International MUON Collider Collaboration

Parametric study for a rapid cycling synchrotron

by Antoine Chance (CEA Paris-Saclay)
Acknowledgements:
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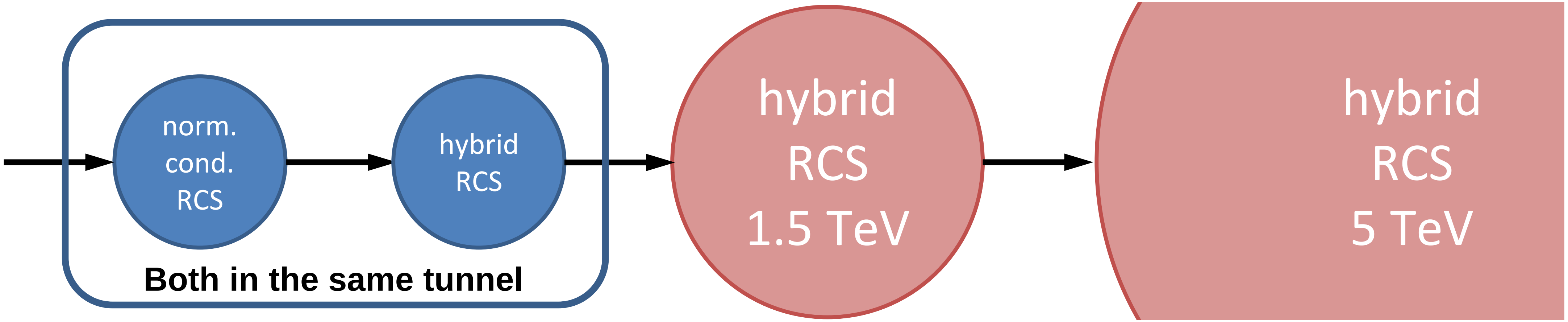
, David Amorim, Scott Berg, Fulvio Boattini, Luca Bottura, Christian Carli,
Alexej Grudiev, Elias Metral, Daniel Schulte

Introduction

- Chain of rapid cycling synchrotrons, counter-rotating μ^+/μ^- beams
  $\rightarrow$ 63 GeV $\rightarrow$ 0.31 TeV $\rightarrow$ 0.75 TeV $\rightarrow$ 1.5 TeV ($\rightarrow$ 5 TeV)

Courtesy: Fabian Batsch

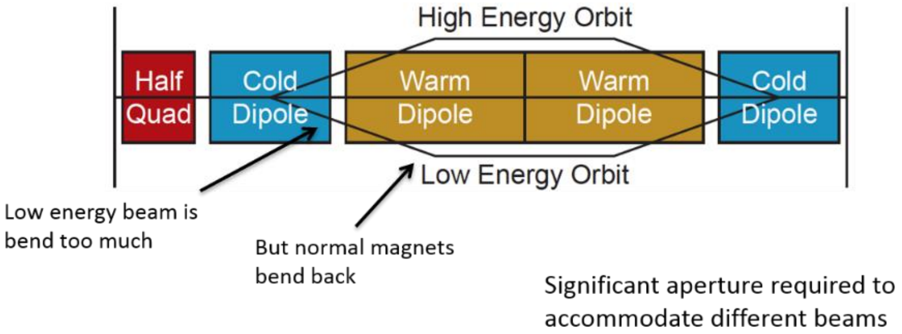


- Hybrid RCSs have intersecting normal conducting (NC) and superconducting (SC) magnets
- The studies presented aim to determine the RF (cavity) and lattice parameter (number of RF stations, momentum compaction factor,…)

A lot of constraints

- Muons decay very fast (Rest lifetime: 2.2 μs):
- We should accelerate as fast: τ_{acc} as low as possible.
  - Muon survival: $\frac{N_{ext}}{N_{inj}} = \left(\frac{E_{ext}}{E_{inj}}\right)^{-\frac{\tau_{acc}}{\tau_\mu(\gamma_{ext}-\gamma_{inj})}}$ for a linear ramp
- To decrease cost operation, we should:
  - Minimize the total voltage and thus energy gain per turn:

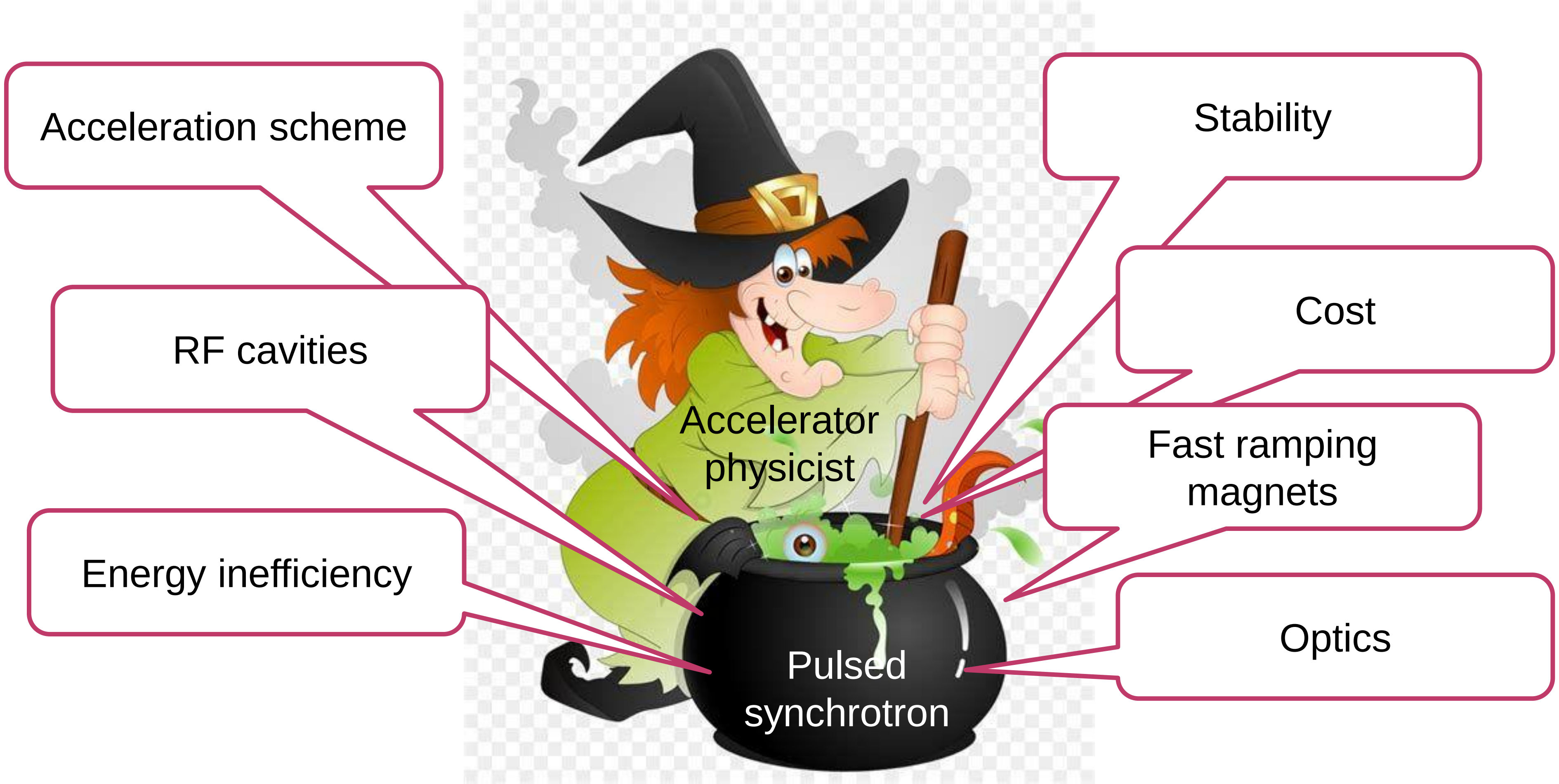
    Energy gain: $\Delta E = \frac{E_{ext}-E_{inj}}{\tau_{acc}}\frac{L_{RCS}}{c}$ ⇒ RCS as small as possible
  - Interest of a hybrid RCS: higher average field ⇒ smaller synchrotron.
    - But different path lengths and orbits.
  - Optimize the dipole ramp to minimize the power consumption (see Fulvio's talk).
- Find the best ratio extraction/injection ratio between the different acceleration stages.



$$L_{NC} = 2\pi\frac{B\rho_{ext} - B\rho_{inj}}{B_{NC,ext} - B_{NC,inj}} = \pi\frac{B\rho_{ext} - B\rho_{inj}}{B_{NC}}$$

$$L_{SC} = 2\pi\frac{B\rho_{inj}B_{NC,ext} - B\rho_{ext}B_{NC,inj}}{B_{SC}(B_{NC,ext} - B_{NC,inj})} = \pi\frac{B\rho_{inj} + B\rho_{ext}}{B_{SC}}$$

How to cook a good RCS ?



Development of a python package to integrate the different constraints

Any help is welcome !

Parameters and tools: General parameter

Courtesy: Fabian Batsch

- **Detailed parameter table: https://cernbox.cern.ch/index.php/s/I9VpITncUeCBtiz**

	RCS1→314 GeV	RCS2→750 GeV	RCS3→1.5 TeV
Circumference, $2\pi R$ [m]	5990	5590	10700
Energy factor, E_{ej}/E_{inj}	5.0	2.4	2.0
Repetition rate, f_{rep} [Hz]	5 (asym.)	5 (asym.)	5 (asym.)
Number of bunches	$1\mu^+$, $1\mu^-$	$1\mu^+$, $1\mu^-$	$1\mu^+$, $1\mu^-$
Bunch population	$2.5*10^{12}$	$2.3*10^{12}$	$2.2*10^{12}$
Survival rate per ring	90%	90%	90%
Acceleration time [ms]	0.34	1.04	2.37
Number of turns	17	55	66
Energy gain per turn, ΔE [GeV]	14.8	7.9	11.4
Acc. gradient for survival [MV/m]	2.4	1.3	1.1
Acc. field in RF cavity [MV/m]	30	30	30
Max. RF voltage for ϕ_s=135° [GV]	20.9	11.2	16.1

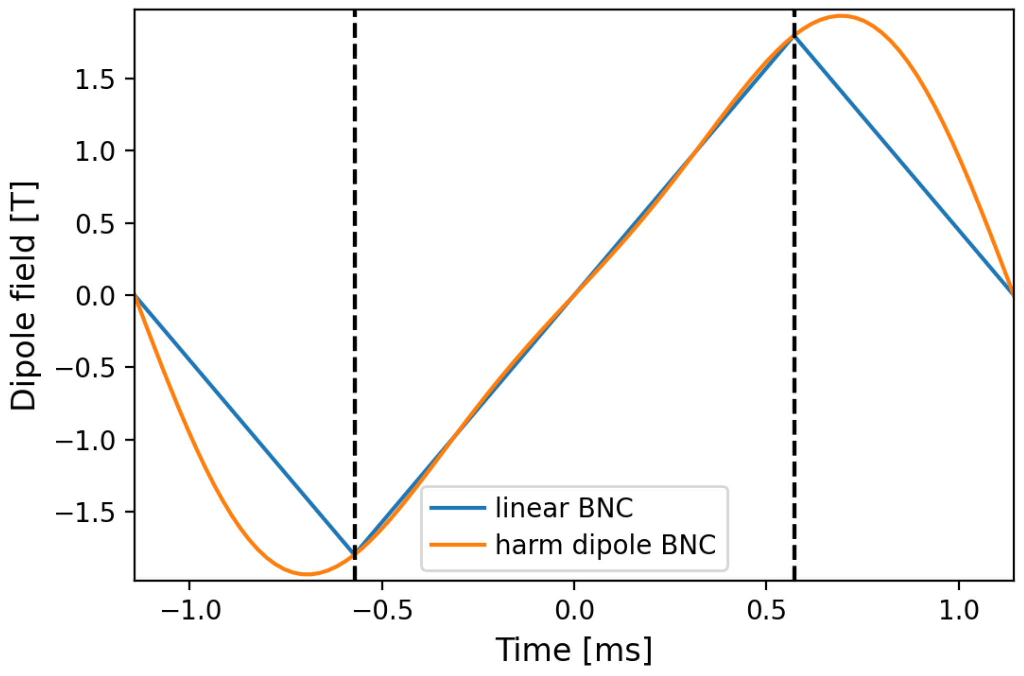
		Symbol	Unit	Stage 1 Value	Details	Stage 2 Value	Details	Stage 3 Value
13				Stage 1		Stage 2		Stage 3
14	Basic data	Symbol	Unit	Value	Details	Value	Details	Value
15	Particles	-	-	μ		μ		μ
16	Costs	-	M€					
17	Type	-	-	RCS		hybrid RCS		hybrid RCS
18								
19	Dynamics							
20	Acceleration time	T_{acc}	[ms]	0.34		1.09704595		2.37
21	Injection energy	E_{inj}	[MeV]/u	63000		313830		750000
22	Ejection energy	E_{ej}	[MeV]/u	313830	defined by n	750000		1500000
23	Energy ratio	E_{ej}/E_{inj}	-	4.98		2.39		2.00
24	Momentum at e_i	p/c	MeV/c	63106		313935		750106
25	Momentum at e_j	p/c	MeV/c	313935		750106		1500106
26	Number of turns	n_{turn}	-	17		55		66
27	Planned Survival rate	N_{ej}/N_{inj}	-	0.9		0.9		0.9
28	Total survival rate	N_{ej}/N_0	-	0.9		0.81		0.729
29	Accel. Gradient, linear for survival	G	[MV/m]	2.44		1.33		1.06
30	Required energy gain per turn	ΔE	[MeV]	14755		7930		11364
31								
32	Transition gamma	γ_{tr}	-	20.41		20.41		~30
33	Injection relativistic mass factor	γ_{inj}	-	597		2971		7099
34	Ejection relativistic mass factor	γ_{ej}	-	2971		7099		14198
35	Injection v/c	β_{inj}	%	0.9999986		0.999999943		0.9999999901
36	Ejection v/c	β_{ej}	%	0.999999943		0.9999999901		0.9999999975
37								
38	Parameter Classical RCS							
39	Radius	R	[m]	953.3		953.3		1703.0
40	Circumference	$2\pi R$	[m]	5990		5990		10700
41	Circumference Ratio	R_{i+1}/R_i	-	-		1		1.79
42	Pack fraction	?	-	0.61		0.61		0.628
43	Bend radius	ρ_B	m	581.8		581.8		1070.2
44	Tot. straight section length	L_{str}	[m]	2334.7		2335.7		3975.7
45	Injection bending field (average)	B_{inj}	[T]	0.36		1.80		2.34
75	RF							
76	Systems	-	-	TESLA		TESLA		TESLA
77	Main RF frequency	f_{RF}	[MHz]	1300		1300		1300
78	Harmonic number	h	-	25957		25957		46367
79	Revolution frequency ej	f_{rev}	[kHz]	50.08		50.08		28.04
80	Revolution period	Trev	[μs]	20.0		20.0		35.7
81	Max RF voltage	V_{RF}	[GV]	20.87		11.22		16.07
82	Max RF power	P_{RF}	[MW]					
83	RF Filling factor	-	-	0.4		0.4		0.45
84	Number RF stations	-	-	Around 50		Around 50		Around 50
85	Cavities	-	-	9-cell		9-cell		9-cell
86	Number of cavities	?	-	696		374		536
87	Peak Impedance		[Ω]					
88	Gradient in cavity	$\Delta E/L$	[MV/m]	30		30		30
89	Average energy gain per total straight	$\Delta E/L$	[MeV/m]	6.3		3.4		2.9
90	Accelerating field per total straight	$\Delta E/L$	[MeV/m]	8.9		4.8		4.0
91	Accelerating field gradient, with FF	$\Delta E/L$	[MV/m]	22.3		12.0		9.0
92	Stable phase	ϕ_s	[°]	45		45		45
93	Conversion factor mm mrad – eVs	-	Vs/mm mra	69.40		165.86		331.72
94	Longitudinal emittance (σE * 4σz)	ε^z_n	[eVs]	0.025	7.5 MeV m	0.025		0.025
95	Longitudinal emittance (phase space area)	ε^z_n	[eVs]	0.079		0.079		0.079
96	Injection bucket area	$A_{B,inj}$	[eVs]	0.62		1.01		1.40
97	Ejection bucket area	$A_{B,ej}$	[eVs]	1.37		1.56		1.97
98	Bucket area reduction factor	$A_B/A_{B,st}$	-	0.172		0.172		0.172
99	Horizontal betatron tune	Q_h	-					
100	Vertical betatron tune	Q_v	-					
101	Average horizontal Twiss beta	βh	[m]	10		10		10
102	Average vertical Twiss beta	βv	[m]	10		10		10
103	Injection synchrotron frequency	$f_{S,inj}$	[kHz]	76.33		25.07		14.53
104	Ejection synchrotron frequency	$f_{S,ej}$	[kHz]	34.20		16.22		10.27
105	Injection synchrotron tune Q_s	$f_{S,inj}/f_{rev}$	-	1.52		0.50		0.52
106	Ejection synchrotron tune Q_s	$f_{S,ej}/f_{rev}$	-	0.68		0.32		0.37

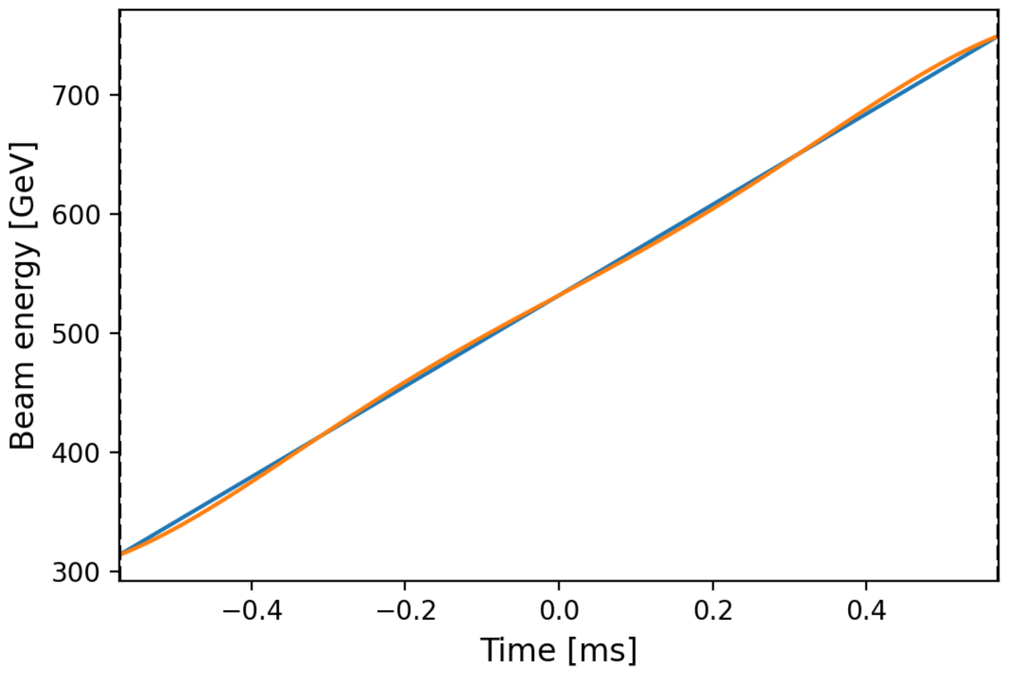
What is new since February

- The user chooses the synchrotron gemetry, injection and extraction phase (the dipoles are oscillating) and defines the dipole families and the dipole patterns.
- The code calculates:
  - The dipole period (compromise between the maximum voltage, the maximum field slope and the minimum acceleration time)
  - The energy and synchronous phase variation
  - The muon survival
  - The optimum dipole lengths for each dipole
  - The path length variation and the needed aperture
  - Some optics parameters (momentum compaction, beam size) by assuming FODO cells (that should be improved in the future).

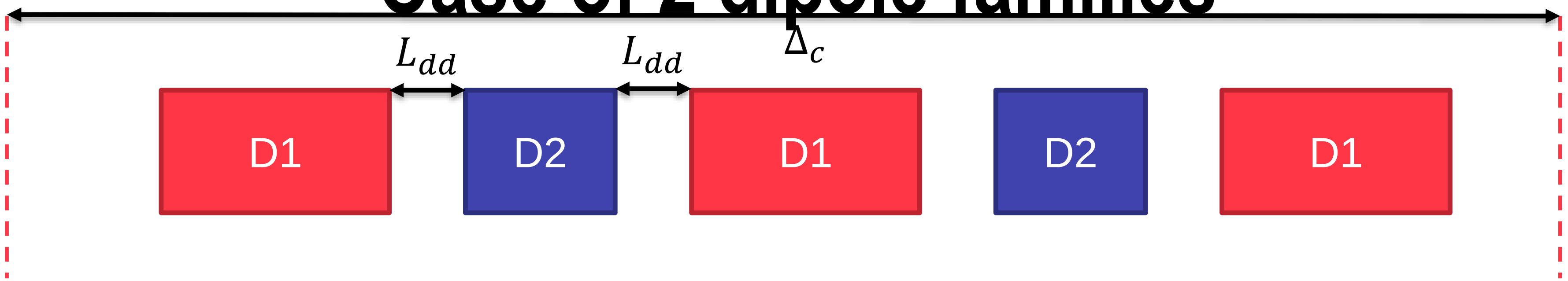
Example of dipole ramp in the RCS ME

- Harmonic dipole: $1.8 \sin \phi - 0.38197 \sin 2\phi$
- Muon survival: 89.60% (linear ramp) against 89.58% (harmonic dipole): negligible difference





Scaling laws
Case of 2 dipole families



n_c	the total number of FODO cells
n_d	the number of D_2 per half-cell
L_{dd}	the distance between 2 dipoles
Δ_c	the chord length of the half-cell
L_1	the straight length of one dipole D_1
L_2	the straight length of one dipole D_2
$L_{T,1} = 2n_c(n_d+1)L_1$	the total length of D_1
$L_{T,2} = 2n_c n_d L_2$	the total length of D_2
$L_T = L_{T,1}+L_{T,2}$	the total length of dipoles
$h^* = 1/\rho^*$	the reference curvature
$h_1 = 1/\rho_1$	the curvature of D_1
$h_2 = 1/\rho_2$	the curvature of D_2

Definition of parameters a, b

$$a = \frac{L_{T,2}}{L_T} \qquad b = \frac{h_2}{h^*}$$

To close the trajectory, we have the constraints:

$$L_1 = \frac{1-a}{2n_c(n_d+1)} L_T \qquad L_2 = \frac{a}{2n_c\, n_d} L_T$$

$$h_1 = \frac{1-a\,b}{1-a} h^* \qquad h_2 = b\, h^*$$

Total path length: analytic formulae

$$\mathcal{S}_T - \mathcal{S}^*_{T,a} = \frac{a(b-1)\pi^2}{3n_d(1+n_d)n_c}\left[L_T\frac{ab+n_d}{4n_c} + L_{dd}\left(n_d(n_d-1)+a(b+1)(1+2n_d)\right)\right] + o(\frac{1}{n_c^2})$$

The total path length difference scales as $\frac{1}{n_c^2}$ when the total dipole length is large compared to the total distance used for the dipole spacing.

For very large n_c we are driven by the spacing between the dipoles.

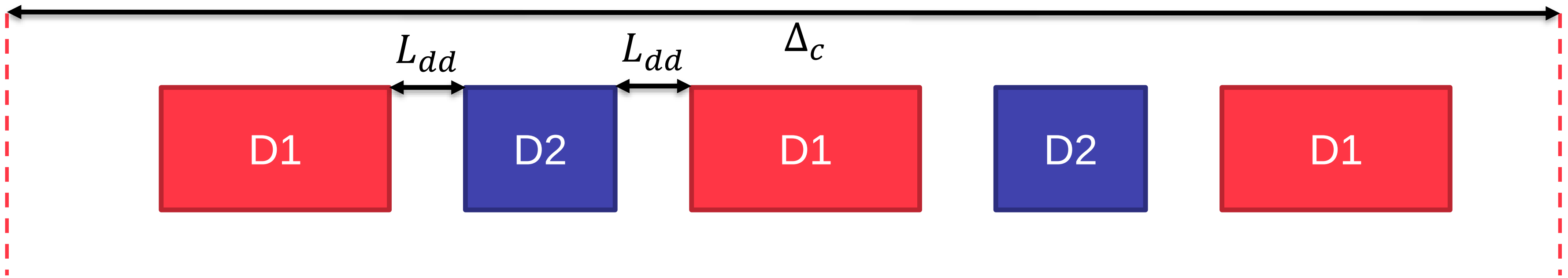
If $4n_c n_d L_{dd} \ll L_T$,

$$\mathbf{S_T - S^*_{T,a} \propto \frac{1}{n_c^2}}$$

$$\mathbf{S_T - S^*_{T,a} \propto \frac{a\,b + n_d}{n_d(1+n_d)}}$$

Clear interest to increase the number of cells and dipoles per half-cell.

Case: NC dipoles first



B_{NC} the field in NC dipole
B_{SC} the field in SC dipole
E_{inj} the injection energy
E_{ext} the extraction energy

$$L_{T,NC} = \frac{\pi(E_{ext} - E_{inj})}{B_{NC}\, c}$$

$$L_{T,SC} = \frac{\pi(E_{ext} + E_{inj})}{B_{SC}\, c}$$

$$L_T = \frac{\pi\left((B_{SC} + B_{NC})E_{ext} - (B_{SC} - B_{NC})E_{inj}\right)}{B_{NC} B_{SC} c} = 2\,\pi\,\rho^*$$

$$a = \frac{B_{NC}(E_{ext} + E_{inj})}{(B_{SC} + B_{NC})E_{ext} - (B_{SC} - B_{NC})E_{inj}}$$

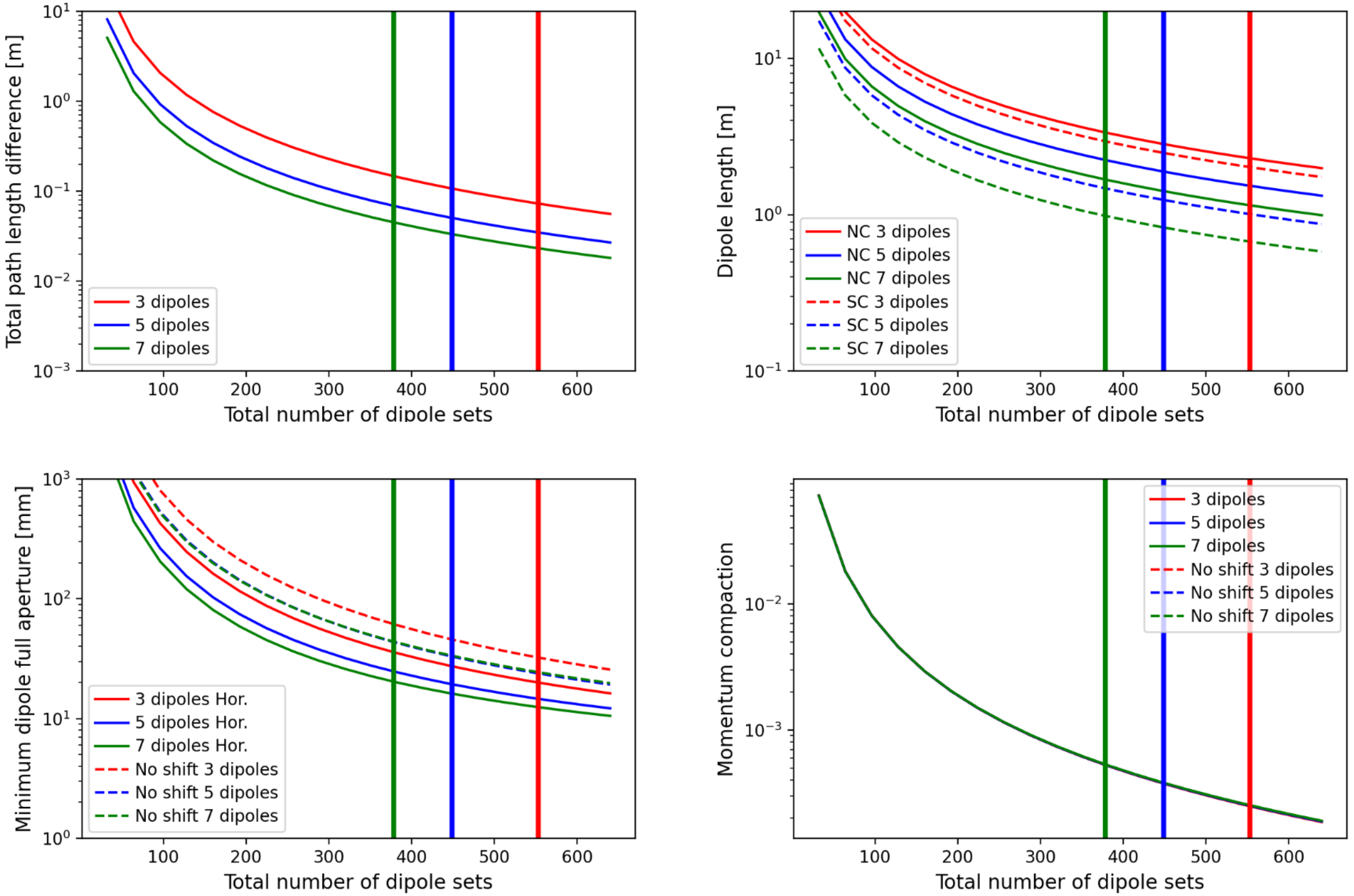
$$h_{2,inj} = \frac{B_{SC}}{E_{inj}} c \quad h_{2,ext} = \frac{B_{SC}}{E_{ext}} c$$

$$b_{inj} = \frac{B_{SC}\, L_T}{2\,\pi\, E_{inj}} c \qquad b_{ext} = \frac{B_{SC}\, L_T}{2\,\pi\, E_{ext}} c$$

$$L_{NC} = \frac{\pi(E_{ext} - E_{inj})}{2\, B_{NC}\, c\ \ n_c(n_d + 1)}$$

$$L_{SC} = \frac{\pi(E_{ext} + E_{inj})}{2\, B_{SC}\, c\ \ n_c\, n_d}$$

RCS ME: NC dipoles first variation with the number of cells nc



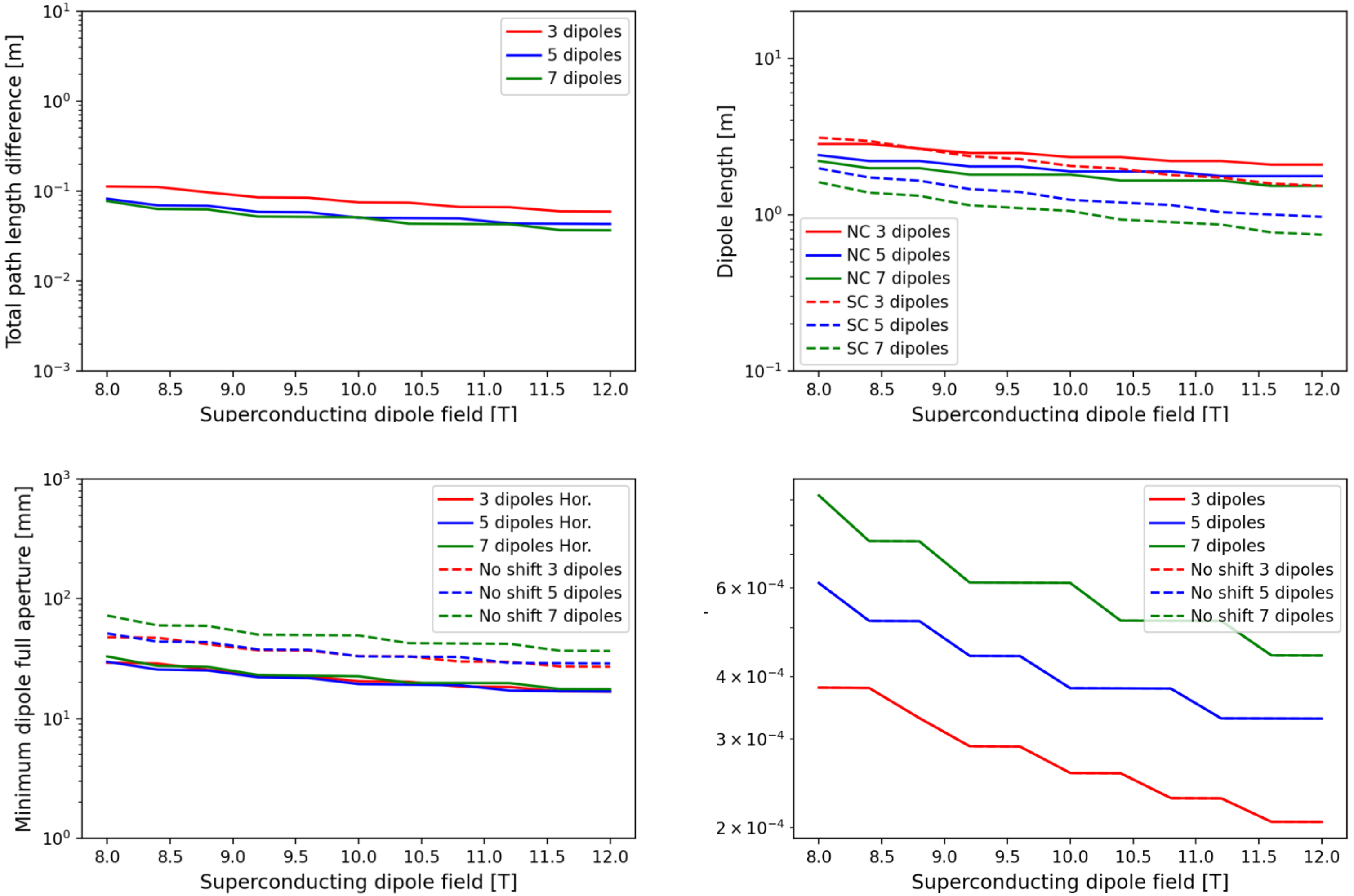
We assume an interconnection length of 0.3 m and dipoles of 2 m.

We consider a beam stay clear of 6 sigmas and FODO cells with a phase advance of 90 degrees.
The magnet screening is not included and should be added to get the inner dipole aperture.

The vertical lines corresponds to the limit value of nc where there ins no space anymore to locate the quadrupoles.

That is better to have 5 or 7 dipoles per cell.

RCS ME: NC dipoles first variation with the SC dipole field



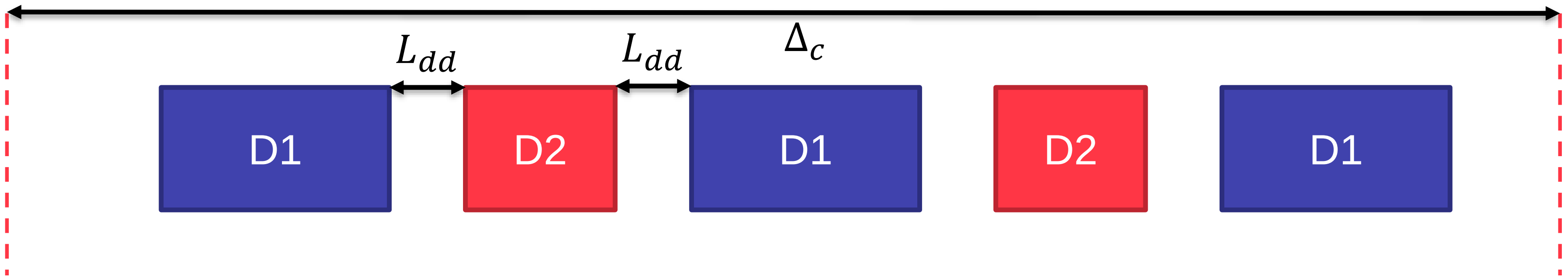
We assume an interconnection length of 0.3 m and dipoles of 2 m.

We consider a beam stay clear of 6 sigmas and FODO cells with a phase advance of 90 degrees.

The number of cells is calculated to maximize the filling ratio of the arcs.

We assume a total number of 16 RF sections (and thus arcs), which explains the steps (number of cells is a multiple of 16).

Case: SC dipoles first



B_{NC}	the field in NC dipole
B_{SC}	the field in SC dipole
E_{inj}	the injection energy
E_{ext}	the extraction energy

$$L_{T,NC} = \frac{\pi\left(E_{ext} - E_{inj}\right)}{B_{NC}\, c}$$

$$L_{T,SC} = \frac{\pi\left(E_{ext} + E_{inj}\right)}{B_{SC}\, c}$$

$$L_T = \frac{\pi\left((B_{SC} + B_{NC})E_{ext} - (B_{SC} - B_{NC})E_{inj}\right)}{B_{NC} B_{SC} c} = 2\,\pi\,\rho^*$$

$$a = \frac{B_{SC}(E_{ext} - E_{inj})}{(B_{SC} + B_{NC})E_{ext} - (B_{SC} - B_{NC})E_{inj}}$$

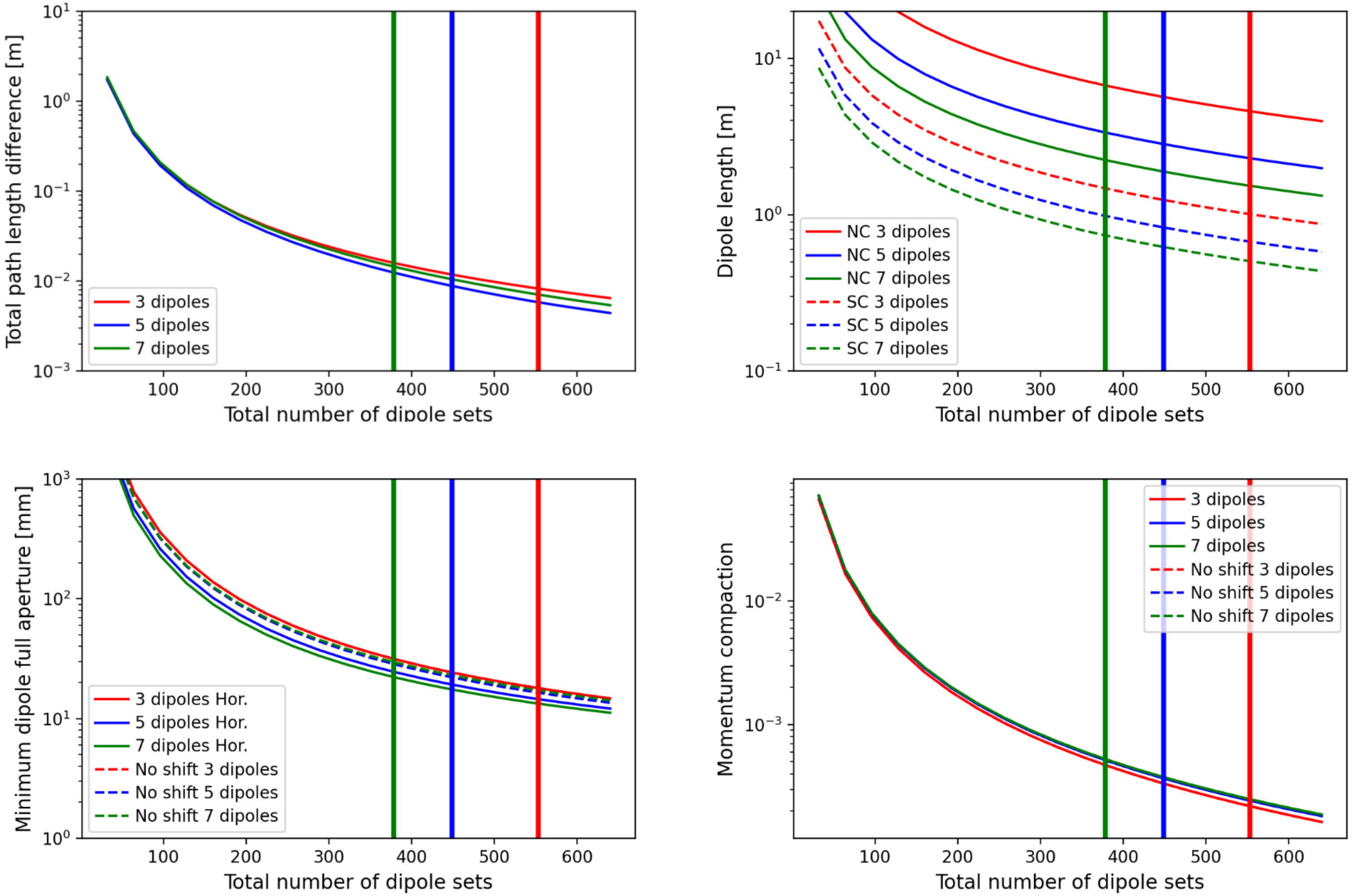
$$h_{2,inj} = -\frac{B_{NC}}{E_{inj}} c \qquad h_{2,ext} = \frac{B_{NC}}{E_{ext}} c$$

$$b_{inj} = -\frac{B_{NC}\, L_T}{2\,\pi\, E_{inj}} c \qquad b_{ext} = \frac{B_{NC}\, L_T}{2\,\pi\, E_{ext}} c$$

$$L_{NC} = \frac{\pi\left(E_{ext} - E_{inj}\right)}{2\, B_{NC}\, c\;\, n_c\, n_d}$$

$$L_{SC} = \frac{\pi\left(E_{ext} + E_{inj}\right)}{2\, B_{SC}\, c\;\, n_c (n_d + 1)}$$

RCS ME: SC dipoles first variation with the number of cells nc



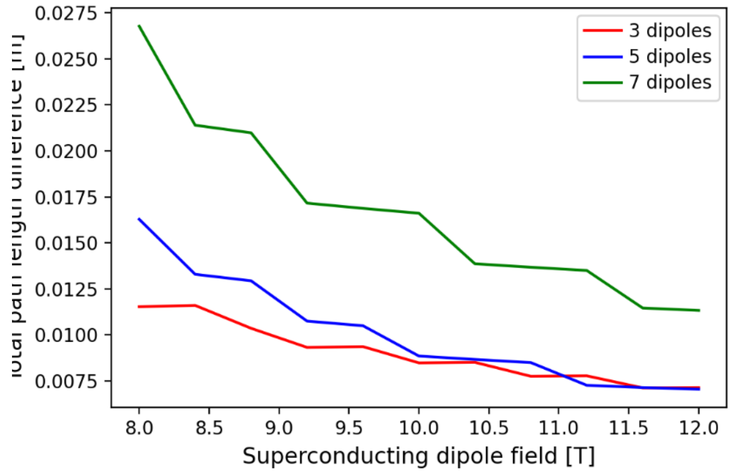
We assume an interconnection length of 0.3 m and dipoles of 2 m.

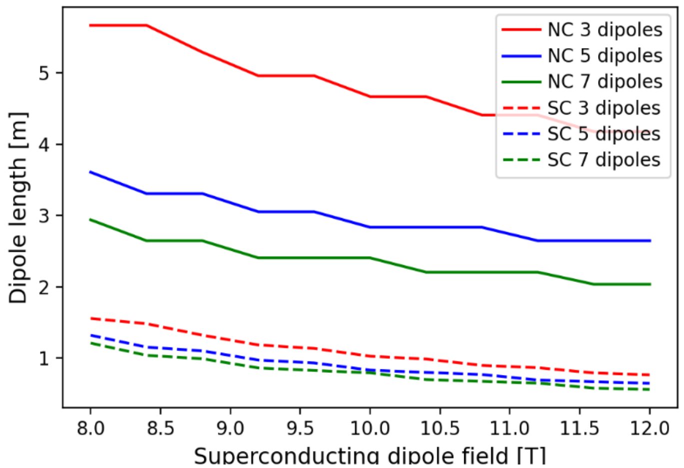
We consider a beam stay clear of 6 sigmas and FODO cells with a phase advance of 90 degrees.
The magnet screening is not included and should be added to get the inner dipole aperture.

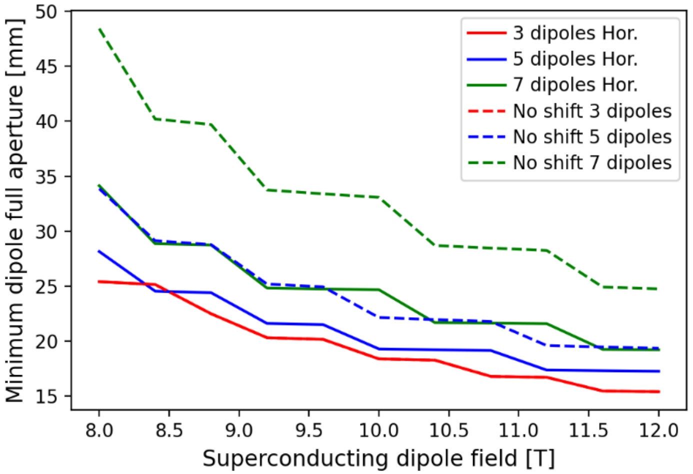
The vertical lines corresponds to the limit value of nc where there ins no space anymore to locate the quadrupoles.

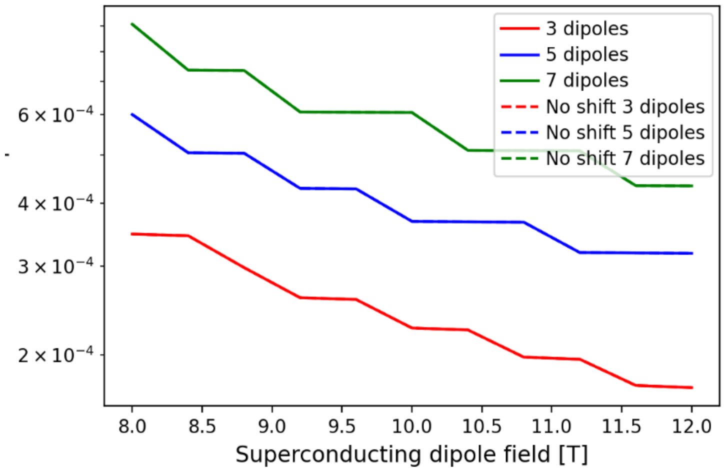
SC dipoles first is better than NC dipoles. No gain to go to 7 dipoles.

RCS ME: SC dipoles first variation with the SC dipole field









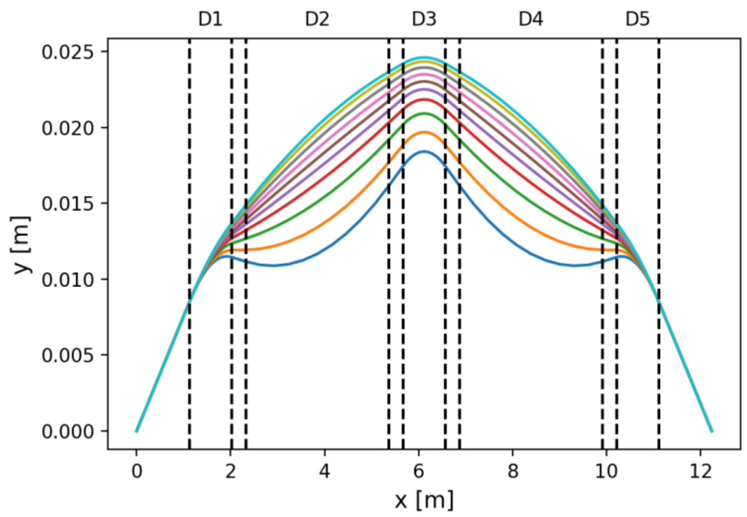
We assume an interconnection length of 0.3 m and dipoles of 2 m.

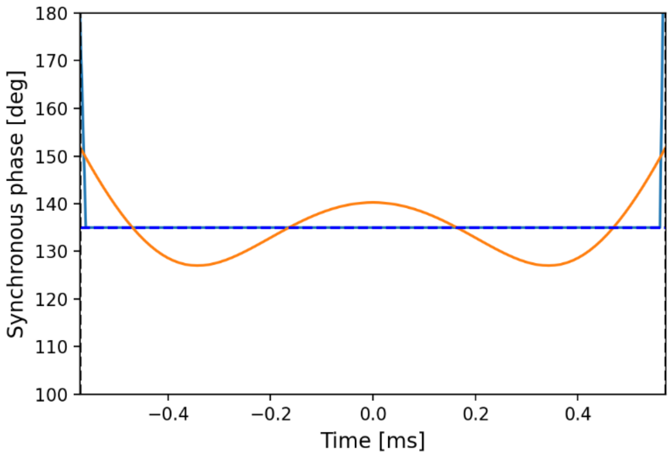
We consider a beam stay clear of 6 sigmas and FODO cells with a phase advance of 90 degrees.

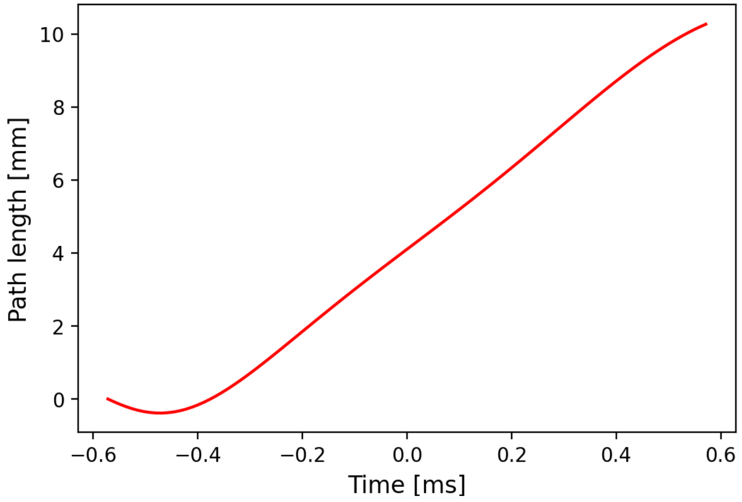
The number of cells is calculated to maximize the filling ratio of the arcs.

We assume a total number of 16 RF sections (and thus arcs), which explains the steps (number of cells is a multiple of 16).

RCS ME:
Case SC first with 5 dipoles and nc=208



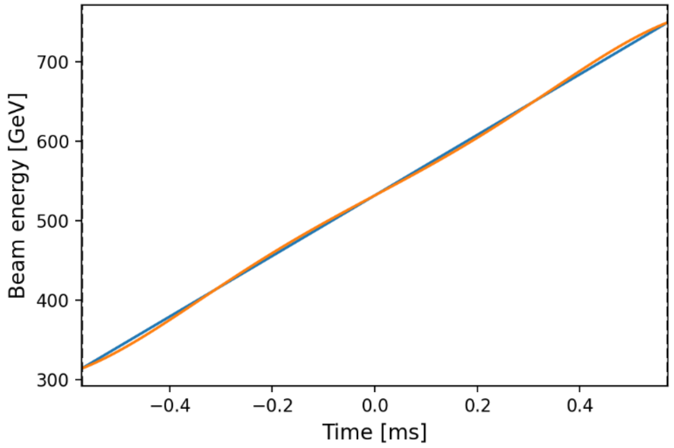




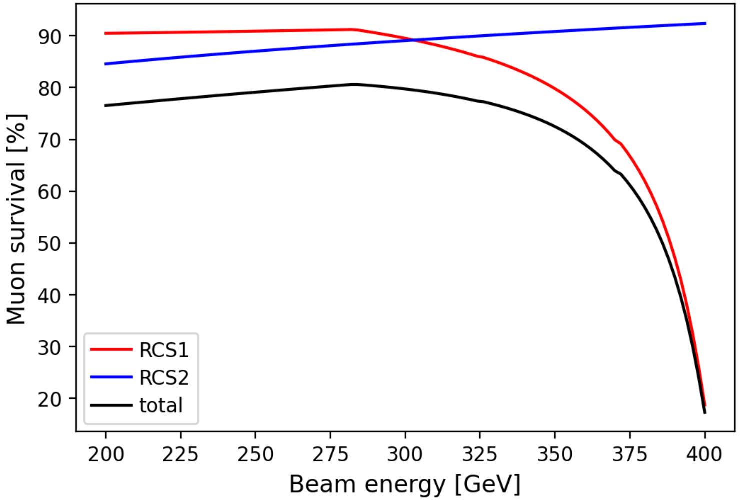
That is possible to get a path length variation of about 1 cm.
However, the cell is very compact.

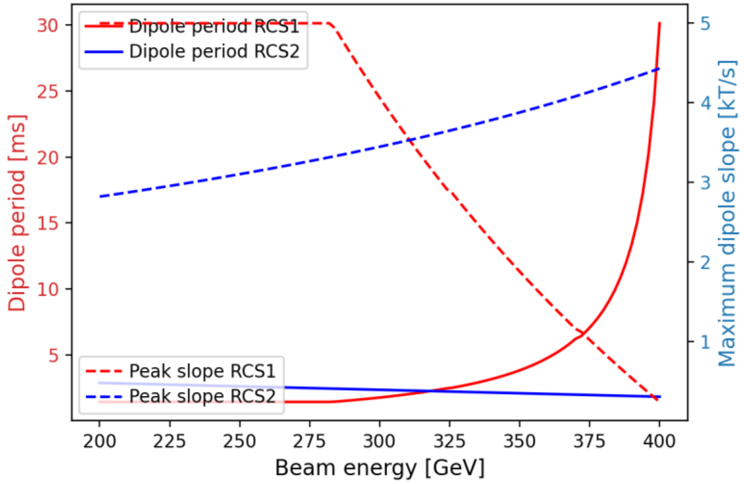
Although the energy ramp is quasi-linear, the synchronous phase varies by more than 10 degrees.

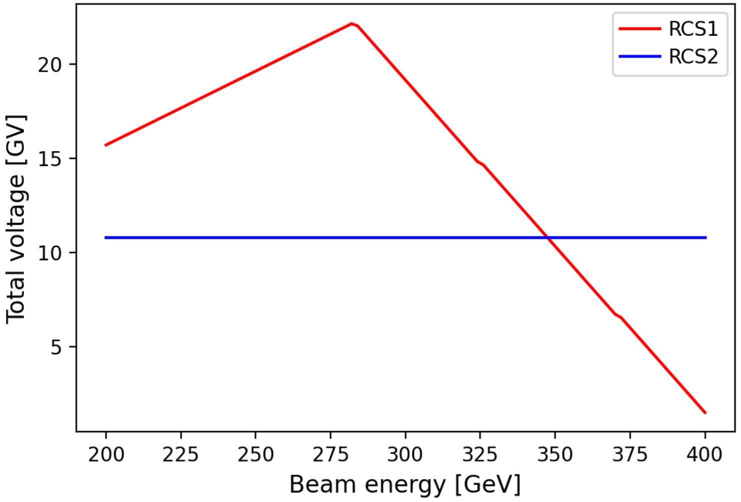
The voltage is assumed to be constant in the cavity.

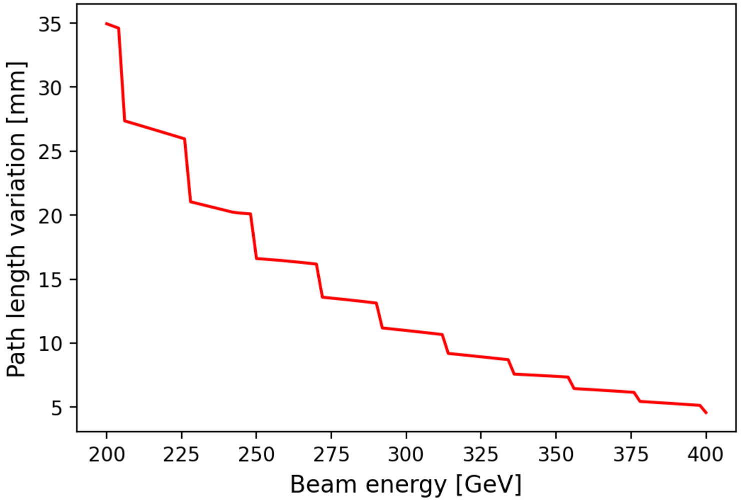


Is 300 GeV optimum as an intermediate energy?









- We make vary the injection energy in RCS ME.
- We adjust the filling factor and number of cells to keep a cell length of about 30 m in the arcs.
- We consider a maximum field slope of 5 kT/s.
- At lower energy, we are limited by the maximum dipole slope.
- At higher energy, we are limited by the maximum dipole field.
- **An energy of 300 GeV is a good starting point.**

Conclusions

- **An analytical model has been developed** to evaluate the path lenth variation and the trajectory difference. We have now scaling rules and the model has been **extended to an arbitrary number of dipole families**.
- A python script has been developed to use these formulae and **generate different RCS geometries**.
- **Concerning the RCS ME, the best dipole patterns are with the SC dipole first and 5 dipoles**. More dipoles does not help because of the needed space for the interconnections.

Next steps

- **The same study can be extended** for the RCS HE and the RCS to go to 5 TeV (the baseline is still preliminary).
- That would be useful to optimize the injection/extraction energy of each step.
- The optics is based on FODO cells but that may be not the good assumption. Indeed, the number of RF sections becomes very large (more than 32, see Fabian Batsch's presentation). The number of FODO cells between 2 RF sections may become very low (5-10). **Other optics may be more appropriate: we need to integrate the RF section in the cell**. The direct impact is a change of the momentum compaction and the beam sizes.
- We can also choose to have more than one dipole block per half-cell. That is also a direction to increase the filling factor of the arcs and thus to reduce the path length variation.
- **Some limitations like the minimum dipole length or the maximum quadrupole gradient are missing**. That would be very useful to get these data to add constraints on the parameter set.

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Thank you for attention