Muon Cooling RF: considerations on beam loading and multi-harmonic RF systems

Alexej Grudiev (CERN)

Acknowledgements: Sergey Arsenyev (CEA), Chris Rogers (STFC)

12/10/2022

Muon Collider Annual meeting

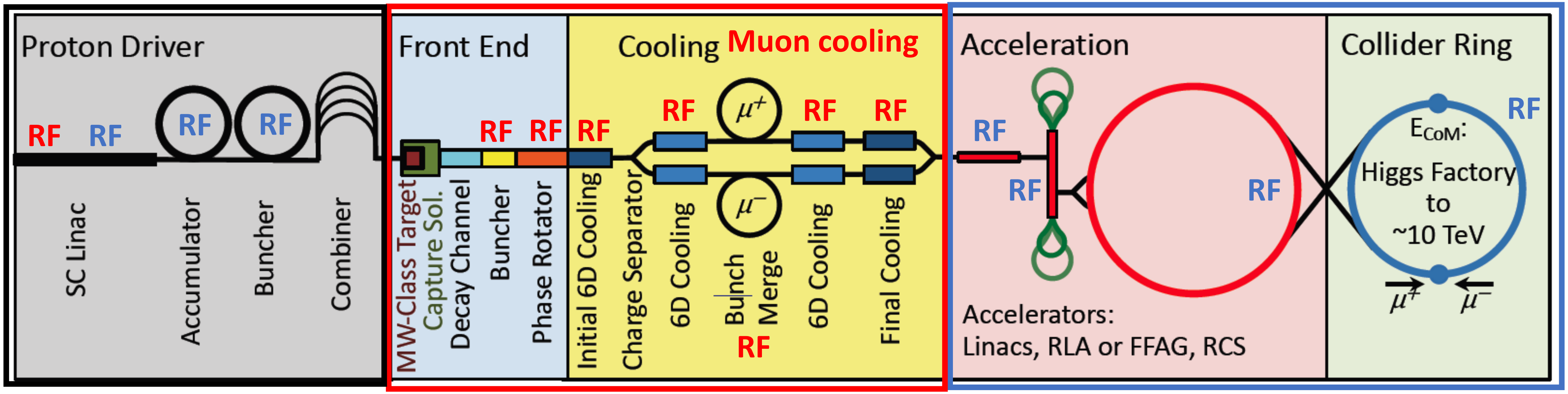
Outline

- Introduction: Muon collider layout and RF
- Effect of beam loading in 6D cooling channel and in the front-end
- Multi-Harmonic RF system for final cooling

Muon Colliders and RF systems

Proton driven Muon Collider Concept (MAP collaboration)

Acceleration and collider rings

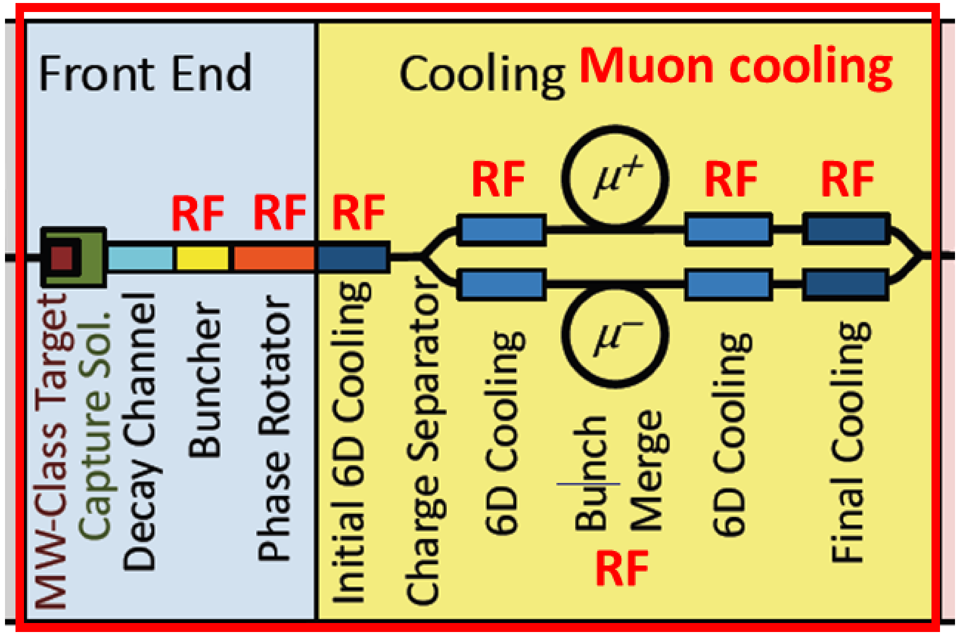


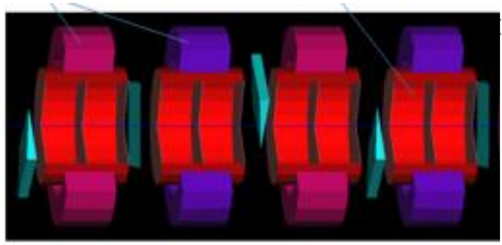
RF system for muon capture and cooling

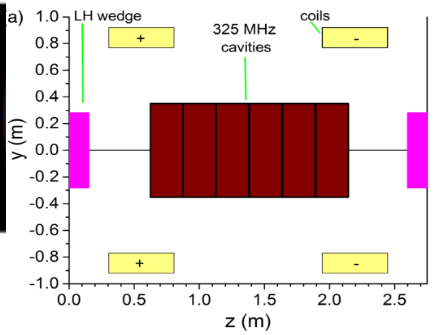
Summarized from:
David Neuffer
Chris Rogers

Region	Length [m]	N of cavities	Frequencies [MHz]	Peak Gradient [MV/m]	Peak RF power [MW/cav.]
Buncher	21	54	490 - 366	**0 - 15**	1.3
Rotator	24	64	366 – 326	**20**	2.4
Initial Cooler	126	360	325	**25**	**3.7**
Cooler 1	400	1605	325, 650	**22, 30**	
Bunch merge	130	26	108 - 1950	**~ 10**	
Cooler 2	420	1746	325, 650	**22, 30**	
Final Cooling	140	96	325 - 20		
Total	**~1300**	**3951**			**=> ~12GW**

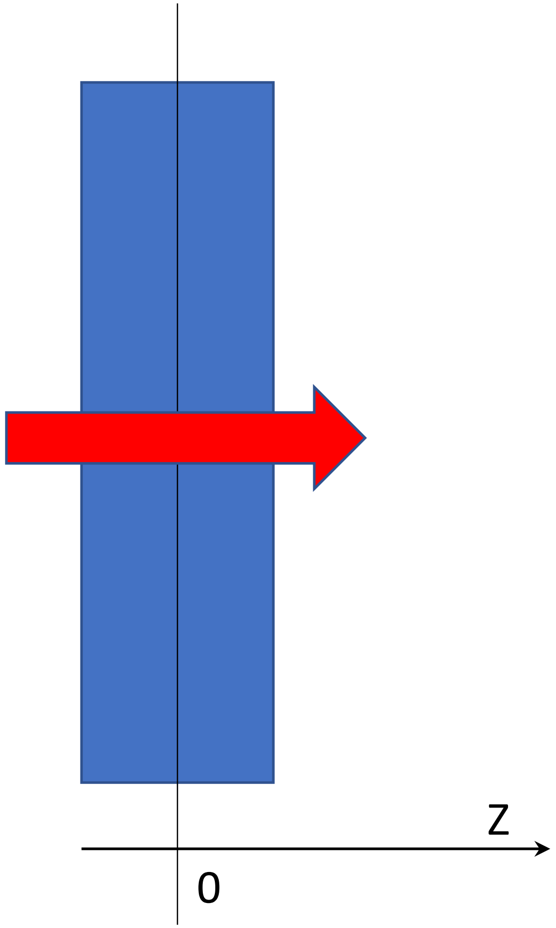
- **It is a very large and complex RF system with high peak power**
- **Very high bunch charge: (~40->4)e12 => Collective effects (?)**





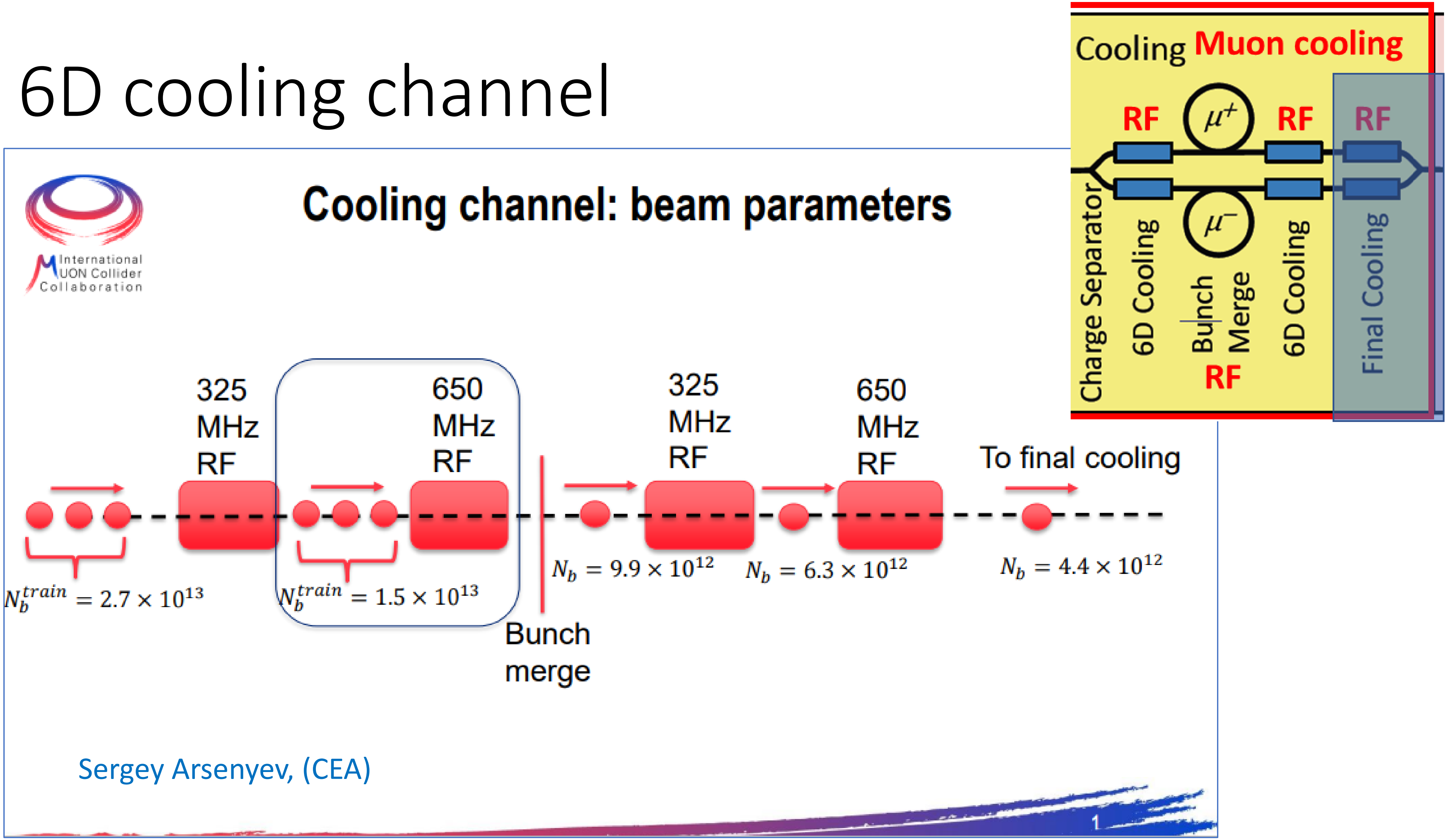


Cavity voltage with beam loading



- Bunch reference: $t(z = 0) = t_0 = \varphi_0/\omega$
- $\mathrm{V}(t) = V_0 \cos(\omega t + \varphi_0) - V_B \cos(\omega t)$
- Beam loaded voltage: $V_B =$
  - $t < 0,\ 0$
  - $t > 0,\ 2qk_{loss} = q\omega \frac{R}{Q} T^2$
- Transit time factor: T is function of bunch velocity

6D cooling channel



Cooling channel: beam parameters

$N_b^{train} = 2.7 \times 10^{13}$

$N_b^{train} = 1.5 \times 10^{13}$

$N_b = 9.9 \times 10^{12}$

$N_b = 6.3 \times 10^{12}$

$N_b = 4.4 \times 10^{12}$

Sergey Arsenyev, (CEA)

Effect of the beam loading, examples

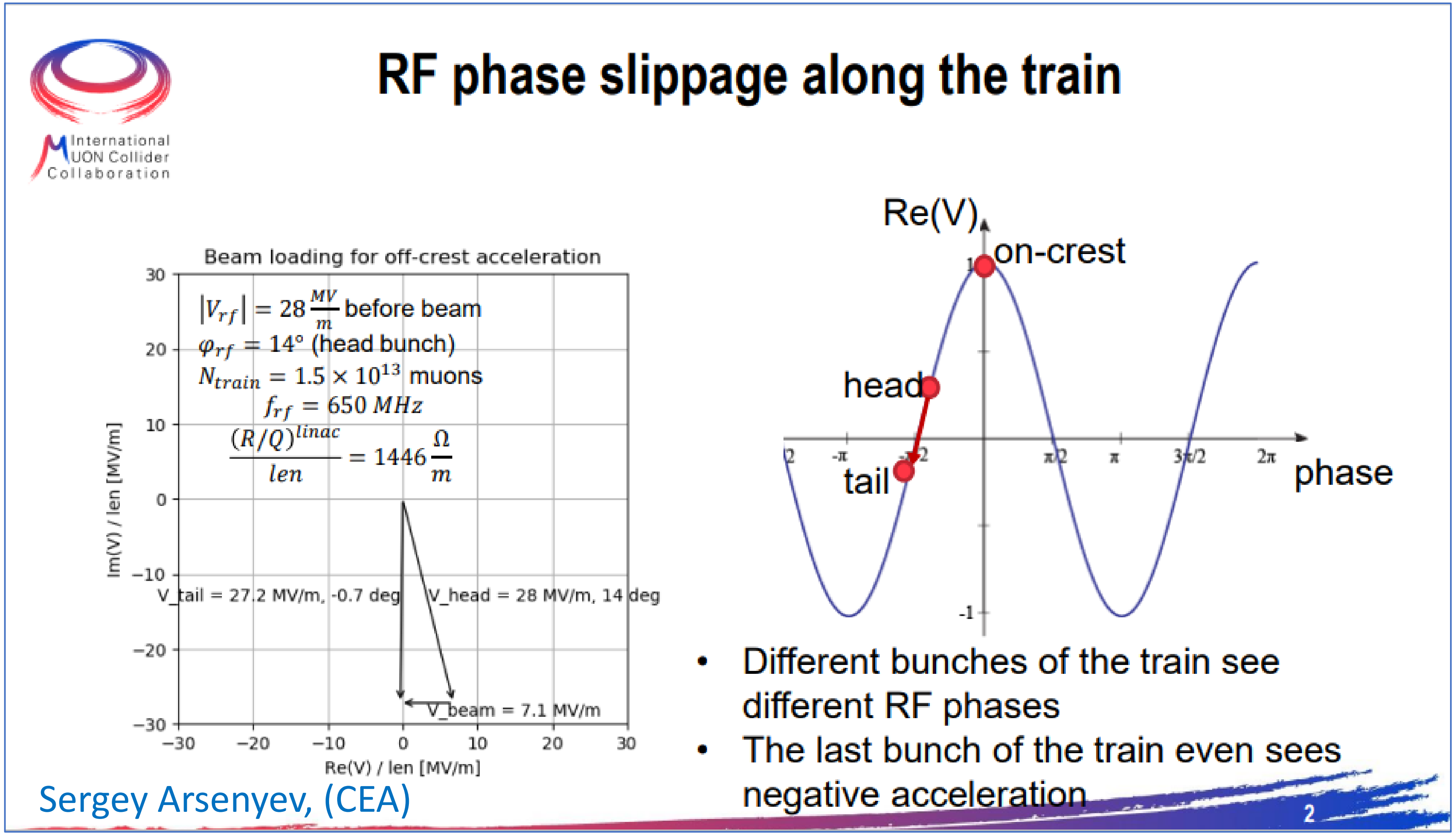
Stratakis, PRAB 18, 031003, (2015)

- A1 stage
- Some parameters from the paper:
  - f = **325 MHz**;
  - h =~ 0.26 m;
  - φ_0 = 90 – 14 = 76 deg;
  - Ez(r=0) = 22 MV/m
- Other parameters assumed:
  - T ~ 1;
  - R/Q ~ 100 Ohm
  - q ~ e*2.7e13 = **4.3 uC**
- Unloaded voltage: V0 ~ 5.7MV
- Accelerating voltage: V0*cos(φ_0) ~ 1.4MV
- Beam loaded voltage: VB ~ **0.89 MV**
- **VB/V0 ~ 16%**

- A3 stage
- Some parameters from the paper:
  - f = **650 MHz**;
  - h =~ 0.13 m;
  - φ_0 = ? deg;
  - Ez(r=0) = 28 MV/m
- Other parameters assumed:
  - T ~ 1;
  - R/Q ~ 100 Ohm
  - q ~ e*1.5e13 = 2.4 uC
- Unloaded voltage: V0 ~ 3.6MV
- Beam loaded voltage: VB ~ **1 MV**
- **VB/V0 ~ 28%**

Non-negligible effect of the beam loading

Effect of beam loading on the cavity voltage



RF phase slippage along the train

- Different bunches of the train see different RF phases
- The last bunch of the train even sees negative acceleration

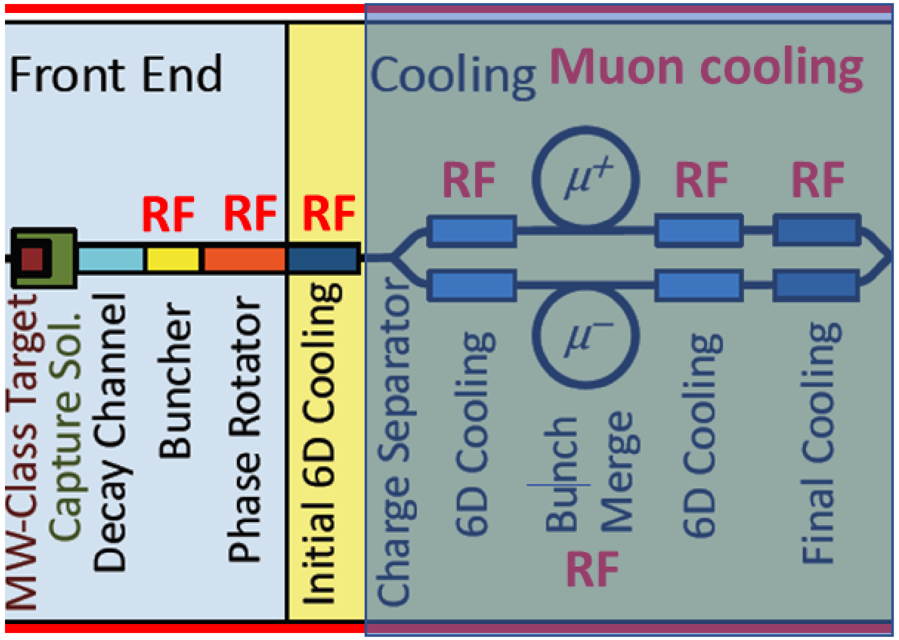
Sergey Arsenyev, (CEA)

Beam loading mitigation

- Higher gradient
- Lower R/Q => Lower frequency
- Mismatch cavity frequency and bunch frequency (detuning)
- …
- Effect of beam loading on the beam dynamics must be **simulated** to apply mitigation measures

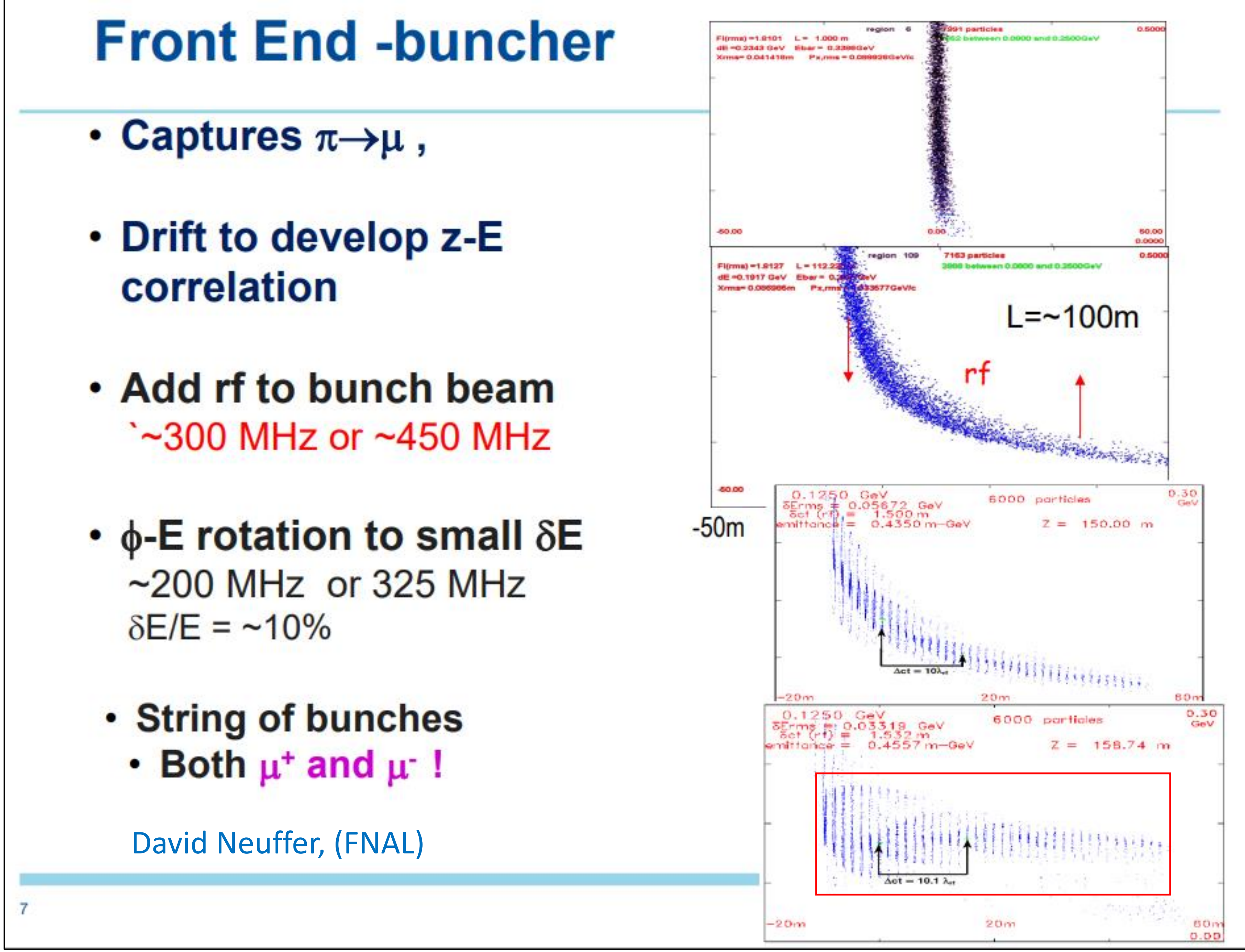
Front-End, worst-case example

- Some example parameters for 325 MHz cavity:
  - f = 325 MHz;
  - h =~ 0.26 m;
  - Ez(r=0) = 0 - 25 MV/m
- Other parameters assumed:
  - T ~ 1;
  - R/Q ~ 100 Ohm
  - q ~ e*4e13 = **6.4 uC**
  - **2** bunches: $\mu^+ + \mu^-$ **=> 12.8 uC**
- Unloaded voltage: V0 ~ 0 – 6 MV
- Beam loaded voltage: VB ~ **5.3 MV**
- **VB/V0 ~ 100% or higher**



Front-End

BL mitigation in Front-End: In addition to RF measures **beam manipulation** measures should be considered



Front End -buncher

- Captures $\pi \rightarrow \mu$,
- Drift to develop z-E correlation
- Add rf to bunch beam
  `~300 MHz or ~450 MHz
- ϕ-E rotation to small δE
  ~200 MHz or 325 MHz
  $\delta E/E$ = ~10%
- String of bunches
  - Both μ^+ and μ^- !

David Neuffer, (FNAL)

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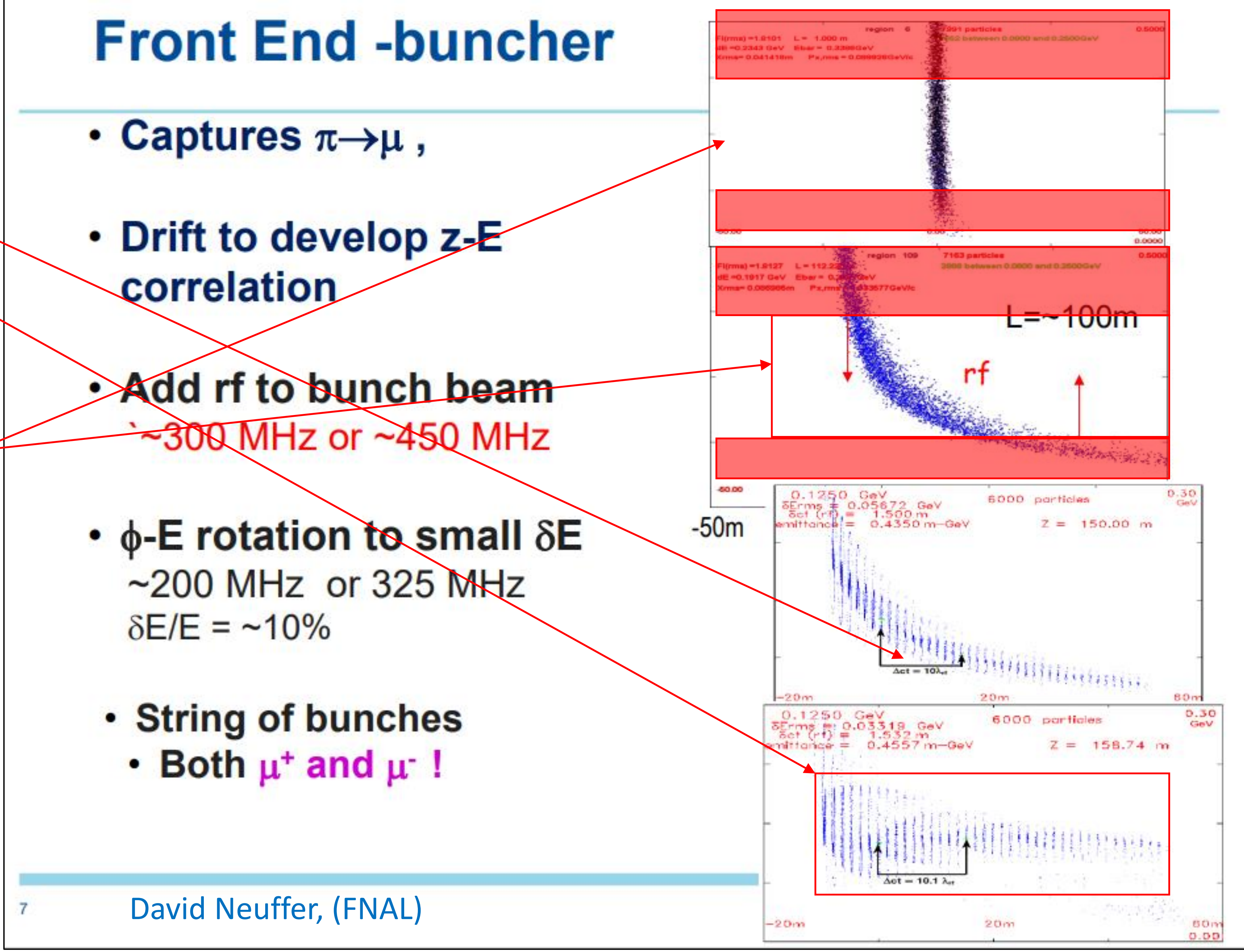
Front-End

- Only fraction of muons is captured but all muons contribute to BL
- Removing useless muons at earliest stage by, for example **energy collimation will reduce BL**
- **Separation μ^+ and μ^- before RF cavities will reduce BL**
- **In addition, Lower RF frequency can both reduce BL and increase bunch intensity (less bunches to recombine)**



Front End -buncher

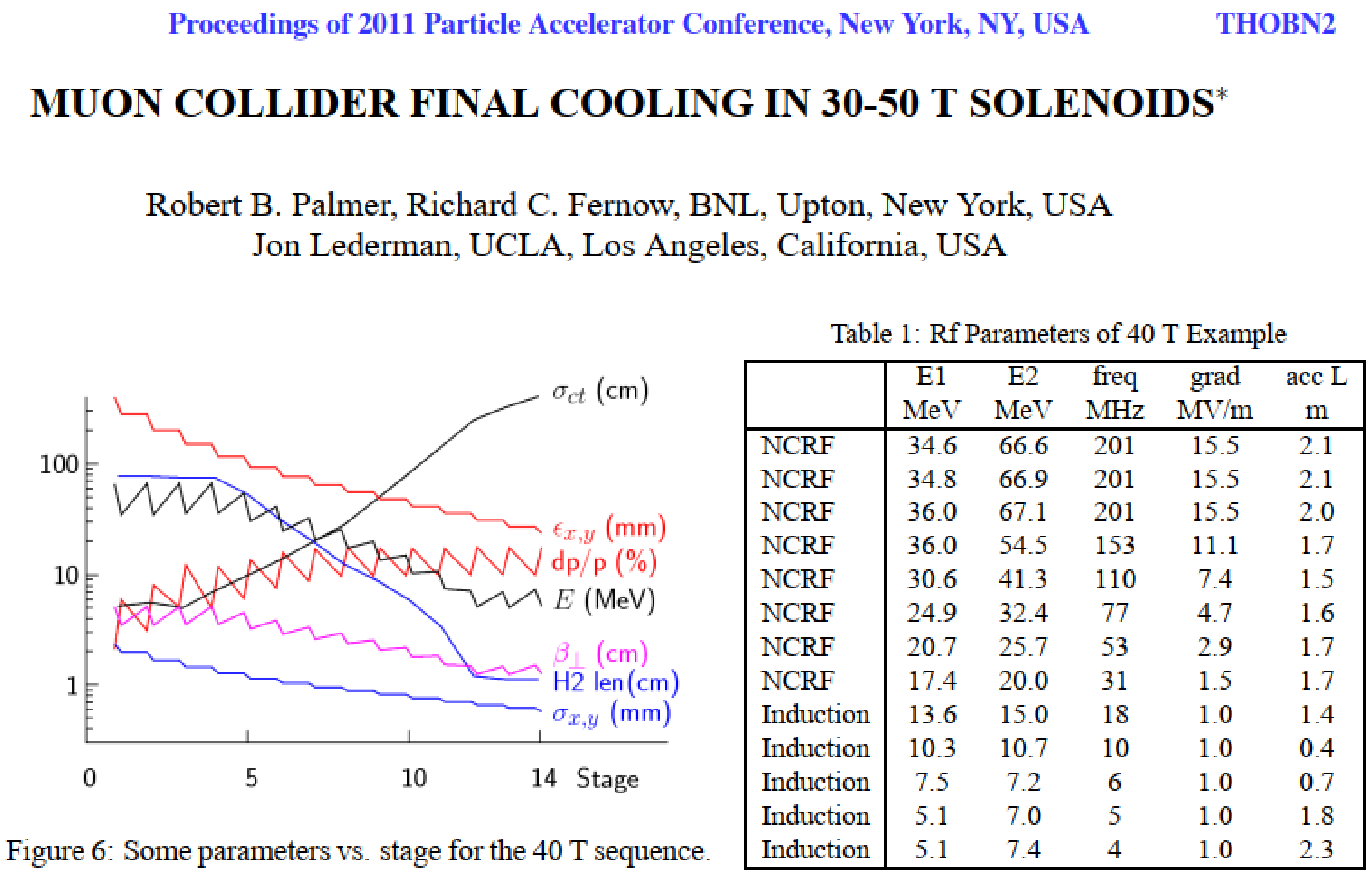
- **Captures $\pi \rightarrow \mu$,**
- **Drift to develop z-E correlation**
- **Add rf to bunch beam**
  `~300 MHz or ~450 MHz
- **ϕ-E rotation to small δE**
  ~200 MHz or 325 MHz
  $\delta E/E$ = ~10%
- **String of bunches**
  - **Both μ^+ and μ^- !**

David Neuffer, (FNAL)

Conclusion on beam loading

- Beam loading (BL) is an important effect and cannot be neglected in the design of the Muon cooling complex
- RF mitigation measures should be applied to compensate the BL effects
- To do this, BL effects on Beam dynamics should be simulated and better understood
- In the worst-case of the front-end, additional mitigation measures by beam manipulations should be considered, for example:
  - Collimation: energy, …
  - Separation of mu+ and mu- before RF cavities,
  - …

Example of final cooling design, (2011)



Proceedings of 2011 Particle Accelerator Conference, New York, NY, USA THOBN2

MUON COLLIDER FINAL COOLING IN 30-50 T SOLENOIDS*

Robert B. Palmer, Richard C. Fernow, BNL, Upton, New York, USA
Jon Lederman, UCLA, Los Angeles, California, USA

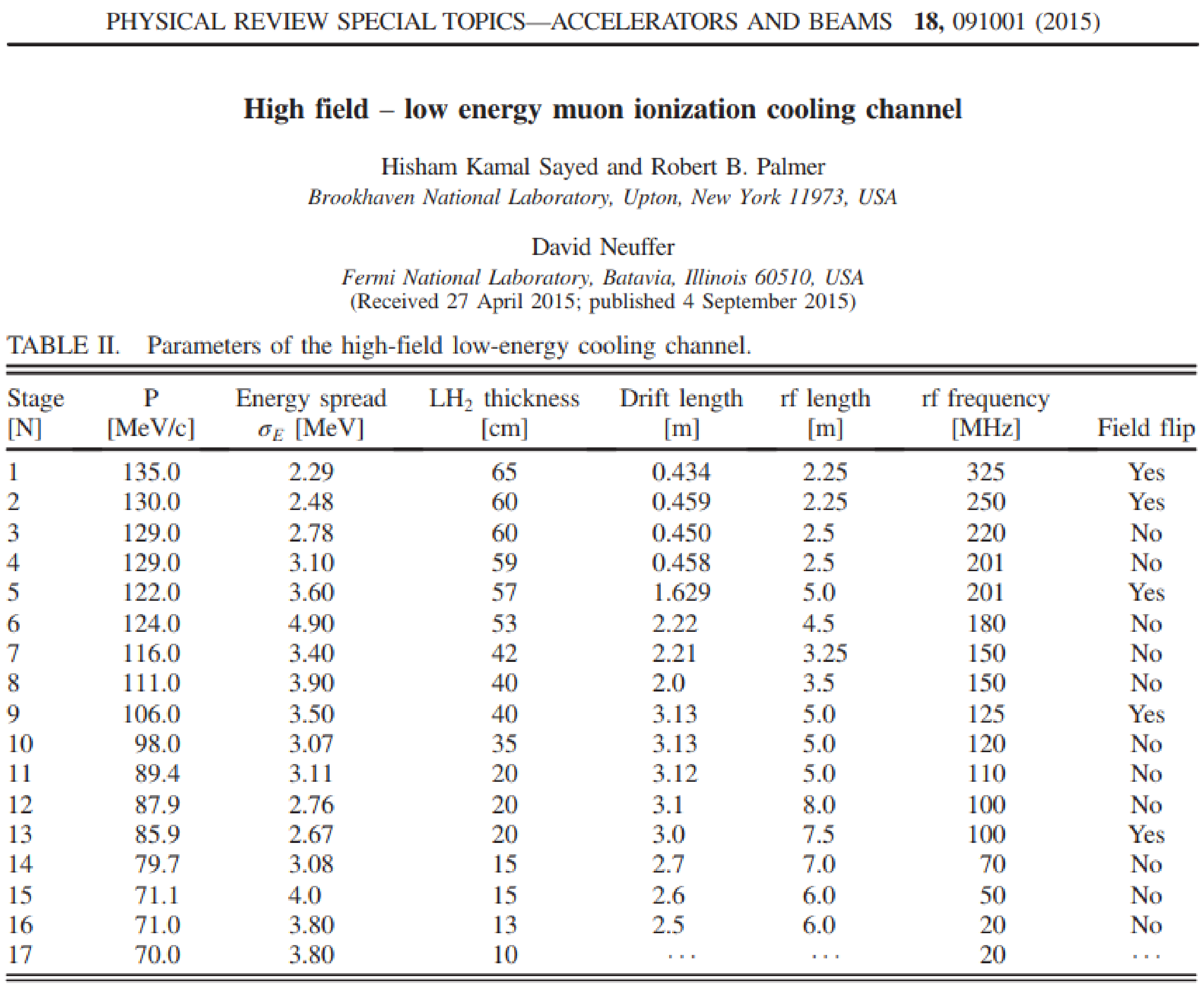
Figure 6: Some parameters vs. stage for the 40 T sequence.

Table 1: Rf Parameters of 40 T Example

	E1 MeV	E2 MeV	freq MHz	grad MV/m	acc L m
NCRF	34.6	66.6	201	15.5	2.1
NCRF	34.8	66.9	201	15.5	2.1
NCRF	36.0	67.1	201	15.5	2.0
NCRF	36.0	54.5	153	11.1	1.7
NCRF	30.6	41.3	110	7.4	1.5
NCRF	24.9	32.4	77	4.7	1.6
NCRF	20.7	25.7	53	2.9	1.7
NCRF	17.4	20.0	31	1.5	1.7
Induction	13.6	15.0	18	1.0	1.4
Induction	10.3	10.7	10	1.0	0.4
Induction	7.5	7.2	6	1.0	0.7
Induction	5.1	7.0	5	1.0	1.8
Induction	5.1	7.4	4	1.0	2.3

- In each stage, RF re-accelerates and phase-rotates the muons providing energy and energy-spread required for the next stage
- To keep energy spread low the bunch length is increased from 5 cm in the 1st stage to 400 cm in the last one
- The RF frequency must follow this trend to make sure that the bucket is large enough and
  - the RF curvature in accelerating mode is not too large compared to energy spread
  - RF slop is linear enough in the phase rotation mode
- The RF curvature is probably more critical leading to criteria for RF frequency choice: $\sigma_{ct} < \lambda/20$ for bunches shorter than 75 cm
- The lowest frequency is 4 MHz assuming induction linac
- The lowest NCRF cavity frequency is 31 MHz

Example of final cooling design, (2015)



PHYSICAL REVIEW SPECIAL TOPICS—ACCELERATORS AND BEAMS **18,** 091001 (2015)

High field – low energy muon ionization cooling channel

Hisham Kamal Sayed and Robert B. Palmer
Brookhaven National Laboratory, Upton, New York 11973, USA

David Neuffer
Fermi National Laboratory, Batavia, Illinois 60510, USA
(Received 27 April 2015; published 4 September 2015)

TABLE II. Parameters of the high-field low-energy cooling channel.

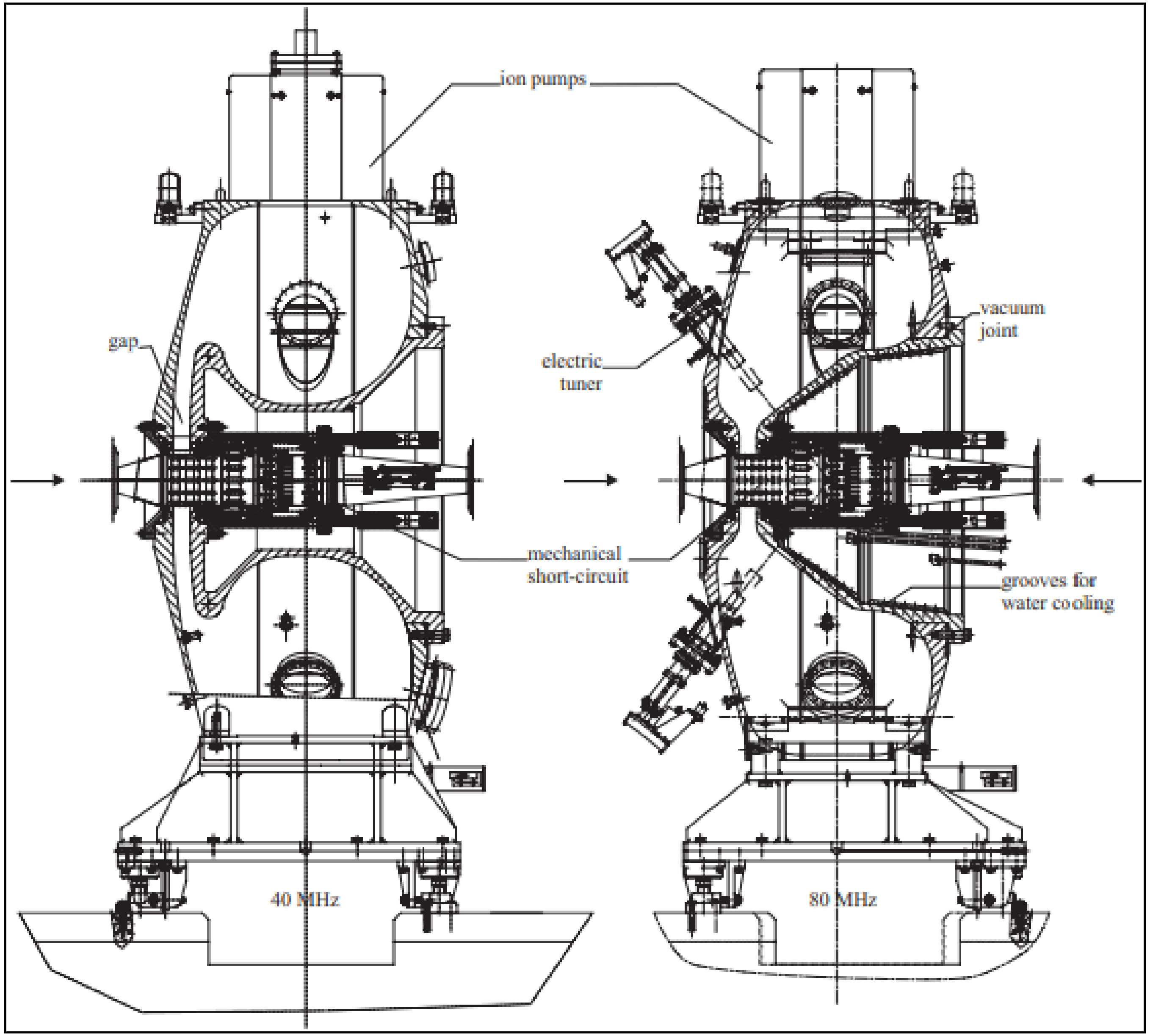
Stage [N]	P [MeV/c]	Energy spread σ_E [MeV]	LH_2 thickness [cm]	Drift length [m]	rf length [m]	rf frequency [MHz]	Field flip
1	135.0	2.29	65	0.434	2.25	325	Yes
2	130.0	2.48	60	0.459	2.25	250	Yes
3	129.0	2.78	60	0.450	2.5	220	No
4	129.0	3.10	59	0.458	2.5	201	No
5	122.0	3.60	57	1.629	5.0	201	Yes
6	124.0	4.90	53	2.22	4.5	180	No
7	116.0	3.40	42	2.21	3.25	150	No
8	111.0	3.90	40	2.0	3.5	150	No
9	106.0	3.50	40	3.13	5.0	125	Yes
10	98.0	3.07	35	3.13	5.0	120	No
11	89.4	3.11	20	3.12	5.0	110	No
12	87.9	2.76	20	3.1	8.0	100	No
13	85.9	2.67	20	3.0	7.5	100	Yes
14	79.7	3.08	15	2.7	7.0	70	No
15	71.1	4.0	15	2.6	6.0	50	No
16	71.0	3.80	13	2.5	6.0	20	No
17	70.0	3.80	10	⋯	⋯	20	⋯

- More recent paper, less details on RF though
- Same criteria is used for RF frequency choice: $\sigma_{ct} < \lambda/20$
- General trend: Higher energies -> shorter bunches -> higher RF frequencies
- The lowest RF frequency is 20 MHz, which might still require induction linac

- There is strong motivation to avoid induction linac due to its rather low gradient: ~1MV/m

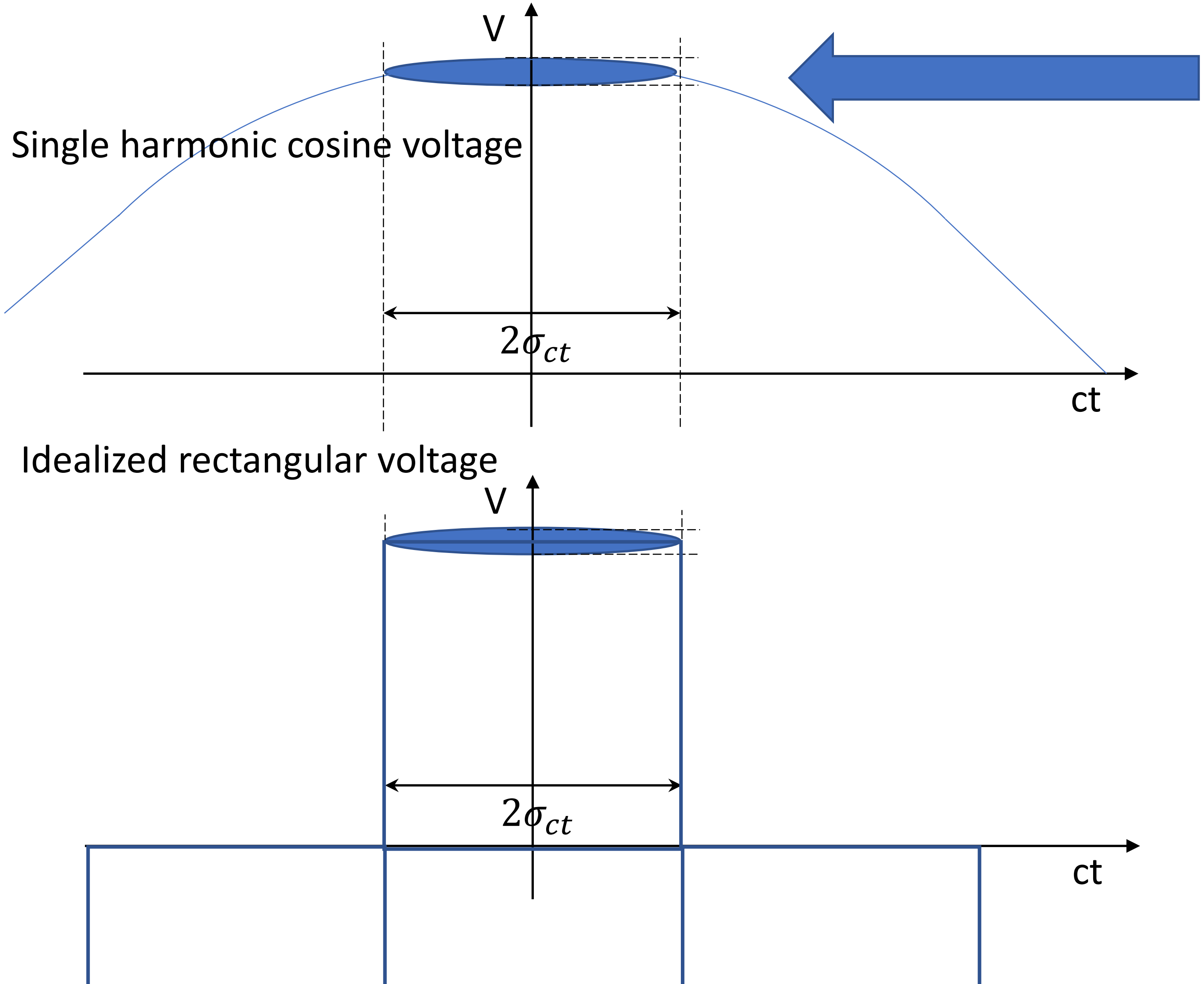
CERN PS RF cavities

- 1.6m diameter, 1m long
- Far from pillbox
- Gap is very small: 5cm, so even for reasonable electric field in the gap of 10MV/m@CW voltage and real estate gradient are very low: 0.5MV and 0.5MV/m, respectively.
- It can potentially be increased in pulsed operation mode…
- => Strong motivation to increase lowest RF frequency required for final cooling



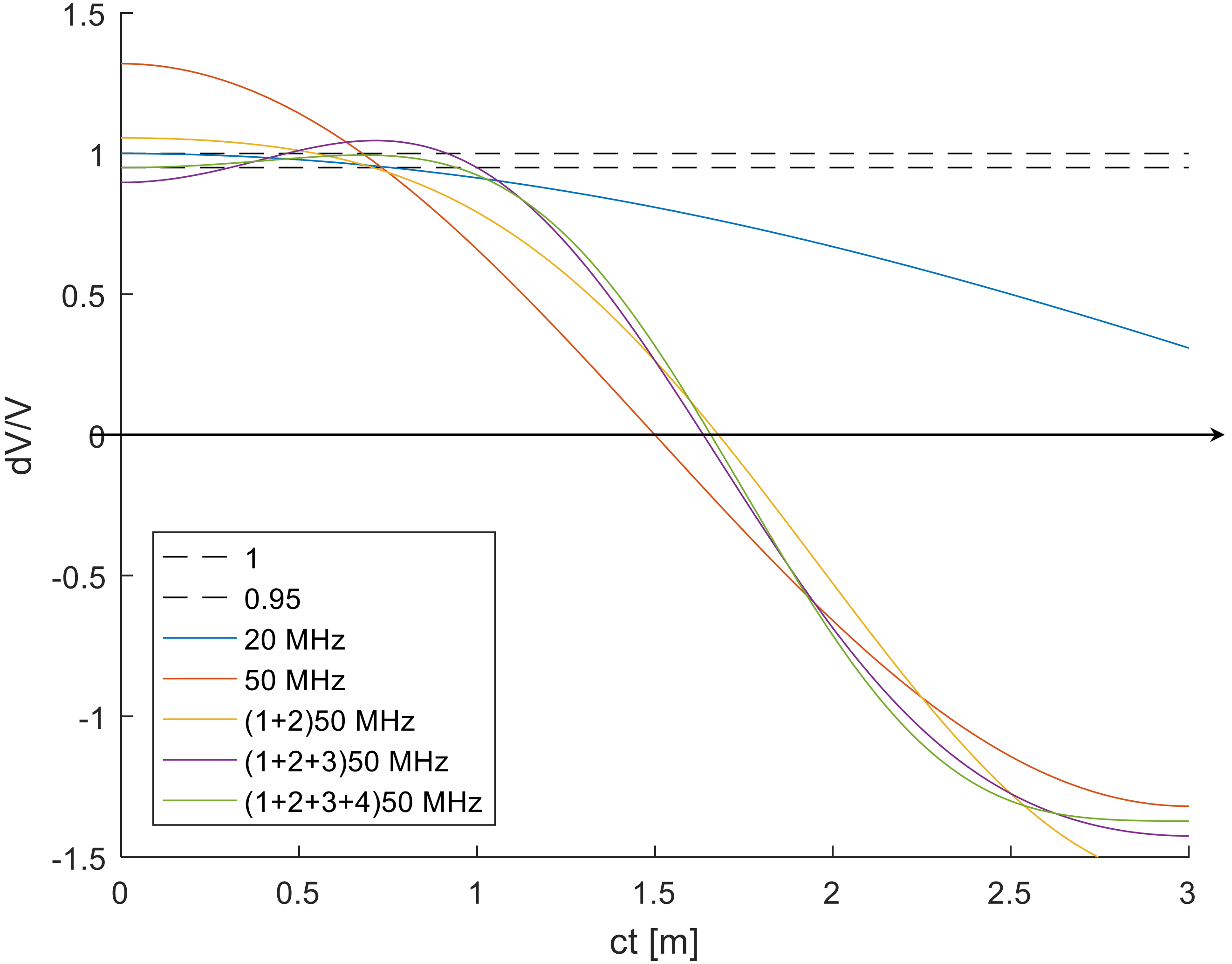
LHC TDR v3, p59 Figure 8.3: Longitudinal section of the 40 MHz (left) and 80 MHz cavities (right).

Idea: Multi-harmonic RF for acceleration



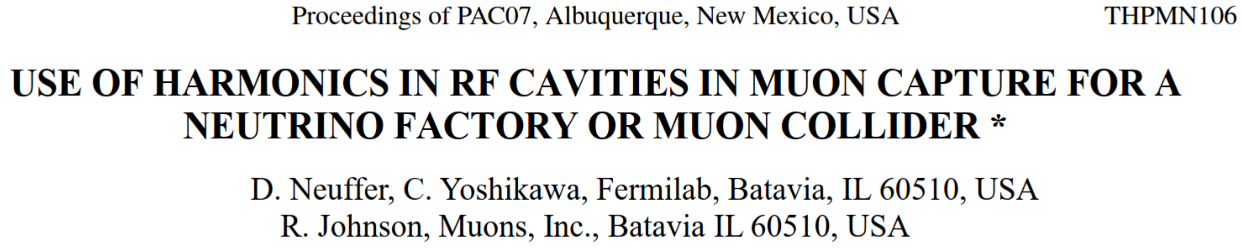
- RF frequency criteria: $\sigma_{ct} < \lambda/20$ for $\sigma_{ct} < 0.75$m related to energy spread introduced by the RF curvature
- For total bunch length of $2\sigma_{ct}$: dV/V = $cos\,\frac{2\pi\sigma_{ct}}{\lambda} \approx \cos\left(\frac{2\pi}{20}\right) \approx$ **5%**
- So, for σ_{ct}=0.75m
  - Single harmonic RF λmax= $20\sigma_{ct} = 15$m -> fmin=**20 MHz**
  - Rectangular meander RF with period of $4\sigma_{ct} = 3$m –> fmin=**100 MHz**
- But **infinite number of higher** harmonics is necessary to approach the ideal rectangular shape

An example: odd harmonics



- In practice **finite number of harmonics** can be used only
- To maintain energy spread of dV/V <5% over σ_{ct} <0.75m, several harmonics of 50 MHz have been used in this example
- 50 MHz provides dV/V <5% over σ_{ct} <0.3m only
- 50, 100, 150, 200 MHz extend the range up to σ_{ct} = 1m
- Minimum frequency can be increased from 50 MHz probably up to 100 MHz
- Using higher harmonics is less and less effective

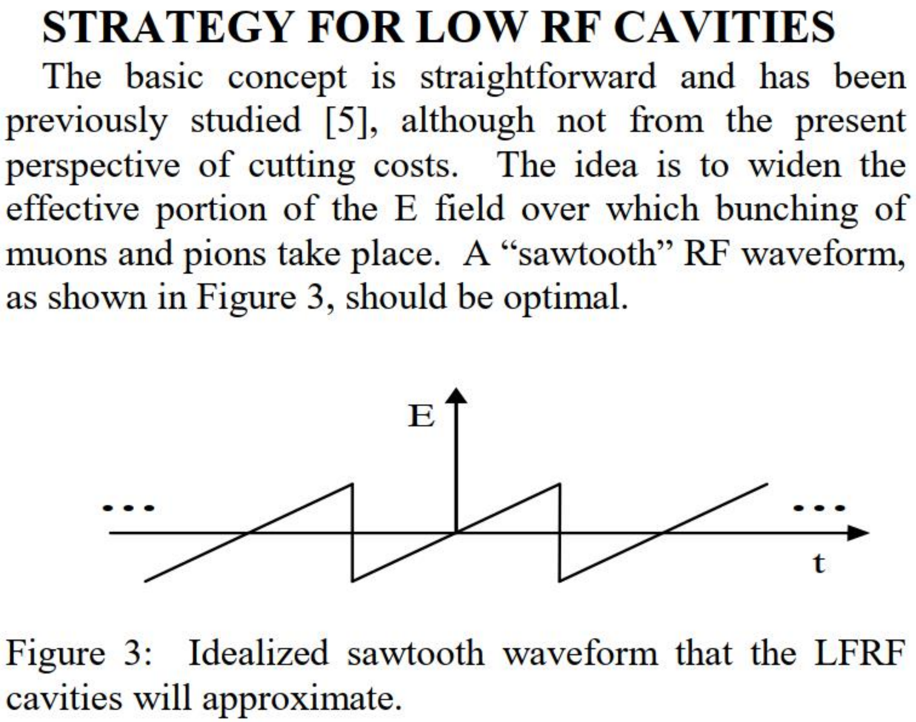
Acknowledgement to earlier work by D. Neuffer



Proceedings of PAC07, Albuquerque, New Mexico, USA THPMN106

USE OF HARMONICS IN RF CAVITIES IN MUON CAPTURE FOR A NEUTRINO FACTORY OR MUON COLLIDER *

D. Neuffer, C. Yoshikawa, Fermilab, Batavia, IL 60510, USA
R. Johnson, Muons, Inc., Batavia IL 60510, USA



STRATEGY FOR LOW RF CAVITIES

The basic concept is straightforward and has been previously studied [5], although not from the present perspective of cutting costs. The idea is to widen the effective portion of the E field over which bunching of muons and pions take place. A "sawtooth" RF waveform, as shown in Figure 3, should be optimal.

Figure 3: Idealized sawtooth waveform that the LFRF cavities will approximate.

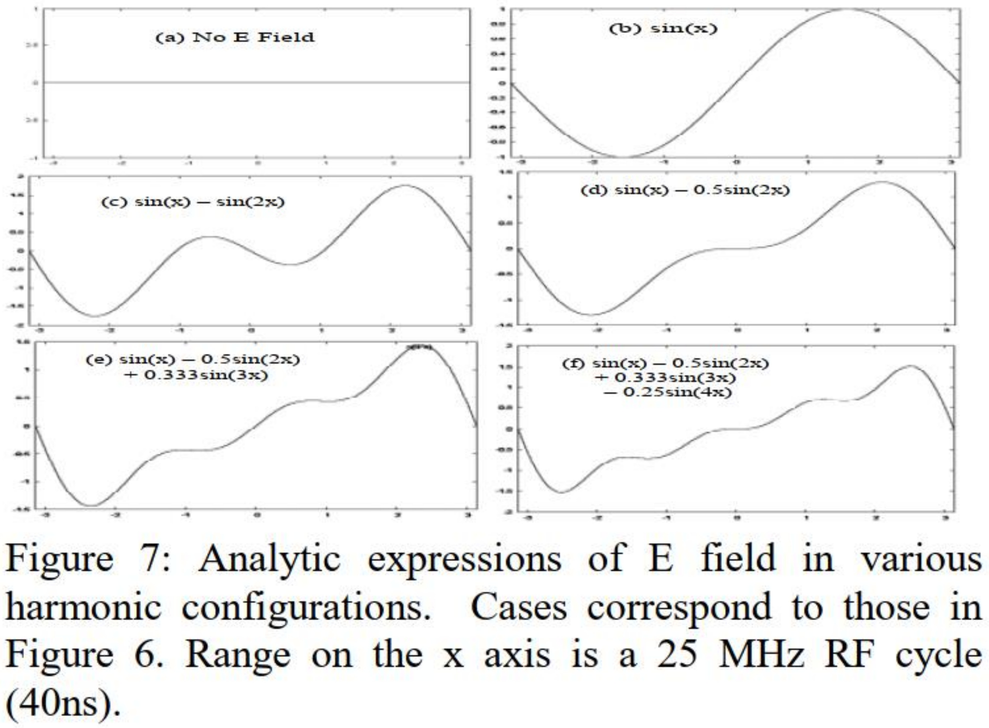


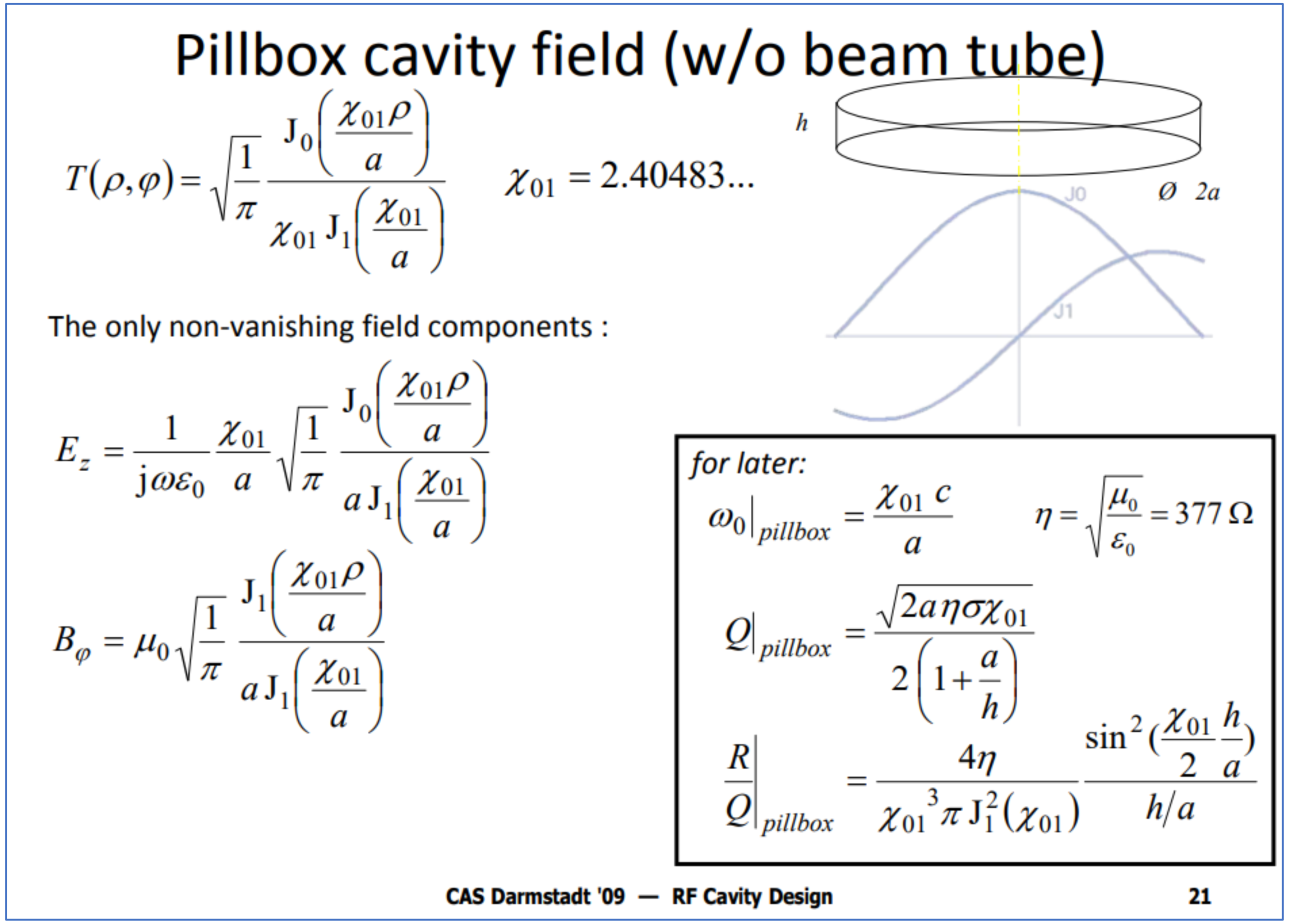
Figure 7: Analytic expressions of E field in various harmonic configurations. Cases correspond to those in Figure 6. Range on the x axis is a 25 MHz RF cycle (40ns).

Conclusions on multi-harmonic RF system for final cooling

- Multi-harmonic RF system for final cooling allow significant increase of the minimum RF frequency required
- For the example from PRAB2015, where 20 MHz is required for the last stage, using multi-harmonic RF system with 4 harmonics from 50 MHz and above provides less energy spread than in the single harmonic 20 MHz RF system
- Following this approach, the minimum frequency can further be increased possibly up to 100 MHz.
- This approach opens the way to potentially avoid using induction linacs and very low frequency RF cavities for the final colling RF system

Spare slides

Pillbox cavity: analytical (E.Jensen, CAS09)



Pillbox cavity field (w/o beam tube)

$$T(\rho,\varphi)=\sqrt{\frac{1}{\pi}}\,\frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01}\,J_1\left(\frac{\chi_{01}}{a}\right)} \qquad \chi_{01}=2.40483\ldots$$

The only non-vanishing field components :

$$E_z=\frac{1}{j\omega\varepsilon_0}\,\frac{\chi_{01}}{a}\sqrt{\frac{1}{\pi}}\,\frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

$$B_\varphi=\mu_0\sqrt{\frac{1}{\pi}}\,\frac{J_1\left(\frac{\chi_{01}\rho}{a}\right)}{a\,J_1\left(\frac{\chi_{01}}{a}\right)}$$

for later:

$$\omega_0\big|_{pillbox}=\frac{\chi_{01}\,c}{a} \qquad \eta=\sqrt{\frac{\mu_0}{\varepsilon_0}}=377\,\Omega$$

$$Q\big|_{pillbox}=\frac{\sqrt{2a\eta\sigma\chi_{01}}}{2\left(1+\frac{a}{h}\right)}$$

$$\frac{R}{Q}\bigg|_{pillbox}=\frac{4\eta}{\chi_{01}^{3}\,\pi\,J_1^2(\chi_{01})}\,\frac{\sin^2\left(\frac{\chi_{01}}{2}\frac{h}{a}\right)}{h/a}$$

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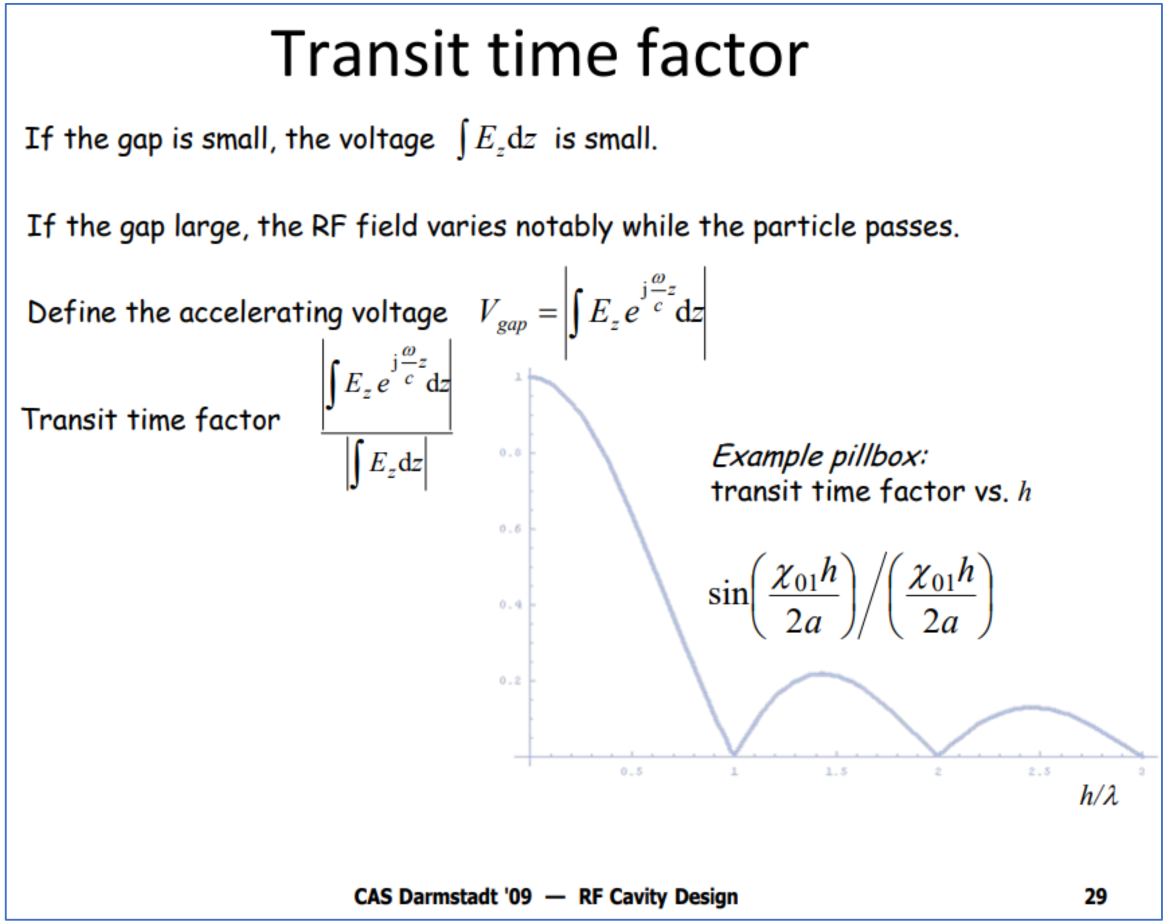

Transit time factor

If the gap is small, the voltage $\int E_z \mathrm{d}z$ is small.

If the gap large, the RF field varies notably while the particle passes.

Define the accelerating voltage $V_{gap}=\left|\int E_z e^{j\frac{\omega}{c}z}\mathrm{d}z\right|$

Transit time factor $\dfrac{\left|\int E_z e^{j\frac{\omega}{c}z}\mathrm{d}z\right|}{\left|\int E_z \mathrm{d}z\right|}$

Example pillbox:
transit time factor vs. h

$$\sin\left(\frac{\chi_{01}h}{2a}\right)\Big/\left(\frac{\chi_{01}h}{2a}\right)$$

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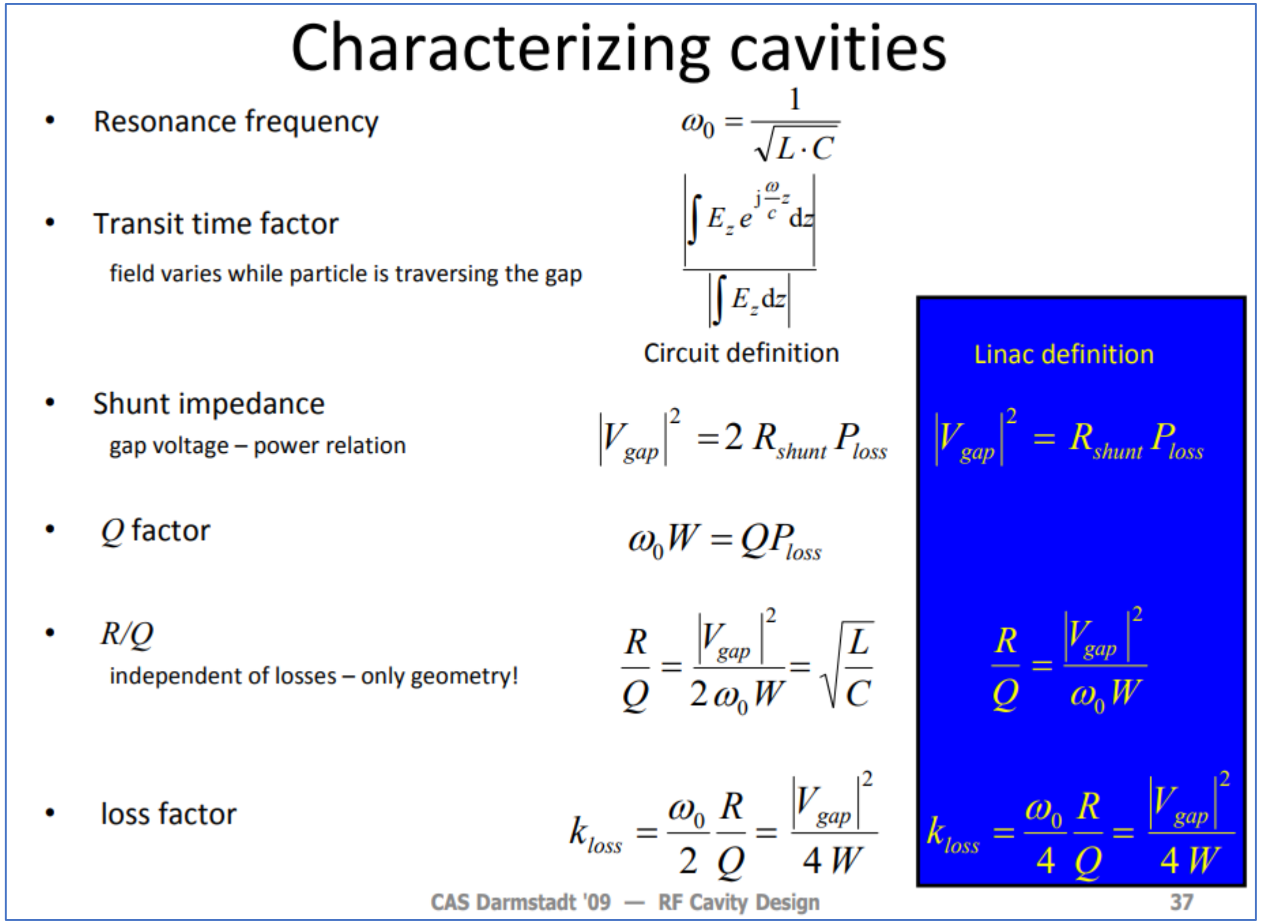
Pillbox cavity: analytical (E.Jensen, CAS09)



Characterizing cavities

- Resonance frequency $\omega_0 = \frac{1}{\sqrt{L \cdot C}}$
- Transit time factor $\frac{\left|\int E_z e^{j\frac{\omega}{c}z} dz\right|}{\int |E_z| dz}$

  field varies while particle is traversing the gap

Circuit definition

Linac definition

- Shunt impedance $|V_{gap}|^2 = 2\, R_{shunt}\, P_{loss}$ | $|V_{gap}|^2 = R_{shunt}\, P_{loss}$

  gap voltage – power relation
- Q factor $\omega_0 W = Q P_{loss}$
- R/Q $\frac{R}{Q} = \frac{|V_{gap}|^2}{2\,\omega_0 W} = \sqrt{\frac{L}{C}}$ | $\frac{R}{Q} = \frac{|V_{gap}|^2}{\omega_0 W}$

  independent of losses – only geometry!
- loss factor $k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{|V_{gap}|^2}{4\,W}$ | $k_{loss} = \frac{\omega_0}{4}\frac{R}{Q} = \frac{|V_{gap}|^2}{4\,W}$

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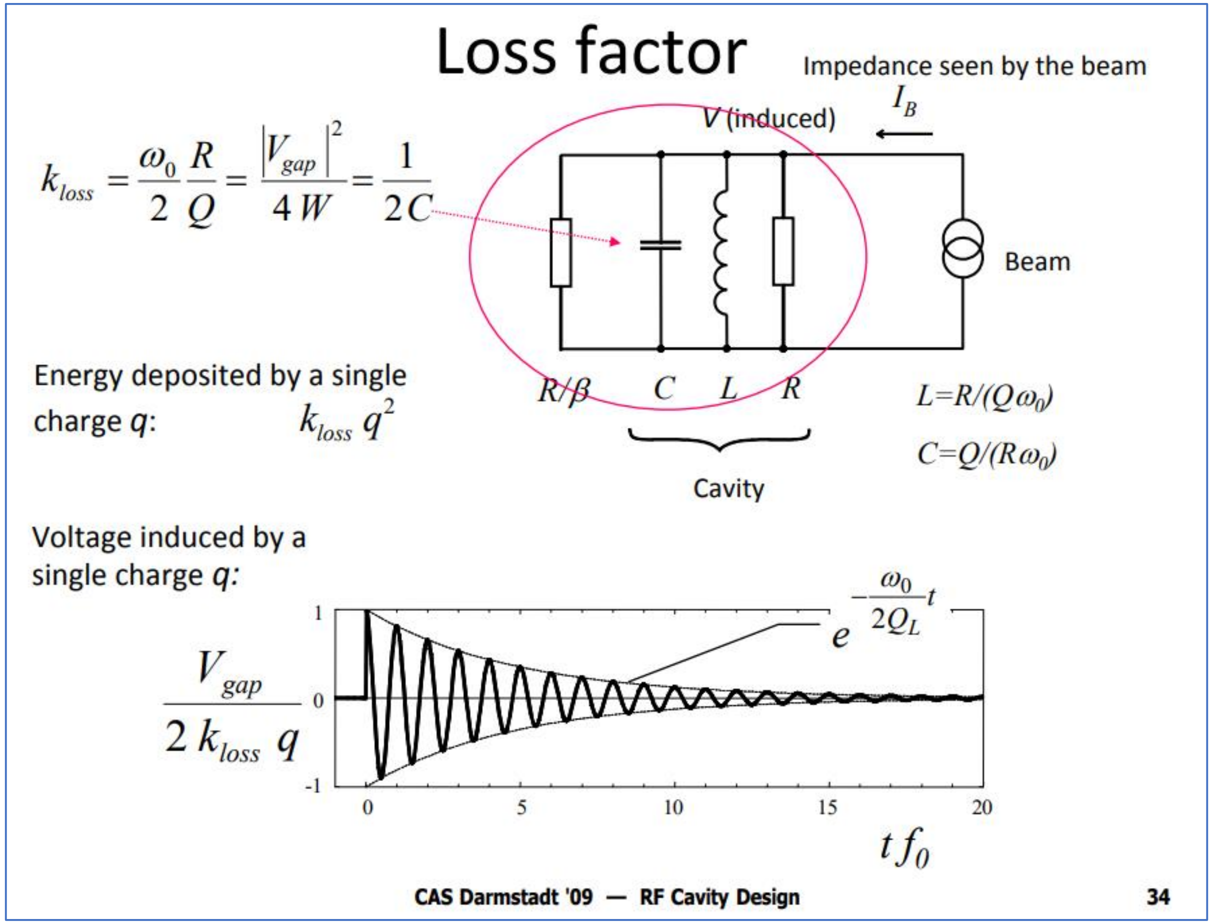

Loss factor

Impedance seen by the beam

$$k_{loss} = \frac{\omega_0}{2}\frac{R}{Q} = \frac{|V_{gap}|^2}{4\,W} = \frac{1}{2C}$$

$L=R/(Q\omega_0)$

$C=Q/(R\omega_0)$

Energy deposited by a single charge q: $k_{loss}\, q^2$

Voltage induced by a single charge q:

$$\frac{V_{gap}}{2\, k_{loss}\, q}$$

$$e^{-\frac{\omega_0}{2Q_L}t}$$

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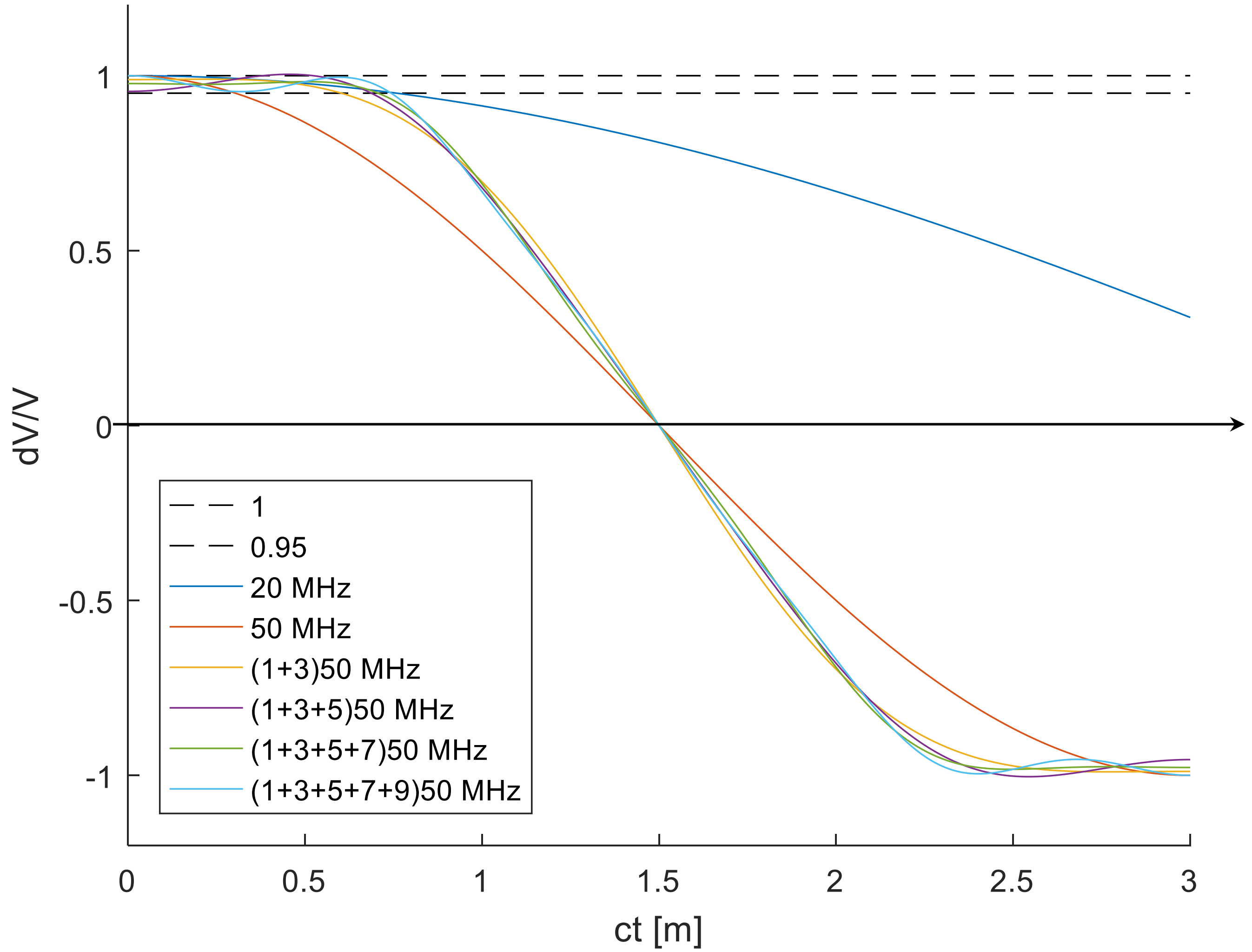
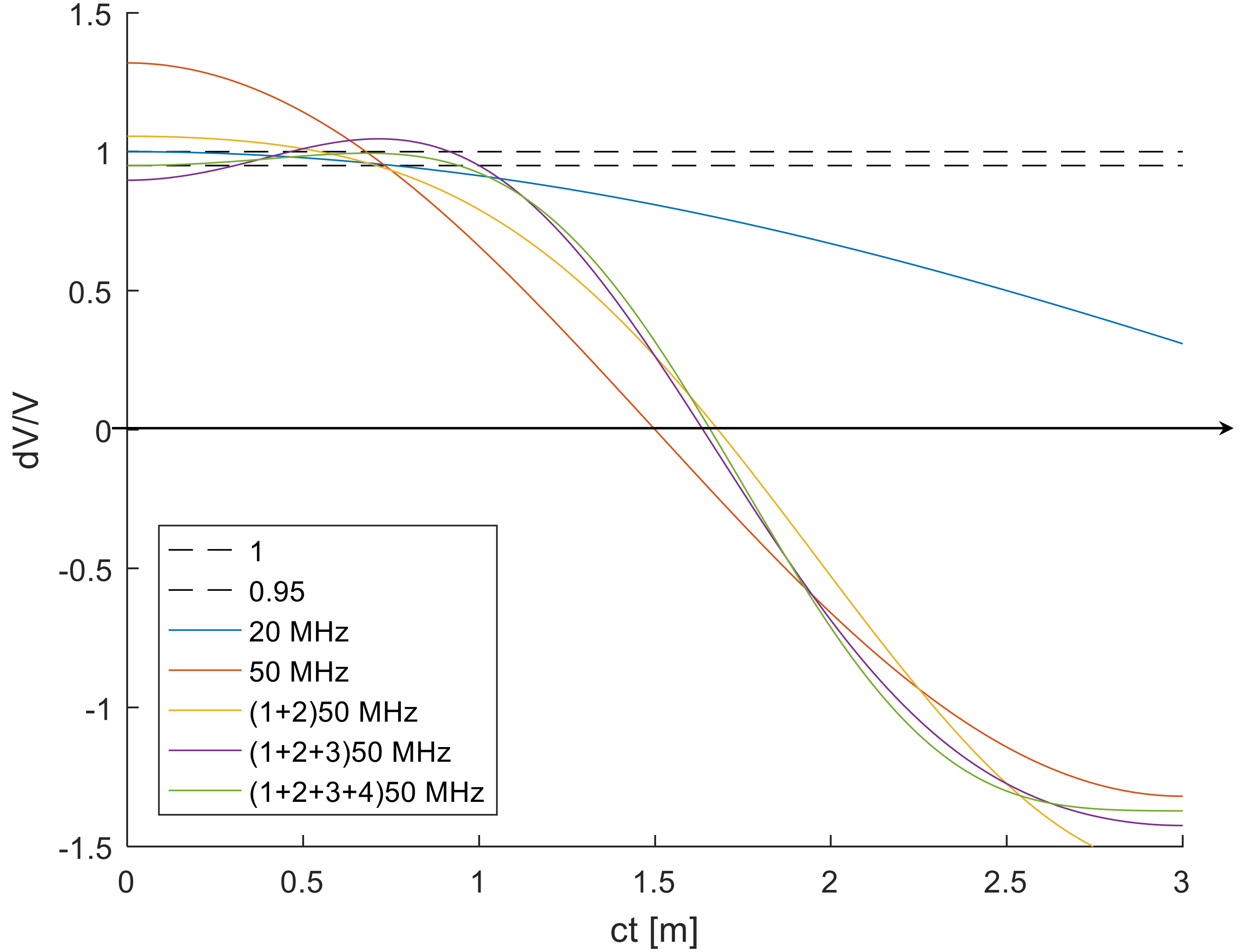
An example: odd harmonics



- In practice **finite number of harmonics** can be used only
- To maintain energy spread of dV/V <5% over σ_{ct} <0.75m, several harmonics of 50 MHz have been used in this example
- 50 MHz provides dV/V <5% over σ_{ct} <0.3m only
- 50, 150, 250 MHz extend the range up to ~0.6m
- 50, 150, 250, 350, 450 MHz make it up to 0.75m satisfying the specs
- Using only odd harmonics keeps voltage +,- symmetric
- Using higher harmonics is less and less effective

An example: odd and even harmonics



- Using even and odd harmonics makes voltage +,- non-symmetric
- Is it an issue for the beam?
- 50, 100, 150, 200 MHz case meets the specs up to σ_{ct} =1m
- Using all harmonics is more effective than using just odd harmonics
- Minimum frequency can be increased from 50 MHz probably up to 100 MHz