Exotic resonances

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Outline

Symmetries of QCD

Brief recall of conventional mesons

Unconventional (exotic) mesons

Toward a nonet of hybrid states

Glueballs: status and recent news

Bound state of glueballs? Bound states of Higgs?

Conclusions

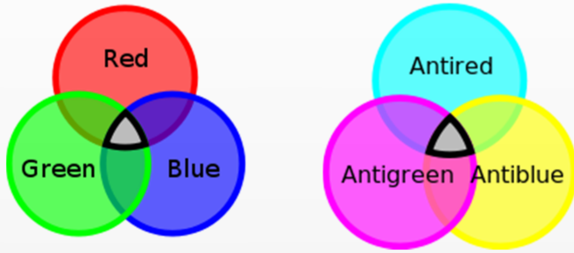
Symmetries of QCD

Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B

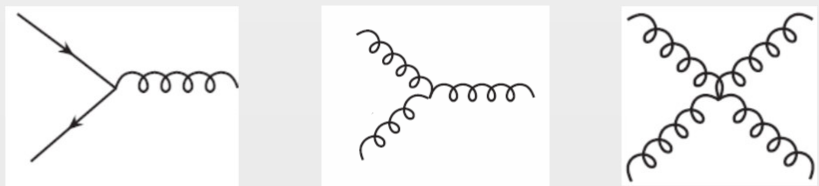


$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix}; \; i = u,d,s\;,...$$

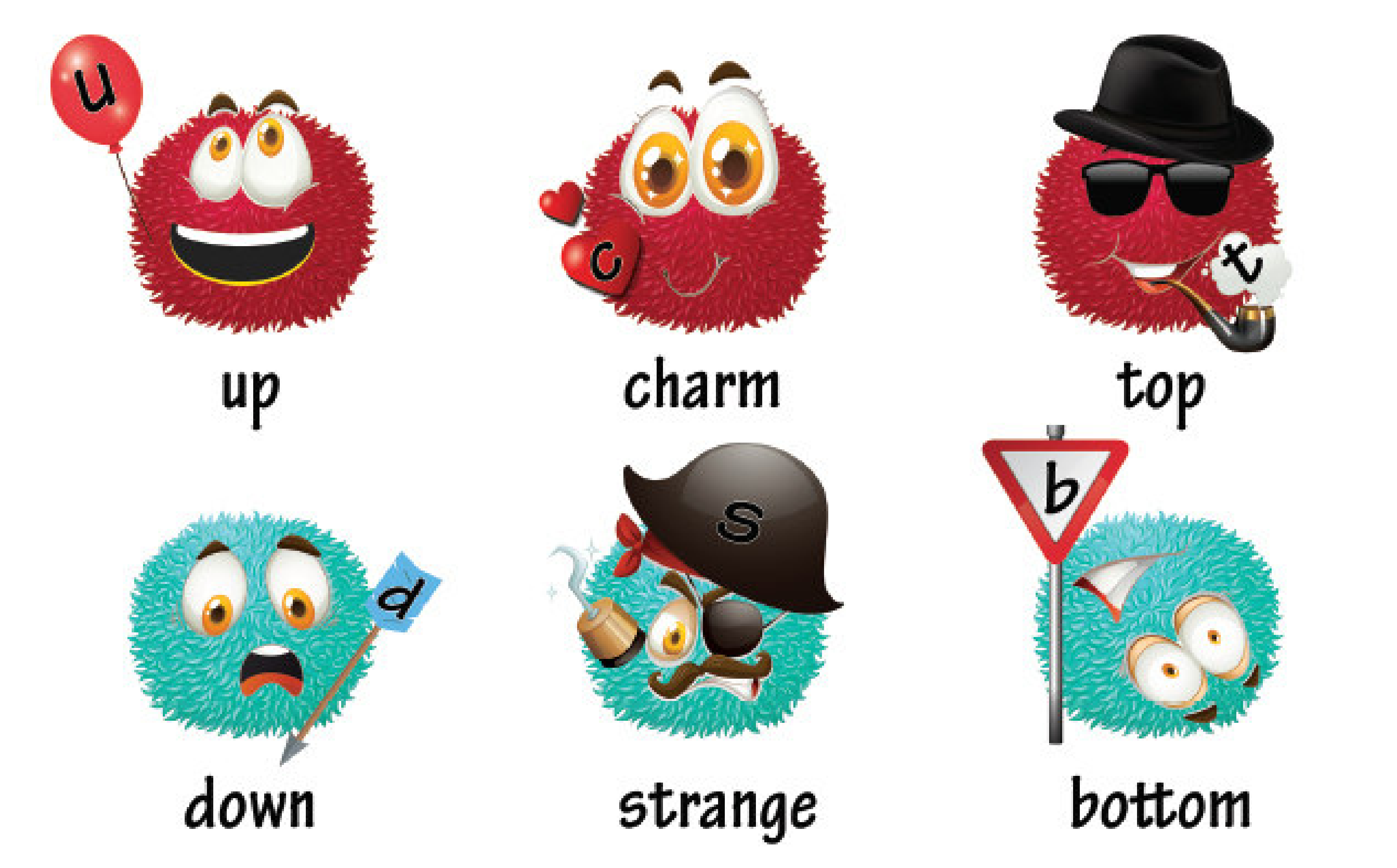
$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

8 type of gluons ($R\overline{G}$, $B\overline{G}$,…)

$$A_\mu^a \; ; \; a = 1,...,8$$



Confinement: quarks never ‘seen’ directly. How they might look like ☺



Picture by Pawel Piotrowski

Trace anomaly: the emergence of a dimension

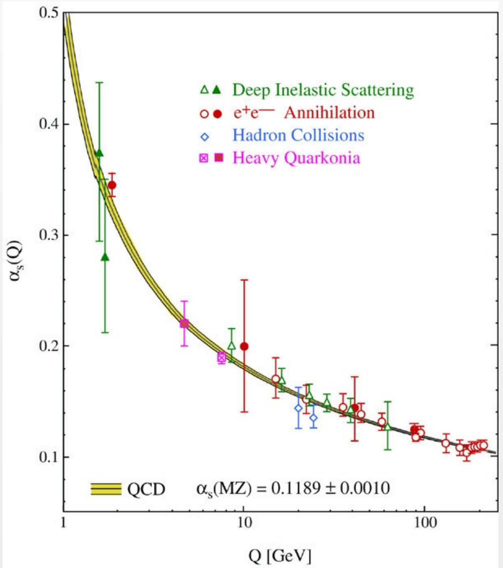
Chiral limit: $m_{\cdot} = 0$

$$x^{\mu} \rightarrow x'^{\mu} = \lambda^{-1} x^{\mu}$$

is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\quad \Lambda_{YM} \approx 250 \text{ MeV}$

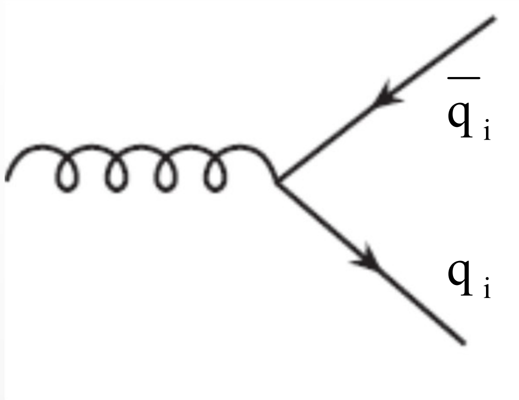
$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \rightarrow m^{*}_{gluon} \approx 500 - 800 \text{ MeV}$

Gluon condensate: $\left\langle G^{a}_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

Flavor symmetry



Gluon-quark-antiquark vertex

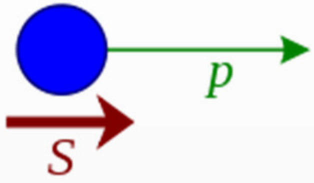
It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

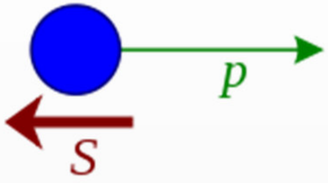
$$U \in U(3)_V \rightarrow U^+ U = 1$$

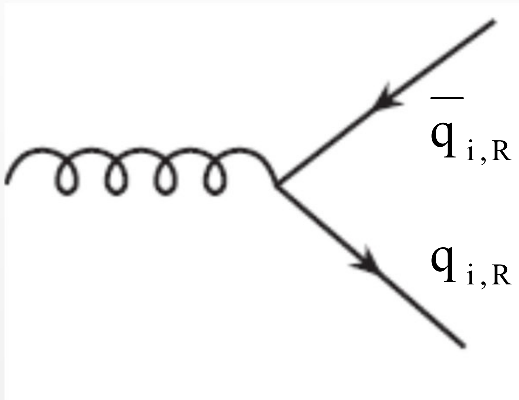
Chiral symmetry

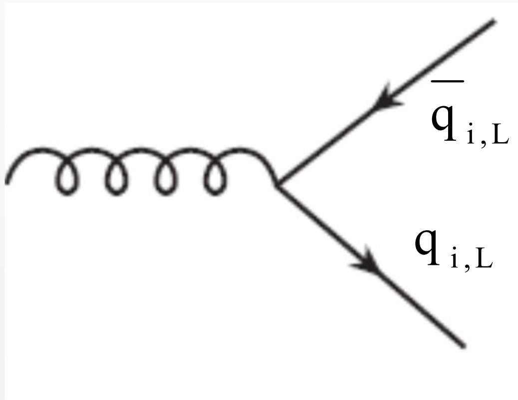
Right-handed:



Left-handed:







$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

In the chiral limit (mi=0) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum

Chiral (or axial) anomaly: explicitely broken by quantum fluctuations

$$\partial^\mu(\bar{q}^i \gamma_\mu \gamma_5 q^i) = \frac{3g^2}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}(G_{\mu\nu}G_{\rho\sigma})$$

Symmetries of QCD and breakings

$SU(3)_{color}$: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (**scale anomaly**) and by quark masses.

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is **spontaneously** broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (**chiral anomaly**)

Conventional mesons: quark-antiquark states

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are “white” and are called hadrons.

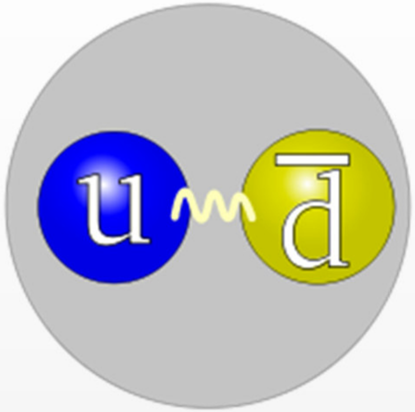
Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Example of conventional quark-antiquark states: the ρ and the π mesons



$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c}\left(|\pi^+\pi^0\rangle + ...\right)$$

where

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

Rho-meson $m_{\rho^+} = 775 \text{ MeV}$

Pion $m_{\pi^+} = 139 \text{ MeV}$

$$m_u + m_d \approx 7 \text{ MeV}$$

Mass generation in QCD is a nonpert. penomenon based on SSB (mentioned previusly).

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	θ_{quad} [°]	θ_{lin} [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	−11.3	−24.5
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		
$3\,^1S_0$	0^{-+}	$\pi(1800)$			$\eta(1760)$		

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	−11.3	−24.5
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		
$3\,^1S_0$	0^{-+}	$\pi(1800)$			$\eta(1760)$		

Some selected nonets

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

TABLE I. Chiral multiplets, their currents, and transformations up to $J = 3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

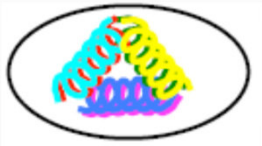
J^{PC}, ${}^{2S+1}L_J$	$I = 1(\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}})$ $I = 1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d)$ $I = 0(\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{**}$	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_L \times SU(3)_R \times \times U(1)_A$
0^{-+}, ${}^1S_0$	π K $\eta, \eta'(958)$	$P^{ij} = \frac{1}{2}\bar{q}^j i\gamma^5 q^i$	$\Phi = S + iP$ $(\Phi^{ij} = \bar{q}^j_R q^i_L)$	$\Phi \to e^{-2i\alpha} U_L \Phi U_R^\dagger$
0^{++}, ${}^3P_0$	$a_0(1450)$ $K_0^*(1430)$ $f_0(1370), f_0(1710)^*$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		
1^{--}, ${}^1S_1$	$\rho(770)$ $K^*(892)$ $\omega(782), \phi(1020)$	$V^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L^{ij}_\mu = \bar{q}^j_L \gamma_\mu q^i_L)$	$L_\mu \to U_L L_\mu U_L^\dagger$
1^{++}, ${}^3P_1$	$a_1(1260)$ $K_{1,A}$ $f_1(1285), f_1(1420)$	$A^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R^{ij}_\mu = \bar{q}^j_R \gamma_\mu q^i_R)$	$R_\mu \to U_R R_\mu U_R^\dagger$
1^{+-}, ${}^1P_1$	$b_1(1235)$ $K_{1,B}$ $h_1(1170), h_1(1380)$	$P^{ij}_\mu = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + iP_\mu$ $(\Phi^{ij}_\mu = \bar{q}^j_R i\overleftrightarrow{D}_\mu q^i_L)$	$\Phi_\mu \to e^{-2i\alpha} U_L \Phi_\mu U_R^\dagger$
1^{--}, ${}^3D_1$	$\rho(1700)$ $K^*(1680)$ $\omega(1650), \phi(?)$	$S^{ij}_\mu = \frac{1}{2}\bar{q}^j i\overleftrightarrow{D}_\mu q^i$		
2^{++}, ${}^3P_2$	$a_2(1320)$ $K_2^*(1430)$ $f_2(1270), f_2'(1525)$	$V^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma_\mu i\overleftrightarrow{D}_\mu + \cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L^{ij}_{\mu\nu} = \bar{q}^j_L(\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i_L)$	$L_{\mu\nu} \to U_L L_{\mu\nu} U_L^\dagger$
2^{--}, ${}^3D_2$	$\rho_2(?)$ $K_2(1820)$ $\omega_2(?), \phi_2(?)$	$A^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R^{ij}_{\mu\nu} = \bar{q}^j_R(\gamma_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_R)$	$R_{\mu\nu} \to U_R R_{\mu\nu} U_R^\dagger$
2^{-+}, ${}^1D_2$	$\pi_2(1670)$ $K_2(1770)$ $\eta_2(1645), \eta_2(1870)$	$P^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(i\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + iP_{\mu\nu}$ $(\Phi^{ij}_{\mu\nu} = \bar{q}^j_R(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i_L)$	$\Phi_{\mu\nu} \to e^{-2i\alpha} U_L \Phi_{\mu\nu} U_R^\dagger$
2^{++}, ${}^3F_2$	$a_2(?)$ $K_2^*(?)$ $f_2(?), f_2'(?)$	$S^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots)q^i$		
3^{--}, ${}^3D_3$	$\rho_3(1690)$ $K_3^*(1780)$ $\omega_3(1670), \phi_3(1850)$	⋮	⋮	⋮

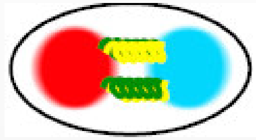
Uniwersytet Jana Kochanowskiego w Kielcach

Table from:

F.G., R. Pisarski, A. Koenigstein Phys.Rev.D 97 (2018) 9, 091901 e-Print: 1709.07454

Non-conventional mesons: beyond qq

1) Glueballs 

2) Hybrids 

3) Four-quark states

Compact diquark-antidiquark states 

Molecular states (a type of dynamical generation) 

Companion poles (another type of dynamical generation)

(Some) novel results for conventional mesons

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Strategy

- For a given nonet , write down the corresponding model-Lagrangian respecting flavor (or if possible chiral) symmetry.
- Consider only C, P, invariant terms
- Calculate decays in all possible channels (first at tree-level, in some selected case including finite width or loop effects;
- Fit free parameters to known experimental value;
- Make postdictions and predictions.

Mesons with J=3

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Phenomenology of $J^{PC} = 3^{--}$ tensor mesons

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We study the strong and radiative decays of the antiquark-quark ground state $J^{PC} = 3^{--}$ ($n^{2S+1}L_J = 1^3D_3$) nonet $\{\rho_3(1690), K_3^*(1780), \phi_3(1850), \omega_3(1670)\}$ in the framework of an effective quantum field theory approach, based on the $SU_V(3)$-flavor symmetry. The effective model is fitted to experimental data listed by the Particle Data Group. We predict numerous experimentally unknown decay widths and branching ratios. An overall agreement of theory (fit and predictions) with experimental data confirms the $\bar{q}q$ nature of the states and qualitatively validates the effective approach. Naturally, experimental clarification as well as advanced theoretical description is needed for trustworthy quantitative predictions, which is observed from some of the decay channels. Besides conventional spin-3 mesons, theoretical predictions for ratios of strong and radiative decays of a hypothetical glueball state $G_3(4200)$ with $J^{PC} = 3^{--}$ are also presented.

J=3: post- and predictions

TABLE V. Decays of $J^{PC} = 3^{--}$ mesons into two pseudoscalars. Experimental data is taken from Ref. [1].

Decay process	Theory Γ/MeV	Experiment Γ/MeV
$\rho_3(1690) \to \pi\pi$	32.7 ± 2.3	38.0 ± 3.2
$\rho_3(1690) \to \bar{K}K$	4.0 ± 0.3	2.54 ± 0.45
$K_3^*(1780) \to \pi\bar{K}$	18.5 ± 1.3	29.9 ± 4.3
$K_3^*(1780) \to \bar{K}\eta$	7.4 ± 0.5	48 ± 22
$K_3^*(1780) \to \bar{K}\eta'(958)$	0.021 ± 0.001	
$\omega_3(1670) \to \bar{K}K$	3.0 ± 0.2	
$\phi_3(1850) \to \bar{K}K$	18.8 ± 1.3	Seen

TABLE VI. Decays of $J^{PC} = 3^{--}$ mesons into a pseudoscalar-vector pair. Experimental data taken from Ref. [1].

Decay process	Theory Γ/MeV	Experiment Γ/MeV
$\rho_3(1690) \to \rho(770)\eta$	3.8 ± 0.8	Seen
$\rho_3(1690) \to \bar{K}^*(892)K$	3.4 ± 0.7	
$\rho_3(1690) \to \omega(782)\pi$	35.8 ± 7.4	25.8 ± 9.8
$\rho_3(1690) \to \phi(1020)\pi$	0.036 ± 0.007	
$K_3^*(1780) \to \rho(770)K$	16.8 ± 3.5	49.3 ± 15.7
$K_3^*(1780) \to \bar{K}^*(892)\pi$	27.2 ± 5.6	31.8 ± 9.0
$K_3^*(1780) \to \bar{K}^*(892)\eta$	0.09 ± 0.02	
$K_3^*(1780) \to \omega(782)\bar{K}$	4.3 ± 0.9	
$K_3^*(1780) \to \phi(1020)\bar{K}$	1.2 ± 0.3	
$\omega_3(1670) \to \rho(770)\pi$	97 ± 20	Seen
$\omega_3(1670) \to \bar{K}^*(892)K$	2.9 ± 0.6	
$\omega_3(1670) \to \omega(782)\eta$	2.8 ± 0.6	
$\omega_3(1670) \to \phi(1020)\eta$	$(7.6 \pm 1.6) \times 10^{-6}$	
$\phi_3(1850) \to \rho(770)\pi$	1.1 ± 0.2	
$\phi_3(1850) \to \bar{K}^*(892)K$	35.5 ± 7.3	Seen
$\phi_3(1850) \to \omega(782)\eta$	0.015 ± 0.003	
$\phi_3(1850) \to \omega(782)\eta'(958)$	0.003 ± 0.001	
$\phi_3(1850) \to \phi(1020)\eta$	3.8 ± 0.8	

TABLE VII. Theoretical predictions for the radiative decays $W_3 \to \gamma P$.

Decay process	Theory Γ/keV
$\rho_3^{\pm/0}(1690) \to \gamma\pi^{\pm/0}$	69 ± 14
$\rho_3^0(1690) \to \gamma\eta$	157 ± 32
$\rho_3^0(1690) \to \gamma\eta'(958)$	20 ± 4
$K_3^{\pm}(1780) \to \gamma K^{\pm}$	58 ± 12
$K_3^0(1780) \to \gamma K^0$	231 ± 48
$\omega_3(1670) \to \gamma\pi^0$	560 ± 120
$\omega_3(1670) \to \gamma\eta$	19 ± 4
$\omega_3(1670) \to \gamma\eta'(958)$	1.4 ± 0.3
$\phi_3(1850) \to \gamma\pi^0$	4 ± 1
$\phi_3(1850) \to \gamma\eta$	129 ± 26
$\phi_3(1850) \to \gamma\eta'(958)$	35 ± 7

$$\begin{pmatrix} \omega_3(1670) \\ \phi_3(1850) \end{pmatrix} = \begin{pmatrix} \cos\beta_{w_3} & \sin\beta_{w_3} \\ -\sin\beta_{w_3} & \cos\beta_{w_3} \end{pmatrix} \begin{pmatrix} \omega_{3,N} \\ \omega_{3,S} \end{pmatrix}$$

$$\beta_{w_3} = 3.5^\circ$$

Tensor and (axial-)tensors

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

PHYSICAL REVIEW D **106**, 036008 (2022)

From well-known tensor mesons to yet unknown axial-tensor mesons

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While the ground-state tensor ($J^{PC} = 2^{++}$) mesons $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, and $f_2'(1525)$ are well known experimentally and form an almost ideal nonet of quark-antiquark states, their chiral partners, the ground-states axial-tensor ($J^{PC} = 2^{--}$) mesons are poorly settled: only the kaonic member $K_2(1820)$ of the nonet has been experimentally found, whereas the isovector state ρ_2 and two isoscalar states ω_2 and ϕ_2 are still missing. Here, we study masses, strong, and radiative decays of tensor and axial-tensor mesons within a chiral model that links them: the established tensor mesons are used to test the model and to determine its parameters, and subsequently various predictions for their chiral partners, the axial-tensor mesons, are obtained. The results are compared to current lattice QCD outcomes as well as to other theoretical approaches and show that the ground-state axial-tensor mesons are expected to be quite broad, the vector-pseudoscalar mode being the most prominent decay mode followed by the tensor-pseudoscalar one. Nonetheless, their experimental finding seems to be possible in ongoing and/or future experiments.

DOI: 10.1103/PhysRevD.106.036008

Postdictions (left) predictions (right)

Uniwersytet
Jana Kochanowskiego w Kielcach

Decay process (in model)	eLSM (MeV)	PDG (MeV)
$a_2(1320) \longrightarrow \bar{K}\,K$	4.06 ± 0.14	$7.0^{+2.0}_{-1.5} \leftrightarrow (4.9 \pm 0.8)\%$
$a_2(1320) \longrightarrow \pi\,\eta$	25.37 ± 0.87	$18.5 \pm 3.0 \leftrightarrow (14.5 \pm 1.2)\%$
$a_2(1320) \longrightarrow \pi\,\eta'(958)$	1.01 ± 0.03	$0.58 \pm 0.10 \leftrightarrow (0.55 \pm 0.09)\%$
$K_2^*(1430) \longrightarrow \pi\,\bar{K}$	44.82 ± 1.54	$49.9 \pm 1.9 \leftrightarrow (49.9 \pm 0.6)\%$
$f_2(1270) \longrightarrow \bar{K}\,K$	3.54 ± 0.29	$8.5 \pm 0.8 \leftrightarrow (4.6^{+0.5}_{-0.4})\%$
$f_2(1270) \longrightarrow \pi\,\pi$	168.82 ± 3.89	$157.2^{+4.0}_{-1.1} \leftrightarrow (84.2^{+2.9}_{-0.9})\%$
$f_2(1270) \longrightarrow \eta\,\eta$	0.67 ± 0.03	$0.75 \pm 0.14 \leftrightarrow (0.4 \pm 0.08)\%$
$f_2'(1525) \longrightarrow \bar{K}\,K$	23.72 ± 0.60	$75 \pm 4 \leftrightarrow (87.6 \pm 2.2)\%$
$f_2'(1525) \longrightarrow \pi\,\pi$	0.67 ± 0.14	$0.71 \pm 0.14 \leftrightarrow (0.83 \pm 0.16)\%$
$f_2'(1525) \longrightarrow \eta\,\eta$	1.81 ± 0.05	$9.9 \pm 1.9 \leftrightarrow (11.6 \pm 2.2)\%$

Decay process (in model)	eLSM (MeV)	PDG-2020 (MeV)
$a_2(1320) \longrightarrow \rho(770)\,\pi$	71.0 ± 2.6	$73.61 \pm 3.35 \leftrightarrow (70.1 \pm 2.7)\%$
$K_2^*(1430) \longrightarrow \bar{K}^*(892)\,\pi$	27.9 ± 1.0	$26.92 \pm 2.14 \leftrightarrow (24.7 \pm 1.6)\%$
$K_2^*(1430) \longrightarrow \rho(770)\,K$	10.3 ± 0.4	$9.48 \pm 0.97 \leftrightarrow (8.7 \pm 0.8)\%$
$K_2^*(1430) \longrightarrow \omega(782)\,\bar{K}$	3.5 ± 0.1	$3.16 \pm 0.88 \leftrightarrow (2.9 \pm 0.8)\%$
$f_2'(1525) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	19.89 ± 0.73	

Decay process (in model)	eLSM (MeV)
$\rho_2(?) \longrightarrow \rho(770)\,\eta$	$\approx 99 \pm 50$
$\rho_2(?) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 85 \pm 43$
$\rho_2(?) \longrightarrow \omega(782)\,\pi$	$\approx 419 \pm 210$
$\rho_2(?) \longrightarrow \phi(1020)\,\pi$	≈ 0.8
$K_{2,A} \longrightarrow \rho(770)\,K$	$\approx 195 \pm 98$
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\pi$	$\approx 316 \pm 158$
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\eta$	≈ 0.01
$K_{2,A} \longrightarrow \omega(782)\,\bar{K}$	$\approx 51 \pm 26$
$K_{2,A} \longrightarrow \phi(1020)\,\bar{K}$	$\approx 50 \pm 25$
$\omega_{2,N} \longrightarrow \rho(770)\,\pi$	$\approx 1314 \pm 657$
$\omega_{2,N} \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 85 \pm 43$
$\omega_{2,N} \longrightarrow \omega(782)\,\eta$	$\approx 93 \pm 47$
$\omega_{2,N} \longrightarrow \phi(1020)\,\eta$	≈ 0.06
$\omega_{2,S} \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 510 \pm 255$
$\omega_{2,S} \longrightarrow \omega(782)\,\eta$	$\approx 1.0 \pm 0.5$
$\omega_{2,S} \longrightarrow \omega(782)\,\eta'(958)$	≈ 0.3
$\omega_{2,S} \longrightarrow \phi(1020)\,\eta$	$\approx 101 \pm 51$

Decay process (in model)	eLSM (MeV)
$\rho_2(?) \longrightarrow a_2(1320)\,\pi$	≈ 88
$K_{2,A} \longrightarrow K_2^*(1430)\,\pi$	≈ 49
$K_{2,A} \longrightarrow a_2(1320)\,K$	≈ 84
$K_{2,A} \longrightarrow f_2(1270)\,K$	≈ 4
$\omega_{2,S} \longrightarrow K_2^*(1430)\,K + \text{c.c.}$	≈ 15

Pseudotensor mesons

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

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THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Phenomenology of pseudotensor mesons and the pseudotensor glueball

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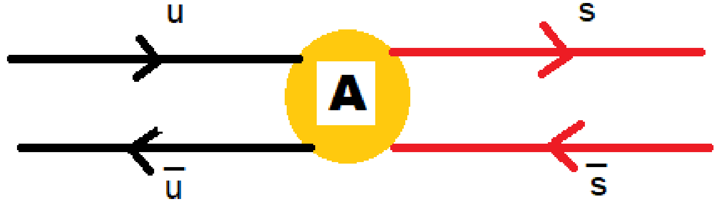
Abstract. We study the decays of the pseudotensor mesons ($\pi_2(1670), K_2(1770), \eta_2(1645), \eta_2(1870)$) interpreted as the ground-state nonet of 1^1D_2 $\bar{q}q$ states using interaction Lagrangians which couple them to pseudoscalar, vector, and tensor mesons. While the decays of $\pi_2(1670)$ and $K_2(1770)$ can be well described, the decays of the isoscalar states $\eta_2(1645)$ and $\eta_2(1870)$ can be brought in agreement with the present experimental data only if the mixing angle between nonstrange and strange states is surprisingly large (about $-42°$, similar to the mixing in the pseudoscalar sector, in which the chiral anomaly is active). Such a large mixing angle is however at odd with all other conventional quark-antiquark nonets: if confirmed, a deeper study of its origin will be needed in the future. Moreover, the $\bar{q}q$ assignment of pseudotensor states predicts that the ratio $[\eta_2(1870) \to a_2(1320)\,\pi]/[\eta_2(1870) \to f_2(1270)\,\eta]$ is about 23.5. This value is in agreement with Barberis *et al.*, (20.4 ± 6.6), but disagrees with the recent reanalysis of Anisovich *et al.*, (1.7 ± 0.4). Future experimental studies are necessary to understand this puzzle. If Anisovich's value is confirmed, a simple nonet of pseudoscalar mesons cannot be able to describe data (different assignments and/or additional states, such as an hybrid state, will be needed). In the end, we also evaluate the decays of a pseudoscalar glueball into the aforementioned conventional $\bar{q}q$ states: a sizable decay into $K_2^*(1430)\,K$ and $a_2(1230)\,\pi$ together with a vanishing decay into pseudoscalar-vector pairs (such as $\rho(770)\,\pi$ and $K^*(892)\,K$) are expected. This information can be helpful in future studies of glueballs at the ongoing BESIII and at the future PANDA experiments.

ArXiv: 1608.08777

Large mixing angle: where does it come from?



PHYSICAL REVIEW D **97**, 091901(R) (2018)

Rapid Communications

How the axial anomaly controls flavor mixing among mesons

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$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos\beta_{pt} & \sin\beta_{pt} \\ -\sin\beta_{pt} & \cos\beta_{pt} \end{pmatrix} \begin{pmatrix} \eta_{2,N} \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \eta_{2,S} \equiv \bar{s}s \end{pmatrix}$$

$$\beta_{pt} = -42^\circ$$

$$\begin{pmatrix} \eta \equiv \eta(547) \\ \eta' \equiv \eta(958) \end{pmatrix} = \begin{pmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_P \simeq -42^\circ$$

For a recent re-analysis with decay widhts partial-wave :
V. Shastry, E. Trotti, F.G., Phys. Rev.D 105 (2022) 5, 054022 • e-Print: 2107.13501

(Excited) vector mesons

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Prediction for ϕ(1930)

Can one find this state?

TABLE XII. Summary table for the putative state $\phi(1930)$.

Meson $\phi(1930)$	
Quark composition	$\approx s\bar{s}$
Old spectroscopy notation	(Predom.) $n^{2S+1}L_J = 1^3D_1$
n	(Predom.) 1
S	(Predom.) $1\uparrow\uparrow$
L	(Predom.) 2
J^{PC}	1^{--}
Mass	$\approx 1930 \pm 40$ MeV
Decays	
Decay channel	Decay width (MeV)
$\phi(1930) \to \bar{K}K$	104 ± 28
$\phi(1930) \to K\bar{K}^*$	260 ± 109
$\phi(1930) \to \Phi(1020)\eta$	67 ± 28
$\phi(1930) \to \Phi(1020)\eta'$	≈ 0
$\phi(1930) \to \gamma\eta$	0.19 ± 0.12
$\phi(1930) \to \gamma\eta'$	0.13 ± 0.08

arXiv: 1708.02593; **it does not fit with ϕ(2170)**

Citation: R.L. Workman *et al.* (Particle Data Group), to be published (2022)

$\phi(2170)$

$I^G(J^{PC}) = 0^-(1^{--})$

See the review on "Spectroscopy of Light Meson Resonances."

$\phi(2170)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
2162 ± 7	**OUR AVERAGE**	Error includes scale factor of 1.1.		

$\phi(2170)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*C*
100 $^{+31}_{-23}$	**OUR AVERAGE**	Error includes scale factor of 2.5.		

Unconventional (exotic) mesons

Toward a nonet of hybrid state/PDG

$\pi_1(1600)$

$I^G(J^{PC}) = 1^-(1^{-+})$

See the review on "Spectroscopy of Light Meson Resonances" and a note in PDG 06, Journal of Physics **G33** 1 (2006).

$\pi_1(1600)$ T-Matrix Pole $\sqrt{s}$

$\pi_1(1600)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1661^{+15}_{-11} OUR AVERAGE		Error includes scale factor of 1.2.		

$\pi_1(1600)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
240± 50 OUR AVERAGE		Error includes scale factor of 1.7. See the ideogram below.		

$\pi_1(1600)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi\pi$	seen
Γ_2	$\rho^0\pi^-$	seen
Γ_3	$f_2(1270)\pi^-$	not seen
Γ_4	$b_1(1235)\pi$	seen
Γ_5	$\eta'(958)\pi^-$	seen
Γ_6	$\eta\pi$	
Γ_7	$f_1(1285)\pi$	seen

$\pi_1(1400)$

$I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1400)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
1354 ±25 OUR AVERAGE		Error includes scale factor of 1.8. See the ideogram below.			

$\pi_1(1400)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
330 ±35 OUR AVERAGE					

$\pi_1(1400)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\eta\pi^0$	seen
Γ_2	$\eta\pi^-$	seen

A unique I=1 hybrid state

PHYSICAL REVIEW LETTERS **122**, 042002 (2019)

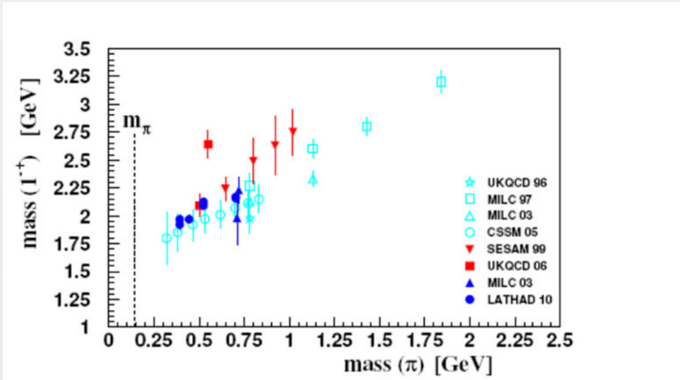
Determination of the Pole Position of the Lightest Hybrid Meson Candidate

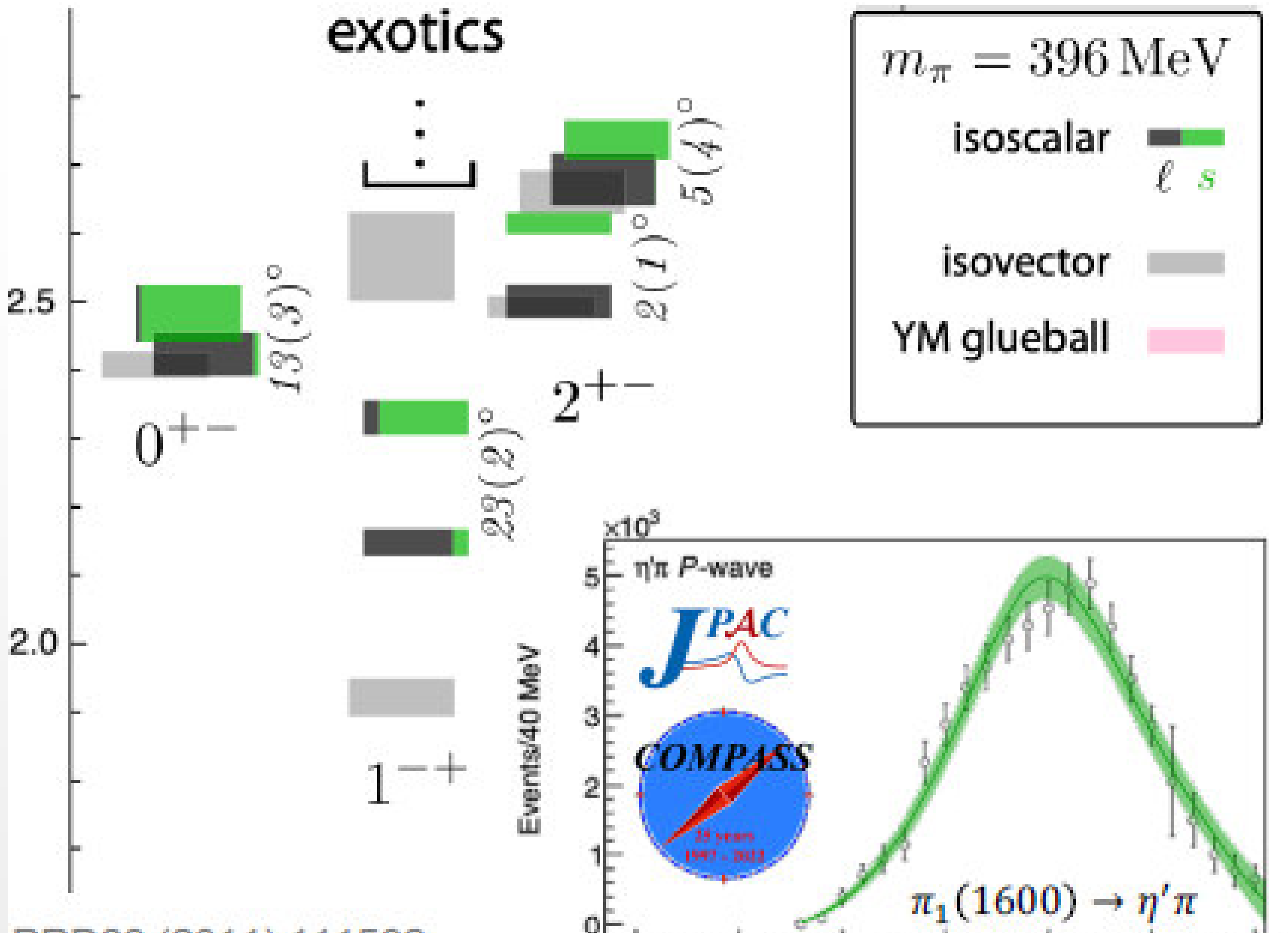
A. Rodas,[1,*] A. Pilloni,[2,3,†] M. Albaladejo,[2,4] C. Fernández-Ramírez,[5] A. Jackura,[6,7] V. Mathieu,[2] M. Mikhasenko,[8] J. Nys,[9] V. Pauk,[10] B. Ketzer,[8] and A. P. Szczepaniak[2,6,7]

Mapping states with explicit gluonic degrees of freedom in the light sector is a challenge, and has led to controversies in the past. In particular, the experiments have reported two different hybrid candidates with spin-exotic signature, $\pi_1(1400)$ and $\pi_1(1600)$, which couple separately to $\eta\pi$ and $\eta'\pi$. This picture is not compatible with recent Lattice QCD estimates for hybrid states, nor with most phenomenological models. We consider the recent partial wave analysis of the $\eta^{(\prime)}\pi$ system by the COMPASS Collaboration. We fit the extracted intensities and phases with a coupled-channel amplitude that enforces the unitarity and analyticity of the S matrix. We provide a robust extraction of a single exotic π_1 resonant pole, with mass and width $1564 \pm 24 \pm 86$ and $492 \pm 54 \pm 102$ MeV, which couples to both $\eta^{(\prime)}\pi$ channels. We find no evidence for a second exotic state. We also provide the resonance parameters of the $a_2(1320)$ and $a_2'(1700)$.

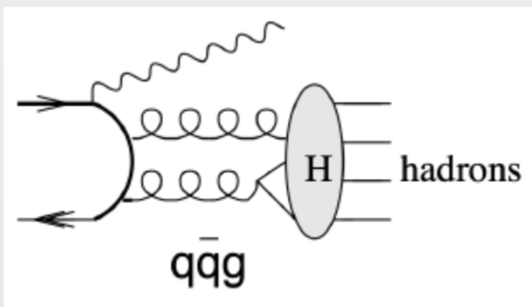
π1(1600) and π1(1400) are the same state
(in agreement with various models and lattice QCD)

C. Meyer and E. Swanson,
Hybrid Mesons,
Prog. Part. Nucl. Phys. 82 (2015) 21
[arXiv:1502.07276 [hep-ph]].





PRD83 (2011) 111502



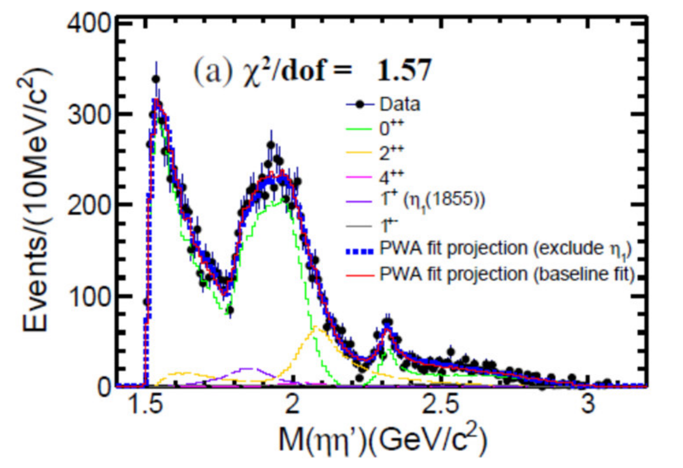
New experimental finding: η1(1855)

Observation of an isoscalar resonance with exotic $J^{PC} = 1^{-+}$ quantum numbers in $J/\psi \to \gamma\eta\eta'$

M. Ablikim[1], M. N. Achasov[10,b], P. Adlarson[68], S. Ahmed[14], M. Albrecht[4], R. Aliberti[28], A. Amoroso[67A,67C], M. R. An[32],

Using a sample of $(10.09\pm0.04)\times10^9$ J/ψ events collected with the BESIII detector operating at the BEPCII storage ring, a partial wave analysis of the decay $J/\psi \to \gamma\eta\eta'$ is performed. The first observation of an isoscalar state with exotic quantum numbers $J^{PC} = 1^{-+}$, denoted as $\eta_1(1855)$, is reported in the process $J/\psi \to \gamma\eta_1(1855)$ with $\eta_1(1855) \to \eta\eta'$. Its mass and width are measured to be $(1855\pm9^{+6}_{-1})$ MeV/c^2 and $(188\pm18^{+3}_{-8})$ MeV, respectively, where the first uncertainties are statistical and the second are systematic, and its statistical significance is estimated to be larger than 19σ.

Phys.Rev.Lett. 129 (2022) 19, 192002 2202.00621 [hep-ex]



A nonet of hybrid states?

Physics Letters B 834 (2022) 137478

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The phenomenology of the exotic hybrid nonet with $\pi_1(1600)$ and $\eta_1(1855)$

Vanamali Shastry [a,*], Christian S. Fischer [b,c], Francesco Giacosa [a,d]

arXiv:2203.04327

Beides π1(1600) and η1(1855), we expect also: K1(1750) and η1(1660). The last two not yet seen.

	M (MeV)	Γ (MeV)
K_1^{hyb}	1761	312 ± 97 170 ± 65
η_1^L	1661	81 ± 15 83 ± 16
η_1^H	1855	259 ± 92 157 ± 68

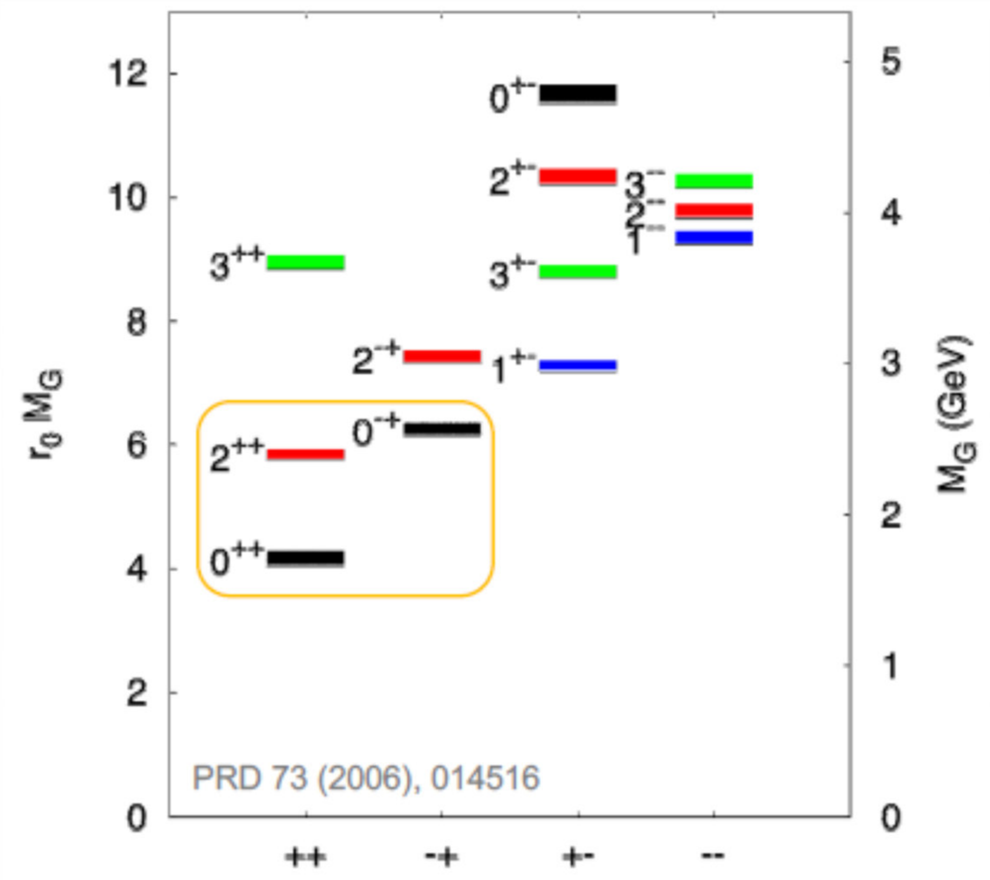
Table 7
The partial widths and branching ratios of various decay channels and the total width for the hybrid kaon $K_1^{hyb}(1750)$. We have assumed the mass of the state to be 1761 MeV [44].

Channel	Width (MeV)		Channel	Width (MeV)	
	Set-1	Set-2		Set-1	Set-2
$\Gamma_{K_1(1270)\pi}$	125 ± 42	48 ± 25	$\Gamma_{\rho K}$	2.18 ± 0.56	2.19 ± 0.57
$\Gamma_{K_1(1400)\pi}$	103 ± 45	98 ± 43	$\Gamma_{\omega K}$	0.82 ± 0.21	0.82 ± 0.21
$\Gamma_{h_1(1170)K}$	1.53 ± 0.28	1.37 ± 0.24	$\Gamma_{\phi K}$	0.49 ± 0.12	0.49 ± 0.13
$\Gamma_{\eta K}$	0.29 ± 0.07	0.29 ± 0.07	$\Gamma_{K^*\pi}$	0.67 ± 0.17	0.67 ± 0.17
$\Gamma_{\eta' K}$	2.77 ± 0.62	2.81 ± 0.62	$\Gamma_{K^*\eta}$	0.30 ± 0.08	0.30 ± 0.08
$\Gamma_{\rho K^*}$	0.045 ± 0.016	0.047 ± 0.016	$\Gamma_{\omega K^*}$	0.011 ± 0.004	0.012 ± 0.004
$\Gamma_{a_1 K}$	11.0 ± 2.32	11.3 ± 2.35	$\Gamma_{b_1 K}$	64 ± 14	3.11 ± 2.88
			Γ_{tot}	312 ± 97	170 ± 65

Table 6
The partial widths and branching ratios of various decay channels and the total width of the η_1^L (left) and the $\eta_1(1855)$ (right) for $\theta_h = 15°$. This corresponds to the "Scenario-2" discussed in the text.

Channel	Width (MeV)		Channel	Width (MeV)	
	Set-1	Set-2		Set-1	Set-2
$\Gamma_{a_1\pi}$	80 ± 15	82 ± 16	$\Gamma_{K_1(1270)K}$	253 ± 92	151 ± 67
Γ_{K^*K}	0.29 ± 0.075	0.29 ± 0.075	Γ_{K^*K}	1.45 ± 0.37	1.46 ± 0.38
$\Gamma_{\eta'\eta}$	0.41 ± 0.09	0.41 ± 0.09	$\Gamma_{\eta'\eta}$	2.28 ± 0.51	2.31 ± 0.51
$\Gamma_{K_1(1270)K}$	0	0	$\Gamma_{a_1\pi}$	0	0
$\Gamma_{\rho\rho}$	0.081 ± 0.028	0.082 ± 0.029	$\Gamma_{\rho\rho}$	0	0
$\Gamma_{K^*K^*}$	0	0	$\Gamma_{K^*K^*}$	0.075 ± 0.027	0.077 ± 0.028
$\Gamma_{\omega\phi}$	0	0	$\Gamma_{\omega\phi}$	$\sim 10^{-4}$	$\sim 10^{-4}$
$\Gamma_{f_1\eta}$	0	0	$\Gamma_{f_1\eta}$	2.15 ± 0.56	2.21 ± 0.57
Γ_{tot}	81 ± 15	83 ± 16	Γ_{tot}	259 ± 92	157 ± 68

Light glueballs



Scalar glueball

PHYSICAL REVIEW D **90**, 114005 (2014)

Is $f_0(1710)$ a glueball?

Stanislaus Janowski,[1] Francesco Giacosa,[1,2] and Dirk H. Rischke[1]
[1]*Institute for Theoretical Physics, Goethe University, Max-von-Laue-Straße 1, 60438 Frankfurt am Main, Germany*
[2]*Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland*
(Received 26 August 2014; published 2 December 2014)

PRL **110,** 021601 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2

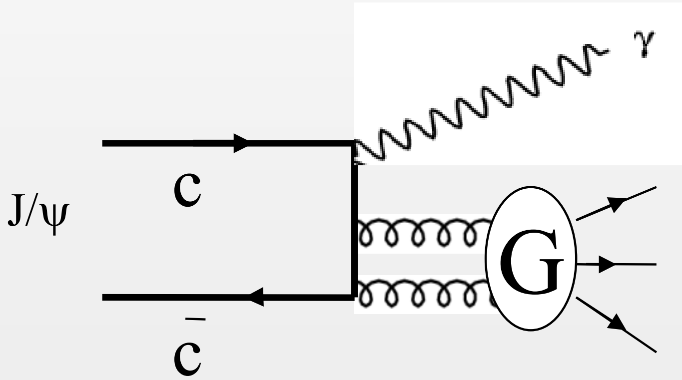
Scalar Glueball in Radiative J/ψ Decay on the Lattice

Long-Cheng Gui,[1,2] Ying Chen,[1,2,*] Gang Li,[3] Chuan Liu,[4] Yu-Bin Liu,[5] Jian-Ping Ma,[6]
Yi-Bo Yang,[1,2] and Jian-Bo Zhang[7]

(CLQCD Collaboration)

[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People's Republic of China*
[2]*Theoretical Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, People's Republic of China*
[3]*Department of Physics, Qufu Normal University, Qufu 273165, People's Republic of China*
[4]*School of Physics and Center for High Energy Physics, Peking University, Beijing 100871, People's Republic of China*
[5]*School of Physics, Nankai University, Tianjin 300071, People's Republic of China*
[6]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*
[7]*Department of Physics, Zhejiang University, Zhejiang 310027, People's Republic of China*
(Received 5 June 2012; published 10 January 2013)

The form factors in the radiative decay of J/ψ to a scalar glueball are studied within quenched lattice QCD on anisotropic lattices. The continuum extrapolation is carried out by using two different lattice spacings. With the results of these form factors, the partial width of J/ψ radiatively decaying into the pure gauge scalar glueball is predicted to be 0.35(8) keV, which corresponds to a branching ratio of $3.8(9) \times 10^{-3}$. By comparing with experiments, out results indicate that $f_0(1710)$ has a larger overlap with the pure gauge glueball than other related scalar mesons.



$\gamma f_0(1710) \to \gamma K \overline{K}$	$(8.5\ ^{+1.2}_{-0.9}) \times 10^{-4}$
$\gamma f_0(1710) \to \gamma \pi \pi$	$(4.0 \pm 1.0) \times 10^{-4}$
$\gamma f_0(1710) \to \gamma \omega \omega$	$(3.1 \pm 1.0) \times 10^{-4}$
$\gamma f_0(1710) \to \gamma \eta \eta$	$(2.4\ ^{+1.2}_{-0.7}) \times 10^{-4}$

$\gamma f_0(1500) \to \gamma \pi \pi$	$(1.01 \pm 0.32) \times 10^{-4}$
$\gamma f_0(1500) \to \gamma \eta \eta$	$(1.7\ ^{+0.6}_{-1.4}) \times 10^{-5}$

Recent BES results

Radiative J/ψ decays

- scalar glueball decays to $\eta\eta'$ expected to be suppressed $\frac{B(G\to\eta\eta')}{B(G\to\pi\pi)} < 0.04$

  PRD 92, 121902 (2015)

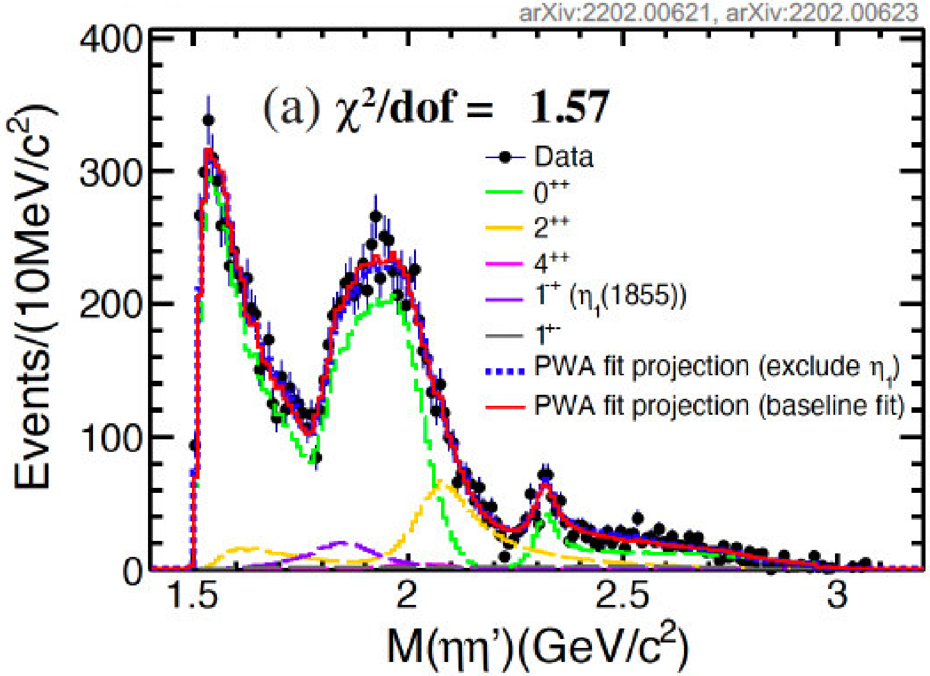
- significant $f_0(1500)$ contribution, but no $f_0(1710)$ (there is a small $f_0(1810)$ in the fit)

- $\frac{B(f_0(1500)\to\eta\eta')}{B(f_0(1500)\to\pi\pi)} = (8.96^{+2.95}_{-2.87})\times10^{-2}$,

- $\frac{B(f_0(1710)\to\eta\eta')}{B(f_0(1710)\to\pi\pi)} < 1.61\times10^{-3}$ (90% CL)

- $\frac{B(f_0(1810)\to\eta\eta')}{B(f_0(1710)\to\pi\pi)} = (1.39^{+0.62}_{-0.52})\times10^{-2}$

arXiv:2202.00621, arXiv:2202.00623



Nils Hüsken
on behalf of the BESIII collaboration

Workshop: Recent results and perspectives in hadron physics
Orsay, October 17th, 2022

Pseudoscalar glueball

PHYSICAL REVIEW LETTERS **129**, 042001 (2022)

Observation of a State $X(2600)$ in the $\pi^+\pi^-\eta'$ System in the Process $J/\psi \to \gamma\pi^+\pi^-\eta'$

$\pi^+\pi^-$ invariant mass spectrum. A simultaneous fit on the $\pi^+\pi^-\eta'$ and $\pi^+\pi^-$ invariant mass spectra with the two η' decay modes indicates that the mass and width of the $X(2600)$ state are $2618.3 \pm 2.0^{+16.3}_{-1.4}$ MeV$/c^2$ and $195 \pm 5^{+26}_{-17}$ MeV, where the first uncertainties are statistical, and the second systematic.

PHYSICAL REVIEW D **87**, 054036 (2013)

Decay of the pseudoscalar glueball into scalar and pseudoscalar mesons

Walaa I. Eshraim,[1] Stanislaus Janowski,[1] Francesco Giacosa,[1] and Dirk H. Rischke[1,2]

Quantity	$M_{\tilde{G}} = 2.6$ GeV
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{\rm tot}_{\tilde{G}}$	0.049
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{\rm tot}_{\tilde{G}}$	0.019
$\Gamma_{\tilde{G}\to \eta\eta\eta}/\Gamma^{\rm tot}_{\tilde{G}}$	0.016
$\Gamma_{\tilde{G}\to \eta\eta\eta'}/\Gamma^{\rm tot}_{\tilde{G}}$	0.0017
$\Gamma_{\tilde{G}\to \eta\eta'\eta'}/\Gamma^{\rm tot}_{\tilde{G}}$	0.00013
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{\rm tot}_{\tilde{G}}$	0.47
$\Gamma_{\tilde{G}\to \eta\pi\pi}/\Gamma^{\rm tot}_{\tilde{G}}$	0.16
$\Gamma_{\tilde{G}\to \eta'\pi\pi}/\Gamma^{\rm tot}_{\tilde{G}}$	0.095

glueball-glueball scattering: a new state?

Eur. Phys. J. C (2022) 82:487
https://doi.org/10.1140/epjc/s10052-022-10403-z

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

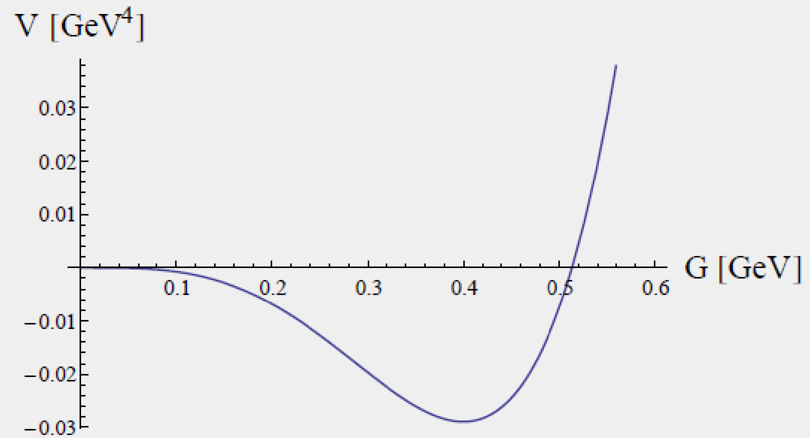
Glueball–glueball scattering and the glueballonium

Francesco Giacosa[1,2], Alessandro Pilloni[3,4], Enrico Trotti[1,a]

$$\mathcal{L}_{\text{dil}} = \frac{1}{2}(\partial_\mu G)^2 - V(G),$$

with

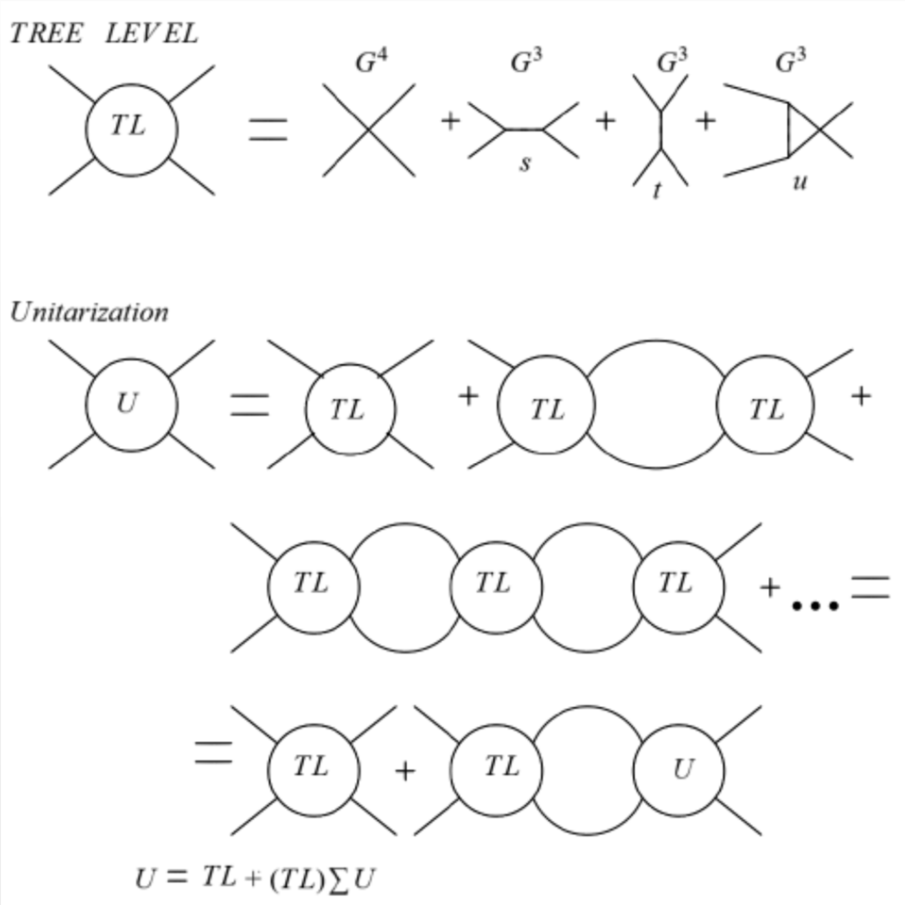
$$V(G) = \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right).$$

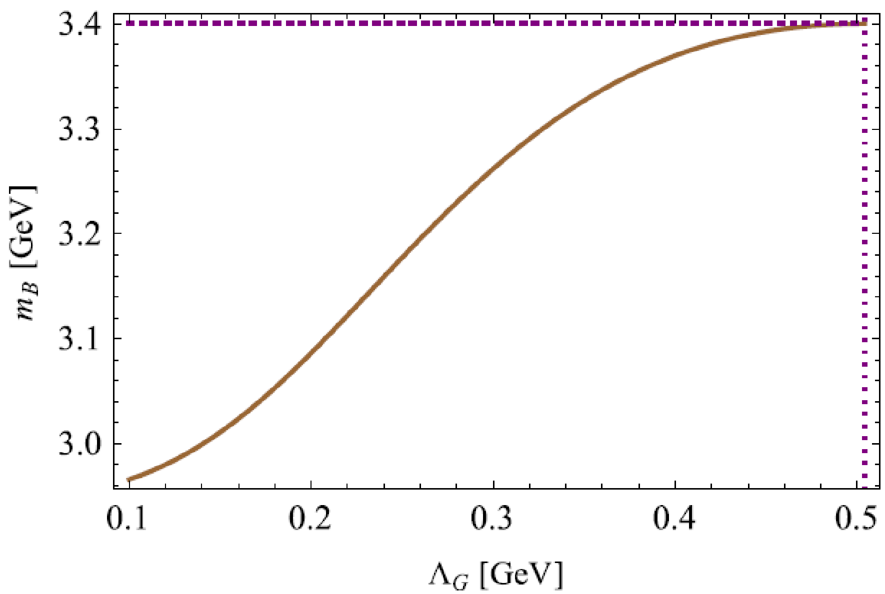


A. A. Migdal and M. A. Shifman, "Dilaton Effective Lagrangian In Gluodynamics," Phys. Lett.B114, 445 (1982)

Glueballonium mass

$$V(G) = V(\Lambda_G) + \frac{1}{2} m_G^2 G^2 + \frac{1}{3!}\left(5\frac{m_G^2}{\Lambda_G}\right) G^3 + \frac{1}{4!}\left(11\frac{m_G^2}{\Lambda_G^2}\right) G^4 + \frac{1}{5!}\left(6\frac{m_G^2}{\Lambda_G^3}\right) G^5 + \ldots$$





Can one see that? In YM-lattice, probably yes.
In experiment? Hard, but…

Dulcis in fundo: scalar sector

Eur. Phys. J. C (2022) 82:487
https://doi.org/10.1140/epjc/s10052-022-10403-z

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

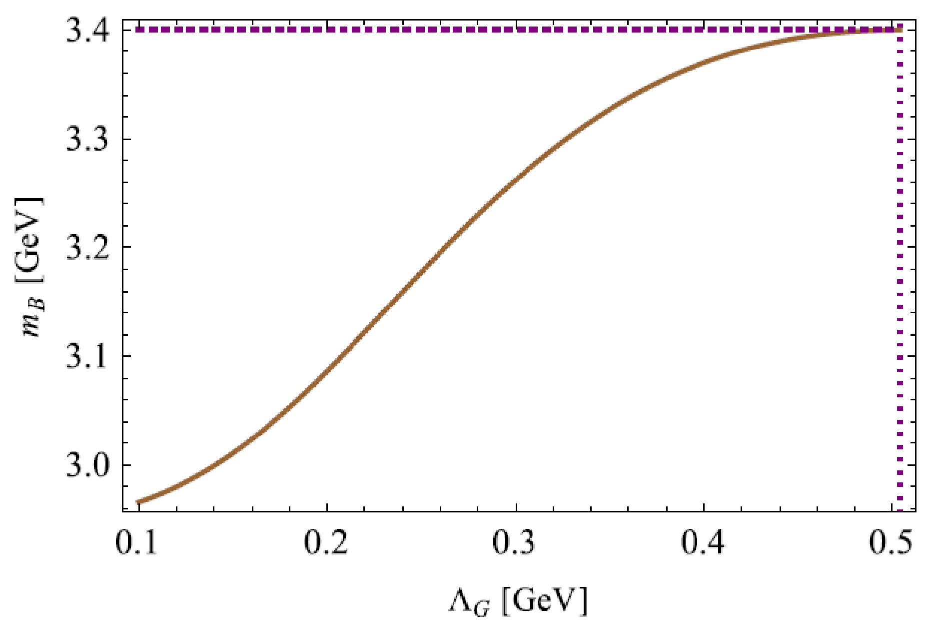
Glueball–glueball scattering and the glueballonium

Francesco Giacosa[1,2], **Alessandro Pilloni**[3,4], **Enrico Trotti**[1,a]

$$\mathcal{L}_{\text{dil}} = \frac{1}{2}(\partial_\mu G)^2 - V(G),$$

with

$$V(G) = \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right).$$

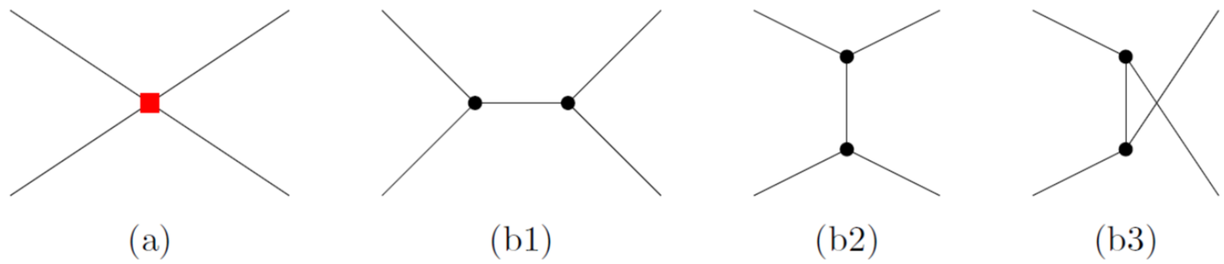


Higgsonium?

$$V(H) = V(v) + \frac{m_H^2}{2!}(H-v)^2 + \frac{g}{3!}(H-v)^3 + \frac{\lambda}{4!}(H-v)^4 + \frac{g_{5H}}{5!}(H-v)^5 + \dots$$
$$= V(v) + \frac{m_H^2}{2!}h^2 + \frac{g}{3!}h^3 + \frac{\lambda}{4!}h^4 + \frac{g_{5H}}{5!}h^5 + \dots$$

$$g = d_3 \frac{3m_H^2}{v} \qquad \lambda = d_4 \frac{3m_H^2}{v^2}$$

$$i\mathcal{M}_a = -i\lambda$$
$$i\mathcal{M}_{b1} = -ig^2 \frac{1}{s - m_H^2 + i\epsilon}$$
$$i\mathcal{M}_{b2} = -ig^2 \frac{1}{t - m_H^2 + i\epsilon}$$
$$i\mathcal{M}_{b3} = -ig^2 \frac{1}{u - m_H^2 + i\epsilon}$$

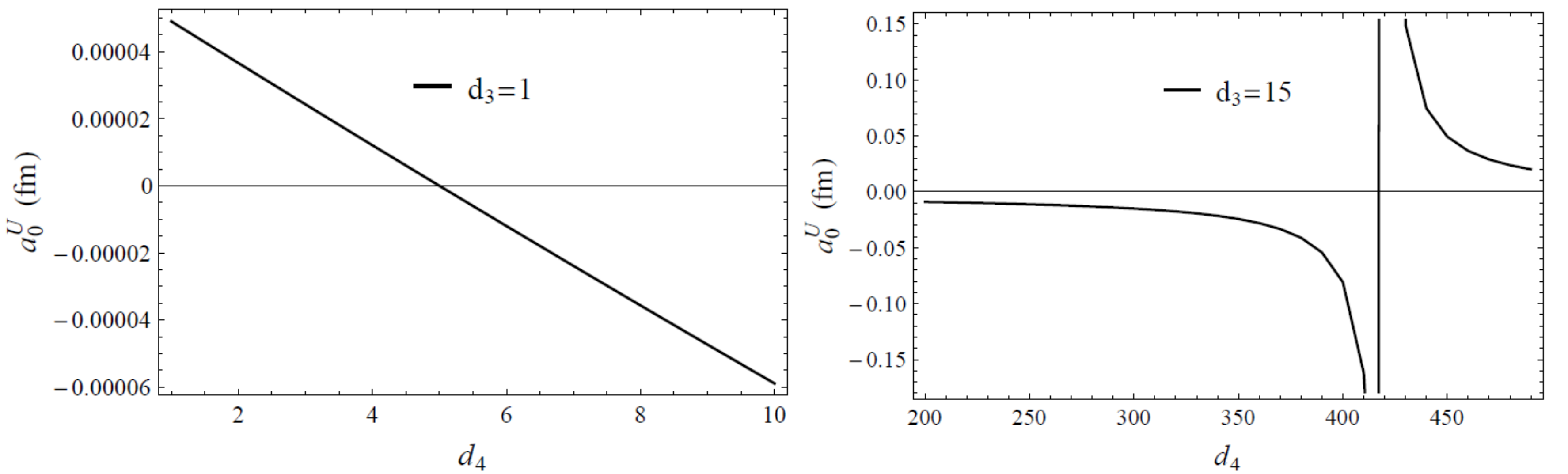


Higgsonium/2

$$a_0^{TL} = \frac{1}{32\pi m_H} A_0(s = 4m_H^2) = \frac{-\lambda + \frac{5g^2}{3m_H^2}}{32\pi m_H} = (4.86 \pm 0.01) \times 10^{-5} \text{ fm.}$$

$$a_2^{TL} = \frac{g^2}{30\pi m_H^7} = \frac{3d_3^2}{10\pi v^2 m_H^3} \stackrel{\text{SM}}{=} 2.4 \times 10^{-16} \text{ fm}^5 ,$$

$$a_4^{TL} = \frac{8g^2}{315\pi m_H^{11}} = \frac{8d_3^2}{35\pi v^2 m_H^7} \stackrel{\text{SM}}{=} 1.1 \times 10^{-27} \text{ fm}^9 .$$



Beyibd Breit-Wigner

Eur. Phys. J. A (2021) 57:336
https://doi.org/10.1140/epja/s10050-021-00641-2

THE EUROPEAN PHYSICAL JOURNAL A

Check for updates

Regular Article - Theoretical Physics

A simple alternative to the relativistic Breit–Wigner distribution

Francesco Giacosa[1,2], Anna Okopińska[1], Vanamali Shastry[1,a]

$$d_S^{\mathrm{BW}}(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}}$$

$$d_S^{\mathrm{rBW}}(E) = \frac{2E}{\pi} \frac{M\Gamma}{(E^2-M^2)^2 + (M\Gamma)^2} \theta(E)$$

$$d_S^{\mathrm{Sill}}(E) = \frac{2E}{\pi} \frac{\sqrt{E^2-E_{th}^2}\,\tilde{\Gamma}}{(E^2-M^2)^2 + (\sqrt{E^2-E_{th}^2}\,\tilde{\Gamma})^2} \theta(E-E_{th})$$

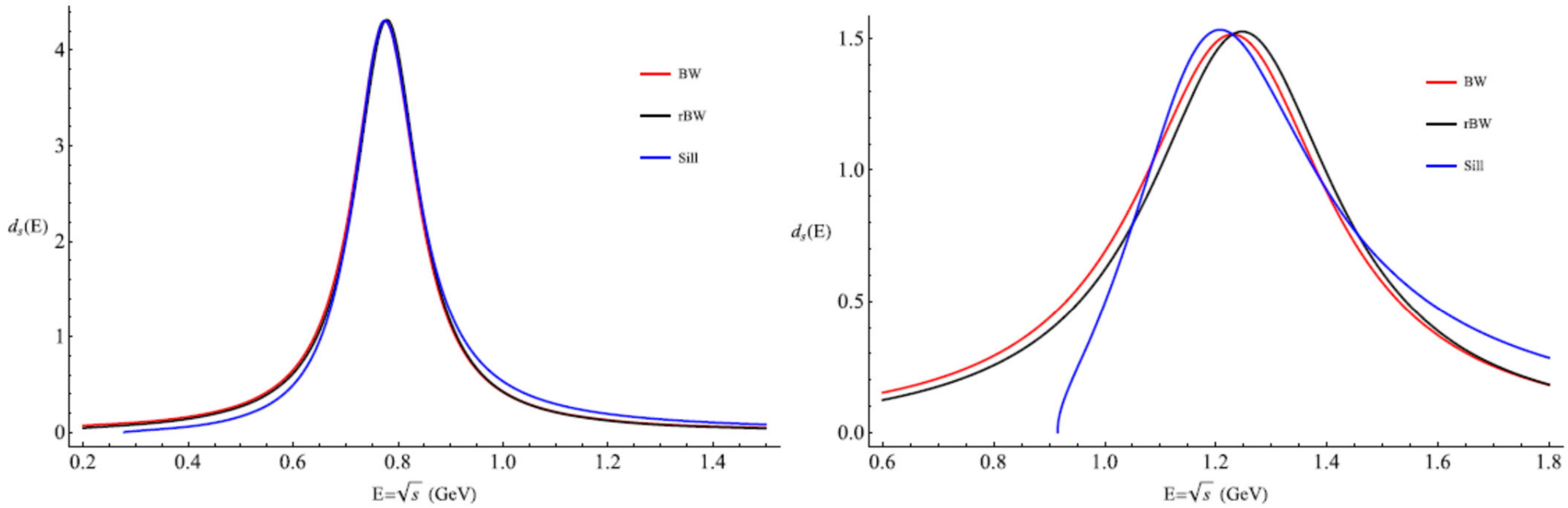
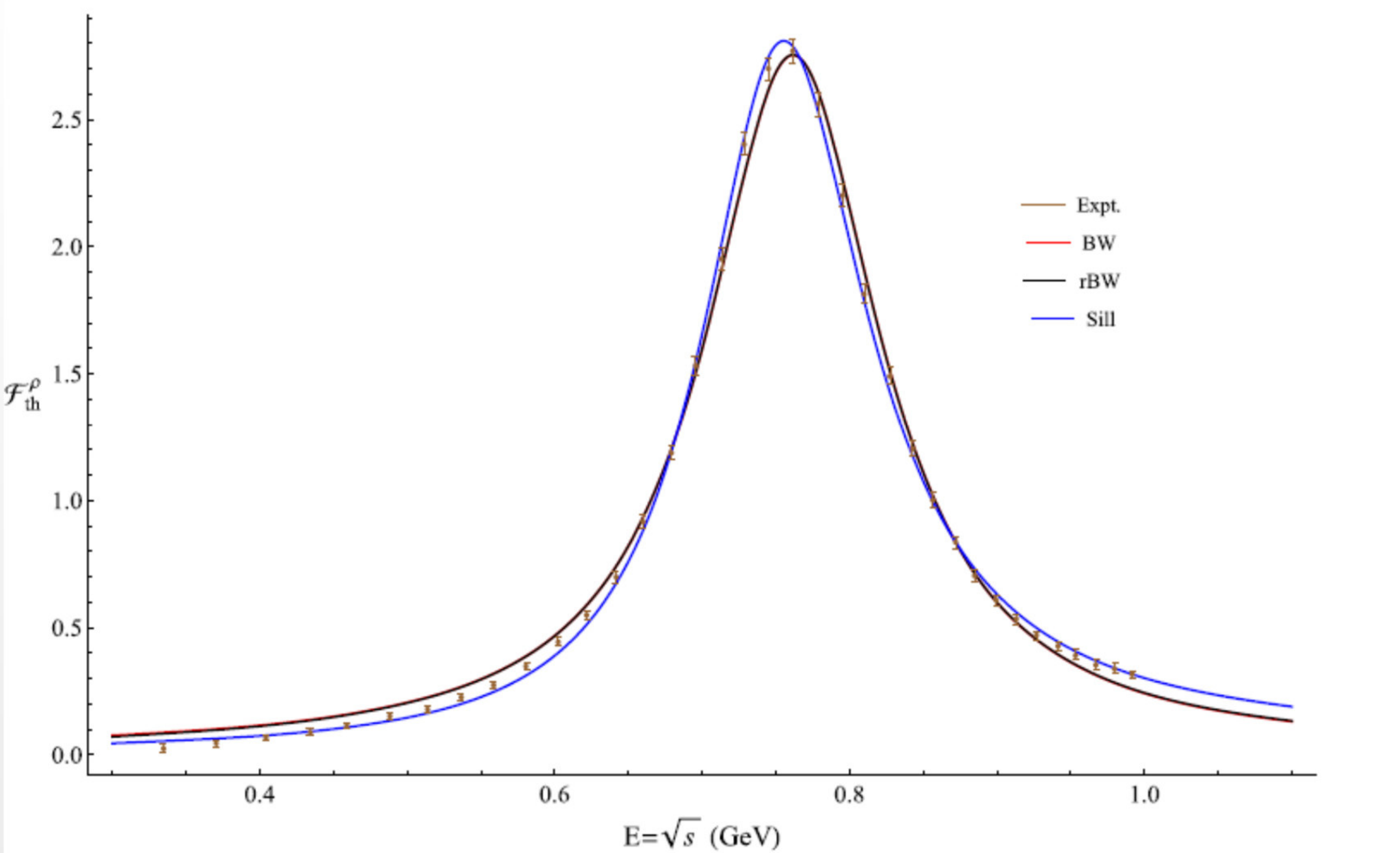
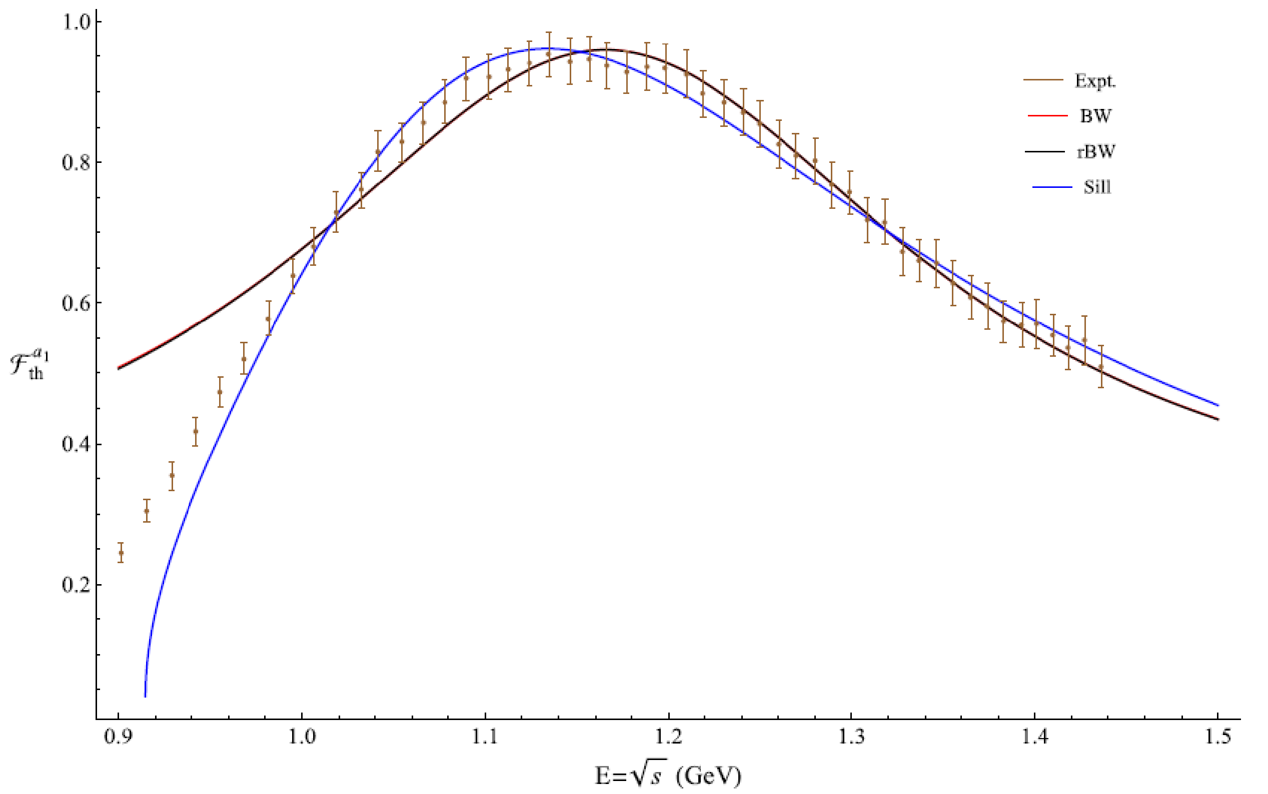


Fig. 1 Illustrative comparison of the four distributions discussed in the paper. Left panel, peak far away from the threshold ($\rho(770)$, $M = 0.775$ GeV, $\Gamma = 0.1478$ GeV, and $E_{th} = 2m_\pi$); right panel, peak near the threshold ($a_1(1260)$, $M = 1.230$ GeV, $\Gamma = 0.5$ GeV, and $E_{th} = m_\rho + m_\pi$)



Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	761.64 ± 0.32	144.6 ± 1.3	10.16	$761.6 - i\,72.3$
Relativistic BW	758.1 ± 0.33	145.2 ± 1.3	9.42	$761.5 - i\,72.3$
Sill	755.82 ± 0.33	137.3 ± 1.1	3.52	$751.7 - i\,68.6$



Distribution	M (MeV)	Γ (MeV)	χ^2/d.o.f	$\sqrt{s_{pole}}$(MeV)
Nonrelativistic BW	1165.6 ± 1.2	415 ± 15	4.31	$1166 - i\,208$
Relativistic BW	1146.5 ± 1.6	424 ± 16	4.25	$1165 - i\,209$
Sill	1181.3 ± 3.4	539 ± 27	3.52	$1046 - i\,250$

Conclusions and outlook

Many nonets fit well in the quark-antiquark picture, but…

- axial-tensor mesons basically unknown;
- pseudotensor mesons, is there a large isoscalar mixing?
- vector mesons: which is the orbitally excited ϕ meson?

Unconventional mesons:

- hybrid mesons: a new nonet?
- Glueballonium (possible), Higgsonium (improbable)

Outlook:

- tensor glueball (ongoing)

Thanks

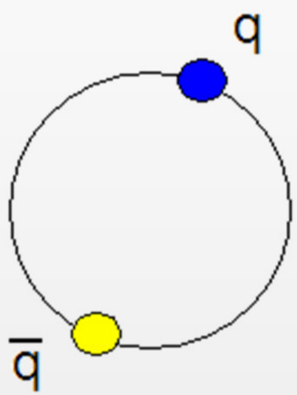
Back-up slides

Conventional mesons

Quark: u,d,s,… R,G,B

Quark-antiquark bound states: conventional mesons

$$\left| color \right\rangle = \sqrt{1/3}\,(\bar{R}R + \bar{B}B + \bar{G}G)$$

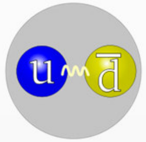


$$P = -(-1)^{L} \qquad C = (-1)^{L+S}$$

L , S → J = L+ S

$$L = S = 0 \rightarrow J^{PC} = 0^{-+} \quad \text{pseudoscalar mesons}$$

$$\left|\pi^{+}\right\rangle = \left|u\bar{d}\right\rangle \left|\text{space}: L = 0\right\rangle \left|\text{spin}: S = 0\right\rangle \left|\bar{R}R + \bar{B}B + \bar{G}G\right\rangle$$



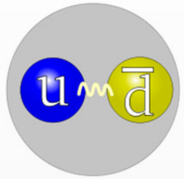
$$\left|K^{+}\right\rangle = \left|u\bar{s}\right\rangle \left|\text{space}: L = 0\right\rangle \left|\text{spin}: S = 0\right\rangle \left|\bar{R}R + \bar{B}B + \bar{G}G\right\rangle$$

...

$$\left|D^{0}\right\rangle = \left|u\bar{c}\right\rangle \left|\text{space}: L = 0\right\rangle \left|\text{spin}: S = 0\right\rangle \left|\bar{R}R + \bar{B}B + \bar{G}G\right\rangle$$

...

$$L=0\ ,\ S=1 \rightarrow J^{PC}=1^{--}$$ vector mesons



$$\left|\rho^{+}\right\rangle=\left|u\bar{d}\right\rangle\left|space:L=0\right\rangle\left|spin:S=1\right\rangle\left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

...

$$\left|K^{*}(892)^{+}\right\rangle=\left|u\bar{s}\right\rangle\left|space:L=0\right\rangle\left|spin:S=1\right\rangle\left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

...

$$\left|D^{*0}\right\rangle=\left|u\bar{c}\right\rangle\left|space:L=0\right\rangle\left|spin:S=1\right\rangle\left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

...

$$\left|j/\Psi\right\rangle=\left|c\bar{c}\right\rangle\left|space:L=0\right\rangle\left|spin:S=1\right\rangle\left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

$$L=S=1 \rightarrow J^{PC}=0^{++}$$ scalar mesons

$$|\sigma\rangle = |u\bar{u}+d\bar{d}\rangle|\text{space}: L=1\rangle|\text{spin}: S=1\rangle|\bar{R}R+\bar{B}B+\bar{G}G\rangle$$

corresponds to the resonance $f_0(1370)$.

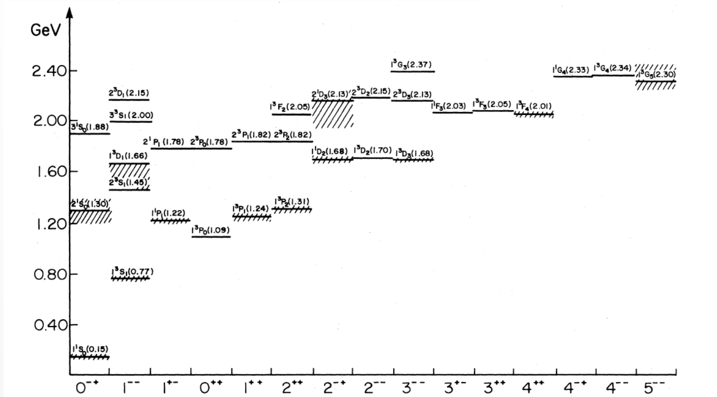
…

…

$$|\chi_{c0}(1S)\rangle = |c\bar{c}\rangle|\text{space}: L=1\rangle|\text{spin}: S=1\rangle|\bar{R}R+\bar{B}B+\bar{G}G\rangle$$

Quark model(s) and their QFT extensions

Mesons in a Relativized Quark Model with Chromodynamics
S. Godfrey, N. Isgur
Phys.Rev. D32 (1985) **189-231**



QCD phenomenology based on a chiral effective Lagrangian
T. Hatsuda, T. Kunihiro
Phys.Rept. **247** (1994) 221-367

NJL: quark-based model with
chiral symmetry and SSB
chiral condensate
Effective quark mass
Mesons as quarkonia (pion: ok)

The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states
R. Alkofer, L. von Smekal
Phys.Rept. **353** (2001) 281

Baryons as relativistic three-quark bound states
G. Eichmann et al.
Progr. Part. Nucl. Phys. **91** (2016) 1

DS:
quarks and gluons propagators from QCD
Condensates
Effective quark and gluon masses
Spectra of mesons as quarkonia
(pion: ok) and baryons as qqq states

Quark-antiquark currents

Meson	$n^{2S+1}L_J$	J^{PC}	S	L	Hermitian quark current operators
pseudoscalar	1^1S_0	0^{-+}	0	0	$P_{ij} = \bar{q}_j\, i\gamma^5\, q_i$
vector	1^3S_1	1^{--}	1		$V_{ij}^\mu = \bar{q}_j\, \gamma^\mu\, q_i$
pseudovector	1^1P_1	1^{+-}	0	1	$P_{ij}^\mu = \bar{q}_j\, \gamma^5 \overleftrightarrow{\partial}^\mu\, q_i$
scalar	1^3P_0	0^{++}	1		$S_{ij} = \bar{q}_j\, q_i$
axial vector	1^3P_1	1^{++}	1		$A_{ij}^\mu = \bar{q}_j\, \gamma^5\gamma^\mu\, q_i$
tensor	1^3P_2	2^{++}	1		$X_{ij}^{\mu\nu} = \bar{q}_j\, i\left[\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu - \frac{2}{3}\tilde{G}^{\mu\nu}\overleftrightarrow{\not\partial}\right] q_i$
pseudotensor	1^1D_2	2^{-+}	0	2	$T_{ij}^{\mu\nu} = \bar{q}_j\, i\left[\gamma^5 \overleftrightarrow{\partial}^\mu \overleftrightarrow{\partial}^\nu - \frac{2}{3}\tilde{G}^{\mu\nu}\overleftrightarrow{\partial}_\alpha \overleftrightarrow{\partial}^\alpha\right] q_i$
excited vector	1^3D_1	1^{--}	1		$S_{ij}^\mu = \bar{q}_j\, \overleftrightarrow{\partial}^\mu\, q_i$
axial tensor	1^3D_2	2^{--}	1		$B_{ij}^{\mu\nu} = \bar{q}_j\, i\left[\gamma^5\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^5\gamma^\nu \overleftrightarrow{\partial}^\mu - \frac{2}{3}\tilde{G}^{\mu\nu}\gamma^5\overleftrightarrow{\not\partial}\right] q_i$
spin-3 tensor	1^3D_3	3^{--}	1		$\ldots$

Decays of J=3-mesons

TABLE III. Effective relativistic interaction terms describing the strong decays of mesons with $J^{PC} = 3^{--}$.

Decay mode	Interaction Lagrangians
$3^{--} \rightarrow 0^{-+} + 0^{-+}$	$\mathcal{L}_{w_3pp} = g_{w_3pp} \mathrm{tr}[W_3^{\mu\nu\rho}[P, (\partial_\mu \partial_\nu \partial_\rho P)]_-]$
$3^{--} \rightarrow 0^{-+} + 1^{--}$	$\mathcal{L}_{w_3v_1p} = g_{w_3v_1p} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}[W_{3,\mu\alpha\beta}\{(\partial_\nu V_{1,\rho}), (\partial^\alpha \partial^\beta \partial_\sigma P)\}_+]$
$3^{--} \rightarrow 0^{-+} + 2^{++}$	$\mathcal{L}_{w_3a_2p} = g_{w_3a_2p} \varepsilon_{\mu\nu\rho\sigma} \mathrm{tr}[W_3{}^{\mu}{}_{\alpha\beta}[(\partial^\nu A_2^{\rho\alpha}), (\partial^\sigma \partial^\beta P)]_-]$
$3^{--} \rightarrow 0^{-+} + 1^{+-}$	$\mathcal{L}_{w_3b_1p} = g_{w_3b_1p} \mathrm{tr}[W_3^{\mu\nu\rho}\{B_{1,\mu}, (\partial_\nu \partial_\rho P)\}_+]$
$3^{--} \rightarrow 0^{-+} + 1^{++}$	$\mathcal{L}_{w_3a_1p} = g_{w_3a_1p} \mathrm{tr}[W_3^{\mu\nu\rho}[A_{1,\mu}, (\partial_\nu \partial_\rho P)]_-]$
$3^{--} \rightarrow 1^{--} + 1^{--}$	$\mathcal{L}_{w_3v_1v_1} = g_{w_3v_1v_1} \mathrm{tr}[W_3^{\mu\nu\rho}[(\partial_\mu V_{1,\nu}), V_{1,\rho}]_-]$

$$W_3^{\mu\nu\rho} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_{3,N}^{\mu\nu\rho} + \rho_3^{0\mu\nu\rho}}{\sqrt{2}} & \rho_3^{+\mu\nu\rho} & K_3^{+\mu\nu\rho} \\ \rho_3^{-\mu\nu\rho} & \frac{\omega_{3,N}^{\mu\nu\rho} - \rho_3^{0\mu\nu\rho}}{\sqrt{2}} & K_3^{0\mu\nu\rho} \\ K_3^{-\mu\nu\rho} & \bar{K}_3^{0\mu\nu\rho} & \omega_{3,S}^{\mu\nu\rho} \end{pmatrix}$$

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

$$V_1^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_{1,N}^\mu + \rho_1^{0\mu}}{\sqrt{2}} & \rho_1^{+\mu} & K_1^{*+\mu} \\ \rho_1^{-\mu} & \frac{\omega_{1,N}^\mu - \rho_1^{0\mu}}{\sqrt{2}} & K_1^{*0\mu} \\ K_1^{*-\mu} & \bar{K}_1^{*0\mu} & \omega_{1,S}^\mu \end{pmatrix}$$

TABLE IV. Decay amplitudes for different decay modes.

Decay mode	$\frac{1}{7}\lvert\mathcal{M}\rvert^2$
$3^{--} \rightarrow 0^{-+} + 0^{-+}$	$g_{w_3pp}^2 \frac{2}{35} \lvert\vec{k}_{p^{(1)},p^{(2)}}\rvert^6$
$3^{--} \rightarrow 0^{-+} + 1^{--}$	$g_{w_3v_1p}^2 \frac{8}{105} \lvert\vec{k}_{v_1,p}\rvert^6 m_{w_3}^2$
$3^{--} \rightarrow 0^{-+} + 2^{++}$	$g_{w_3a_2p}^2 \frac{2}{105} \lvert\vec{k}_{a_2,p}\rvert^4 \frac{m_{w_3}^2}{m_{a_2}^2} (2\lvert\vec{k}_{a_2,p}\rvert^2 + 7m_{a_2}^2)$
$3^{--} \rightarrow 0^{-+} + 1^{+-}$	$g_{w_3b_1p}^2 \frac{2}{105} \lvert\vec{k}_{b_1,p}\rvert^4 (7 + 3\frac{\lvert\vec{k}_{b_1,p}\rvert^2}{m_{b_1}^2})$
$3^{--} \rightarrow 0^{-+} + 1^{++}$	$g_{w_3a_1p}^2 \frac{2}{105} \lvert\vec{k}_{a_1,p}\rvert^4 (7 + 3\frac{\lvert\vec{k}_{a_1,p}\rvert^2}{m_{a_1}^2})$
$3^{--} \rightarrow 1^{--} + 1^{--}$	$g_{w_3v_1v_1}^2 \frac{1}{105} (m_{v_1^{(1)}}^2 m_{v_1^{(2)}}^2)^{-1} \lvert\vec{k}_{v_1^{(1)},v_2^{(2)}}\rvert^2 [6\lvert\vec{k}_{v_1^{(1)},v_1^{(2)}}\rvert^4$ $+35 m_{v_1^{(1)}}^2 m_{v_1^{(2)}}^2 + 14\lvert\vec{k}_{v_1^{(1)},v_1^{(2)}}\rvert^2 (m_{v_1^{(1)}}^2 + m_{v_1^{(2)}}^2)]$

Tensor and axial-tensor: the Lagrangian

2^{++}, $^3P_2$	$\begin{cases} a_2(1320) \\ K_2^*(1430) \\ f_2(1270), f_2'(1525) \end{cases}$	$V_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma_\mu i\overleftrightarrow{D}_\mu + \cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L_{\mu\nu}^{ij} = \bar{q}_L^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q_L^i)$	$L_{\mu\nu} \to U_L L_{\mu\nu} U_L^\dagger$
2^{--}, $^3D_2$	$\begin{cases} \rho_2(?) \\ K_2(1820) \\ \omega_2(?), \phi_2(?) \end{cases}$	$A_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R_{\mu\nu}^{ij} = \bar{q}_R^j(\gamma_\mu \overleftrightarrow{D}_\nu + \cdots)q_R^i)$	$R_{\mu\nu} \to U_R R_{\mu\nu} U_R^\dagger$

$$\mathcal{L}_{g_2^{\rm ten}} = \frac{g_2^{\rm ten}}{2}\left(\mathrm{Tr}\left[\mathbf{L}_{\mu\nu}\{L^\mu, L^\nu\}\right] + \mathrm{Tr}\left[\mathbf{R}_{\mu\nu}\{R^\mu, R^\nu\}\right]\right)$$

$$2^{++} \longrightarrow 0^{-+} + 0^{-+} \; ; \qquad 2^{--} \longrightarrow 0^{-+} + 1^{--} \; .$$

Also in this case: small isoscalar mixing angle

$$\begin{pmatrix} f_2(1270) \\ f_2'(1525) \end{pmatrix} = \begin{pmatrix} \cos\beta_T & \sin\beta_T \\ -\sin\beta_T & \cos\beta_T \end{pmatrix} \begin{pmatrix} f_{2,N} \\ f_{2,S} \end{pmatrix}$$

$$\beta_T = (3.16 \pm 0.81)^\circ$$

Citation: R.L. Workman *et al.* (Particle Data Group), to be published (2022)

$\phi(2170)$

$$I^G(J^{PC}) = 0^-(1^{--})$$

Uniwersytet Jana Kochanowskiego w Kielcach

See the review on "Spectroscopy of Light Meson Resonances."

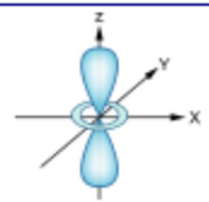
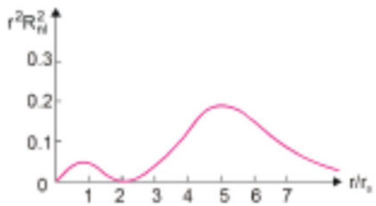
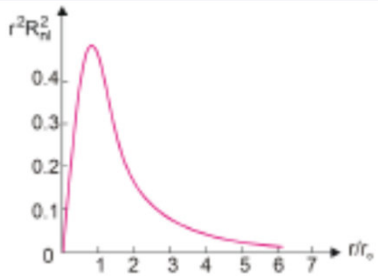
$\phi(2170)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
2162 ± 7 OUR AVERAGE	Error includes scale factor of 1.1.			

$\phi(2170)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*C*
100^{+31}_{-23} OUR AVERAGE	Error includes scale factor of 2.5.			

Excited vector mesons: properties

Type of excitation	Radially excited vector mesons	Angular momentum excited vector mesons
Quantum numbers	$n\ ^{2S+1}L_J = 2^3S_1$	$n\ ^{2S+1}L_J = 1^3D_1$
Notation	V_E	V_D
S	1 $\uparrow\uparrow$	1 $\uparrow\uparrow$
n	2	1
L	0	2
orbital		
Radial function		
Associated states	$\rho(1450), K^*(1410),$ $\phi(1680), \omega(1420)$	$\rho(1700), K^*(1680),$ $\phi_P, \omega(1650)$
Decay types	$V_E \to PP$ $V_E \to VP$ $V_E \to \gamma P$	$V_D \to PP$ $V_D \to VP$ $V_D \to \gamma P$

Radially excited vector mesons: some results

TABLE X. Decays widths of (predominantly) orbitally excited vector mesons into a pseudoscalar meson and a ground-state vector meson ($V_D \to VP$).

Decay process $V_D \to VP$	Theory (MeV)	Experiment (MeV)
$\rho(1700) \to \omega\pi$	140 ± 59	Seen (see text)
$\rho(1700) \to K^*(892)K$	56 ± 23	83 ± 66 MeV (see text)
$\rho(1700) \to \rho\eta$	41 ± 17	68 ± 42 MeV (see text)
$\rho(1700) \to \rho\eta'$	≈ 0	Not listed in PDG
$K^*(1680) \to K\rho$	64 ± 27	101 ± 35 by PDG
$K^*(1680) \to K\phi$	13 ± 6	Not listed in PDG
$K^*(1680) \to K\omega$	21 ± 9	Not listed in PDG
$K^*(1680) \to K^*(892)\pi$	81 ± 34	96 ± 33 by PDG
$K^*(1680) \to K^*(892)\eta$	0.5 ± 0.2	Not listed in PDG
$K^*(1680) \to K^*(892)\eta'$	≈ 0	Not listed in PDG
$\omega(1650) \to \rho\pi$	370 ± 156	~ 205, 154 ± 44, ~ 273, 120 ± 18 (see text)
$\omega(1650) \to K^*(892)K$	42 ± 18	Not listed in PDG
$\omega(1650) \to \omega(782)\eta$	32 ± 13	~ 100, 56 ± 30 (see text)
$\omega(1650) \to \omega(782)\eta'$	≈ 0	Not listed in PDG
$\phi(1930) \to K\bar{K}^*$	260 ± 109	Resonance not yet known
$\phi(1930) \to \phi(1020)\eta$	67 ± 28	Resonance not yet known
$\phi(1930) \to \phi(1020)\eta'$	≈ 0	Resonance not yet known

A previous work on hybrids (decay ratios only)

Eur. Phys. J. Plus (2020) 135:945
https://doi.org/10.1140/epjp/s13360-020-00900-z

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

Hybrid phenomenology in a chiral approach

Walaa I. Eshraim[1,2], Christian S. Fischer[1,3], Francesco Giacosa[2,4,a],
Denis Parganlija[5,6]

The eLSM: a chiral model of QCD

PHYSICAL REVIEW D **87,** 014011 (2013)

Meson vacuum phenomenology in a three-flavor linear sigma model with (axial-)vector mesons

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(Received 7 August 2012; published 8 January 2013)

PHYSICAL REVIEW D **90,** 114005 (2014)

Is $f_0(1710)$ a glueball?

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(Received 26 August 2014; published 2 December 2014)

Other considerations on pseudotensor mesons

Our model

- couples pseudotensor mesons to pseudoscalar, vector and tensor mesons.
- reproduces present experimental data for $\pi_2(1670)$ and $K_2(1770)$.
- identifies $\eta_2(1870)$ and $\eta_2(1645)$ with the $\bar{q}q$ pseudotensor meson nonet, if non-strange-strange mixing is large.
- predicts a large non-strange-strange mixing angle $\beta_{pt} \approx -40°$ in the isoscalar sector.
- contributes to the discussion on conflicting experimental results for the branching ratios of $\eta_2(1870)$.

Results for I = 1 and I = ½ (pseudotensor)

Decay process	Theory (MeV)	Experiment (MeV)
$\pi_2(1670) \to \rho(770)\,\pi$	80.6 ± 10.8	80.6 ± 10.8
$\pi_2(1670) \to f_2(1270)\,\pi$	146.4 ± 9.7	146.4 ± 9.7
$\pi_2(1670) \to \bar{K}^*(892)\,K + c.c.$	11.7 ± 1.6	10.9 ± 3.7
$\pi_2(1670) \to \bar{K}_2^*(1430)\,K + c.c.$	0	
$\pi_2(1670) \to f_2'(1525)\,\pi$	0.1 ± 0.1	
$\pi_2(1670) \to a_2(1320)\,\pi$	0	not seen
$\pi_2(1670) \to a_2(1320)\,\eta$	0	
$\pi_2(1670) \to a_2(1320)\,\eta'(958)$	0	
$K_2(1770) \to \rho(770)\,K$	22.2 ± 3.0	
$K_2(1770) \to \bar{K}^*(892)\,\pi$	25.5 ± 3.4	seen
$K_2(1770) \to \bar{K}^*(892)\,\eta$	10.5 ± 1.4	
$K_2(1770) \to \bar{K}^*(892)\,\eta'(958)$	0	
$K_2(1770) \to \omega(782)\,K$	8.3 ± 1.1	seen
$K_2(1770) \to \phi(1020)\,K$	4.2 ± 0.6	seen
$K_2(1770) \to a_2(1320)\,K$	0	
$K_2(1770) \to \bar{K}_2^*(1430)\,\pi$	84.5 ± 5.6	dominant
$K_2(1770) \to \bar{K}_2^*(1430)\,\eta$	0	
$K_2(1770) \to \bar{K}_2^*(1430)\,\eta'(958)$	0	
$K_2(1770) \to f_2(1270)\,K$	5.8 ± 0.4	seen
$K_2(1770) \to f_2'(1525)\,K$	0	

Table 4: Decays of $I = 1$ and $I = 1/2$ pseudotensor states. The first two entries were used to determine the coupling constants of the model, see Eq. (3.2). The total decay widths are $\Gamma^{tot}_{\pi_2(1670)} = (260 \pm 9)$ MeV and $\Gamma^{tot}_{K_2(1770)} = (186 \pm 14)$ MeV.

ArXiv: 1608.08777

Results in the isoscalar (large isoscalar mixing!)

Decay process	Theory (MeV) ($\beta_{pt} = -42°$)	Experiment (MeV)
$\eta_2(1645) \to \bar{K}^*(892)\, K + c.c.$	24.7	seen
$\eta_2(1645) \to a_2(1320)\, \pi$	186.5	
$\eta_2(1645) \to \bar{K}_2^*(1430)\, K + c.c.$	0	
$\eta_2(1645) \to f_2(1270)\, \eta$	0	not seen
$\eta_2(1645) \to f_2(1270)\, \eta'(958)$	0	
$\eta_2(1645) \to f_2'(1525)\, \eta$	0	
$\eta_2(1645) \to f_2'(1525)\, \eta'(958)$	0	
$\eta_2(1870) \to \bar{K}^*(892)\, K + c.c.$	3.3	
$\eta_2(1870) \to a_2(1320)\, \pi$	221.0	
$\eta_2(1870) \to \bar{K}_2^*(1430)\, K + c.c.$	0	
$\eta_2(1870) \to f_2(1270)\, \eta$	9.4	
$\eta_2(1870) \to f_2(1270)\, \eta'(958)$	0	
$\eta_2(1870) \to f_2'(1525)\, \eta$	0	
$\eta_2(1870) \to f_2'(1525)\, \eta'(958)$	0	

Table 6: Decays of $I = 0$ pseudotensor states. The total decay widths are $\Gamma^{tot}_{\eta_2(1645)} = (181 \pm 11)$ MeV and $\Gamma^{tot}_{\eta_2(1870)} = (225 \pm 14)$ MeV.

ArXiv: 1608.08777

For a recent re-analysis with decay widhts partial-wave :
V. Shastry, E. Trotti, F.G., Phys. Rev.D 105 (2022) 5, 054022 • e-Print: 2107.13501

Considerations

If new experimental data confirms our results,

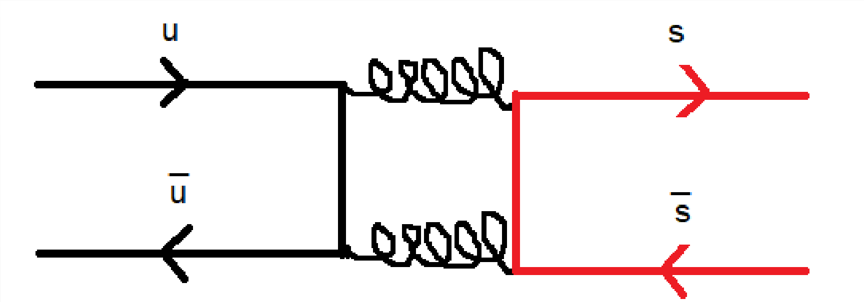
- we have good candidates for a ground-state pseudotensor meson nonet.
- the large mixing angle $\beta_{pt} \approx -40^{\circ}$ would be a mystery which deserves a detailed study.
- the current phenomenological study should be redone, including higher order corrections.

If new experimental data is at odd with our results,

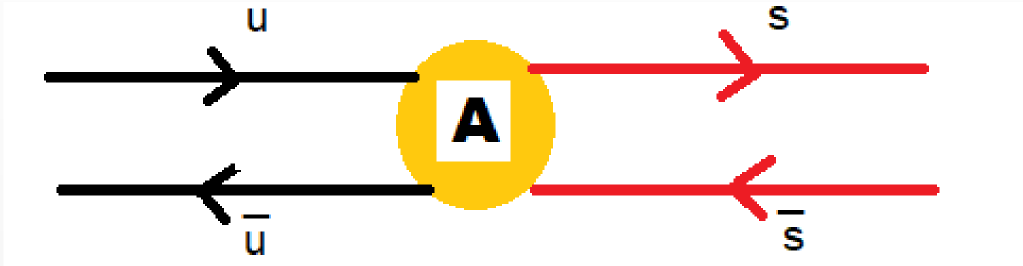
- an understanding of the lowlying pseudotensor states as a standard quark-antiquark nonet would be hard.
- $\eta_2(1870)$ could be wrongly assigned as a $\bar{q}q$-state.
- possible further mixings with (hybrid) states could be included in the model.

Large mixing angle: where does it come from?

Such a mixing is suppressed...



But this can be large



- For pseudoscalar mesons: η(547) and η'(958). Θmix = -42° Large mixing caused by the axal anomaly.
- For vector mesons: ω(782) and φ(1020). Θmix = -3° Very small mixing.
- For tensor mesons: f2(1270) and f'2(1525). Θmix = 3° Also very small mixing. Why?
- Pseudotensor mesons: also large, but confirmation is needed.

Details in: **1709.07454**

Excited vectors: Lagrangians

The Lagrangian of the model is:

$$\mathcal{L} = \mathcal{L}_{1,E} + \mathcal{L}_{1,D} + \mathcal{L}_{2,E} + \mathcal{L}_{2,D},$$

where:

$$\mathcal{L}_{1,E} = ia_E Tr[\partial^\mu P, V_{E,\mu}]P \qquad \mathcal{L}_{1,D} = ia_D Tr[\partial^\mu P, V_{D,\mu}]P$$

$$\mathcal{L}_{2,E} = b_E Tr[\tilde{V}_E^{\mu\nu}\{V_{\mu\nu}, P\}] \qquad \mathcal{L}_{2,D} = b_D Tr[\tilde{V}_D^{\mu\nu}\{V_{\mu\nu}, P\}]$$

a_E, a_D, b_E, b_D- coupling constants of the different decay types.

- $R \to \gamma P$ through „vector meson dominance"

$$V_{\mu\nu} \to V_{\mu\nu} + \frac{e_0}{g_\rho} Q F_{\mu\nu}$$

$F_{\mu\nu}-$ field strength tensor for photons
$e_0 = \sqrt{4\pi\alpha} \quad \alpha \approx 1/137 \quad g_\rho \approx 5.5 \pm 0.5 \quad Q = diag(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$

Strong and radiative decay widths

TYPE OF DECAY

- $R \to PP$

$$\Gamma_{R\to PP} = S\frac{|\vec{k}|^3}{6\pi m_R^2}[\frac{a_i}{2}\lambda_{RPP}]^2$$

- $R \to VP, R \to \gamma P$

$$\Gamma_{R\to VP} = S\frac{|\vec{k}|^3}{12\pi}[\frac{b_i}{2}\lambda_{RVP}]^2$$

EXAMPLES

- $K^*(1410) \to K\eta$

$$\Gamma_{K^*(1410)\to K\eta} = \frac{|\vec{k}|^3}{6\pi m^2_{K^*(1410)}}[\frac{a_E}{2}\frac{1}{2}(cos\theta_p - \sqrt{2}sin\theta_p)]^2$$

- $\phi(1680) \to \phi(1020)\eta$

$$\Gamma_{\phi(1680)\to\phi(1020)\eta} = \frac{|\vec{k}|^3}{12\pi}[\frac{b_E}{2}\frac{sin\theta_p}{\sqrt{2}}]^2$$

where:

$|\vec{k}| = \frac{\sqrt{m_R^2+(m_a^2-m_b^2)^2-2(m_a^2+m_b^2)m_R^2}}{2m_R}$;

m_R- mass of the decaying resonance;

a_i, b_i- coupling constants $(i = E, D)$;

m_a, m_b- masses of decay products;

$S-$ symmetry factor;

Matrices of fields

$$P=\begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \qquad V^\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega^\mu+\rho^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} & K_i^{\mu\star +} \\ \rho^{\mu -} & \frac{\omega^\mu-\rho^{\mu 0}}{\sqrt{2}} & K^{\mu\star 0} \\ K^{\mu\star -} & \bar{K}^{\mu\star 0} & \phi^\mu \end{pmatrix}$$

$$V_E^\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_E^\mu+\rho_E^{\mu 0}}{\sqrt{2}} & \rho_E^{\mu +} & K_E^{\mu\star +} \\ \rho_E^{\mu -} & \frac{\omega_E^\mu-\rho_E^{\mu 0}}{\sqrt{2}} & K_E^{\mu\star 0} \\ K_E^{\mu\star -} & \bar{K}_E^{\mu\star 0} & \phi_E^\mu \end{pmatrix} \qquad V_D^\mu=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_D^\mu+\rho_D^{\mu 0}}{\sqrt{2}} & \rho_D^{\mu +} & K_D^{\mu\star +} \\ \rho_D^{\mu -} & \frac{\omega_D^\mu-\rho_D^{\mu 0}}{\sqrt{2}} & K_D^{\mu\star 0} \\ K_D^{\mu\star -} & \bar{K}_D^{\mu\star 0} & \phi_D^\mu \end{pmatrix}$$

- $P=\{\pi, K, \eta, \eta'\}$
- $V=\{\rho(770), K^*(892), \phi(1020), \omega(782)\}$
- $V_E=\{\rho(1450), K^*(1410), \phi(1680), \omega(1420)\}$
- $V_D=\{\rho(1700), K^*(1680), \phi_p, \omega(1650)\}$

Which mass for the missing state?

TABLE I. Mass differences between the members of the two nonets of excited vector mesons.

V_E	$\rho(1450)$	$K^*(1410)$	$\omega(1420)$	$\phi(1680)$
V_D	$\rho(1700)$	$K^*(1680)$	$\omega(1650)$	$\phi(???)$
Difference	250 MeV	270 MeV	230 MeV	?

Hence, we can estimate the mass of $\phi(???)$ as

$$m_{\phi(???)} \simeq (m_{\phi(1680)} + 250 \pm 20)\ \text{MeV} = 1930 \pm 20\ \text{MeV}.$$

From now on we shall call this hypothetical state

$$\phi(???) \equiv \phi(1930).$$