

Amplitude quantization





- · Quantization replaces a range of continuous values by a set of discrete ones.
- · Usually the number of levels is a power of 2.
- The difference between the original signal and the discrete representation is the quantization error

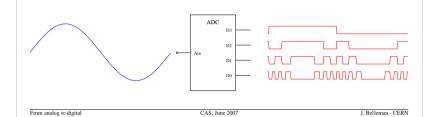
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What does an ADC do?



Analog to Digital Converter

An ADC converts a continuously variable signal, a voltage or a current, into a sequence of numbers, represented by logic levels on a group of wires.



Amplitude quantization



Power in original signal:

$$P_{s} = \frac{A^{2}}{T} \int_{0}^{T} \sin^{2} \omega t \, dt = \frac{A^{2}}{2}$$

Quantized to n bits, one quantum is:

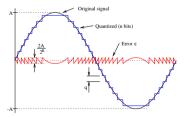
$$q = \frac{2A}{2^n} = A \cdot 2^{-(n-1)}$$

Maximum quantization error:

$$\varepsilon = \frac{\pm q}{2} = \pm A \cdot 2^{-n} (with \ p(\varepsilon) = \frac{1}{q})$$

Power in quantization error:

$$P_{\varepsilon} = \int_{0}^{q/2} p(\varepsilon) \varepsilon^{2} d\varepsilon \approx \frac{A^{2} \cdot 2^{-2n}}{3}$$



Thus
$$SNR = \frac{P_s}{P_s} = 1.5 \cdot 2^{2n}$$

In dB:
$$10 \cdot \log_{10} \frac{P_s}{P} = 1.76 + 6.02 \text{ m}$$

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Time quantization or sampling



Multiplication of the signal by a train of impulses w(t) with period $T_c (= 1/F_c)$:

$$g(t)=u(t)\cdot w(t)$$

$$g(t) = u(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$g(t) = \sum_{n=-\infty}^{\infty} u(nT_s) \delta(t - nT_s)$$

Fourier transform of w(t):

$$W(f) = \int_{-\infty}^{\infty} w(t) e^{-j2\pi f t} dt$$



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Nyquist



If the sampling rate F_s is less than twice the signal bandwidth, the spectral images overlap. The avoid this, the following condition must be fulfilled:

$$F_{s} > 2 \cdot BW$$

This is the Nyquist criterion

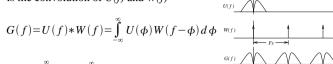
One way this condition can be fulfilled is by filtering the analogue signal prior to digitizing it, using what is called an anti-aliasing filter. Since brick-wall filters cannot be made, the sampling rate should usually be quite a bit greater than twice the signal bandwidth.

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Time quantization or sampling



The spectrum of the sampled signal is the convolution of U(f) and W(f)



$$G(f) = \int_{-\infty}^{\infty} U(\phi) \sum_{n=-\infty}^{\infty} \delta(f - \frac{n}{T_s} - \phi) d\phi$$

$$G(f) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} U(\phi) \, \delta(f - \frac{n}{T_s} - \phi) \, d\phi$$

$$G(f) = \sum_{n=-\infty}^{\infty} U(f - \frac{n}{T_s})$$

After sampling, the signal spectrum repeats for all multiples of $F_{\rm s}$

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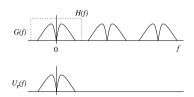
Reconstruction of the original signal



Each of the images in the spectrum of the sampled signal contains all the information needed to reconstruct the original. They are *aliases*.

We might reconstruct the original signal with a filter that rejects everything except the original frequency band. After filtering, the spectrum is exactly that of the original signal, in other words, no information is lost. We have recovered the original signal exactly.

This is Shannon's theorem



Reconstruction of the original signal



Filter the baseband using a rectangular filter H(f). The filter time-domain response is the inverse Fourier transform of its frequency-domain shape:

$$h(t) = \mathcal{F}^{-1}\{H(f)\} = \int_{-\infty}^{\infty} H(f) e^{j2\pi f t} df = \int_{-F/2}^{F/2} e^{j2\pi f t} df = F_s \frac{\sin \pi F_s t}{\pi F_s t}$$

Convolution of filter with sample stream:

$$u_r(t) = g(t) * h(t)$$

$$u_r(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} u(\tau) \cdot \delta(\tau - nT_s) \cdot h(t - \tau) d\tau$$

$$u_r(t) = \sum_{n=-\infty}^{\infty} u(nT_s) \cdot h(t - nT_s)$$

$$u_r(t) = \sum_{n=-\infty}^{\infty} u(nT_s) \cdot F_s \frac{\sin \pi F_s(t - nT_s)}{\pi F_s(t - nT_s)}$$

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Frequency conversion & sub-sampling



We could also choose a different spectral image to (re)construct the signal:

First work out the time-domain representation of the filter:

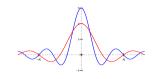
$$H(f)=1$$
 for $F_s < |f| < \frac{3}{2}F_s$

$$H(f)=0$$
 everywhere else

$$h(t) = \mathscr{F}^{-1}{H(f)} = \int_{-\infty}^{\infty} H(f) e^{j2\pi f t} df$$

$$h(t) = \int_{\frac{-3}{2}F_{s}}^{\frac{3}{2}F_{s}} e^{j2\pi ft} df - \int_{-F_{s}}^{F_{s}} e^{j2\pi ft} df$$

$$h(t) = 3F_s \operatorname{sinc}(3\pi t F_s) - 2F_s \operatorname{sinc}(2\pi t F_s)$$

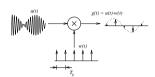


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Frequency conversion & sub-sampling



Note that exactly the same spectrum results for any signal frequency band displaced by $m \cdot F$ (for integer m)

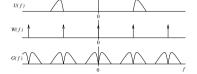


This goes by the name of *sub-sampling*

$$G(f) = \int_{0}^{\infty} U(\Phi + mF_{s})W(f - \Phi) d\Phi$$

$$G(f) = \sum_{n=-\infty}^{\infty} U(f - \frac{n}{T_s} + mF_s)$$

$$G(f) = \sum_{n=-\infty}^{\infty} U(f + \frac{m-n}{T})$$



$$G(f) = \sum_{n=-\infty}^{\infty} U(f - \frac{n}{T_s})$$

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Frequency conversion & sub-sampling



Then convolve the sample stream with the filter function:

$$u_r(t) = g(t) * h(t)$$

$$u_r(t) = \int_{-\infty}^{\infty} g(\tau) \cdot h(t-\tau) d\tau$$



$$u_r(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} u(\tau) \cdot \delta(\tau - nT_s) \cdot h(t - \tau) d\tau$$

$$u_r(t) = \sum_{n=-\infty}^{\infty} u(nT_s) \cdot h(t - nT_s)$$

$$u_r(t) = \sum_{n=-\infty}^{\infty} u(nT_s) \{ 3F_s \operatorname{sinc} (3\pi(t-nT_s)F_s) - 2F_s \operatorname{sinc} (2\pi(t-nT_s)F_s) \}$$

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Couldn't get my hands on that one. Everyone refers to him, so I mention him as well.

1928: H. Nyquist Telegraph transmission theory

Classic! Deals with signal distortion in transmission channels like undersea cables, which were a hot subject, at the time.

1933: V.A. Kotelnikov : Carrying capacity of the ether

Detailed demonstration that band-limited signals can be represented by a sum of sinc functions, apparently independently from Nyquist and Whittaker.

1949: C. Shannon Communication in the presence of noise

Classic! Gives transmission capacity of a channel as a function of bandwidth and signal to noise ratio. The sampling theorem is dealt with in section II.

Other names: R.V.L Hartley, J.M. Whittaker, C-J. de la Vallée Poussin, ...

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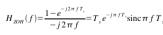
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Practical signal reconstruction

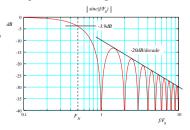


As a consequence, the reconstructed signal spectrum is convolved with a sinc(f/F) function and some energy from adjacent spectral images leaks into the desired band. Note that at the Nyquist frequency, $F_N = F_s/2$, the response is down by 3.9dB.

If this is a problem, the reconstruction filter may be designed to compensate. (You can also pre-compensate in the digital domain.)







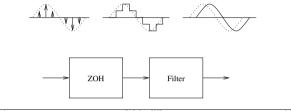
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Practical signal reconstruction



Although mathematically Dirac deltas, brick-wall filters and infinite sums are quite nice to handle, in real electronic circuitry, you can't have them.

The Dirac δ is replaced by an (almost) rectangular pulse of one sampling period duration, and filters are described by finite polynomials, with finite-slope band edges. So, the output is held constant during each sampling period, which is functionally called a zero-order hold, and a low-pass filter smooths over the steps.



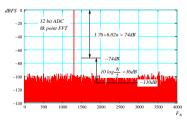
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Spectrum of quantization error



For 'large enough' and 'busy enough' signals, the quantization error is a random variable with a flat distribution.

 \rightarrow Quantization noise is white and spread out evenly over 0 < f < F/2.



FFT Noisefloor:
$$-\left|1.76+6.02\,n+10\log_{10}\frac{N}{2}\right|dBFS$$

(N = number of samples) From analog to digita J. Belleman - CERN

Spurious Free Dynamic Range (SFDR)

12 bit ADC



Unfortunately, quantization noise isn't always white:

- · Simple ratios between F_{in} and F_{s} cause some of the quantization noise power to concentrate in discrete spectral lines
- · ADC non-linearities cause harmonics of the input signal

Spurs appear in the spectrum:

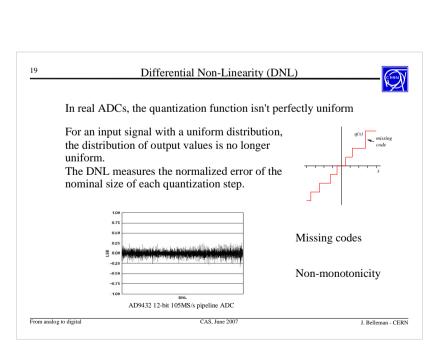
SFDR is the distance between the input signal and the greatest spur.

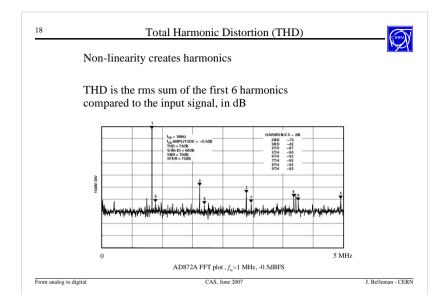
A little bit of dither can help to reduce spurs.

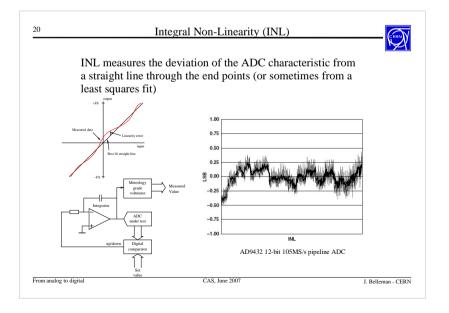
(Dither is the intentional injection of a little bit of noise.)

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Effective Number Of Bits (ENOB):



Apply a nearly full-scale sinusoidal signal. Measure P_{c} , as the rms sum over all frequencies, ignoring DC and the first five harmonics, and solve for *n*:

$$SNR = \frac{P_s}{P_s} = 1.5 \cdot 2^{2n}$$

If you choose to also add in all harmonics into the calculation of P_s , you would get the SINAD. (SIgnal over Noise-And-Distortion) (Which looks a little bit worse, of course)

SNR and SINAD are usually expressed in dBc or dBFS

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Clock jitter

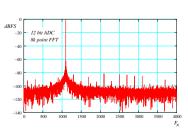


Importance of clock jitter depends on rate of change of analogue input signal

A clock timing error Δt yields an amplitude error:

$$\Delta U = \frac{d u(t)}{dt} \cdot \Delta t$$

This is a severe condition!



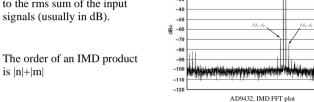
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Inter-Modulation Distortion



Non-linearity also causes inter-modulation distortion. (Creating sum and difference frequencies from two applied tones $f_{imd} = \pm n f_1 \pm m f_2$.)

IMD is the rms sum of the intermodulation products compared to the rms sum of the input signals (usually in dB).



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_ AIN1 = 70.3MHz (-7dBFS) AIN2 = 71.3MHz (-7dBFS) - ENCODE = 105MSPS

is |n|+|m|

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The effect of clock jitter



Tolerable jitter:

$$\Delta t = \frac{1}{\frac{du(t)}{dt} \cdot 2^n}$$

Ex: Suppose we digitize a 100MHz sinusoid to 10 bits:

$$u(t) = \sin 2\pi 10^8 t$$
 $\Rightarrow \frac{du(t)}{dt} = 2\pi 10^8 \cos 2\pi 10^8 t$

So at the steepest slope:

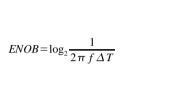
$$\Delta t = \frac{1}{2\pi 10^8 \cdot 2^{10}} \approx 1.6 \, ps \implies \text{A good quartz or ceramic resonator}$$
oscillator is needed

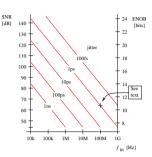
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²⁵ Clock jitter limits vs. SNR and f



$$SNR = 20\log_{10}\frac{1}{2\pi f \Delta T}$$





With 1 ps of jitter, a 100 MHz signal can only be digitized to 10.5 bits Relaxed by root(decimation ratio) for Σ - Δ converters

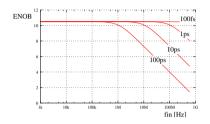
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RC and logic gate oscillators Fair Jitter 10 - 100ps Pierce oscillator Good. Usually better than 1ps From analog to digital CAS, June 2007 Typical jitter specs of common clock sources RC and logic gate Poor Jitter >100ps Fair Jitter 10 - 100ps J. Belleman - CERN

Clock jitter limits vs. SNR and f



Even using the best clock, the resolution reaches a limit. For example, for an actual 12-bit ADC, the ENOB vs. F_{in} plot might look like this:



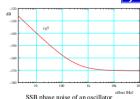
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Oscillator jitter & phase noise



Upconverted thermal, schottky and 1/f noise

$$\Delta t_{ms} = \frac{T_0}{2\pi} \sqrt{\int_0^{\infty} S_{\varphi}(f) \cdot 4\sin^2(\pi f \tau) df}$$



 $S_{\varphi}(f)$ is the spectral density of the phase noise

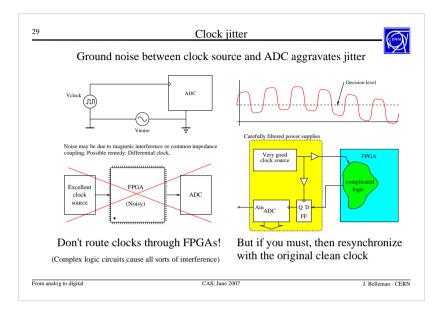
Se (Function of Fourier frequency f and sum of both sidebands)

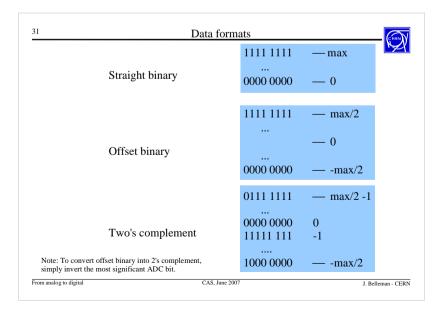
 $\sin^2(\pi f \tau)$ is a weighting function

(For low frequencies of *S*, the phase can't drift very far, when $\tau = n/f$, the contribution cancels, and there are

 τ is the time between two events (Usually $\tau = T_0$)

The term $\frac{T_0}{2\pi}$ converts phase into time





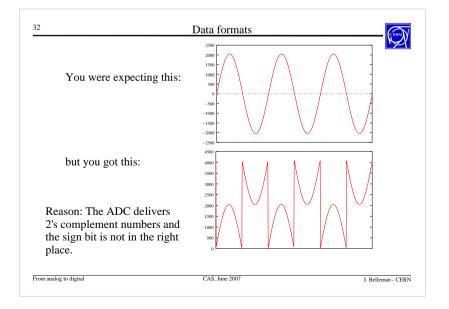
Clock jitter summary

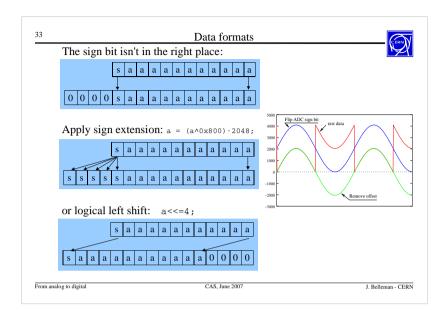


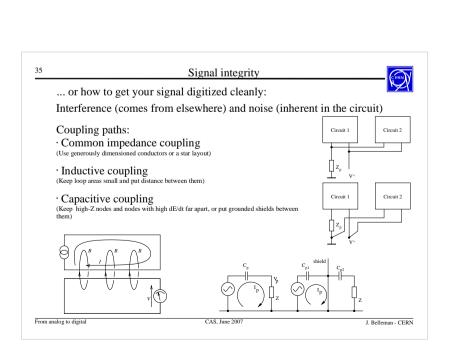
- · ADCs (and DACs too!) need good quality clock sources
- · Digital electronics is not optimized for low crosstalk
- · PLLs in FPGAs usually have *very* poor jitter specs

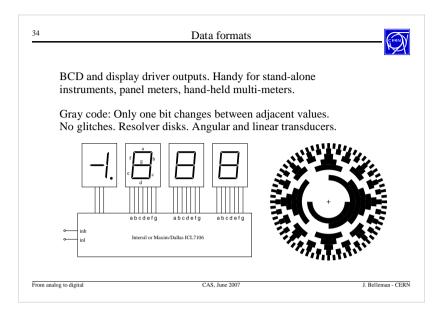
Treat your clock oscillator like a sensitive analogue circuit

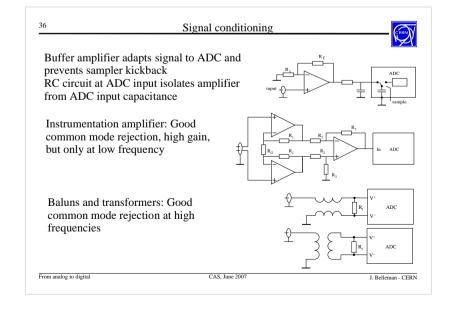
- Filter and bypass clock generation & distribution power supply extra carefully
- · Keep PCB layout tight and compact, minimize loop areas
- · Refer clock source to the same GND as the ADC
- · Do not route an ADC clock through an FPGA
- · Don't use left-over gates in clock buffer package for other purposes









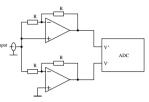


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Signal conditioning

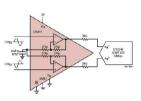


Single-ended to differential conversion (Often used for high-performance or low voltage ADCs)



Baluns and transformers can also be used for this.

Or use monolithic differential buffer amplifiers (ADA4941, AD8351, LT6411, THS4503, etc.)



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ADC types



Architecture	Speed	Resolution	Linearity	Applications
Flash	Very fast (GS/s)	Poor (8 bits)	Poor	Oscilloscopes
				Transient recorders
Successive approximation	Fast (MS/s)	Fair (14 bits)	Fair	DSP, video, digital receivers, instrumentation
Σ-Δ	Slow (kS/s)	Excellent (24 bits)	Excellent	Process control, audio, weight, pressure, temperature measurement
Dual-slope Integration	Very slow (S/s)	Very good (18 bits)	Very good	Bench-top and hand-held measuring instruments, battery powered devices

Other architectures:

- · Mixed forms (E.g. flash with SA, or flash with Σ - Δ)
- · Tracking ADC
- · Voltage-to-frequency converters

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Input signal conditioning



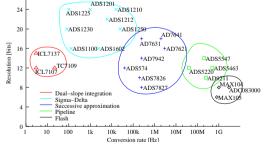
Summary:

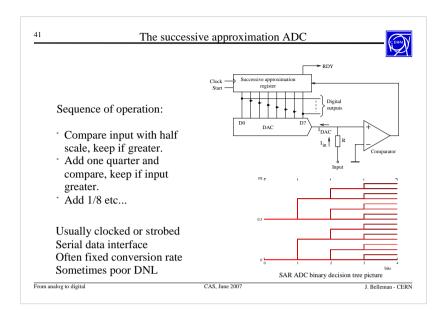
- · Adapting signal range to ADC input range (Scaling & level shifting)
- * Conversion between single-ended and differential signals
- · Protecting the ADC input from overload
- * Terminate long cables into their characteristic impedance...
- · ...or, to the contrary, provide a high impedance to avoid loading the source
- · Rejecting interference
- · Filtering out-of-band frequencies (Anti-alias filter, noise reduction)
- · Holding input constant while conversion takes place

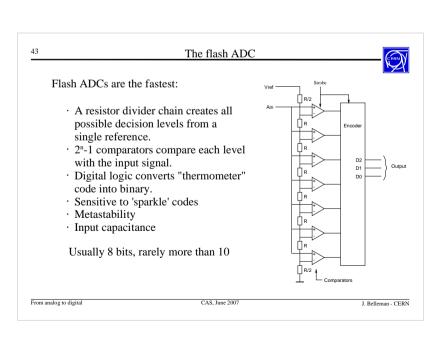
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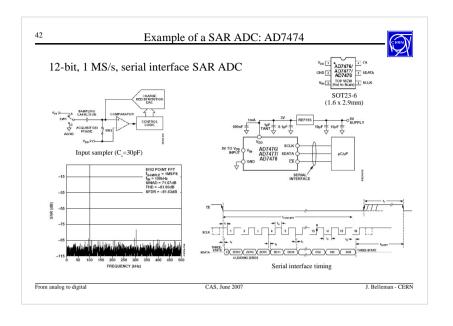
The ADC landscape

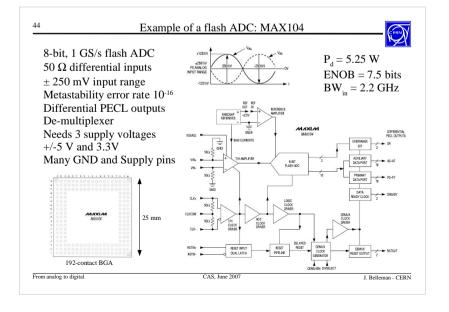










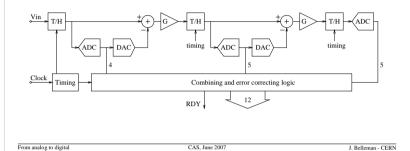


45 Mixed architecture ADCs



Segmented or pipelined ADC:

- · Sample rate comparable to flash ADCs, but with several clock periods of latency.
- · Resolution comparable to successive approximation architecture.

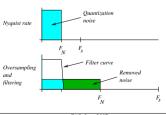


The effect of oversampling



Increase sample rate:

- → Quantization noise is spread over a larger BW.
- Numerically low-pass filter the sample stream:
- → Out-of-band quantization noise power is removed. Conclusion:
- → SNR gets better by 3dB/octave of oversampling rate. Dither may be necessary for very quiet ADCs.



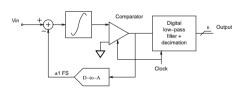
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12-bit, 250 MS/s, 5-stage pipeline ADC Differential LVDS or de-multiplexed outputs 2pF sampling capacitor Differential analogue input, BW 1.2 GHz 12-bit, 250 MS/s, 5-stage pipeline ADC Differential LVDS or de-multiplexed outputs 2pF sampling capacitor Differential analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline ADC Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline Differential Analogue input, BW 1.2 GHz 15-bit, 250 MS/s, 5-stage pipeline Differential Analogue input, BW 1.2 GHz 15-b

The Σ/Δ ADC



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Average duty cycle of comparator output reflects input value.

Digital low-pass decimation filter trades sample rate for resolution.

Good DNL, good resolution, slow.

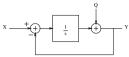
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The Σ/Δ ADC



For the input: $\frac{Y}{X} = \frac{1}{s+1}$

$$\frac{Y}{X} = \frac{1}{s+1}$$



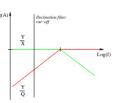
Pretend that quantizer contributes random uncorrelated noise:

For the noise:

$$\frac{Y}{Q} = \frac{s}{s+1}$$

Input signal is low-pass filtered Quantization noise is high-pass filtered Decimation filter rejects high frequency

→ Resolution is improved



Noise shaping!

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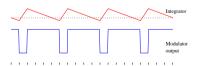
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Σ/Δ ADC ailments



Side tones, idling patterns, birdies

- * For some input values, the Σ/Δ modulator can produce repeating patterns with repetition rates well below the sampling frequency
- These may leak though the decimation filter, causing a side tone or 'birdie'



· Possible remedies include using higher order modulators and dither, to randomize things

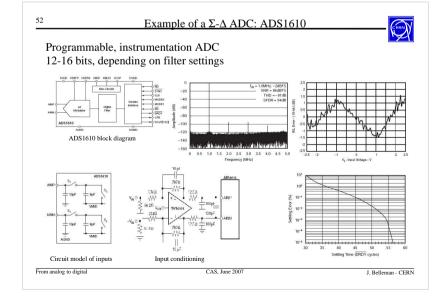
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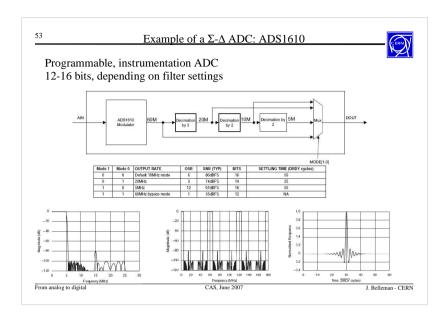
The Σ/Δ ADC Higher order loops and noise shaping, E.g., a 2nd order modulator: For the input: For the quantization noise:

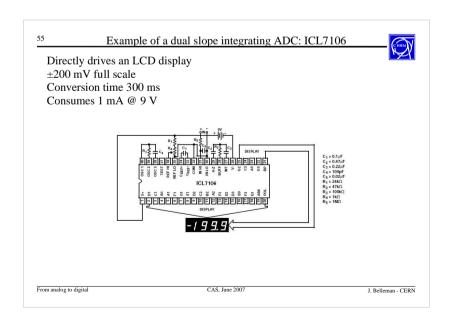
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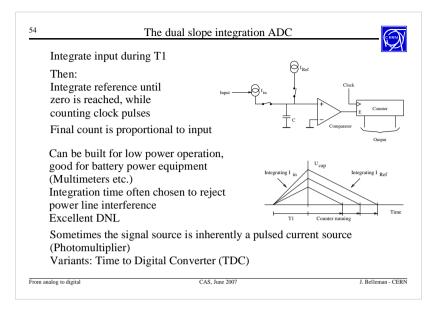
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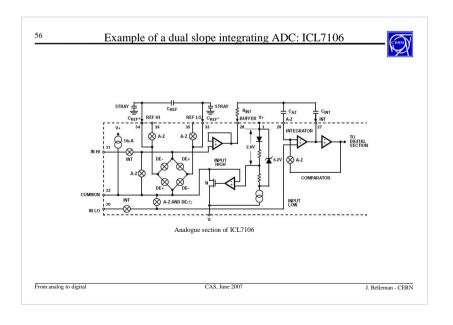
Decimating filter rejects noise









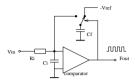


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Voltage-to-Frequency converters



Input voltage is integrated onto Ci A fixed-size packet of charge is removed each time the switch connects Cf to the input.



Applications:

Process control

Easy to use as integrating converter (Just add a counter)

In combination with F-to-V converter: Cheap isolation amplifier

Low-cost

Slow!

Signal easy to transmit over large distances

Very good linearity

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Special converters



Non-linear converters, A-law, μ -law, used in telephony.



Digital potentiometers (DS1669, MAX5438, AD5259, etc. Digital output sensors, temperature, acceleration (AD7414, AD16006) Capacitance-to-digital converters (AD7745)

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8-pin DIP Linearity 0.003% Fmax 10 kHz From analog to digital Example of a V-to-F converter: LM331 Dual-in-Line Package Consequence Promatory Promat

DAC architectures



Architecture			
Kelvin-Varley divider	Accurate, monotonous. Mainly as building block in integrated DACs		
Thermometer DAC	Monotonous. Limited number of bits		
Binary weighted ladder	Very common, but subject to glitches		
R-2R ladder	Widely used. Not very power efficient		
Σ-Δ	Linear, accurate, but complex.		

Mixed architectures:

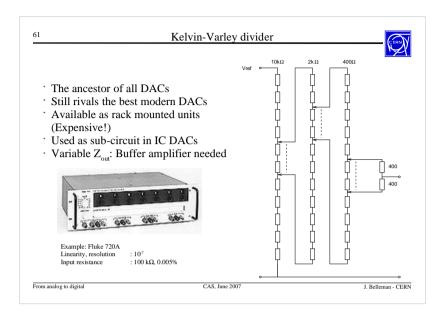
Variants:

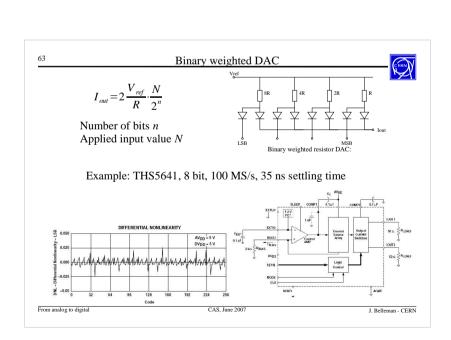
Segmented DAC
Interpolating DAC

- Multiplying DACs
- · Current or voltage output
 - · Differential or single-ended

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Thermometer' DAC

Decode

Prom analog to digital

'Thermometer' DAC

Tout

Lout

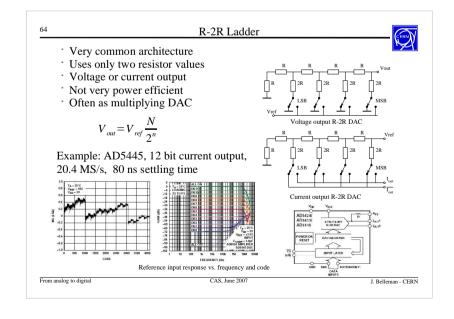
Lout

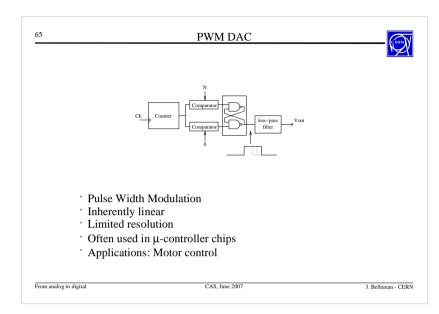
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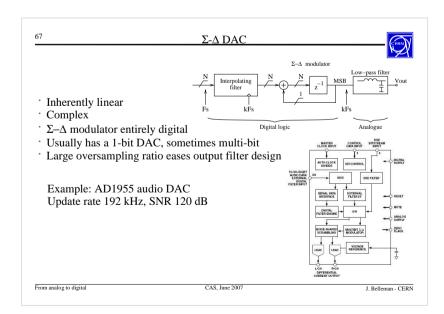
CAS, June 2007

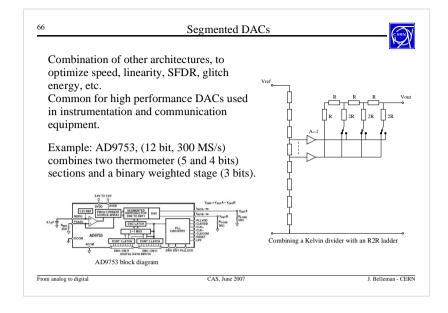
Jout

Lout









Conclusions



- · There are ADCs and DACs for almost any imaginable application
- · Performance is ever getting better
- $\cdot \ Prices \ keep \ going \ down \ \hbox{\scriptsize (15\,years ago, a 12\,bit 10\,MS/s ADC cost 1\,k\$. \ Now it's around 10\,\$)}\\$

Digital is here to stay!