom.  
 dim.

Feyn. Rules

$$\langle \bar{n} \rangle = \frac{i\alpha}{2} \frac{\Delta(\bar{n} \cdot p)}{n \cdot p + \frac{p_\perp^2}{\bar{n} \cdot p} + i\alpha} + \frac{i\alpha}{2} \frac{\Delta(-\bar{n} \cdot p)}{n \cdot p + \frac{p_\perp^2}{\bar{n} \cdot p} - i\alpha} = \frac{i\alpha}{2} \frac{\bar{n} \cdot p}{p^2 + i\alpha}$$

particle                          anti-particle

$$\langle \bar{n} \rangle = \dots$$

$$\langle \bar{n} \rangle = \dots, \quad \langle \bar{n} \rangle = \dots, \quad \langle \bar{n} \rangle = \dots$$

$$\begin{aligned} \langle \bar{n}^\mu \rangle &= \langle \bar{n}^\mu \rangle, \\ \langle \bar{n}^\mu \rangle &= \langle \bar{n}^\mu \rangle, \quad [\text{Feyn. Gauge for collinear}] \\ \langle \bar{n}^\mu \rangle &\propto n^\mu \text{ too} \end{aligned}$$

Usofts have eikonal coupling  $\propto n^\mu$  to collinears

$$\begin{aligned} \langle \bar{n}^\mu \rangle &\propto \frac{\bar{n} \cdot p}{\bar{n} \cdot p \cdot n \cdot (p+k) + p_\perp^2 + i\alpha} = \frac{\bar{n} \cdot p}{\bar{n} \cdot p \cdot n \cdot k + p^2 + i\alpha} = \frac{1}{n \cdot k + i\alpha} \\ &\quad \text{on-shell } p^2=0 \quad \underline{\text{eikonal propagator}} \end{aligned}$$

[Usofts do not change  $p_n^+$ ,  $\bar{n} \cdot p_n$ , neither soft nor collinear can change direction  $n$  ]

$$\begin{aligned} \langle \bar{n}^\mu \rangle &\propto \frac{\bar{n} \cdot (p+q)}{(p+q)^2 + i\alpha} \quad \text{for collinears} \\ &\quad n \rightarrow q \quad n \rightarrow q \end{aligned}$$

## Ultrasoft - Collinear Factorization

put n·Aus into ultrasoft Wilson lines

$$\Upsilon_n(x) = \rho \exp \left( ig \int_{-\infty}^0 ds n \cdot \text{Aus}(x+s) \right)$$

$$[n \cdot \text{Aus } \Upsilon_n] = 0, \quad \Upsilon_n^+ \Upsilon_n = \mathbb{1} = \Upsilon_n \Upsilon_n^+$$

Field Redefinition:  $\xi'_n(x) = \Upsilon_n(x) \xi'_n(x)$

$$A_n^\mu(x) = \Upsilon_n(x) A_n'(x) \Upsilon_n^+(x) \quad \begin{array}{l} \text{some for} \\ \text{ghost } c_n \end{array}$$

$$w_n = \sum_{\text{perms}} \exp \left( \frac{-g}{i\pi \cdot 2n} \bar{n} \cdot A_n \right) \xrightarrow{\substack{\text{use} \\ \text{multipole} \\ \text{expn}}} \Upsilon_n w_n' \Upsilon_n^+$$

$$\text{Also: } \chi_n \rightarrow \Upsilon_n \chi'_n, \quad {}^0 B_{n\perp} \rightarrow \Upsilon_n {}^0 B_{n\perp}' \Upsilon_n^+$$

$$\begin{aligned} \mathcal{L}_{n\perp}^{(0)} &= \bar{\xi}'_n \frac{i\cancel{k}}{2} \left[ \Upsilon_n^+ i n \cdot \text{Aus} \Upsilon_n + \Upsilon_n^+ (g_n n \cdot A_n' \Upsilon_n^+) \Upsilon_n + \dots \right] \xi'_n \\ &= \bar{\xi}'_n \frac{i\cancel{k}}{2} \left[ i n \cdot \cancel{d} + g_n n \cdot A_n' + i \cancel{B}_{n\perp} \frac{1}{i\pi \cdot \bar{D}_n'} i \cancel{B}_{n\perp}' \right] \xi'_n \end{aligned}$$

$$\mathcal{L}_{n\perp}^{(0)}(\xi_n, A_n, n \cdot \text{Aus}) = \mathcal{L}_{n\perp}^{(0)}(\xi'_n, A_n', 0)$$

some for  $\mathcal{L}_{n\parallel}^{(0)}$ , so decoupled in  $\mathcal{L}^{(0)}$

Reappear in currents:

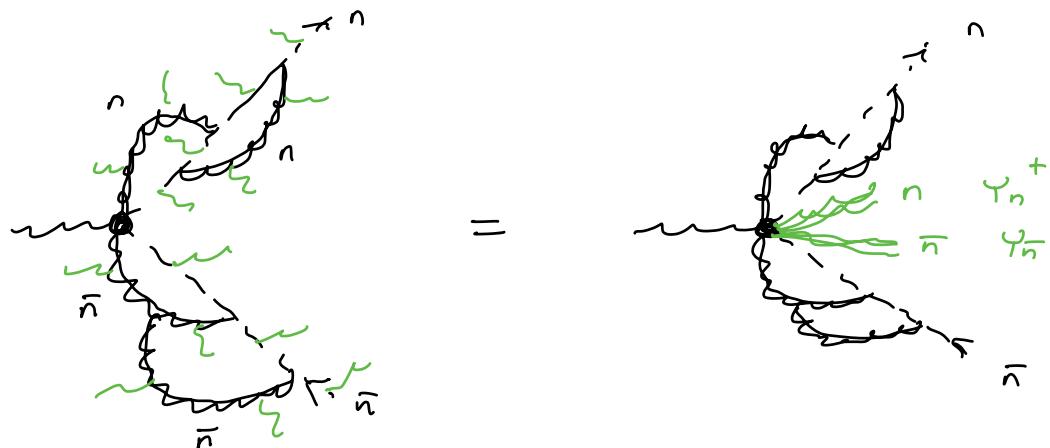
$$\text{eg 1 } (\bar{\chi}_n \cap \chi_{\bar{n}}) \rightarrow \bar{\chi}'_n (\Upsilon_n^+ \Upsilon_{\bar{n}}) \cap \chi'_{\bar{n}} \\ (\text{n-collin}) \text{ (ultrasoft) } (\bar{n}-\text{collin})$$

factorized up to global color & spin indices

$$\text{eg 2 } (\bar{\chi}_n \cap \chi_n) \rightarrow \bar{\chi}'_n (\cancel{\Upsilon_n^+ \Upsilon_n}) \cap \chi'_n \\ \text{cancel here}$$

Sums up  $\infty$  class of diagrams

- 27 -



### 1-loop Matching Example

$e^+e^- \rightarrow \text{dijets}$

[Feyn. Gauge again]

$$\begin{array}{c} \text{QCD} \\ \downarrow \\ \text{SCET} \end{array}$$

$$\mathcal{L}_{\text{QCD}} + \mathcal{J}^\mu = \bar{\psi} \gamma^\mu \psi$$

$$\mathcal{L}_{\text{SCET}}^{(1)} + \mathcal{L}_{\text{hard}}^{(1)} = C \bar{\chi}_n \gamma_\mu \chi_n$$

find  $C$  at  $\mathcal{O}(\alpha_s)$

$$(\text{1-loop ren. QCD}) - (\text{1-loop ren. SCET}) = C^{(\text{1-loop})} \langle \mathcal{O}_{\text{SCET}}^{(1)} \rangle$$

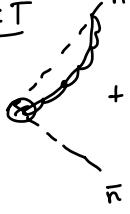
- Must use some IR regulator in QCD & SCET
- Result for  $C$  will be independent of IR reg. choice.

$$p^2 = \bar{p}^2 \neq 0$$

$$\overbrace{\text{Feyn.}}^{} = z_4 - 1 = z_3 - 1 = \overbrace{\text{Feyn.}}^{} = \frac{\alpha_s C_F}{4\pi} \left[ -\frac{1}{\epsilon} - \ln \frac{\mu^2}{-p^2} - 1 \right]$$

$$\overbrace{\text{Feyn.}}^{\text{QCD}} + \overbrace{\text{Feyn.}}^{\perp \epsilon_{\mu\nu}} = \frac{\alpha_s(\mu) C_F}{4\pi} \left[ -2 \ln^2 \left( \frac{p^2}{Q^2} \right) - 3 \ln \left( \frac{p^2}{Q^2} \right) - \frac{2\pi^2}{3} - 1 \right]$$

$\perp \epsilon_{\mu\nu}$        $\perp \epsilon_{\mu\nu}$  ← cancel since curr. current

S<sub>EFT</sub> +  +  +  + 

$$= \frac{\alpha_s(\mu) C_F}{4\pi} \left[ \underbrace{2 \ln^2\left(\frac{\mu^2}{-\rho^2}\right)}_{\text{collinear graphs}} + 3 \ln\left(\frac{\mu^2}{-\rho^2}\right) - \underbrace{\ln^2\left(\frac{\mu^2 Q^2}{-\rho^4}\right)}_{\text{soft graph}} + 7 - \frac{5\pi^2}{6} \right] \text{ both}$$

$$= \frac{\alpha_s(\mu) C_F}{4\pi} \left[ \underbrace{\ln^2\left(\frac{\mu^2}{-Q^2}\right)}_{\ln \rho^2 \text{ IR divergences agree}} - \underbrace{2 \ln^2\left(\frac{\rho^2}{Q^2}\right)}_{\ln \rho^2 \text{ IR divergences agree}} - 3 \ln\left(\frac{\rho^2}{Q^2}\right) + 3 \ln\left(\frac{\mu^2}{-Q^2}\right) + 7 - \frac{5\pi^2}{6} \right]$$

$$\text{QCD} - \text{S}_{\text{EFT}} = \frac{\alpha_s(\mu) C_F}{4\pi} \left[ - \ln^2\left(\frac{\mu^2}{-Q^2_{\text{cusp}}}\right) - 3 \ln^2\left(\frac{\mu^2}{-Q^2_{\text{cusp}}}\right) - 8 + \frac{\pi^2}{6} \right]$$

matching:

$$C(Q, \mu) = 1 + \frac{\alpha_s(\mu) C_F}{4\pi} \left[ \text{I} \quad \text{II} \quad \text{III} \quad \text{IV} \quad \text{V} \right]$$

Dim. Reg. Trick [useful for many EFT matching calculations]

use  $\gamma_{\text{cusp}}$  for IR divergences &  $\gamma_{\text{ew}}$  for UV

$$\text{---} = \overset{\text{QCD}}{\text{---}} \propto (\frac{1}{\epsilon_{\text{ew}}} - \gamma_{\text{cusp}}), \quad \text{---} + \text{---} + \text{---} \propto \frac{\gamma_{\text{ew}}^2 - \gamma_{\text{cusp}}^2}{\frac{1}{\epsilon_{\text{ew}}} - \gamma_{\text{cusp}}}$$

$$(z_c^{ms} - 1) \text{---} = \frac{\alpha_s C_F}{4\pi} \left[ -\frac{2}{\epsilon_{\text{ew}}^2} - \frac{2}{\epsilon_{\text{ew}}} \ln\left(\frac{\mu^2}{-Q^2}\right) - \frac{3}{\epsilon_{\text{ew}}} \right] \text{ as before}$$

$$\therefore \left[ \text{---} + \text{---} + \text{---} - \text{---} \right]_{\text{S}_{\text{EFT}}}^{\text{ren}} = \frac{\alpha_s C_F}{4\pi} \left[ -\frac{2}{\epsilon_{\text{cusp}}^2} - \frac{2}{\epsilon_{\text{cusp}}} \ln\left(\frac{\mu^2}{-Q^2}\right) - \frac{3}{\epsilon_{\text{cusp}}} \right]$$

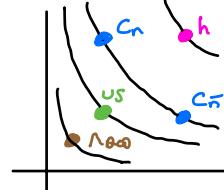
$$\text{QCD} \text{---} = \frac{\alpha_s C_F}{4\pi} \left[ -\frac{2}{\epsilon_{\text{cusp}}^2} - \frac{2}{\epsilon_{\text{cusp}}} \ln\left(\frac{\mu^2}{-Q^2}\right) - \frac{3}{\epsilon_{\text{cusp}}} - \frac{\ln^2\left(\frac{\mu^2}{-Q^2}\right)}{-Q^2} - 3 \ln\left(\frac{\mu^2}{-Q^2}\right) - 8 + \frac{\pi^2}{6} \right]$$

• set  $\epsilon_{\text{cusp}} = \epsilon_{\text{ew}}$  & assume  $\gamma_{\text{cusp}} = \gamma_{\text{ew}}$  - Don't need EFT calc.

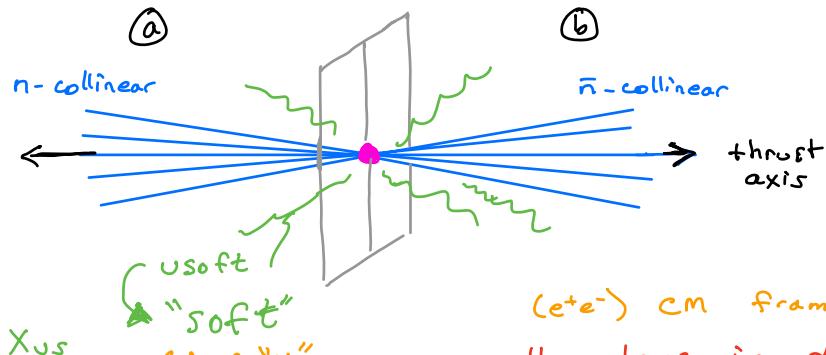
⇒ Result for  $C(Q, \mu)$  is IR finite part of pure dim. reg QCD result (agrees with earlier  $p^2 \neq 0$  result as expected)

$e^+e^- \rightarrow \text{dijets}$

SCET<sub>I</sub>



~~2G~~



$$e^+e^- \rightarrow \gamma^* \text{ or } Z^* \rightarrow X_n X_{\bar{n}} X_{us}$$

$q^{\mu}$

Scales

- hard scale  $\mu_h \sim Q = \sqrt{q^2}$  good up to  $\mathcal{O}(\lambda^2)$

- $P_x^{\mu} = P_{x_a}^{\mu} + P_{x_b}^{\mu}$  Hemisphere invariant mass

$$M_a^2 \equiv (P_{x_a}^{\mu})^2 = \left( \sum_{i \in a} P_i^{\mu} \right)^2 \ll Q^2$$

$$M_b^2 \equiv \left( \sum_{i \in b} P_i^{\mu} \right)^2 \ll Q^2$$

Jets (jet masses)

$$\text{let } M_J^2 = M_a^2 + M_b^2 \\ M_a \sim M_b \sim M_J$$

$n$ -collinear

$$Q(\lambda^2, 1/\lambda)$$

$$\lambda \sim \frac{M_J}{Q}$$

$$\mu_J \sim M_J$$

$\bar{n}$ -collinear

$$Q(1, \lambda^2, \lambda)$$

- soft radiation

- uniform - eikonal - jets communicate

$$M_J^2/Q > \Lambda_{QCD}$$

soft perturbative

$$\mu_h > \mu_J > \mu_s > \Lambda_{QCD}$$

"tail" region

$$\text{Perturbative} + \mathcal{O}\left(\frac{\Lambda_{QCD}}{\mu_s}\right)$$

constant parameter

$$M_J^2/Q \sim \Lambda_{QCD}$$

non-perturbative soft fn.

$$\mu_h > \mu_J > \mu_s \sim \Lambda_{QCD}$$

"peak" region

$$\text{Perturbative} \times \left( \frac{\Lambda_{QCD}}{\mu_s} \right)^k \sim 1$$

any K

$\Lambda_{QCD}$  function

Current  $J^\mu = \bar{\psi} \Gamma^\mu \psi \rightarrow \int d\omega d\bar{\omega} C(\omega\bar{\omega}) (\bar{\xi}_n \omega_n)_\omega \Gamma^\mu (\gamma_n^\dagger \gamma_{\bar{n}}) (\omega_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}}$

skipped

$$\text{Kinematics } q^\mu = P_{x_n}^\mu + P_{x_{\bar{n}}}^\mu + P_{x_s}^\mu = Q \left( \frac{n^\mu + \bar{n}^\mu}{2} \right)$$

$$\bar{n} \cdot q = Q = \bar{n} \cdot P_{x_n} + \dots$$

$$\omega = Q$$

$$n \cdot q = Q = n \cdot P_{x_n} + \dots$$

$$\bar{\omega} = Q$$

$\Gamma$  small

after field redefinition

momentum conservation  
strong enough that no convolutions in  $\omega, \bar{\omega}$

### Factorize the Cross-Section

$$\text{QCD} \quad \sigma = \sum_{\substack{x \\ \text{dijet}}} (2\pi)^4 \delta^4(q - p_x) L_{\mu\nu} \langle \sigma | J^\mu(o) | x \rangle \langle x | J^\nu(o) | 0 \rangle$$

$\hookrightarrow e^+e^- \rightarrow \gamma^*$   
 & restrict to dijet  $x$  states. SCET allows us to move restrictions into operators

$$\bullet |x\rangle = |x_n\rangle |x_{\bar{n}}\rangle |x_{us}\rangle$$

$$\mathcal{L} = \mathcal{L}_n + \mathcal{L}_{\bar{n}} + \mathcal{L}_{us}$$

so Hilbert Space factorizes

$$\sigma = N_0 \sum_n \sum_{x_n, x_{\bar{n}}, x_{us}} (2\pi)^4 \delta^4(q - p_{x_n} - p_{x_{\bar{n}}} - p_{x_{us}}) \langle \sigma | \gamma_n^+ \gamma_{\bar{n}} | x_{us} \rangle \langle x_{us} | \gamma_{\bar{n}}^+ \gamma_n | 0 \rangle$$

$$* |\mathcal{C}(o)|^2 \langle \sigma | \not{x_n} | x_n \rangle \langle x_n | \not{\bar{x}_n} | 0 \rangle$$

$$* \langle \sigma | \not{x_{\bar{n},a}} | x_{\bar{n}} \rangle \langle x_{\bar{n}} | \not{x_{\bar{n}}} | 0 \rangle$$

insert measurement we want \*  $\int dM_a^2 dM_b^2 \delta(M_a^2 - (p_{x_n} + p_{x_{us}}^a)^2) \delta(M_b^2 - (p_{x_{\bar{n}}} + p_{x_{us}}^b)^2)$   $\approx 1$

### Factorize Measurement , Simplify, --

$$\frac{d\sigma}{dM_a^2 dM_b^2} = \sigma_0 |\mathcal{C}(o)|^2 \int dk^+ d\ell^+ dk^- d\ell^- \delta(M_a^2 - Q(k^+ + \ell^+)) \delta(M_b^2 - Q(\ell^- + \ell^-))$$

$$* \sum_{x_n} \frac{1}{2\pi} \int d^4x e^{ik^+ x^+ / 2} + \langle \sigma | \not{x_n} | x_n \rangle \langle x_n | \not{\bar{x}_n} | 0 \rangle$$

$$* \sum_{x_{\bar{n}}} \frac{1}{2\pi} \int d^4y e^{i k^- y^+ / 2} + \langle \sigma | \not{x_{\bar{n},a}} | x_{\bar{n}} \rangle \langle x_{\bar{n}} | \not{\bar{x}_{\bar{n}}} | 0 \rangle$$

$$* \sum_{x_{us}} \frac{1}{N_c} \delta(\ell^+ - p_{x_{us}}^{a+}) \delta(\ell^- - p_{x_{us}}^{b-}) + \langle \sigma | \gamma_n^+ \gamma_{\bar{n}} | x_{us} \rangle \langle x_{us} | \gamma_{\bar{n}}^+ \gamma_n | 0 \rangle$$

$$= \sigma_0 H(Q, \mu) \int d\ell^+ d\ell^- J(M_a^2 - Q\ell^+, \mu) J(M_b^2 - Q\ell^-, \mu) S(\ell^+, \ell^-, \mu)$$

dijet factorization theorem for hemisphere masses

- soft function encodes both  $\ell^\pm \sim M_J^2/Q$  and  $\ell^\pm \sim \Lambda_{\text{QCD}}$

$$\bullet \frac{d\sigma}{dM_J^2} = \int dM_a^2 dM_b^2 \delta(M_J^2 - M_a^2 - M_b^2) \frac{d\sigma}{dM_a^2 dM_b^2}$$

1-variable = simpler

$\tau = M_J^2/Q^2 = 1 - \text{Thrust}$

$$= \sigma_0 H(Q, \mu) \int d\ell J_\tau(M_J^2 - Q\ell, \mu) S_\tau(\ell, \mu)$$

for  $\tau \ll 1$

Bare  $\rightarrow$  Renormalized

$$H(Q) = z_H H(q, \mu)$$

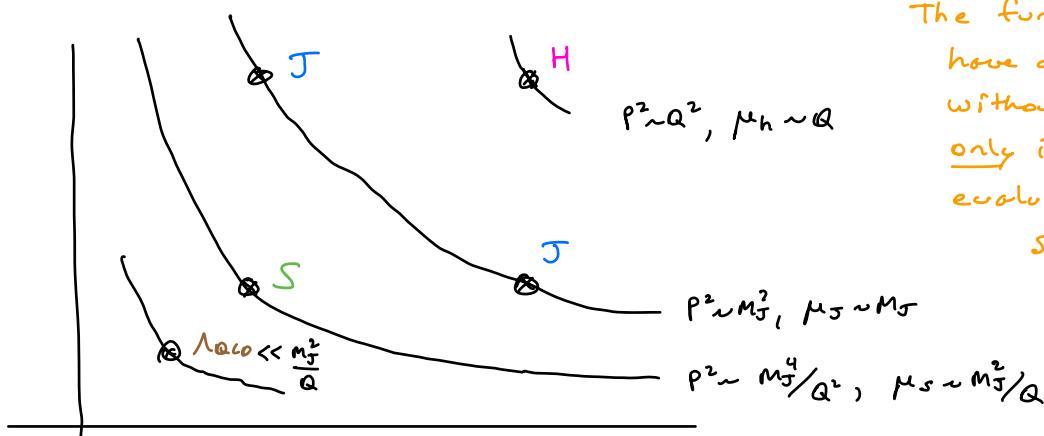
$$J_r(M^2) = z_J \otimes J_r(M'^2, \mu^2)$$

$$S_r(l') = z_S \otimes S_r(l, \mu)$$

$\infty$  integrals, like for PDF example

$$H(Q, \epsilon) \int dl' J_r(M^2 - Ql', \epsilon) S_r(l', \epsilon) \quad \leftarrow \text{bare}$$

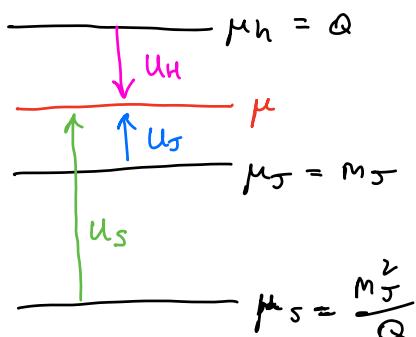
$$= H(Q, \mu) \int dl J_r(M^2 - Ql, \mu) S_r(l, \mu) \quad \leftarrow \text{renormalized}$$



The functions  $H, J, S$   
have ds expansions  
without large logs  
only if each is  
evaluated at different  
scale  $\mu$ !

RGE Coefficient =  $\begin{pmatrix} \text{Operator} \\ \text{Ren.} \end{pmatrix}^{-1}$  "consistency conditions"

$\rightarrow$  relates  $\gamma_{H, J, S}$  anomalous dimensions



- large logs all in evolution factors  $U_H, U_J, U_S$
- can pick any  $\mu$

$\star$  Sudakov Form Factor  $\otimes = \text{integral}$

$$\frac{d\sigma}{ds} = H(Q, \mu_h) U_H(Q, \mu_h, \mu) J_r(M_J^2 - s', \mu_J) \otimes^S U_J(s' - Ql, \mu_J, \mu) \otimes^L S_r(l - l', \mu_S) \otimes^L U_S(l', \mu_S, \mu)$$

pick  $\mu = \mu_S$ :  $U_S(l', \mu_S, \mu_S) = \delta(l')$   $\rightarrow$  only need  $U_J, U_H$  ✓

$$\mathcal{J}^{\text{bare}}(s) = \int ds' \gamma_J(s-s') \mathcal{J}(s', \mu)$$

-32-  
invariant mass evolution

$$\mu \frac{d}{d\mu} \mathcal{J}(s, \mu) = \int ds' \gamma_J(s-s', \mu) \mathcal{J}(s', \mu)$$

$$\gamma_J(s, \mu) = -2 \Gamma^{\text{susp}}[\alpha_s] \frac{1}{\mu^2} \left[ \frac{\mu^2 \alpha(s)}{s} \right]_+ + \gamma[\alpha_s] \delta(s) \quad \begin{matrix} \text{all orders} \\ \text{in } \alpha_s \end{matrix}$$

Solve by Fourier transform  $y = y - i\alpha$

$$\mathcal{J}(y) = \int ds e^{-isy} \mathcal{J}(s) \quad \mu \frac{d}{d\mu} \mathcal{J}(y, \mu) = \gamma_J(y, \mu) \mathcal{J}(y, \mu)$$

simple

skipped but  
review for

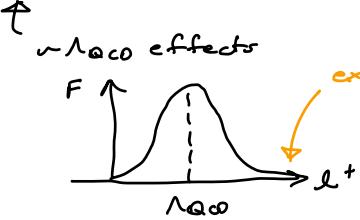
Hmwk #4 Calculate  $\mathcal{J}(s, \mu)$  at 1-loop

Soft Fn OPE

$$S(\ell, \mu) = \int \ell' \hat{S}(\ell - \ell', \mu) F(\ell')$$

↑  
power law terms

$$\sim \frac{(g_F \ell/\mu)^K}{\ell}$$



tail  $\frac{\Lambda_{QCD}}{\ell} \ll 1$  expansion:

$$S(\ell) = \hat{S}(\ell) \underbrace{1}_{\sim} - \left[ \frac{\partial}{\partial \ell} \hat{S}(\ell) \right] \underbrace{\ell_1}_{\sim} + \dots = \int_0^\infty dk k F(k)$$

Wilson Coeff. for power corr.

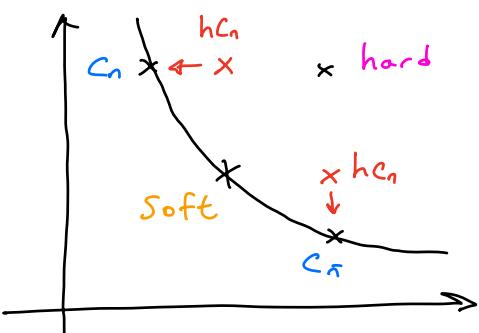
$$\ell_1 = \langle 0 | \bar{Y}_n^+ Y_n^+ \hat{E}_T Y_n \bar{Y}_n | 0 \rangle \sim \Lambda_{QCD}$$

hadronization parameter,  
universal across dijet  
event shapes

$\uparrow L4$   
 $\downarrow L5$

Recap Fact  $e^+e^- \rightarrow \text{dijet}$  & how modes communicate

SCET<sub>II</sub>



$$q = p_1 + p_5 \sim Q(2, 1, 2)$$

$$q^2 = Q^2 2 \gg Q^2 2^2 \sim p_1^2 !$$

offshell

$q \sim Q(2, 1, \sqrt{2})$  on-shell scaling  
hard-collinear mode

Constructing  $SCET_{\text{II}}$  operators using  $SCET_{\text{I}}$ :

- 1) Match QCD  $\rightarrow SCET_{\text{I}}$  ( $h c_n, h \bar{c}_{\bar{n}}, \text{soft}$ )
- 2) Factorize (field redefinition)
- 3) Match  $SCET_{\text{I}}$   $\rightarrow SCET_{\text{II}}$  ( $c_n, c_{\bar{n}}, \text{soft}$ )

e.g.  $e^+ e^- \rightarrow \text{dijet } p_{\perp}$

$$\mathcal{J}_{SCET_{\text{I}}} = \bar{\chi}_n^{hc} (\gamma_n^\perp \gamma_{\bar{n}}) \Gamma \chi_{\bar{n}}^{hc}$$

$$\downarrow$$

$$\mathcal{J}_{SCET_{\text{II}}} = \bar{\chi}_n (S_n^\perp S_{\bar{n}}) \Gamma \chi_{\bar{n}}$$

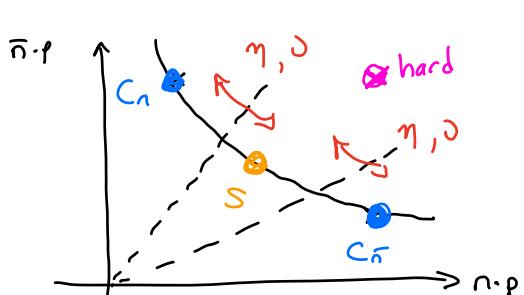
soft Wilson lines

- can also be obtained by matching QCD  $\rightarrow SCET_{\text{II}}$ , but more work
- with  $\geq 2$   $SCET_{\text{I}}$  operators having usoft & collinear fields, can get  $\int d\mathbf{r} \cdot dk^+ J(\mathbf{r}, k^+) C_n(\mathbf{r}) S(k^+)$ ,  $J = h.c.$  matching,

$$\mathcal{L}_{SCET_{\text{II}}}^{(0)} = \mathcal{L}_{\text{soft}}^{(0)} + \sum_n (\mathcal{L}_{c_n}^{(0)} + \mathcal{L}_{\bar{c}_n}^{(0)}) + \mathcal{L}_G^{(0)}$$

already decoupled

$SCET_{\text{I}} = SCET_{\text{II}}$  same



- $SCET_{\text{II}}$  also has 0-bin subtr.
- modes distinguished by rapidity

$$e^{2y} = \frac{p^-}{p^+} \sim \lambda^{-2}, \lambda^0, \lambda^2$$

$C_n \quad S \quad C_{\bar{n}}$

- And can have "rapidity divergences" not regulated by  $\epsilon$

$\epsilon \leftrightarrow$  regulates  
 $k^2$  offshellness

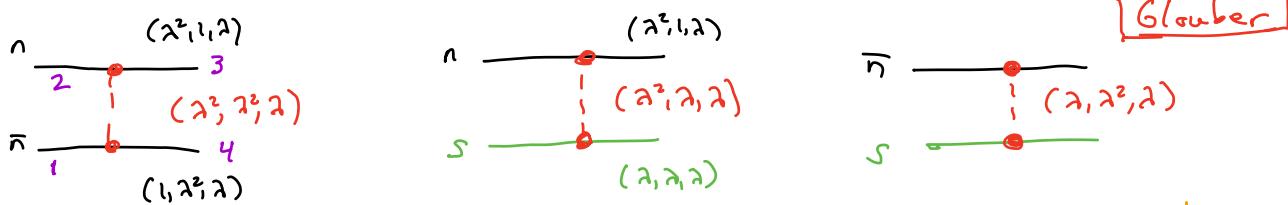
$$\int \frac{dk^+}{k^+} R(k, \eta, \zeta)$$

$\uparrow \quad \uparrow$  scale like  
reg- like  $\epsilon \quad \mu$

$k^+ = \frac{k_\perp^2}{k^+}$ , along hyperbola

## Glauber Exchange $\mathcal{L}_G^{(0)}$ see arXiv: 1601.04695 -34-

- modes with  $p^+ p^- \ll \vec{P}_\perp^2 \sim \lambda^2$  offshell
- history: CSS '88 cancel in DY, also etc.  $\rightarrow$  dijets, ...
- needed? (not seen in standard matching colors) add it
- mediates Forward scattering  $s \gg -t$   
small- $x$  phenomena (Regge, BFKL, ...)



- $\frac{1}{\vec{P}_\perp^2}$  potentials, instantaneous in  $z \neq t$  Coulomb
- Forward:  $\bar{n} \cdot p_2 = \bar{n} \cdot p_3$ ,  $n \cdot p_1 = n \cdot p_4$

Match from QCD, integrating Glauber out:

$$\mathcal{L}_G^{(0)} = \sum_n \sum_{i,j=8,9} O_n^{iB} \frac{1}{P_\perp^2} O_s^{j_n B} + \sum_{n,n'} \sum_{i,j=8,9} O_n^{iB} \frac{1}{P_\perp^2} O_s^{B_n C} \frac{1}{P_\perp^2} O_{n'}^{j_n C}$$

(2-rapidities) (3-rapidities)

$$O_n^{iB} = \bar{\chi}_n T^B \frac{\not{x}}{2} \chi_n, \quad O_n^{j_n B} = \frac{i}{2} f^{BCD} \not{B}_{n\perp\mu} \frac{\not{n}}{2} \cdot (\not{\epsilon}_n - \not{\epsilon}_{n'}) \not{B}_{n'\perp}^D$$

similar  $O_{\bar{n}}$ 's

$$O_s^{j_n B} = 8\pi \alpha_S \bar{\psi}_S^n T^B \frac{\not{x}}{2} \psi_S^n, \quad O_s^{B_n C} = 8\pi \alpha_S \frac{i}{2} f^{BCD} \not{B}_{S\perp\mu} \frac{\not{n}}{2} \cdot (\not{\epsilon}_S - \not{\epsilon}_{S'}) \not{B}_{S'\perp}^D$$

$$O_s^{B_n C} = 8\pi \alpha_S \left\{ P_\perp^\mu S_n^T S_{\bar{n}} P_{\perp\mu} - P_{\perp\mu} \not{g} \widetilde{B}_{S\perp}^{\mu\nu} S_n^T S_{\bar{n}} - S_n^T S_{\bar{n}} \not{g} \widetilde{B}_{S\perp}^{\mu\nu} P_{\perp\mu} \right. \\ \left. - \not{g} \widetilde{B}_{S\perp}^{\mu\nu} S_n^T S_{\bar{n}} \not{B}_{S\perp\mu}^{\bar{\nu}} - \frac{\alpha_S \bar{n}_\mu}{2} S_n^T i \not{g} \widetilde{G}_S^{\mu\nu} S_{\bar{n}} \right\}^{BC}$$

Here

$$\psi_S^n = S_n^+ q_S \rightarrow \not{B}_{S\perp}^{\mu\nu} = \frac{1}{2} [S_n^+ \{ D_{S\perp}^\mu S_n \}]$$

Determined by top-down  
matching & bottom-up basis

tildes:  $\widetilde{B}_{S\perp}^{\mu\nu AB} = -i f^{ABC} \not{B}_{S\perp}^{AC}$



$S_n$  adjoint wilson line

2 gluons enough to fix  
all terms

- suppressed: rapidity regulation  $|k_\perp|^{-n}$ , multipole expansion.
- 0-bin subtractions

Note

- construction involves using SCET p.c. theorem
- universal for  $i, j = g, g$
- no hard coefficient (loop corrections) for  $\mathcal{L}_G^{(0)}$
- 1 or 2 collinear directions in  $\mathcal{L}_G^{(0)}$   
others are T-products
- breaks factorization  $\mathcal{L}_G^{(0)}(\{\mathbf{q}_{ni}, \mathbf{A}_{ni}\}, q_S, A_S)$   
couples  $n, \bar{n}$  modes at  $O(\beta^0)$
- encodes known examples of fact. violation  
(Wilson line directions,  $i\pi'$ 's, ...)
- SCET vs. CSS Glauber

SCET: expand first, defined as contribution that can be independently calculated

CSS: deform contour, see where we are trapped  
make soft expn once out of trapped region

- one-gluon Feyn. Rule of  $\mathcal{O}_S^{AB}$  is Lipatov Vertex
- rapidity RGE for  $\mathcal{L}_G^{(0)}$  (Amplitude level) or   
gives amplitude level  $\left(\frac{Q^2}{Q_0^2}\right)^{-\gamma_{n0}} = \left(\frac{s}{-t}\right)^{-\gamma_{n0}}$   $\gamma_{n0} = \gamma_{\bar{n}0} = -\frac{\gamma_{S0}}{2}$   
"gluon reggeization" sums logs  $(ds \ln \frac{s}{-t})^k$

rapidity RGE for  $\mathcal{O}_{\text{forward}}$

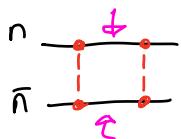
$$\sqrt{\frac{2}{20}} S(q_\perp, q'_\perp, v) = \int d^2 k_\perp \gamma^{BFKL}(q_\perp, k_\perp) S(k_\perp, q'_\perp, v)$$

useful for small- $x$  resummation

## - Glauber Loops give $i\pi$

rapidity regulator  
& scale

-36-



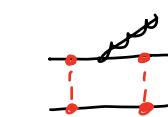
$$\int \frac{d^d k}{k_\perp^2} |2k^z|^{-\eta} e^{2ik} \\ \frac{1}{(k_\perp^2 - \bar{k}_\perp^2)^2 (k^+ - \Delta_1(k_\perp) + i\alpha)(-k^- - \Delta_2(k_\perp) + i\alpha)}$$

$$= \left(\frac{-i}{4\pi}\right) \int \frac{d^{d-2} k_\perp}{k_\perp^2 (k_\perp^2 - \bar{k}_\perp^2)^2} [-i\pi + O(\alpha)]$$

effectively eikonal

$$= 0 \quad \text{with regulator}$$

- Pure unitarity result
- $\Delta$ 's do not matter here



$$= 0$$

can't collapse to equal  $t \neq z$  !

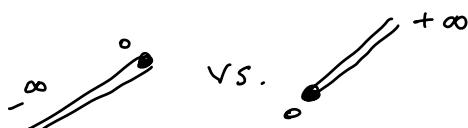
collapse  $\leftrightarrow$  shockwave picture  
for high energy scattering



$$\neq 0$$

not eikonal

effectively eikonal



## - Wilson Line Directions

$$\text{in } \omega_n(\pm\infty) \quad \frac{1}{n \cdot k \pm i\alpha}$$

sign matters for  $\delta(n \cdot k)$

not collinear

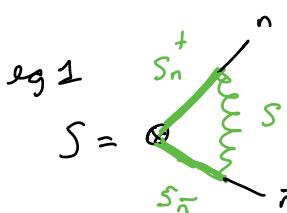
$$\text{in } S_n(\pm\infty) \quad \frac{1}{n \cdot k \pm i\alpha}$$

.. ..

$$\delta(n \cdot k)$$

not soft

- actually Glauber

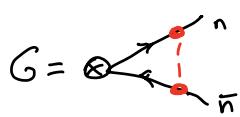


$$\text{naive } \tilde{S} = \int \frac{d^d k}{(k^2 - m^2_{\text{soft}})(n \cdot k + i\alpha)(\bar{n} \cdot k + i\alpha)}$$

"active  
- active"

$$= (\dots) + i\pi \left( \frac{1}{G} + \ln \frac{\mu^2}{m^2} \right)$$

$$\text{true } S = \tilde{S} - S^{(G)} = (\dots) \text{ only (0-bin subt.)}$$

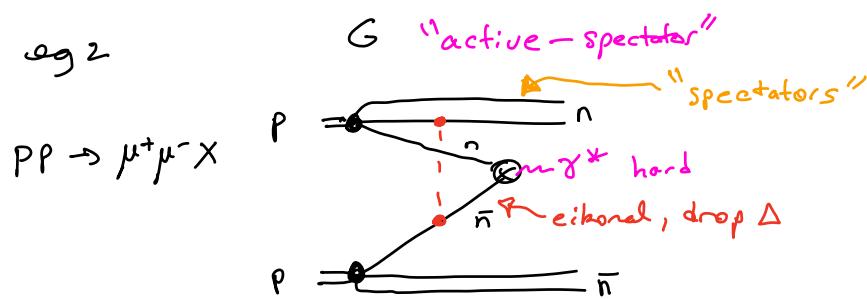


$$G = S^{(G)} \text{ here, pure } i\pi$$

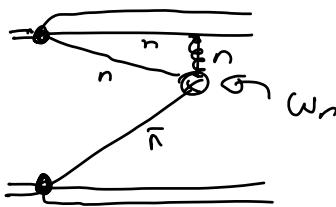
$$S + G = \tilde{S}$$

- $G$  carries info about soft Wilson line directions
- can absorb  $G$  into soft if we take proper directions for  $S_n$  lines

eg 2



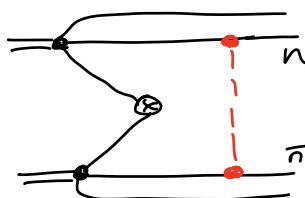
$$C_n = \tilde{C}_n - C_n^{(G)} \quad -37-$$



- direction dependence in  $G$  not  $C_n$ ,  $C_n^{(G)} = G$
  - can absorb  $G$  into  $C_n$ , but then  $W_n$  direction is fixed
- $\Rightarrow$  Physical Manifestation in TMD PDFs

Sivers:  $(f_{1T}^\perp)^{\text{Sivers}} = - (f_{1T}^\perp)^{\text{DY}}$  T-pol. proton, unpol. quark  
 Boer-Mulders:  $(h_1^\perp)^{\text{Sivers}} = - (h_1^\perp)^{\text{DY}}$  T-pol. quark, unpol. proton

eg 3



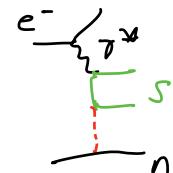
"Spectator-Spectator"

no soft or collinear analogs  
at leading power

cancel for  $|A|^2$  with  $\int d\Delta p_\perp^{\text{spectators}}$

[Also cancel integr. of  $\int d\Delta p_\perp$  for final states  $\rightarrow$  gen. to arb. graphs]

- $\mathcal{L}_G^{(0)}$  cancels in inclusive DIS (just  $n$ -collinear)
- $e^+e^- \rightarrow$  dijets  ~~$\mathcal{L}_G^{(0)}$~~  due to collapse role & final state active-active cancellation
- $\mathcal{L}_G^{(0)}$  cancels in Drell-Yan due to combination of Unitarity  $\sum x <x| = 1$  (final state cancellation), simplicity of measurement, ...
- $\mathcal{L}_G^{(0)}$  important for small- $x$  resummation, forward scattering, diffractive scattering
- $\mathcal{L}_G^{(0)}$  can be used to study fact. violation



## Higgs $P_\perp$ -distribution from Gluon Fusion

SCET<sub>II</sub>

Physical reg.  
in SCET<sub>II</sub>, also J-RGE

$$pp \rightarrow H(p_\perp) + X$$

$$Q \sim M_H \rightarrow p_\perp \ll M_H$$

$$O_\perp = \vec{P}_\perp^H + \vec{P}_\perp^X$$

$$\vec{P}_\perp^H = -\vec{P}_{n\perp} - \vec{P}_{\bar{n}\perp} - \vec{P}_{S\perp}$$

Fourier  
transform  $\rightarrow \vec{b}_\perp$

$$p_\perp \sim \frac{1}{b_\perp} \sim M_H \lambda$$

$$p_n \sim M_H (2^2 l, \lambda)$$

$$n \curvearrowleft \curvearrowright \overset{s}{\curvearrowright} p_S^\mu \sim M_H \lambda$$

$$p_{\bar{n}} \sim M_H (l, 2^2 \lambda)$$

$$\bar{n} \curvearrowleft \curvearrowright \bar{n}$$

Soft radiation

SCET<sub>II</sub>

$$\frac{d\sigma}{dy} = N_0 H_{gg}^{(M_H, \mu)} \int d^2 b_\perp e^{i \vec{b}_\perp \cdot \vec{q}_\perp}$$

$\leftarrow$  wants  $\mu \sim b_\perp^{-1}, \lambda \sim M_H$

$\leftarrow B_{g/p}^{(\mu)} \left( \frac{M_H}{\sqrt{s}} e^y, \vec{b}_\perp, \mu, \frac{\nu}{M_H e^y} \right)$

$\leftarrow \begin{cases} \bullet B_{g/p} \sim \langle p_n | B_{n\perp}^{A\mu}(b_\perp, \vec{b}_\perp) W_r B_{n\perp}^{AJ}(0) | p_n \rangle \\ \bullet \text{has 0-bin subtraction} \end{cases}$

$\leftarrow B_{g/p}^{(\mu)} \left( \frac{M_H}{\sqrt{s}} e^y, \vec{b}_\perp, \mu, \frac{\nu}{M_H e^y} \right)$

$\leftarrow \tilde{f}_{g/p}^{\text{naive}} = \frac{f_{g/p}}{S^{\text{0-bin}}}$

$\leftarrow S(b_\perp, \mu, \gamma_\mu) \leftarrow$  wants  $\omega \mu \sim b_\perp^{-1}$

$\leftarrow \langle 0 | (\gamma_n \gamma_\perp^+) (b_\perp) \gamma_r (\gamma_{\bar{n}} \gamma_\perp^+) (0) | 0 \rangle$  "Soft Function"

Note:  $X_\alpha = \frac{\bar{n} \cdot \vec{p}_n}{\bar{n} \cdot \vec{p}_p} = \frac{\bar{n} \cdot \vec{q}}{\bar{n} \cdot \vec{p}_p} = \frac{M_H e^{-y}}{\sqrt{s}}$

Here:  $B_{g/p}^{(\mu)}(x, \vec{b}_\perp, \mu, \frac{\nu}{Q})$  is "beam function"

- For  $p_T \gg \Lambda_{QCD}$  it contains parton distribution  $f_{i/p}(\xi, \mu)$  & perturbative collinear radiation at scale  $\mu \sim p_T$

$$B_{g/p}^{(\mu)}(x, \vec{b}_\perp, \mu, \frac{\nu}{Q}) = \sum_i \int \frac{d\xi}{\xi} C_{gi}^{(\mu)} \left( \frac{x}{\xi}, \vec{b}_\perp, \mu, \frac{\nu}{Q} \right) f_{i/p}(\xi, \mu)$$

$$+ \mathcal{O}(\Lambda_{QCD}^2 b_\perp^2)$$

$$b_\perp \sim \frac{1}{p_\perp}$$

- For  $p_T \sim \Lambda_{QCD}$  the  $Bg_\rho$  &  $S$  become non-perturbative.

Often define single "Transverse Momentum Distribution" (TMD)

$$f_{TMD}^{\mu\nu}(x, \vec{b}_\perp, \mu, Q) = \tilde{B}_{g/\rho}^{\mu\nu}(x, \vec{b}_\perp, \mu, \frac{Q}{\mu}) \overbrace{SS(\vec{b}_\perp, \mu, \frac{Q}{\mu})}^{S}$$

$$\frac{d\sigma}{dp_T dy} = N_0 H(M_H, \mu) \int d^2 b_\perp e^{i \vec{p}_T \cdot \vec{b}_\perp} f_{TMD}^{\mu\nu}(x_e, \vec{b}_\perp, \mu, M_H e^{-y}) f_{\mu\nu}(x_b, \vec{b}_\perp, \mu, M_H e^y)$$

### Rapidity Divergences

Sometimes (but not always) we may have rapidity divergences from our separation of modes.

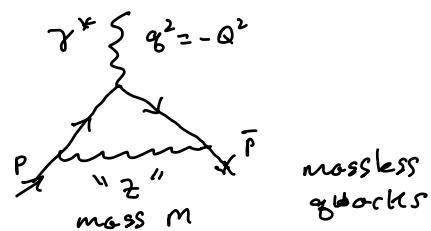
Simple Example: Massive Sudakov Form Factor

**[QCD]**  $J^\mu = \bar{q} \gamma^\mu q$      $Q^2 \gg m^2$ ,     $\lambda = \frac{m}{Q}$

$$\langle g(\vec{p}) | J^\mu | g(p) \rangle = F(Q^2, m^2) \bar{u} \gamma^\mu u$$

$z$  can be:

(Full) theory



$$\begin{aligned} c_n &= Q(z^2, 1, \lambda) \\ c_{\bar{n}} &= Q(1, \bar{z}^2, \lambda) \\ S &= Q(z, \bar{z}, \lambda) \end{aligned}$$

$$\text{e.g. } \int \frac{d^4 k}{(k^+ - m^2)(k^- + k^+ p^-)(k^- + k^- p^+)} \quad p^- = \bar{p}^+ = Q$$

$$J_{SCET_{II}}^\mu = C(Q) (\bar{q}_n w_n) (S_{\bar{n}}^+ S_n^-) \gamma^\mu (w_n^+ \bar{q}_n)$$



$$\hookrightarrow \text{expect } F(Q^2, m^2) = C J_{\bar{n}} J_n S$$

Add regulator to Wilson lines

$$S_n = \sum_{\text{perm}} \exp \left( - \frac{g}{c_n \cdot \partial_n} \frac{w \circ \omega}{|z \cdot \partial_z|^{n/2}} n \cdot A_S \right)$$

$$|z \cdot \partial_z| = |\bar{n} \cdot \partial_n|$$

$$W_n = \sum_{\text{perm}} \exp \left( - \frac{g}{i \bar{n} \cdot \partial_n} \frac{w^2 \omega^n}{|\bar{n} \cdot \partial_n|^n} \bar{n} \cdot A_n \right)$$

up to power corr for  $W_n$

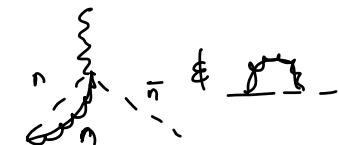
Dim. reg like rapidity regulator  $\frac{1}{n}$  like  $\frac{1}{\epsilon}$  -40-  
 div  $\ln \omega$   $\ln \mu$

$$\text{where } \omega(n, \omega) \propto n^{1/2}, \quad \frac{\partial}{\partial \omega} \omega(n, \omega) = -\frac{n}{2} \omega(n, \omega) \quad \omega(0, \omega) \equiv 1$$

$\omega$  is book keeping parameter >

- Renormalize by  $1^{\text{st}}$   $n \rightarrow 0$ , add  $\frac{f(\epsilon)}{n}$  counterterm,  
 then  $\epsilon \rightarrow 0$  &  $\frac{1}{\epsilon}$  counterterms

most IR div. integral (scalar) integral is



eg.  $\int d^4 k \frac{p^-}{(k^2 - m^2)(k^+ + p^-) k^-} \frac{\omega^2 \omega^n}{|k^-|^n}$

$$C_n^{\text{full}} = \frac{ds G_F \omega^2}{\pi} \left[ \underbrace{\frac{e^{\epsilon \gamma_E} \Gamma(\epsilon) (\mu/m)^{2\epsilon}}{2^n}}_{\equiv (z_n - 1)} + \frac{1}{2\epsilon} \ln \frac{\omega}{p^-} + \frac{3}{8\epsilon} + \ln \left( \frac{\mu}{m} \right) + \ln \left( \frac{\omega}{p^-} \right) \ln \left( \frac{\mu}{m} \right) + \text{constant} \right]$$

$\ln \frac{\omega}{p^-}$

same  $\leftrightarrow -$

$$C_n(m, \mu, \omega) = z_m^{-1} z_n^{-1} C_n^{\text{bare}}$$



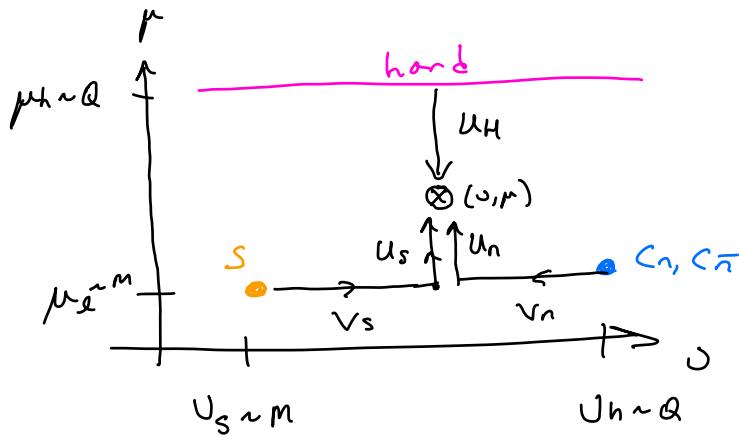
eg.  $\int d^4 k \frac{1}{(k^2 - m^2)(k^+) (k^-)} \frac{\omega^2 \omega^n}{|2k_\perp|^n}$

$$C_n^{\text{full}} = \frac{ds G_F \omega^2}{\pi} \left[ \underbrace{-\frac{e^{\epsilon \gamma_E} \Gamma(\epsilon) (\mu/m)^{2\epsilon}}{2^n}}_{\equiv (z_s - 1)} + \frac{1}{\epsilon} \ln \left( \frac{\mu}{\omega} \right) + \frac{1}{2\epsilon^2} + \ln^2 \frac{\mu}{m} - 2 \ln \frac{\omega}{m} \ln \frac{\mu}{m} + \text{constant} \right]$$

$$\text{Sum} = \frac{ds G_F}{\pi} \left[ \underbrace{\frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu}{Q} + \frac{1}{\epsilon}}_{\text{cancel between sectors}} + \ln^2 \frac{\mu}{m} + 2 \ln \frac{\mu}{m} \ln \frac{\omega}{Q} + 2 \ln \frac{\mu}{m} + \text{constant} \right]$$

- $\frac{1}{\eta}, \ln \omega$  cancel between sectors
- some overall counterterm  $z_c$  as  $SCT_I$  ( $Q^2 \rightarrow -Q^2$  here)
- logs in  $C_n$  minimized for single  $\mu, \omega$  choice,  
 & same for  $S$

$\mu$ -RGE &  $\nu$ -RGE to sum logs



$$\mu \frac{d}{d\mu} S = \gamma_\mu^S S$$

$$\nu \frac{d}{d\nu} S = \gamma_\nu^S S$$

etc.

• path independence

$$\left[ \frac{1}{2} \frac{d}{d\mu} \left( \frac{d}{d\mu} \right) \frac{d}{d\nu} \right] = 0$$

$\boxed{\mu}$   $\gamma_\mu^S = - z_S^{-1} \mu \frac{d}{d\mu} z_S = \frac{ds(\mu) C_F}{\pi} 2 \ln \frac{\mu}{\mu_0}$

$$\gamma_\mu^\nu = - z_\nu^{-1} \mu \frac{d}{d\mu} z_\nu = \frac{ds(\mu) C_F}{\pi} \left[ \ln \frac{\nu}{\nu_0} + \frac{3}{4} \right] = \gamma_\mu^\nu$$

$\boxed{\nu}$   $\gamma_\nu^S = - z_S^{-1} \nu \frac{d}{d\nu} z_S = - \frac{ds(\nu) C_F}{\pi} 2 \ln \frac{\nu}{\nu_0}$

$$\gamma_\nu^\nu = - z_\nu^{-1} \nu \frac{d}{d\nu} z_\nu = \frac{ds(\nu) C_F}{\pi} \ln \frac{\nu}{\nu_0} = \gamma_\nu^\nu$$

$$\gamma_\mu^S + \gamma_\mu^\nu + \gamma_\nu^\nu = - \gamma_H \quad , \quad \gamma_\nu^S + \gamma_\nu^\nu + \gamma_\nu^\nu = 0$$

since  $z_S^{-1} \left[ \frac{d}{d\mu} \left( \frac{d}{d\mu} \right) \frac{d}{d\nu} \right] z_S = 0 \rightarrow \mu \frac{d}{d\mu} \gamma_\nu^S = \nu \frac{d}{d\nu} \gamma_\mu^S$  etc  
which we can check

solutions are evolution kernels  $U_S, U_N, V_S, V_N$

$$\text{eg. } U_S^{LL}(\mu, \mu_S; \nu_S) = \exp \left[ - \frac{8\pi C_F}{\rho_0^2} \left( \frac{1}{ds(r)} - \frac{1}{ds(\mu_S)} - \frac{1}{ds(\nu_S)} \ln \frac{ds(\mu)}{ds(\mu_S)} \right) \right]$$

$$V_S^{LL}(\nu, \nu_S; \mu) = \exp \left[ \frac{2C_F}{\rho_0} \ln \left( \frac{ds(\mu)}{ds(\nu_S)} \right) \ln \left( \frac{\nu^2}{\nu_S^2} \right) \right]$$

see arXiv: 1202.0814 for further details

## Conclusions

SCET provides powerful framework for analyzing hard scattering  $\sigma$

$\Rightarrow$  Factorization : universal non-pert. functions  
universal perturbative functions

$\Rightarrow$  Resummation : large  $\frac{\text{double logs}}{\text{single}}$  via RGE  
both  $\mu$  (inv. mass) &  $\lambda$  (rapidity)

$\Rightarrow$  Manifest Power Counting  $\mathcal{L}^{(0)} \sim \lambda^0$   
 $\mathcal{L}^{(1)}, \mathcal{L}^{(2)} \sim \lambda^2$

can also handle multi-scale problems

$$Q \gg M_T \gg M_S^2/Q \gg \Lambda_{\text{QCD}}$$

can study power corrections with some methods

$\Rightarrow$  Provides universal description of factorization violation in hard-scattering  $\sigma$  :  $\mathcal{L}_G^{(0)}$   
Interestingly the same  $\mathcal{L}_G^{(0)}$  mediates phenomena in Forward scattering Kinematics  $s \gg t$ , small  $x$

$\Rightarrow$  Fun mathematical structure  $W_n, Y_n$

$\Rightarrow$  Multi IR-Mode EFT, prototype for other EFTs with more complicated Kinematics