Morse Graphs can effectively estimate the **Regions of Attraction** (RoAs) of dynamical systems, including closed-box ones.

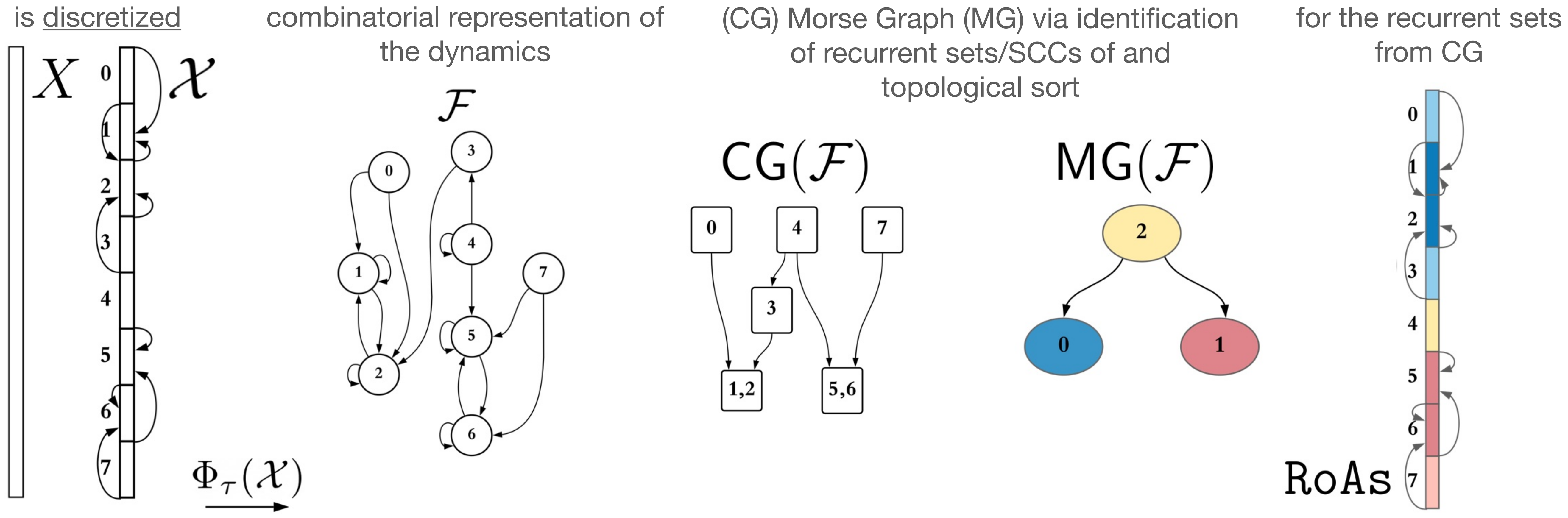
Morse Graphs: Topological Tools for Analyzing the Global Dynamics of Robot Controllers (WAFR 2022)

MOTIVATION

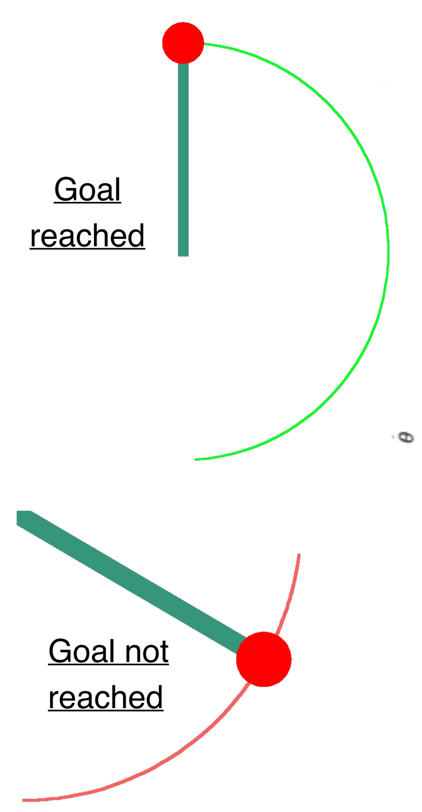
- RoA estimation is essential for understanding the conditions under which a controller can be safely applied.
- RoAs can be used for controller composition that work from a wider swath of the underlying state space.

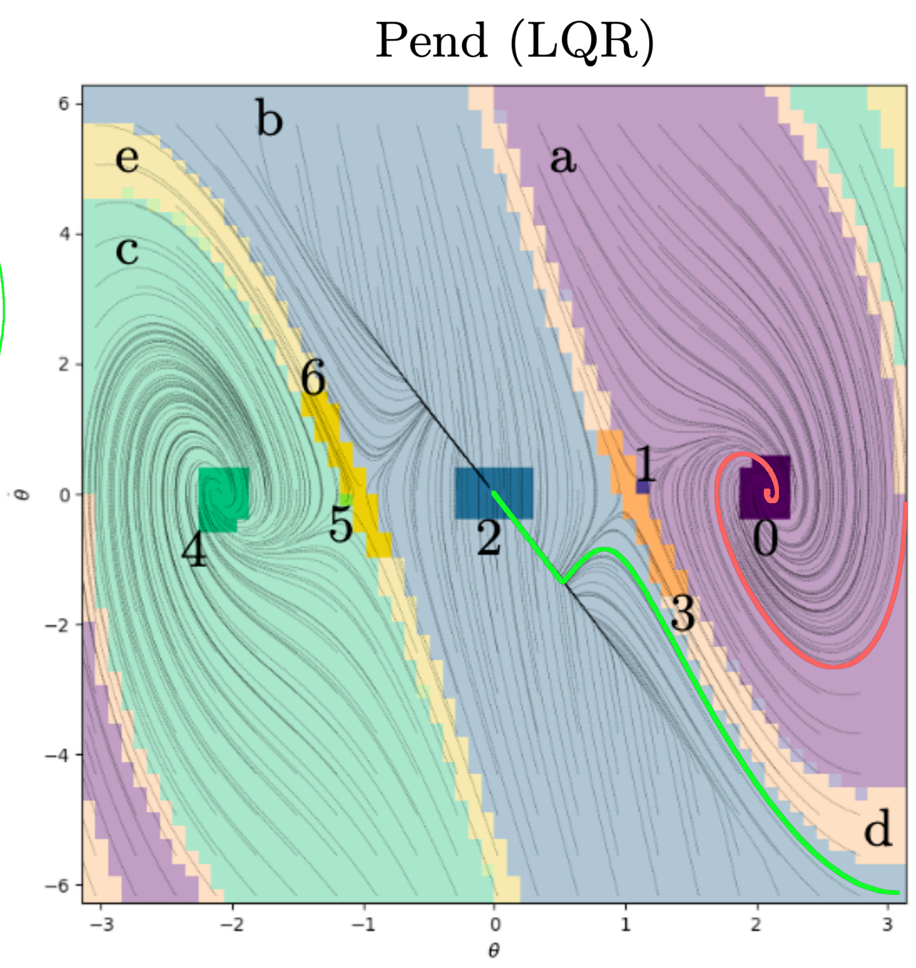
METHOD (TopMG)

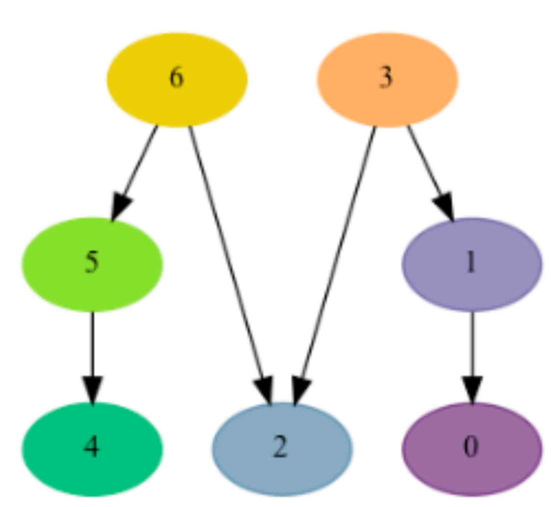
1. System's state space is discretized
2. Construction of the combinatorial representation of the dynamics
3. Computation of Condensation Graph (CG) Morse Graph (MG) via identification of recurrent sets/SCCs of and topological sort
4. Derivation of RoAs for the recurrent sets from CG

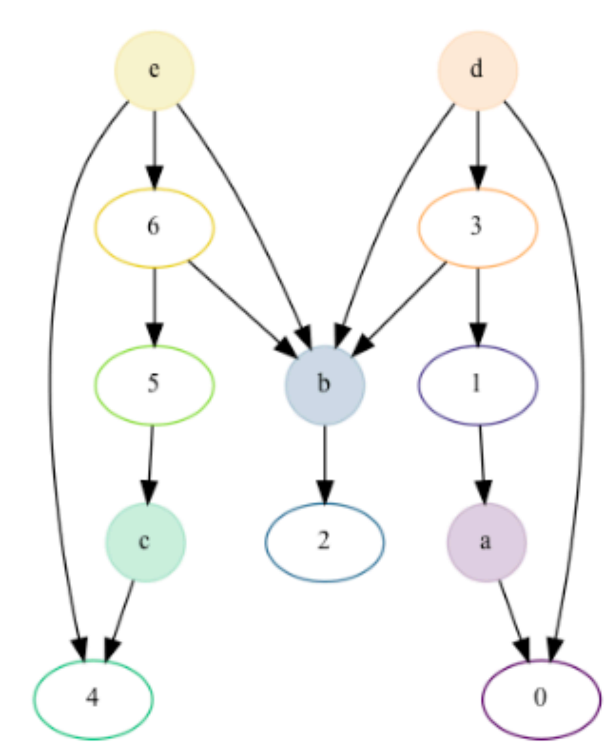


RESULTS









- Table 1 - The morse graph is applied in a variety of systems (controllers) and compared to alternative methods
- Table 2 - Data requirements for the Morse Graph are lower than the alternative for most cases

Table 1

Benchmark	L-NN[1]	L-LQR	L-SOS	Ours: MG
Pend (LQR)	61.33%	61.33%	61.33%	**1.66%**
Pend (Learned)	81.44%			**1.25%**
Acro (LQR)	10.94%	73.16%	74.34%	**3.64%**
Acro (Hybrid)	86.21%			**1.25%**
Ack (LQR)	83.03%	**82.87%**	**82.87%**	100%
Ack (Corke)	27.81%		29.35%	**24.5%**
Ack (Learned)	8.53%			**0.00%**

Table 2

Benchmark	L-NN	Ours: MG
Pend (LQR)	667.1M	**6.6M**
Pend (Learned)	341.9M	**6.6M**
Acro (LQR)	5.7B	**1.1B**
Acro (Hybrid)	**533M**	2.1B
Ack (LQR)	**9.9M**	520M
Ack (Corke)	37.5M	**13M**
Ack (Learned)	704.6M	**520M**

[1] Richards, Spencer M., Felix Berkenkamp, and Andreas Krause. "The lyapunov neural network: Adaptive stability certification for safe learning of dynamical systems." *Conference on Robot Learning*. PMLR, 2018.

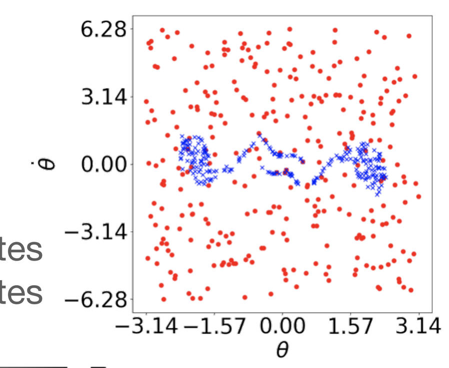
Data-Efficient Characterization of the Global Dynamics of Robot Controllers with Confidence Guarantees (ICRA 2023)

MOTIVATION

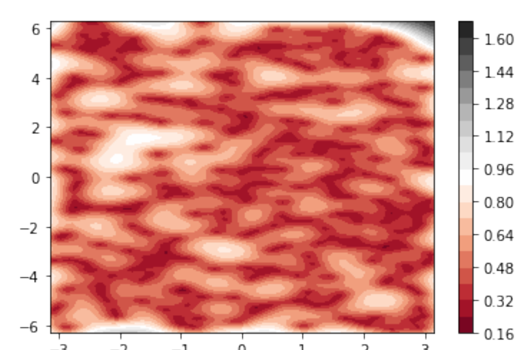
- Reduce data requirements of Morse Graphs using Gaussian Processes as a surrogate model

METHOD (GPMP)

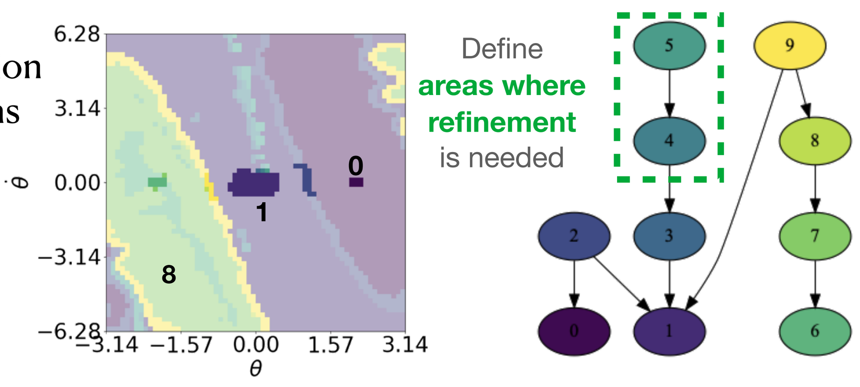
Step 1: Collect trajectories of the system of interest.



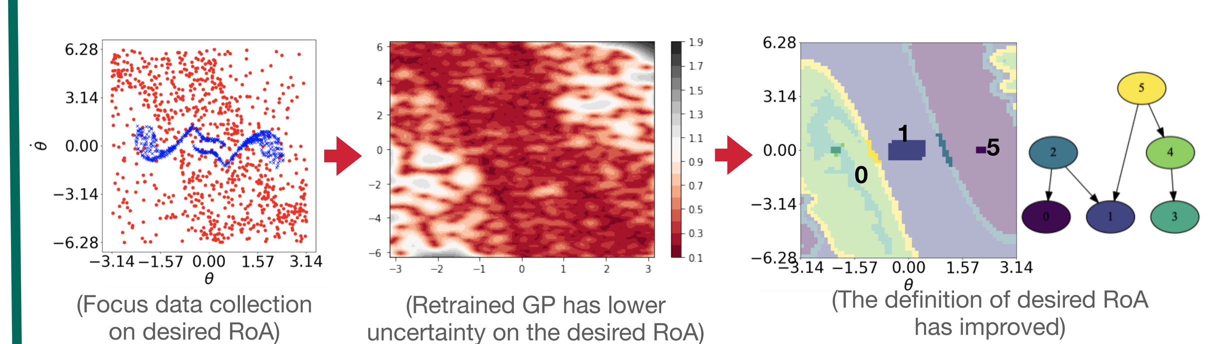
Step 2: Train a surrogate model using Gaussian Processes (GPs)



Step 3: Verification via Morse Graphs and RoAs



Step 4: Repeat



RESULTS

- Morse Graphs can successfully identify RoAs using surrogate models if data is expensive to acquire. The required amount of data is significantly smaller (Table 2) with minimal sacrifice on accuracy (Table 1).

Table 1

Benchmark	L-NN[1]	L-LQR/SOS	TopMG	GPMG
Quad (Learned)	-	N.A.	-	**1.0**
Pend (LQR)	**0.98**	0.7 / 0.03	0.97	0.91
Car (Learned)	-	N.A.	**1.0**	**1.0**
Land (TOC)	-	N.A.	**1.0**	0.79
Ack (Learned)	0.91	N.A.	**1.0**	**1.0**
Acro (LQR)	0.89	0.27 / 0.26	0.96	**1.0**
Acro (Hybrid)	0.14	N.A.	0.99	**1.0**

Table 2

Benchmark	L-NN[1]	TopMG	Ours: GPMG	Dim
Quad (Learned)	-	-	**25,000**	2
Pend (LQR)	667.1M	6.6M	**120,000**	2
Car (Learned)	-	6.6M	**3,000**	3
Land (TOC)	-	1M	**300,000**	3
Ack (Learned)	704.6M	520M	**10,000**	3
Acro (LQR)	5.7B	1.1B	**100,000**	4
Acro (Hybrid)	533M	2.1B	**2.5M**	4

RUTGERS

DATA-INSPIRE

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