Digital Signal Processing in RF Applications

Part II

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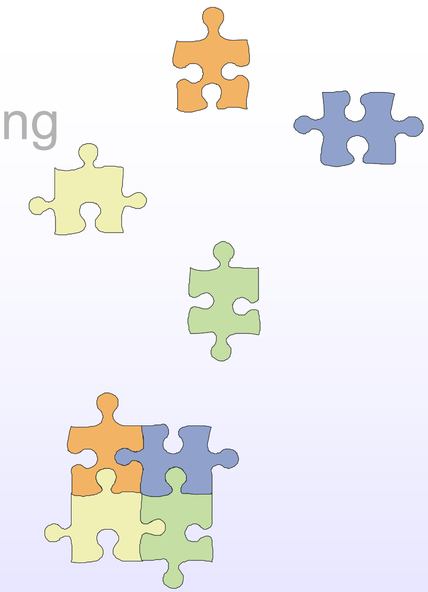
Outline

1. signal conditioning / down conversion
2. detection of amp./phase by digital I/Q sampling
   - I/Q sampling
   - non I/Q sampling
   - digital down conversion (DDC)
3. upconversion
4. **algorithms in RF applications**
   - **feedback systems**
     - **cavity amplitude and phase**
     - radial and phase loops
   - adaptive feedforward
   - system identification



RF cavity: amplitude and phase feedback

task:	maintain phase and amplitude of the accelerating field within given tolerances to accelerate a charged particle beam

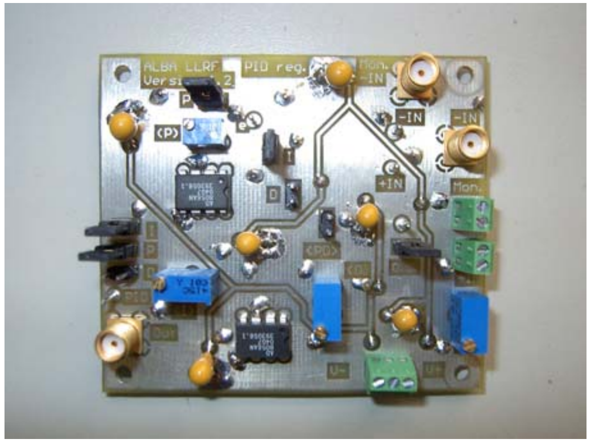
- **operating frequency:**
  few MHz / ~50 MHz (cyclotrons) – 30 GHz (CLIC)
- **required stability:**
  10^{-2} – 10^{-4} in amplitude
  (1% - 0.01%),
  1° - 0.01° (10^{-2} – 10^{-4} rad) in phase
  (0.01° @ 1.3 GHz corresponds to 21 fs)
- **often: additional tasks required** like exception handling, built-in diagnostics, automated calibration, …

- **design choices:**
  - analog / digital / combined
  - amplitude/phase versus IQ control
- **control of**
  - single cell/multicell cavity with one RF amplifier (klystron, IOT,…)
  - string of several cavities with single klystron (vector sum control)
  - pulsed / CW operation
  - normal / superconducting cavities

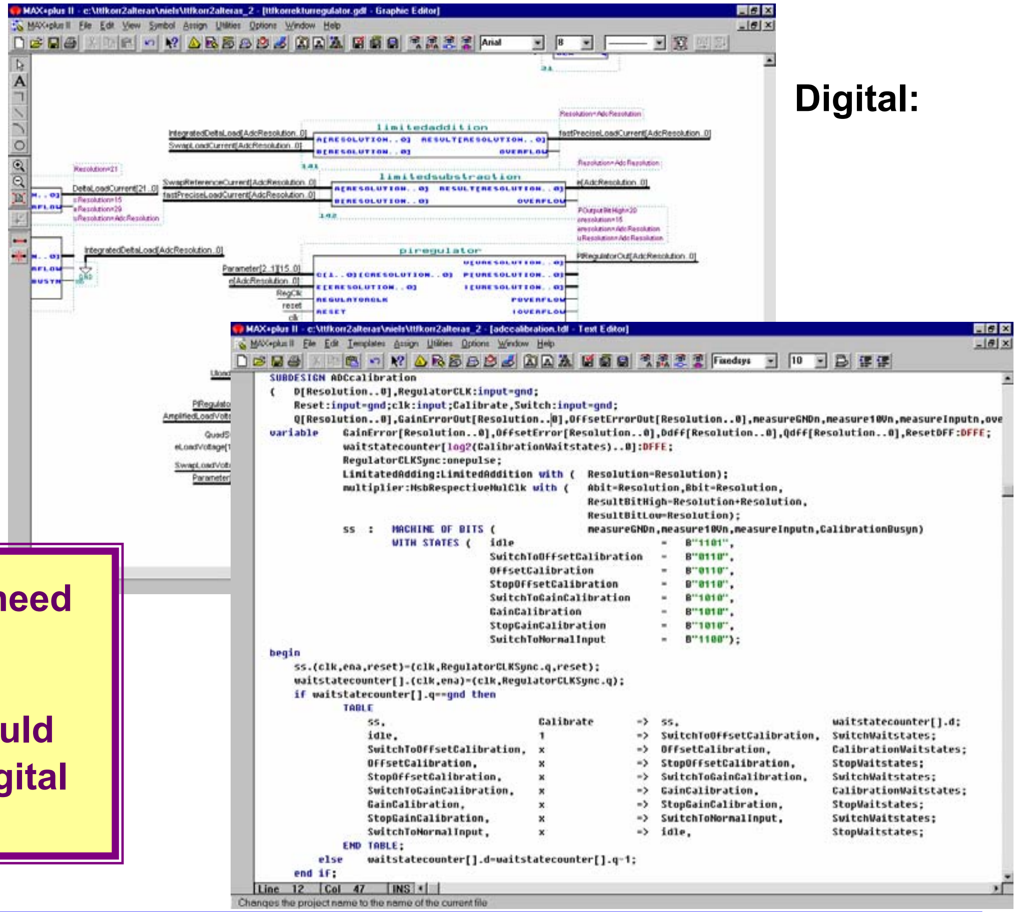
RF cavity: amplitude and phase feedback (2)

Analog/Digital LLRF comparison – Flexibility (ALBA)

Analog:



Digital:

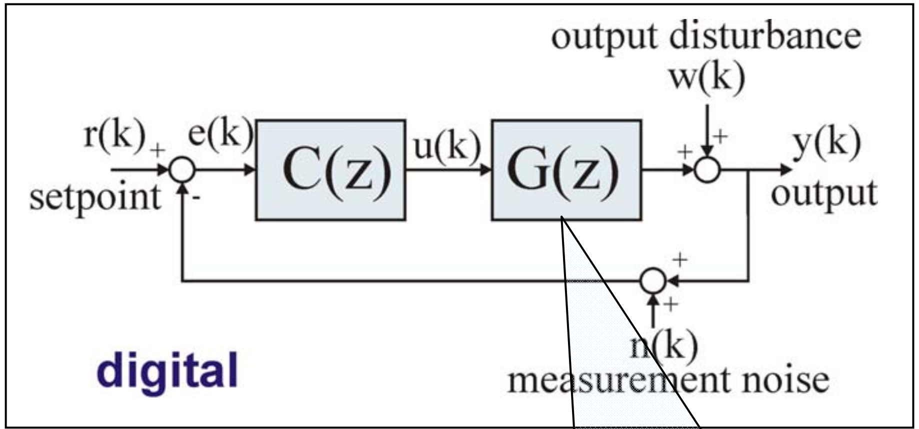


- ***Analog:*** **any major change would need a new PCB design.**
- ***Digital:*** **most of future changes would be a matter of reprogramming the digital processor.**

H. Hassanzadegan (CELLS)

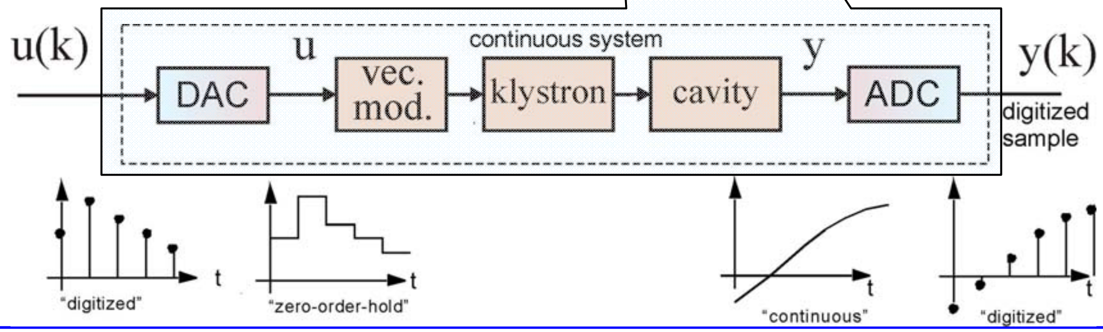
RF cavity: amplitude and phase feedback (3)

basic feedback loop:



r: setpoint
y: plant output
n: measurement noise
w: output disturbance
e: control error
u: plant input
C: controller
G: plant (klystron, cavity, …)

analog → digital: $G(z) = \frac{z-1}{z}\mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{G(s)}{s}\right\}\Big|_{t=kT_S}\right\}$ (due to zero order hold function of DAC)



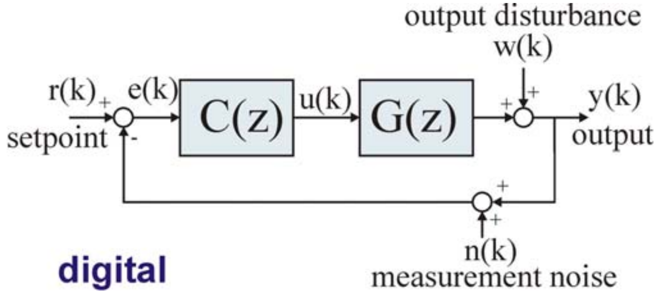
task:
model the plant
to find *G(s)* and
transform it into Z-space

RF cavity: amplitude and phase feedback (4)

$$y = \underbrace{\boxed{\frac{GC}{1+GC}}}_{\boldsymbol{T}}[r-n] + \underbrace{\boxed{\frac{1}{1+GC}}}_{\boldsymbol{S}}w$$

complementary sensitivity function — **sensitivity function**

$$\boxed{\boldsymbol{T} + \boldsymbol{S} = 1}$$



digital

$$e_T = \boxed{\frac{GC}{1+GC}}n + \boxed{\frac{1}{1+GC}}[r-w]$$

$e_T = r - y$ (tracking error)

- ***GC*: open loop transfer function**
- for output y :
  measurement error $\boldsymbol{n}$ behaves like a change in the setpoint $\boldsymbol{r}$
  (e.g. I/Q sampling error…) ➡ $\boldsymbol{T}$ should be small (robustness)
- output y should be insensitive for low frequencies output disturbances w
  (→high gain with the controller to get GC>>1) ➡ $\boldsymbol{S}$ should be small (performance)

trade-off between performance and robustness

RF cavity: amplitude and phase feedback (5)

LTI feedback: Bode integral theorem - waterbed effect

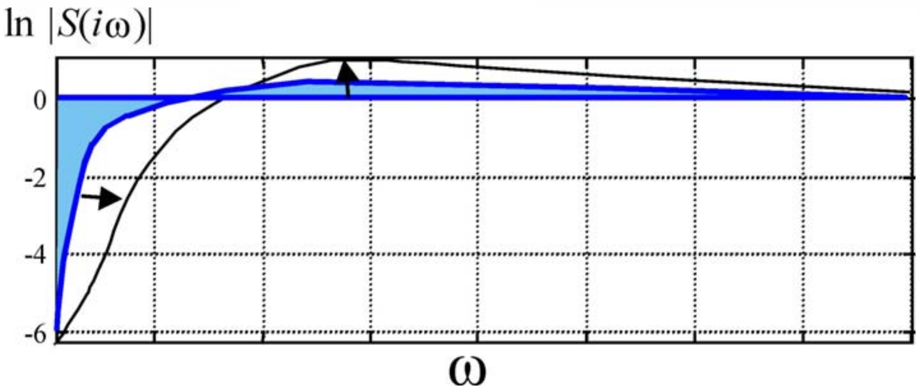
- if GC has no unstable poles and there are two or more poles than zeros:
  (continuous: no poles in the right hand plane; discrete: no poles outside unity circle)

continuous:

$$\int_0^\infty \ln|S(i\omega)|d\omega = 0$$

discrete:

$$\int_{-\pi}^{\pi} \ln|S(e^{i\omega})|d\omega = 0$$

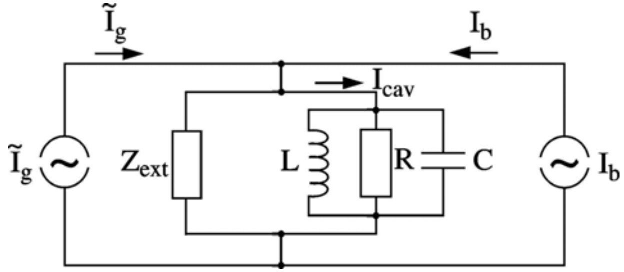


- Small sensitivity at low frequencies must be "paid" by a larger than 1 sensitivity at some higher frequencies **"waterbed effect"**

RF cavity: amplitude and phase feedback (6)

representation of RF cavity (transfer function / state space)

simplified model: LCR circuit



$\mathbf{V}(t)$: cavity voltage
$\mathbf{I}(t)$: driving current (from generator and beam)
ω_0: resonance frequency of undamped cavity
Q_L: loaded quality factor of cavity
R_L: cavity resistance incl. external load

differential equation for driven LCR circuit:

$$\ddot{\mathbf{V}}(t) + \frac{\omega_0}{Q_L}\dot{\mathbf{V}}(t) + \omega_0^2\mathbf{V}(t) = \frac{\omega_0 R_L}{Q_L}\dot{\mathbf{I}}(t)$$

stationary solution for a harmonic driven cavity:

$$\boxed{V(t) = \hat{V} \cdot \sin(\omega t + \psi)}$$

ampl.:

$$\hat{V} \approx \frac{R_L I_0}{\sqrt{1 + (2Q_L \frac{\Delta\omega}{\omega})^2}}$$

detuning angle:

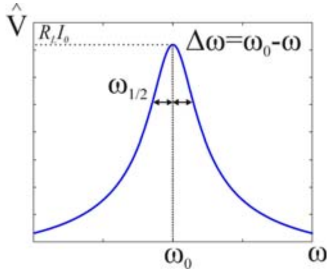
$$\tan\psi \approx 2Q_L \frac{\Delta\omega}{\omega}$$

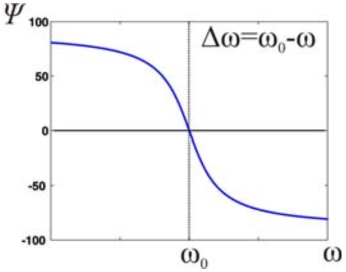
bandwidth:

$$\omega_{1/2} = \frac{\omega_0}{2Q_L}$$

detuning:

$$\Delta\omega = \omega_0 - \omega$$





RF cavity: amplitude and phase feedback (7)

separate fast RF oscillations from the **slowly** changing amplitude/phases:

(slowly: compared to time period of RF oscillations)

$$\mathbf{V}(t) = \begin{pmatrix} V_r(t) \\ V_i(t) \end{pmatrix} \cdot e^{i\omega t} \qquad \mathbf{I}(t) = \begin{pmatrix} I_r(t) \\ I_i(t) \end{pmatrix} \cdot e^{i\omega t}$$

(notation:
real and imaginary parts
instead of I/Q values)

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} V_r \\ V_i \end{pmatrix} = \begin{pmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} V_r \\ V_i \end{pmatrix} + \begin{pmatrix} R_L\omega_{1/2} & 0 \\ 0 & R_L\omega_{1/2} \end{pmatrix} \cdot \begin{pmatrix} I_r \\ I_i \end{pmatrix}}$$

Laplace transformation:

$$\underbrace{\begin{pmatrix} V_r(s) \\ V_i(s) \end{pmatrix}}_{V(s)} = \underbrace{\frac{\omega_{1/2}}{\Delta\omega^2 + (s+\omega_{1/2})^2} \begin{pmatrix} s+\omega_{1/2} & -\Delta\omega \\ \Delta\omega & s+\omega_{1/2} \end{pmatrix}}_{H_{cav}(s)} \cdot \underbrace{\begin{pmatrix} R_L \cdot I_r(s) \\ R_L \cdot I_i(s) \end{pmatrix}}_{U(s)}$$

cavity transfer matrix (continuous)

z transformation **(continuous → discrete with zero order hold):**

$$H(z) = \frac{\omega_{12}}{\Delta\omega^2 + \omega_{12}{}^2} \cdot \begin{bmatrix} \omega_{12} & -\Delta\omega \\ \Delta\omega & \omega_{12} \end{bmatrix} - \left(\frac{\omega_{12}}{\Delta\omega^2 + \omega_{12}{}^2} \cdot \frac{z-1}{z^2 - 2ze^{\omega_{12}T_s}.\cos(\Delta\omega T_s) + e^{2\omega_{12}T_s}} \right)$$
$$. \left\{ \left((z - e^{\omega_{12}T_s}.\cos(\Delta\omega T_s)).\begin{bmatrix} \omega_{12} & -\Delta\omega \\ \Delta\omega & \omega_{12} \end{bmatrix} \right) - e^{\omega_{12}T_s}.\sin(\Delta\omega T_s).\begin{bmatrix} \Delta\omega & \omega_{12} \\ -\omega_{12} & \Delta\omega \end{bmatrix} \right\}$$

let matlab do the job for you!

state space:

$$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B} \cdot \mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x}(t)$$

$$\mathbf{x}(t) = \begin{pmatrix} V_r(t) \\ V_i(t) \end{pmatrix} \qquad \mathbf{u}(t) = \begin{pmatrix} I_r(t) \\ I_i(t) \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} -\omega_{1/2} & -\Delta\omega \\ \Delta\omega & -\omega_{1/2} \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} R_L\omega_{1/2} & 0 \\ 0 & R_L\omega_{1/2} \end{pmatrix} \qquad \mathbf{y}(t) = \mathbf{x}(t)$$

RF cavity: amplitude and phase feedback (8)

properties of cavity transfer functions:

$$H_{cav}(s) = \frac{\omega_{1/2}}{\Delta\omega^2 + (s+\omega_{1/2})^2} \begin{pmatrix} s+\omega_{1/2} & -\Delta\omega \\ \Delta\omega & s+\omega_{1/2} \end{pmatrix} = \begin{pmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{pmatrix}$$

$\Delta\omega=0$ (cavity **on** resonance)

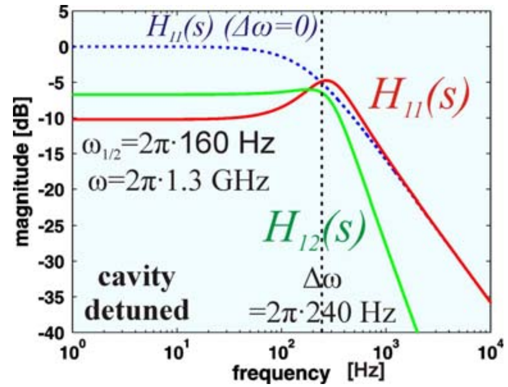
- cavity behaves like a first order low pass filter **(20 dB roll off per decade)**
- I/Q (or amplitude and phase) are decoupled

$$H_{11}(s) = H_{22}(s) = \frac{\omega_{1/2}}{s+\omega_{1/2}}$$

$$H_{12}(s) = H_{21}(s) = 0$$

$\Delta\omega\neq 0$ (cavity **off** resonance)

- I/Q are coupled



- if $\Delta\omega=\Delta\omega(t)$

⇒ cavity models are time variant, control is more complex

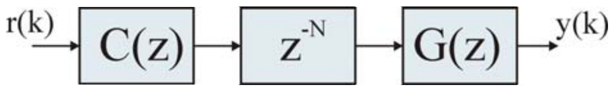
RF cavity: amplitude and phase feedback (9)

example: loop analysis in frequency domain (simplified model !)

superconducting cavity: $Q_L=3\cdot 10^6$ $\Delta\omega=0$ Hz $f_S=10$ MHz

$f_0=1.3$ GHz $\omega_{1/2}=216$ Hz

$t_d=1$ μs



controller cavity

loop delay

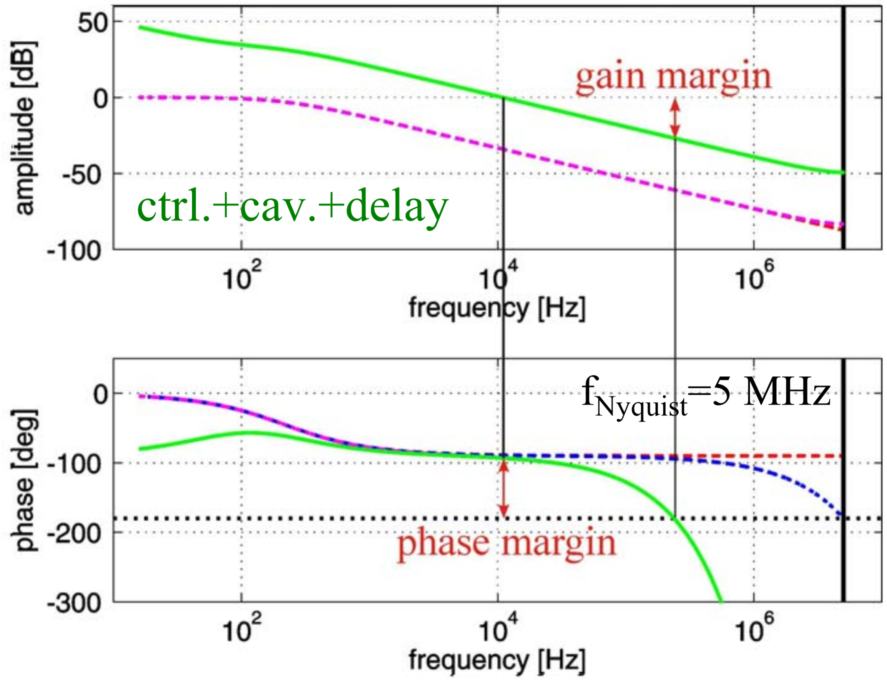
PID:
$$
\begin{aligned}
u(k) &= u(k-1) \\
&+ (K_p + K_i + K_d)\cdot e(k) \\
&- (K_p + 2K_d)\cdot e(k-1) \\
&+ K_d \cdot e(k-2)
\end{aligned}
$$

rule of thumb:

Gain margin at least between 6 and 8dB

Phase margin between 40° and 60°

open loop transfer function



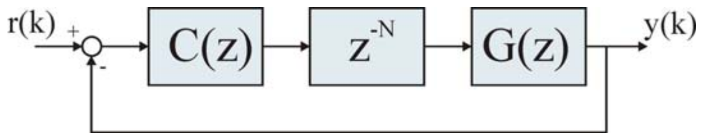
RF cavity: amplitude and phase feedback (10)

example: loop analysis in frequency domain

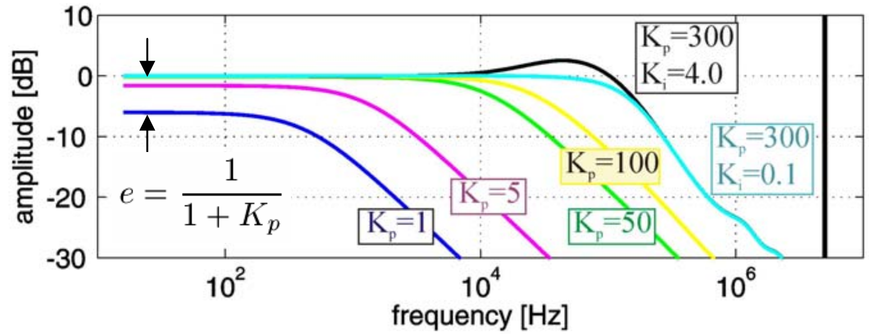
superconducting cavity: $Q_L=3\cdot10^6$ $t_D = 1\ \mu s$ $f_S=10$ MHz

$f_0=1.3$ GHz $\omega_{1/2}=216$ Hz

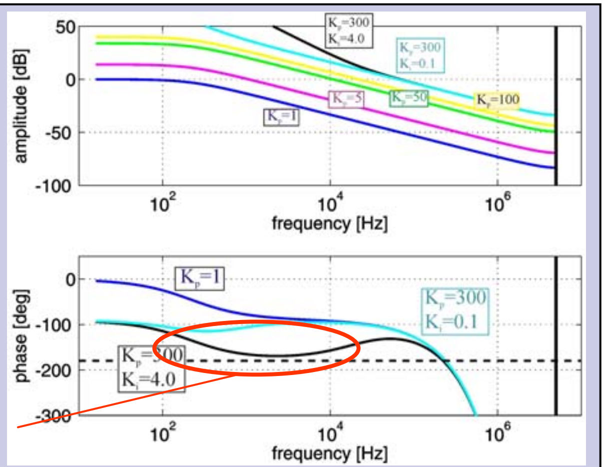
closed loop:



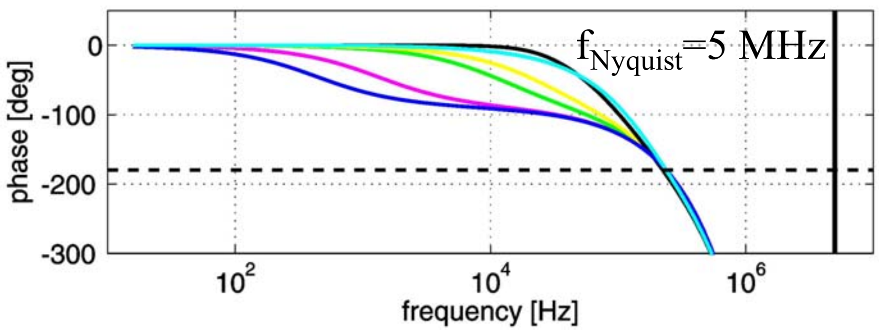
complementary sensitivity function



open loop:



phase margin!



RF cavity: amplitude and phase feedback (11)

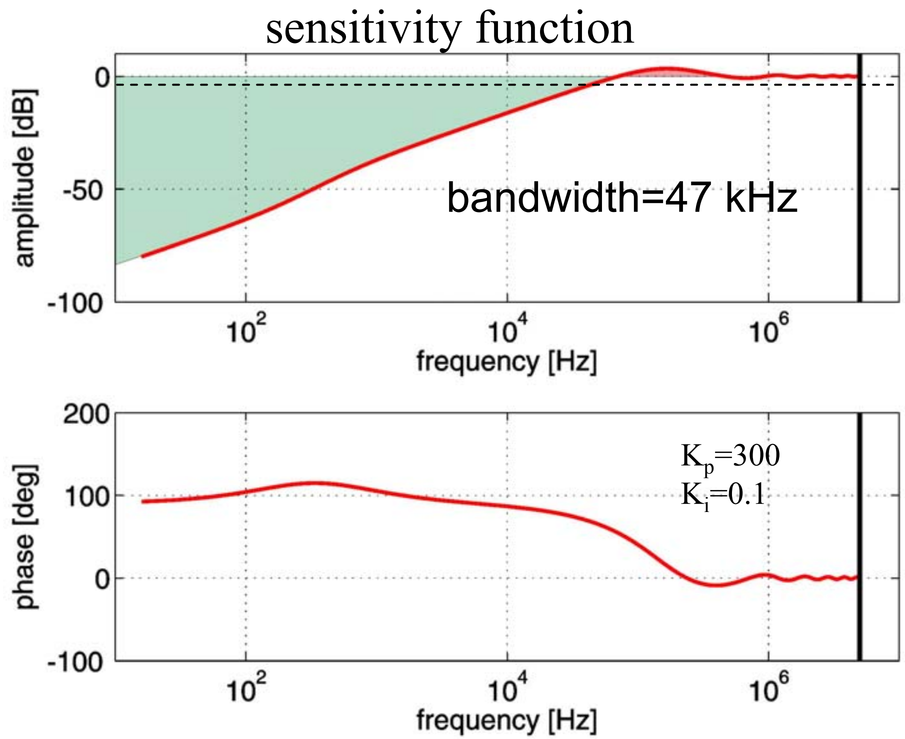
example: loop analysis in frequency domain

superconducting cavity: $Q_L=3\cdot 10^6$ $t_D= 1\ \mu s$ $f_S=10$ MHz

$f_0=1.3$ GHz $\omega_{1/2}=216$ Hz

choose parameter such that

- dominant disturbance frequencies are suppressed
- no dangerous lines show up in the range where the feedback can excite
- system performance will not be spoiled by sensor noise due to increasing loop gain



RF cavity: amplitude and phase feedback (12)

example: loop analysis in frequency domain

superconducting cavity: $Q_L=3\cdot10^6$ $\Delta\omega=0$ Hz $f_S=10$ MHz

$\omega_{1/2}=216$ Hz

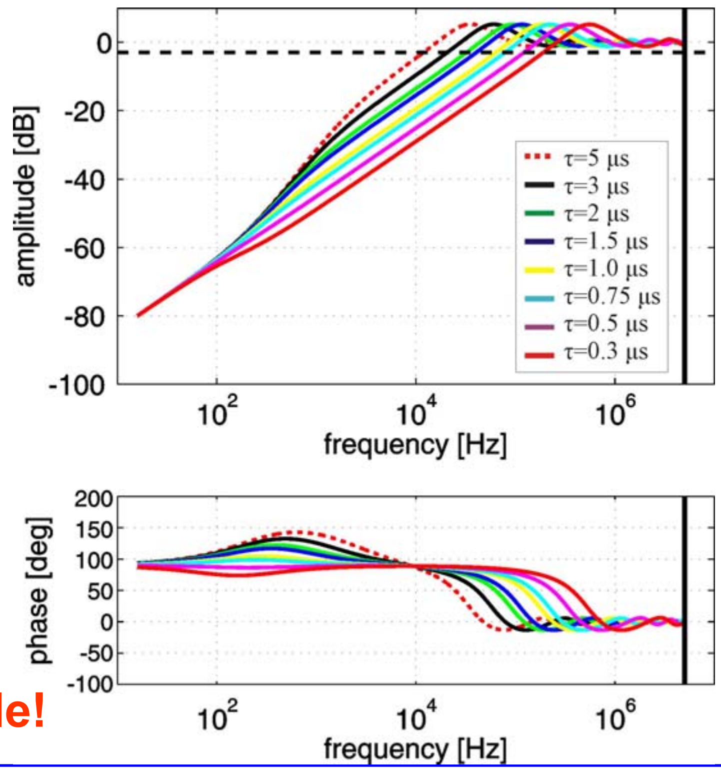
variation of the loop delay

(boundary condition:

keep gain margin constant at 8 dB; $K_i=0.1$)

t_D	K_p	loop bandwidth (-3 dB)
5 μs	87	11.9 kHz
3 μs	145	20.6 kHz
2 μs	223	32.2 kHz
1.5 μs	278	40.3 kHz
1.0 μs	436	63.6 kHz
0.75 μs	539	78.6 kHz
0.5 μs	832	121 kHz
0.3 μs	1303	190 kHz

sensitivity function



total loop delay is an important parameter; keep it as small as possible!

RF cavity: amplitude and phase feedback (13)

so far: loop analysis done for **fixed** detuning ($\Delta\omega=0$) only!
things get much more complicated if $\Delta\omega = \Delta\omega(t)$

➡ time varying control

example: pulsed superconducting cavity with high gradient

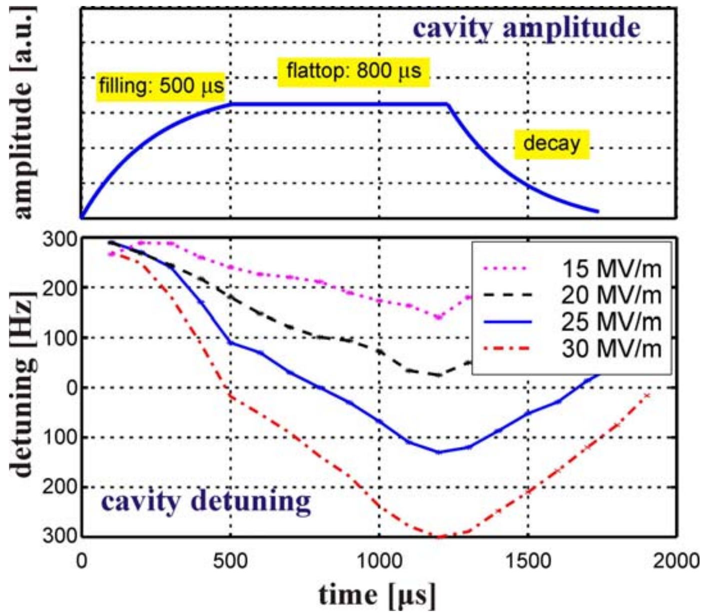
➡ strong Lorentz forces

➡ cavity detunes during the pulse

remember: $$H_{cav}(s) = \frac{\omega_{1/2}}{\Delta\omega^2 + (s+\omega_{1/2})^2} \begin{pmatrix} s+\omega_{1/2} & -\Delta\omega \\ \Delta\omega & s+\omega_{1/2} \end{pmatrix}$$

$\Delta\omega$: function of the gradient (time varying)

pulsed operation of superconducting cavity



➡ stability of the feedback loop has to be guaranteed under these parameter changes!

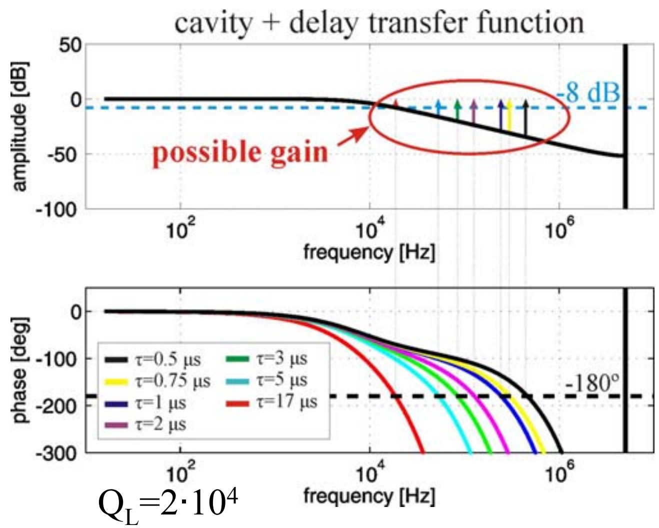
➡ this might limit the feedback gain in contrast to the simple analysis !

➡ design of "optimal" controller under study at many labs…

RF cavity: amplitude and phase feedback (14)

cavities

superconducting	normal conducting
◆ Q_L : **~few 10^5 - 10^7** cavity time constants $\tau_{cav} = Q_L/(\pi f_{RF})$: **~few 100 µs** bandwidth $f_{1/2} = f_{RF}/(2Q_L)$: **~few 100 Hz** ◆ feedback loop delay **small** compared to τ_{cav}	◆ Q_L : **~10 – 10^5** cavity time constants τ_{cav}: **~few µs** bandwidth $f_{1/2}$: **~100 kHz** ◆ feedback loop delay **in the order** of τ_{cav}



$Q_L = 2 \cdot 10^4$
$f_{RF} = 324$ MHz

loop latency limits high feedback gain
for high bandwidth cavities!

if the gains/bandwidths achieved by digital feedback systems are not sufficient

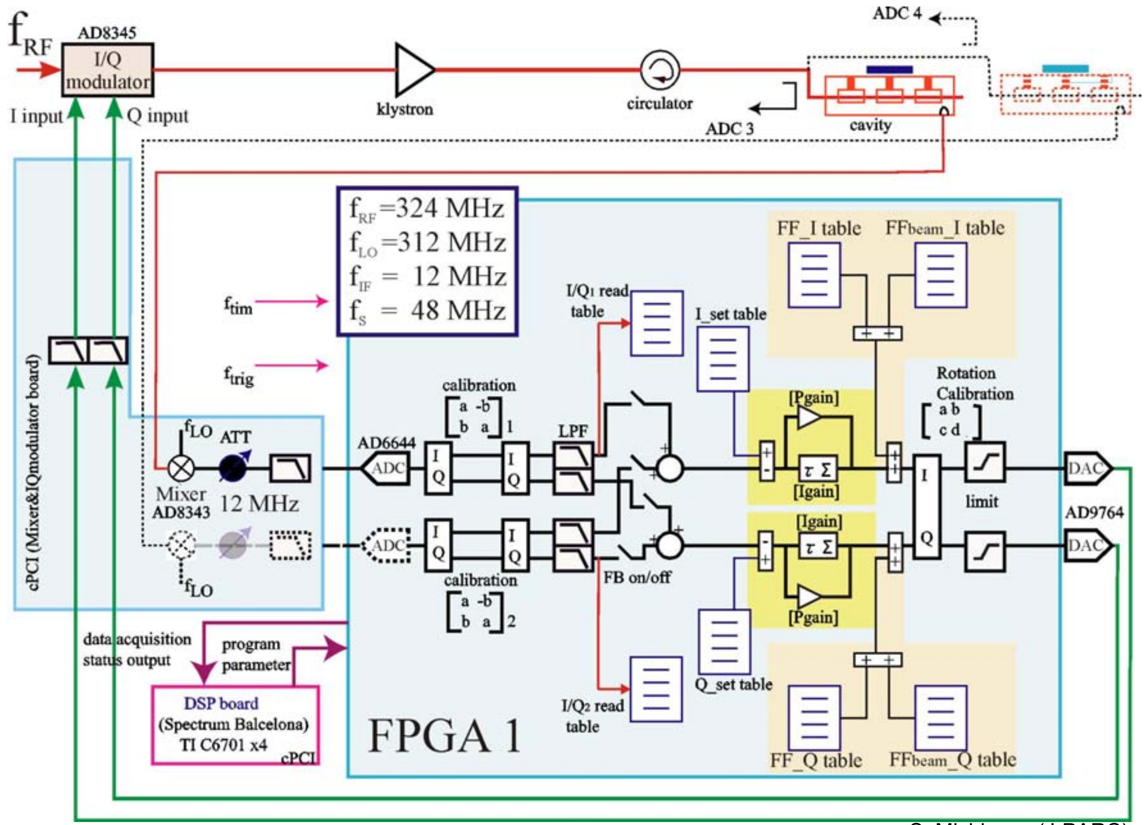
➡ analog/digital hybrid system might be an alternative !?

amplitude and phase feedback: example

LLRF: J-PARC linac

(RFQ, DTL, SDTL)

- 400 MeV proton linac
- pulsed operation; rep. rate: 12.5/25 Hz; pulse length: ~500-650 μs
- vector sum control
- normal conducting cavities; Q_L~8'000-300'000 τ_{cav}~100 μs

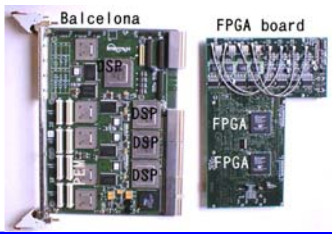


S. Michizono (J-PARC)

requirements / achieved:

- **amplitude: < +-1% / < +-0.15%**
- **phase: < +-1° / < +-0.15°**

combined DSP/FPGA board



performance:

- **loop delay: 500 ns**
- **gains: proportional~10, integral~0.01**
- **bandwidth: ~100 kHz**

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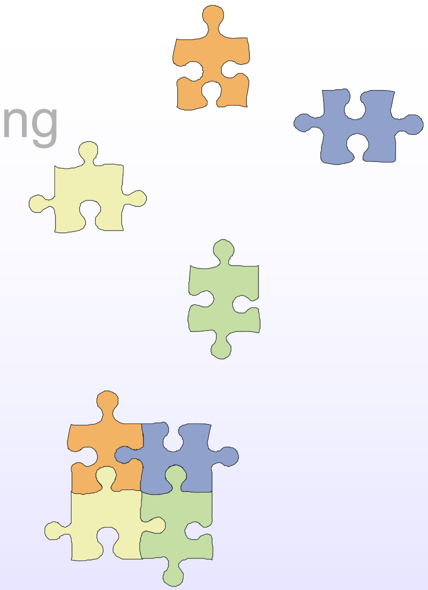


Feedbacks in hadron/ion synchrotrons

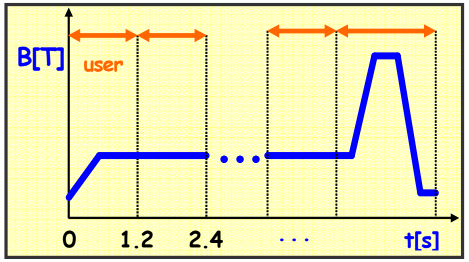
booster synchrotrons:

capture and adiabatically rebunch the beam and accelerate to the desired extraction energy.

Beam Control System

task: control of

- RF frequency during the ramp
  (large frequency swings of up to a factor of ten, usually from several 100 kHz to several 10 MHz)
- cavity amplitude and phase
  (ampl. can follow a pattern during acceleration)
- mean radial position of the beam
- phase between beam and cavity RF (synchronous phase Φ_S) → deviations from Φ_S will lead to synchrotron oscillations
- synchronization to master RF phase
  (to synchronize the beam transport to other accelerator rings)

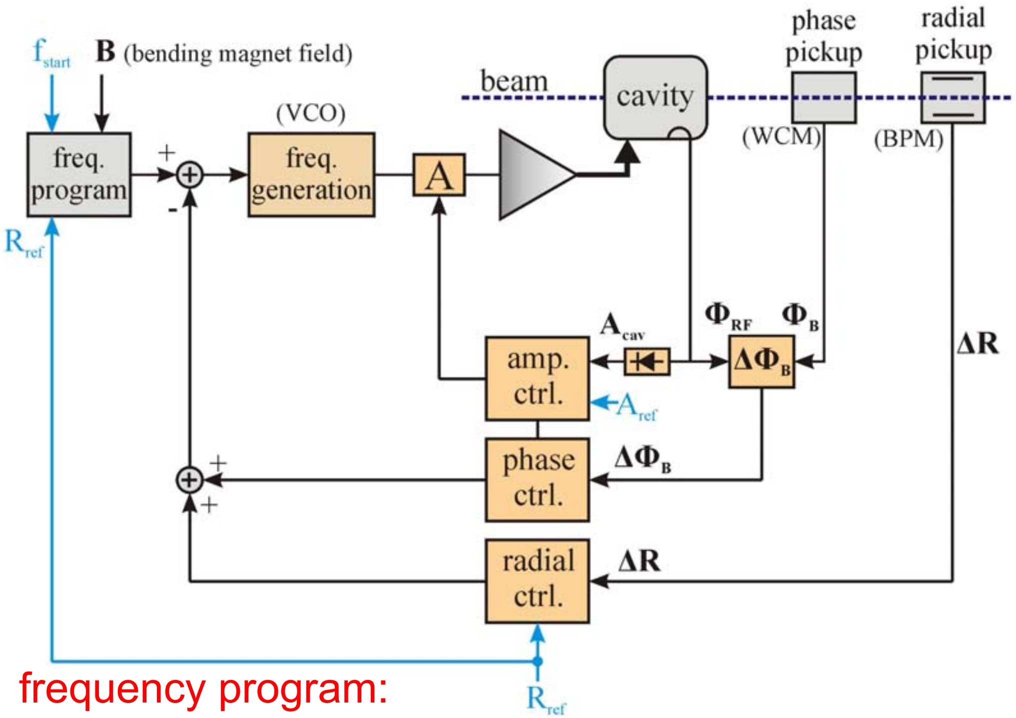


Typical LEIR commissioning cycle.

$$\frac{dB}{B} = \gamma^2 \frac{df}{f} + (\gamma^2 - \gamma_{tr}^2)\frac{dR}{R}$$

in reality: errors due to phase noise, B field errors, power supply ripples, … ➡ **feedbacks are required**

Beam Control System



frequency program:

1) calculate frequency based on the B field, desired radial position
2) optimize the freq. ramp to improve injection efficiency
3) generate dual harmonic RF signals for cavities (bunch shaping)

beam phase loop

damps coherent synchrotron oscillations from

1) injection errors (energy, phase)
2) bending magnet noise
3) frequency synthesizer phase noise

radial loop

keeps the beam to its design radial position during acceleration

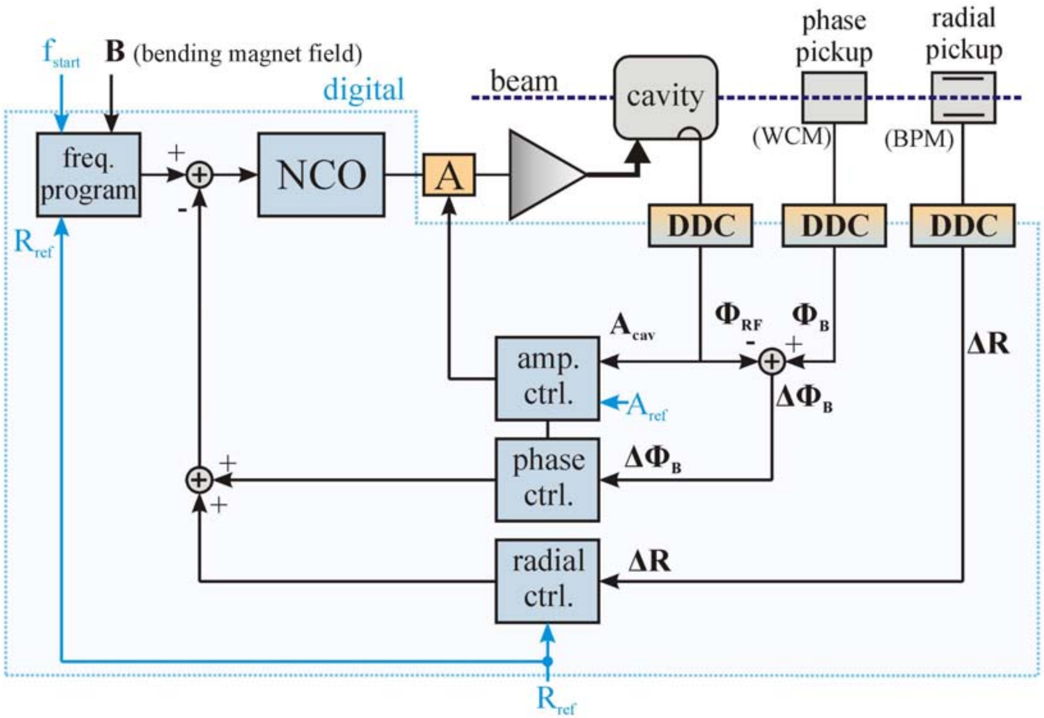
cavity amplitude loop

1) compensates imperfections in the cavity amplifier chain
2) amplitude has to follow a ramping function

synchronization loop (not shown)

locks the phase to a master RF

Beam Control System: from analog to digital



How do we setup the control loops?

➡ model required

in '80s: **DDS/NCO replace VCO**

(VCO:

lack of absolute accuracy, stability limitations if freq. tuning is required over a broad range)

in recent years (LEIR, AGS, RHIC):

fully digital beam control system

- digitize RF signals (I/Q, DDC)
- all control loops are purely digital
- feedback gains: function of the beam parameters (keep the same loop performances through the acceleration cycle)

Radial and phase loops

beam dynamics delivers the differential equations ➡ transfer functions

transfer functions without derivation:

RF freq. (NCO output) to phase deviation of the beam from the synchronous phase

$$H_{\phi_B}(s) = \frac{\Delta\phi_B}{\Delta\omega_{RF}} = \frac{s}{s^2 + \omega_S^2}$$

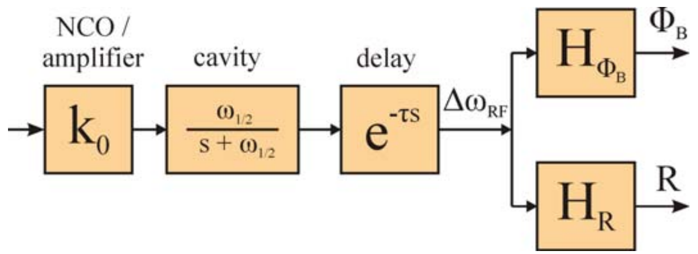
RF freq. (NCO output) to radial position R

$$H_R(s) = \frac{R}{\Delta\omega_{RF}} = \frac{b}{s^2 + \omega_S^2}$$

$\omega_S = \omega_S$ (E): synchrotron frequency, depend on the beam energy

$b=b(E,\Phi_S)$: function of energy, synchronous phase

model of the system:



since energy varies along the ramp

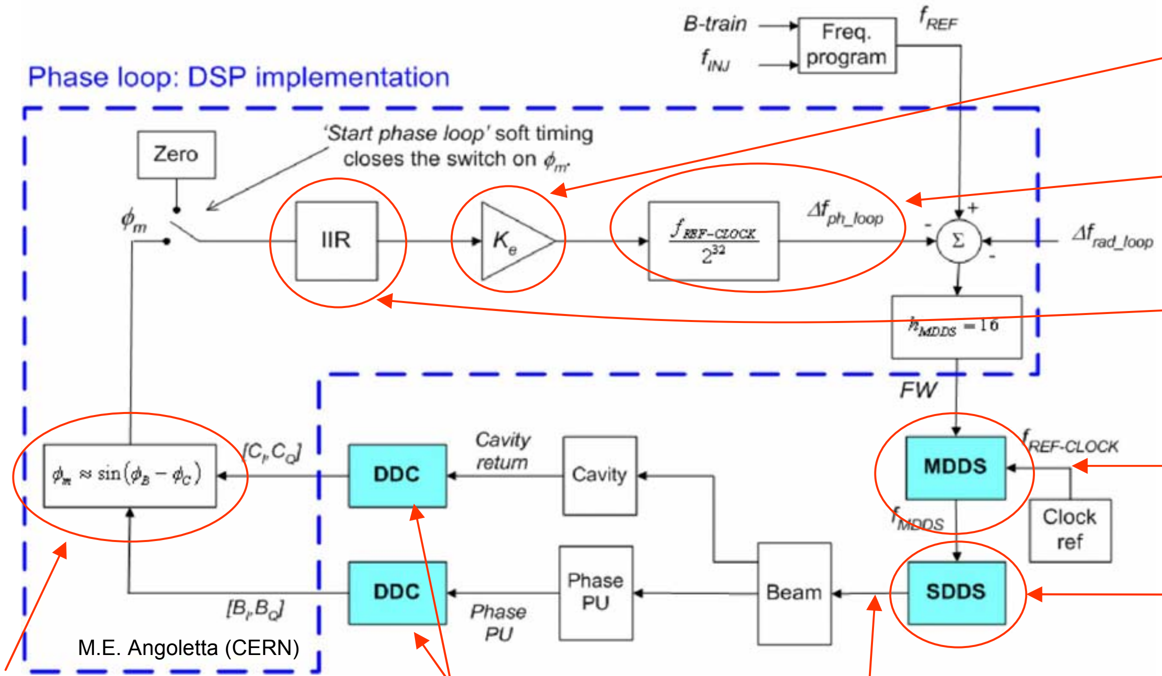
➡ time varying model !

LPV: linear parameter varying model

design of the controller: parameters have to be adjusted over time to meet the changing plant dynamics (guarantee constant loop performance and stability)

Phase loop: example

implementation example (test system for LEIR): **PS Booster** @ CERN



programmable proportional gain

phase-to-freq. conversion

IIR: 1st order high pass
cut-off: 10 Hz
AC coupling

1 master DDS (MDDS)

several slave DDS (SDDS)
- cavity control (2 harmonics)
- RF signal for diagnostics

$$\Delta\phi_B \approx \sin(\phi_B - \phi_{RF}) = \frac{I_c Q_b - I_b Q_c}{\sqrt{I_c^2 + Q_c^2}\sqrt{I_b^2 + Q_b^2}}$$

$$|\sin\varphi| = \frac{|\vec{u} \times \vec{v}|}{|\vec{u}||\vec{v}|}$$

$f_S = f_{MDDS}$ = 9.6 - 27.9 MHz

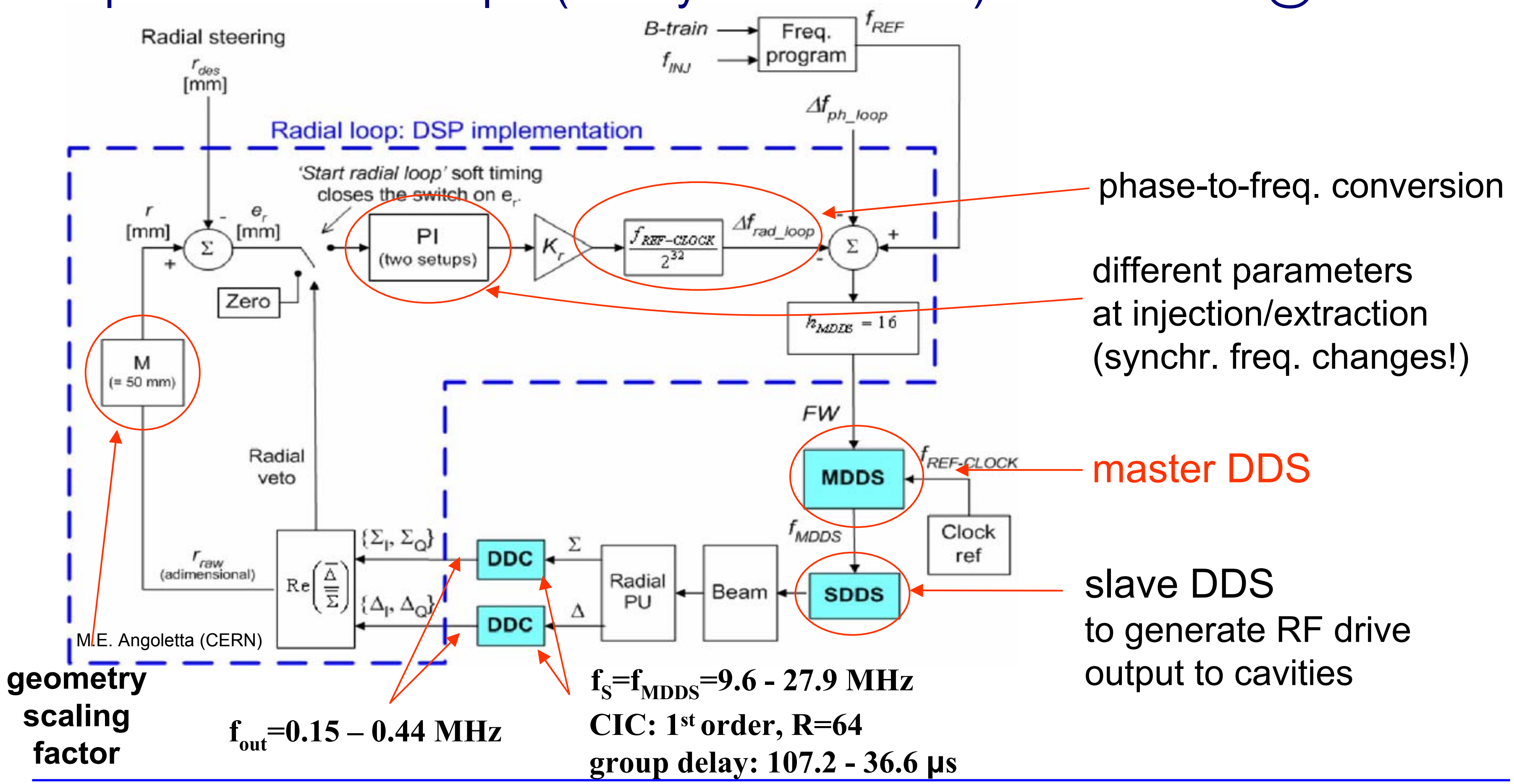
f_{rev} = 0.599-1.746 MHz (inj. / ext.)

CIC: 1st order, R=1
group delay: 1.67 - 0.57 µs

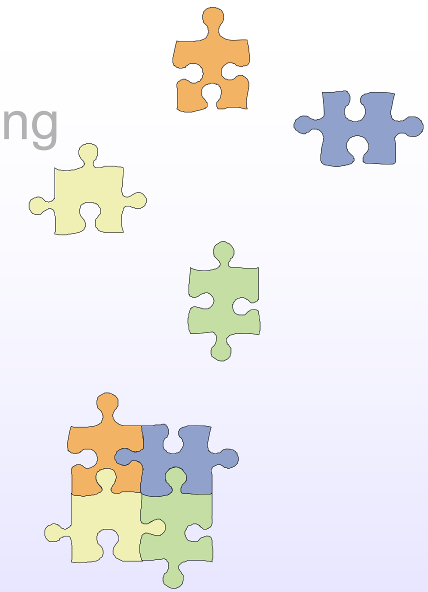
loop bandwidth: 7 kHz

Radial loop: example

implementation example (test system for LEIR): **PS Booster** @ CERN



Outline

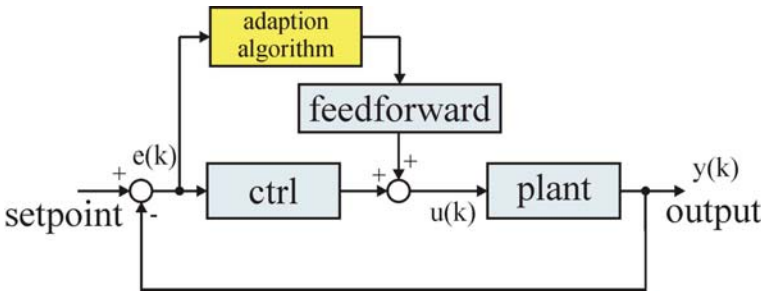


1. signal conditioning / down conversion
2. detection of amp./phase by digital I/Q sampling
   - ❑ I/Q sampling
   - ❑ non I/Q sampling
   - ❑ digital down conversion (DDC)
3. upconversion
4. **algorithms in RF applications**
   - ❑ feedback systems
     - ■ cavity amplitude and phase
     - ■ radial and phase loops
   - ❑ **adaptive feedforward**
   - ❑ system identification

Adaptive Feedforward

goal:

- suppress repetitive errors by feedforward in order to disburden the feedback
- cancel well known disturbances where feedback is not able to (loop delay!)
- adapt feedforward tables continuously to compensate changing conditions



warning:
adding the error (loop delay corrected) to system input **does not work**!
(dynamics of plant is not taken into account)

How to obtain feedforward correction?

we need to calculate the proper input which generates output signal –*e(k)*

→ **inverse system model needed!**

Adaptive Feedforward (2)

in reality: model for plant not well known enough

- system identification ➡ model
- measure system response (e.g. by step response measurements)

linear system (SISO):

$$u(k) \rightarrow \boxed{G(z)} \rightarrow y(k)$$

$$\underbrace{\begin{pmatrix} \Delta y(\tau_1) \\ \vdots \\ \Delta y(\tau_p) \end{pmatrix}}_{\textbf{system output}} = \underbrace{\begin{pmatrix} R_{11} & & 0 \\ \vdots & \ddots & \\ R_{p1} & \dots & R_{pp} \end{pmatrix}}_{\textbf{system response matrix}} \cdot \underbrace{\begin{pmatrix} \Delta u(t_1) \\ \vdots \\ \Delta u(t_p) \end{pmatrix}}_{\textbf{test inputs}}$$

$\tau_k = t_k + \tau_d$

loop delay

in successive measurements: apply $\Delta u(t_k)$ and measure response $\Delta\vec{y}$

➡ results in $\boldsymbol{R}$ (with some math depending on the test input)

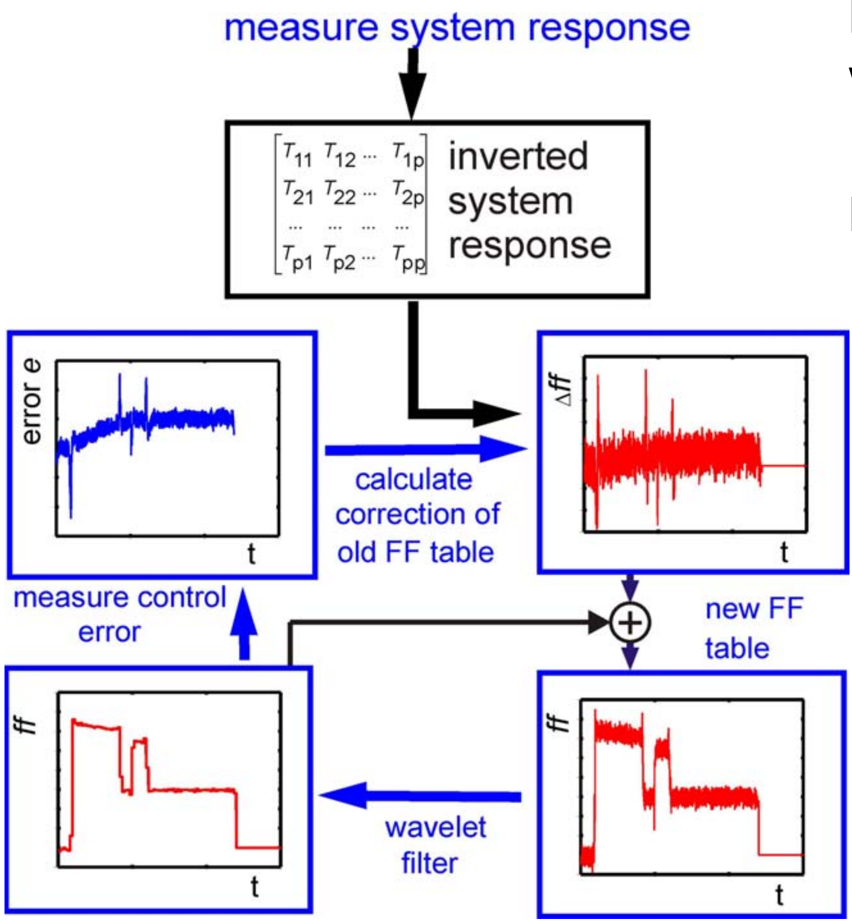
➡ invert response matrix $\boldsymbol{T}=\boldsymbol{R}^{-1}$ (possible due to definition of sampling time $\tau_k = t_k + \tau_d$)

$$\Delta\vec{\mathbf{u}} = \mathbf{T} \cdot \Delta\vec{\mathbf{y}}$$

feedforward for error correction:

$$\boxed{\Delta\vec{\mathbf{ff}} = \mathbf{T} \cdot \vec{\mathbf{e}} = \mathbf{T} \cdot (\vec{\mathbf{r}} - \vec{\mathbf{y}})}$$

Adaptive Feedforward (3)



pulsed superconducting 1.3 GHz cavity:
works fine in principle

but:

- remeasure T when operating point changes (amplitude/phase) **(non-linearities in the loop)**
- response measurement could not be fast enough

➡ need for a fast and robust adaptive feedforward algorithm!

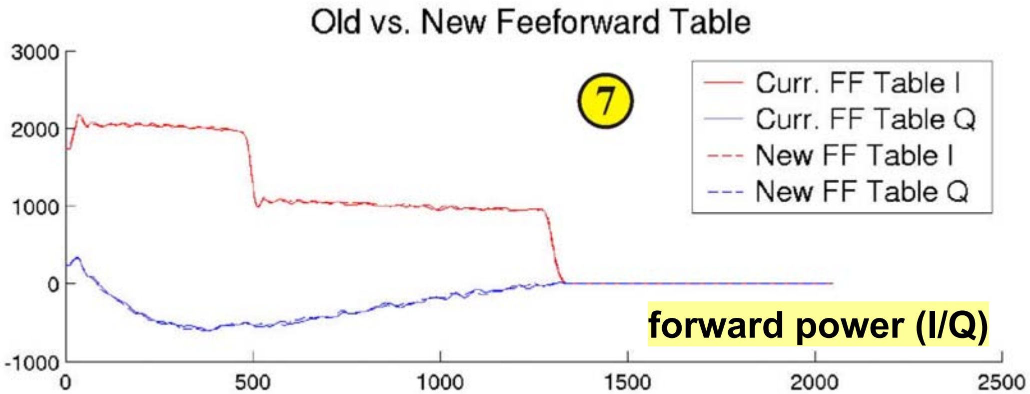
Adaptive Feedforward (4)

“time reversed” filtering:

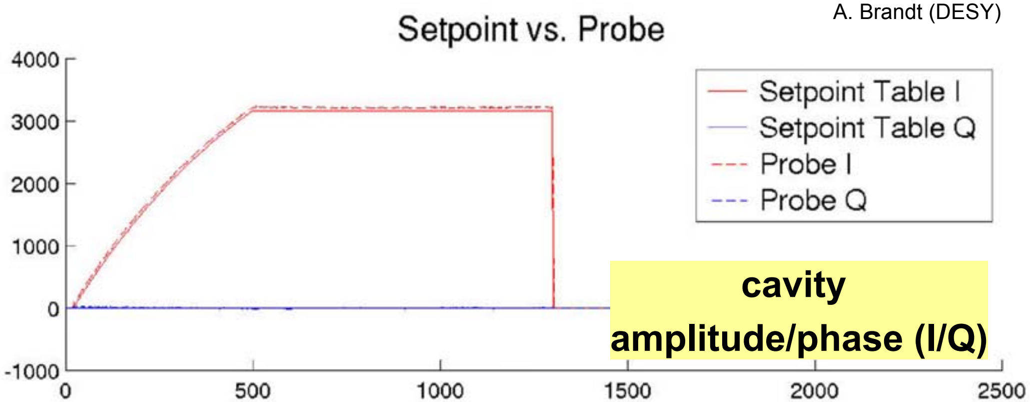
- developed for FLASH, in use at FLASH/tested at SNS
- works only for pulsed systems
- not really understood but it works within a few iterations!

recipe:

- record feedback error signal $e(t)$
- time reverse $e(t) \rightarrow e(-t)$
- lowpass filter $e(-t)$ with ω_{LP}
- reverse filtered signal in time again
- shift signal in time (Δt_{AFF}) to compensate loop delay
- add result to the previous FF table



A. Brandt (DESY)



best results: $\omega_{LP} \approx$ closed loop bandwidth
$\Delta t_{AFF} \approx$ loop delay

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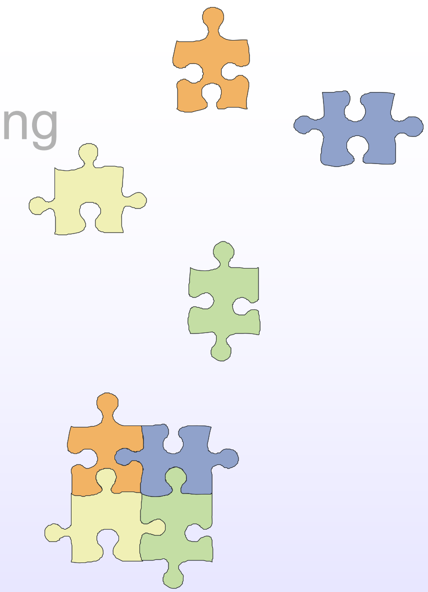
   - **system identification**

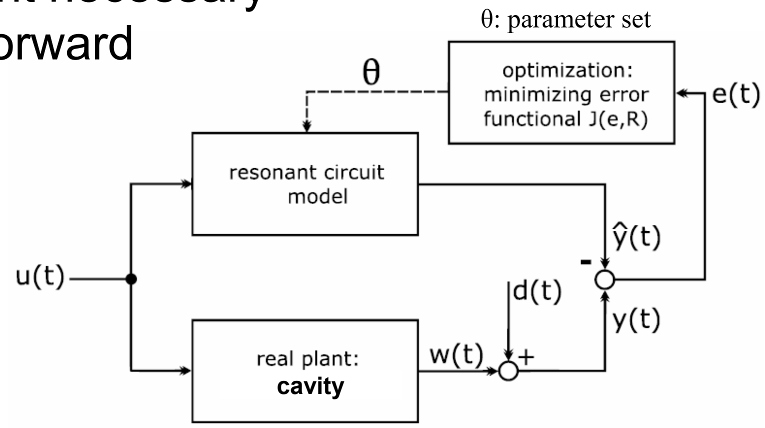


System Identification in RF plants

goal:
- design (synthesis) of high performance cavity field controllers is model based;
  ➡ mathematical model of plant necessary
- model required for efficient feedforward

➡ **system identification**

output error (OE) model structure:



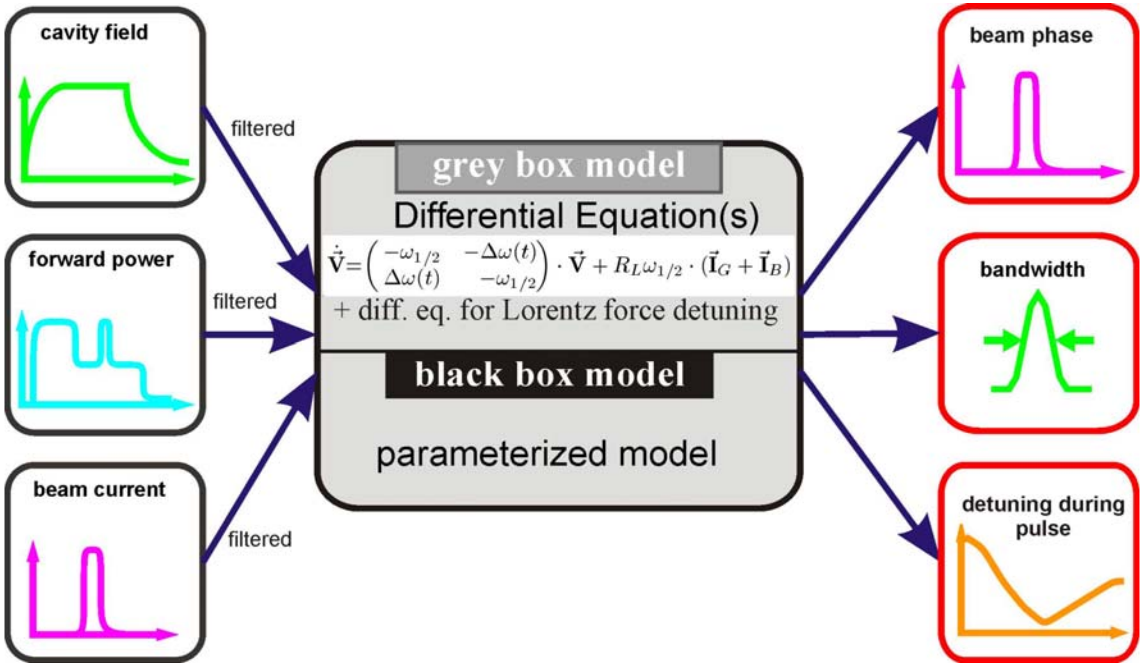
system identification steps
- record output data with proper input signal (step, impulse, white noise)
- choose model structure
  - grey box (preserves known physical structures with a number of unknown free parameters)
  - black box (no physical structure, parameters have no direct physical meaning)
- estimate model parameter (minimize $e(t)$)
- validate model with a set of data not included in the identification process

System Identification in RF plants (2)

example:

pulsed high gradient superconducting cavities with Lorentz force detuning

LPV: linear parameter varying model



System Identification in RF plants (3)

example: identification of Lorentz force detuning in high gradient cavity

differential equation for cavity ampl./phase (polar coordinates)

measure cavity ampl./phase; derive dV/dt, dφ/dt

LMS fit for constant parameters (e.g. $\omega_{1/2}$, shunt impedance R, …); solve remaining equation for $\Delta\omega$

derived data: $\Delta\omega$

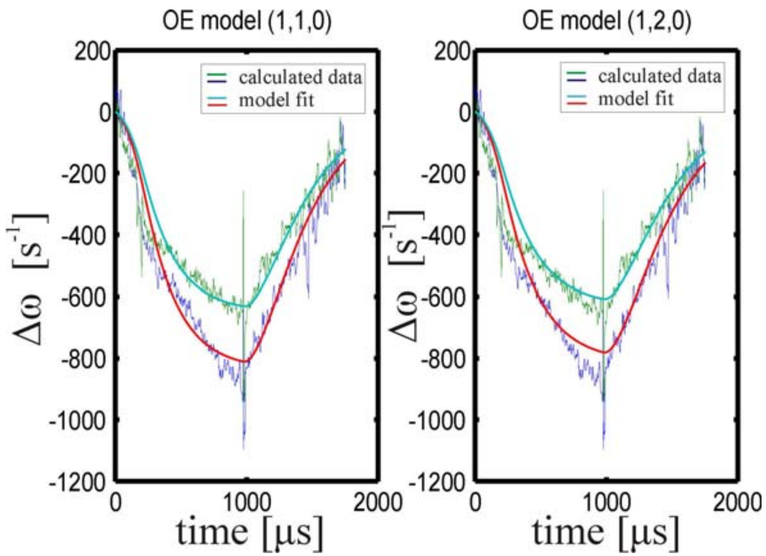
system identification (OE) for $\Delta\omega$
(grey box model)

different parametric models:

OE(1,1,0): $\Delta\dot{\omega}(t) = -\frac{1}{\tau_m}(\Delta\omega(t) - \Delta\omega_T) - 2\pi\frac{K}{\tau_m}E_{acc}^2$

OE(1,2,0): $\Delta\ddot{\omega}(t) = -\frac{1}{\tau_m}\Delta\dot{\omega}(t) - \omega_m^2(\Delta\omega(t) - \Delta\omega_T) - 2\pi K\omega_m^2 E_{acc}^2$

loop delay
order of denominator polynomal in transfer function
order of numerator polynomal in transfer function



M. Hüning (DESY)

Conclusion/ Outlook

- performance is very often dominated by systematic errors and non-linearities of sensors and analog components
- digital LLRF does not look very different from other RF applications (beam diagnostics…) ➡ common platforms?
- extensive diagnostics in digital RF systems allow automated procedures and calibration for complex systems (finite state machines…)
- digital platforms for RF applications provide playground for sophisticated algorithms

Now it's your turn to contribute to this exciting field!