RF Parameter Choices and Longitudinal Stability

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***Acknowledgements*: David Amorim, Scott Berg, Fulvio Boattini, Luca Bottura, Christian Carli, Antoine Chancé, Alexej Grudiev, Elias Metral, Daniel Schulte**

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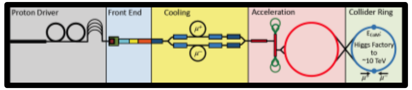
Outline

- The TESLA cavity and RCS parameter
- Intensity effects & longitudinal beam dynamics:

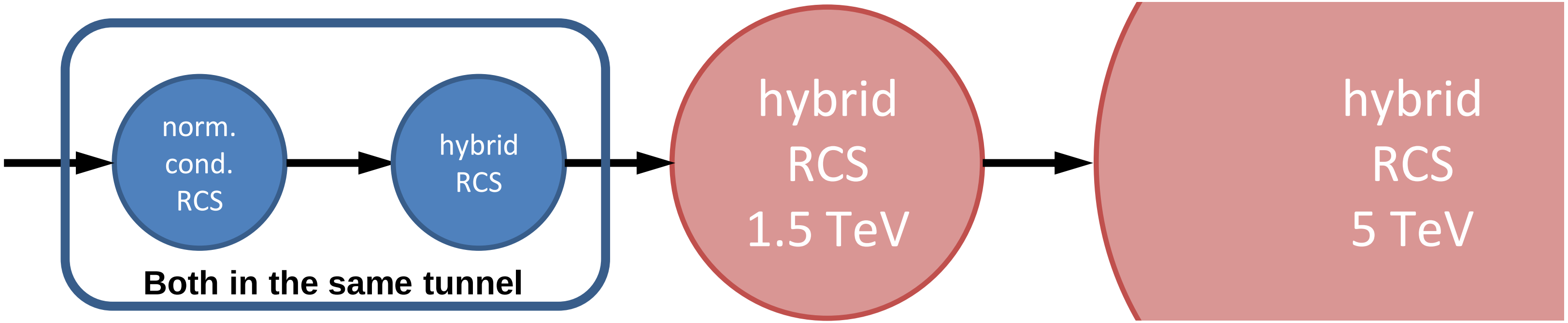
  Short-range wakefield and beam loading compared to the cavity voltage
- Synchrotron tune & Number of RF stations versus emittance
- Emittance evolution during the acceleration stages
- Outlook and Summary

Muon acceleration to high energies



- **Chain of rapid cycling synchrotrons, counter-rotating μ^+/μ^- beams $\rightarrow$ 63 GeV $\rightarrow$ 314 GeV $\rightarrow$ 750 GeV $\rightarrow$ 1.5 TeV ($\rightarrow$ 5 TeV)**

Details on RCS: See talk by H. Damerau



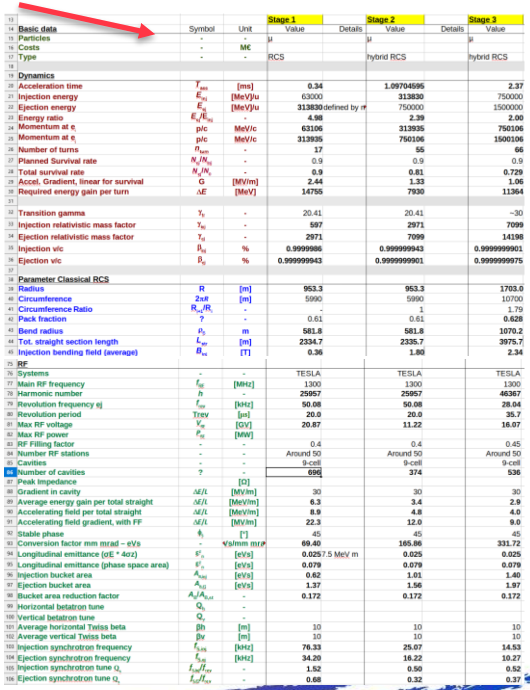
- **Hybrid RCSs have intersecting normal conducting (NC) and superconducting (SC) magnets**
- **Studies presented aim to determine the RF (cavity) and lattice parameters (number of RF stations, momentum compaction factor,…)**

H. Damerau

Parameters and tools: General parameter

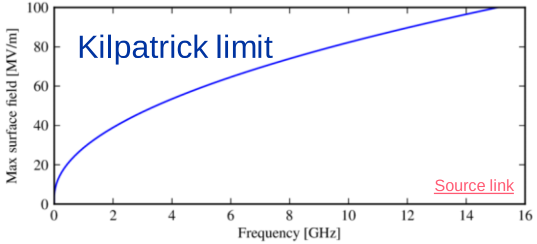
- **Detailed parameter table: https://cernbox.cern.ch/index.php/s/I9VpITncUeCBtiz**

	RCS1→314 GeV	RCS2→750 GeV	RCS3→1.5 TeV
Circumference, $2\pi R$ [m]	5990	5590	10700
Energy factor, E_{ej}/E_{inj}	5.0	2.4	2.0
Repetition rate, f_{rep} [Hz]	5 (asym.)	5 (asym.)	5 (asym.)
Number of bunches	$1\mu^+$, $1\mu^-$	$1\mu^+$, $1\mu^-$	$1\mu^+$, $1\mu^-$
Bunch population	2.5×10^{12}	2.3×10^{12}	2.2×10^{12}
Survival rate per ring	90%	90%	90%
Acceleration time [ms]	0.34	1.04	2.37
Number of turns	17	55	66
Energy gain per turn, ΔE [GeV]	14.8	7.9	11.4
Acc. gradient for survival [MV/m]	2.4	1.3	1.1
Acc. field in RF cavity [MV/m]	30	30	30
Max. RF voltage for ϕ_s=45° [GV]	20.9	11.2	16.1



				Stage 1		Stage 2		Stage 3
14	Basic data	Symbol	Unit	Value	Details	Value	Details	Value
15	Particles	-	-	μ		μ		μ
16	Costs	-	M€					
17	Type	-	-	RCS		hybrid RCS		hybrid RCS
18								
19	Dynamics							
20	Acceleration time	T_{acc}	[ms]	0.34		1.09704595		2.37
21	Injection energy	E_{inj}	[MeV]/u	63000		313830		750000
22	Ejection energy	E_{ej}	[MeV]/u	313830	defined by m	750000		1500000
23	Energy ratio	E_{ej}/E_{inj}	-	4.98		2.39		2.00
24	Momentum at e_i	p/c	MeV/c	63106		313935		750106
25	Momentum at e_j	p/c	MeV/c	313935		750106		1500106
26	Number of turns	n_{turn}	-	17		55		66
27	Planned Survival rate	N_{ej}/N_{inj}	-	0.9		0.9		0.9
28	Total survival rate	N_{ej}/N_0	-	0.9		0.81		0.729
29	Accel. Gradient, linear for survival	G	[MV/m]	2.44		1.33		1.06
30	Required energy gain per turn	ΔE	[MeV]	14755		7930		11364
31								
32	Transition gamma	γ_{tr}	-	20.41		20.41		~30
33	Injection relativistic mass factor	γ_{inj}	-	597		2971		7099
34	Ejection relativistic mass factor	γ_{ej}	-	2971		7099		14198
35	Injection v/c	β_{inj}	%	0.9999986		0.999999943		0.9999999901
36	Ejection v/c	β_{ej}	%	0.999999943		0.9999999901		0.9999999975
37								
38	Parameter Classical RCS							
39	Radius	R	[m]	953.3		953.3		1703.0
40	Circumference	$2\pi R$	[m]	5990		5990		10700
41	Circumference Ratio	R_{i+1}/R_i	-	-		1		1.79
42	Pack fraction	?	-	0.61		0.61		0.628
43	Bend radius	ρ_B	m	581.8		581.8		1070.2
44	Tot. straight section length	L_{str}	[m]	2334.7		2335.7		3975.7
45	Injection bending field (average)	B_{inj}	[T]	0.36		1.80		2.34
75	RF							
76	Systems	-	-	TESLA		TESLA		TESLA
77	Main RF frequency	f_{RF}	[MHz]	1300		1300		1300
78	Harmonic number	h	-	25957		25957		46367
79	Revolution frequency ej	f_{rev}	[kHz]	50.08		50.08		28.04
80	Revolution period	Trev	[μs]	20.0		20.0		35.7
81	Max RF voltage	V_{RF}	[GV]	20.87		11.22		16.07
82	Max RF power	P_{RF}	[MW]					
83	RF Filling factor	-	-	0.4		0.4		0.45
84	Number RF stations	-	-	Around 50		Around 50		Around 50
85	Cavities	-	-	9-cell		9-cell		9-cell
86	Number of cavities	?	-	696		374		536
87	Peak Impedance		[Ω]					
88	Gradient in cavity	$\Delta E/L$	[MV/m]	30		30		30
89	Average energy gain per total straight	$\Delta E/L$	[MeV/m]	6.3		3.4		2.9
90	Accelerating field per total straight	$\Delta E/L$	[MeV/m]	8.9		4.8		4.0
91	Accelerating field gradient, with FF	$\Delta E/L$	[MV/m]	22.3		12.0		9.0
92	Stable phase	ϕ_s	[°]	45		45		45
93	Conversion factor mm mrad – eVs	-	Vs/mm mra	69.40		165.86		331.72
94	Longitudinal emittance (σE * 4σz)	ε^l_n	[eVs]	0.025	7.5 MeV m	0.025		0.025
95	Longitudinal emittance (phase space area)	ε^l_n	[eVs]	0.079		0.079		0.079
96	Injection bucket area	$A_{b,inj}$	[eVs]	0.62		1.01		1.40
97	Ejection bucket area	$A_{b,ej}$	[eVs]	1.37		1.56		1.97
98	Bucket area reduction factor	$A_B/A_{B,st}$	-	0.172		0.172		0.172
99	Horizontal betatron tune	Q_h	-					
100	Vertical betatron tune	Q_v	-					
101	Average horizontal Twiss beta	βh	[m]	10		10		10
102	Average vertical Twiss beta	βv	[m]	10		10		10
103	Injection synchrotron frequency	$f_{S,inj}$	[kHz]	76.33		25.07		14.53
104	Ejection synchrotron frequency	$f_{S,ej}$	[kHz]	34.20		16.22		10.27
105	Injection synchrotron tune Q_s	$f_{S,inj}/f_{rev}$	-	1.52		0.50		0.52
106	Ejection synchrotron tune Q_s	$f_{S,ej}/f_{rev}$	-	0.68		0.32		0.37

Parameters and tools: RF – The TESLA cavity



- **Common f_{RF}, from H. Damerau's talk & citation from design report:**

Frequency	Accelerator	Remark
352 MHz	LEP	Moderate gradient
400 MHz	LHC	Moderate gradient
800 MHz	ERL, FCC	Alternative for µRCS
1.3 GHz	TESLA, ILC, FELs (XFEL)	µRCS
1.5 GHz	JLab-CEBAF	

3.2.1 Choice of frequency

The losses in a microwave cavity are proportional to the product of conductor area and surface resistance. For a given length of a multicell resonator, the area scales with $1/f$ while the surface resistance of a superconducting cavity scales with f^2 for $R_{BCS} \gg R_{res}$ and is independent of f for $R_{BCS} \ll R_{res}$. At an operating temperature $T = 2$ K the BCS term dominates above 3 GHz and hence the losses grow linearly with frequency whereas for frequencies below 300 MHz the residual resistance dominates and the losses grow with $1/f$. To minimize the dissipation in the cavity wall one should therefore select f in the range 300 MHz to 3 GHz.

Cavities in the 350 to 500 MHz regime are in use in electron-positron storage rings. Their large size is advantageous to suppress wake field effects and higher order mode losses. However, for a linac of several 10 km length the niobium and cryostat costs for these bulky cavities would be prohibitive, hence a higher frequency has to be chosen. Considering material costs $f = 3$ GHz might appear the optimum but there are compelling arguments for choosing about half this frequency.

- The wake fields losses scale with the second to third power of the frequency ($W_{\|} \propto f^2$, $W_{\perp} \propto f^3$). Beam emittance growth and beam-induced cryogenic losses are therefore much higher at 3 GHz.
- The f^2 dependence of the BCS resistance sets an upper limit[5] of about 30 MV/m at 3 GHz, hence choosing this frequency would definitely preclude a possible upgrade of TESLA to 35–40 MV/m [17].

The choice for 1.3 GHz was motivated by the availability of high power klystrons.

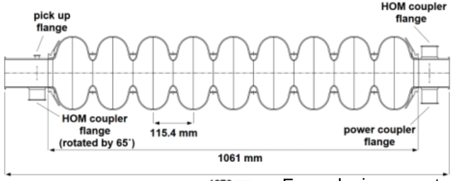
Phys. Rev. ST Accel. Beams 3, 092001, 2000

- ***f_{RF}* & *h* impact bucket area and gradient!**

$$A_B(t) = \frac{8\sqrt{2}}{2\pi h f_{rev}} \cdot \sqrt{\frac{E(t)V_{RF}(t)}{\pi h \eta}} \quad \sim 1/h^{3/2}$$

- **→ 1.3 GHz TESLA cavity as base for muon collider RCS**

Parameters and tools: RF – The TESLA cavity



From design report

- **Studies are based on the 1.3 GHz Tesla cavity (design report: Phys. Rev. ST Accel. Beams 3, 092001, 2000)**
  → see talk by A. Yamamoto
- **Relevant beam parameter**
  - **Bunch population 2.54x10^{12}, ε_L=0.01 eVs** → large intensity effects
  - **Bunch current 20.4 / 18.8 / 10.0 mA → 2x430 kW per cavity**
  - **700 / 374 / 532 cavities** in ring, distributed over n_{RF} RF stations (with 30 MV/m accelerating gradient)
  - **Synchronous phase 45°** (above transition: γ_{tr} = 20.41, 600 < γ < 14200)
- **TESLA Cavity parameter (9 cells, L=1.06 m):**
  - f_{RF} **= 1.3 GHz → harmonic number h = 25957 to 46367**
  - **R/Q = 518** Ω, total R_s = 306 GΩ
  - **Gradient 30 MV/m**
  - **Q_L = 2.2e6** (for beam loading compensation with Δf = 320 Hz)

Table 2: TTF cavity design parameters.[a]

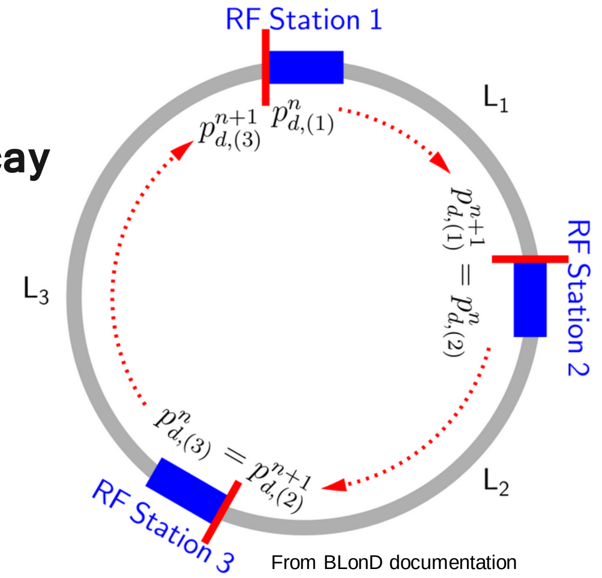
type of accelerating structure	standing wave
accelerating mode	TM_{010} , π mode
fundamental frequency	1300 MHz
design gradient E_{acc}	25 MV/m
quality factor Q_0	$> 5 \cdot 10^9$
active length L	1.038 m
number of cells	9
cell-to-cell coupling	1.87 %
iris diameter	70 mm
geometry factor	270 Ω
R/Q	518 Ω
E_{peak}/E_{acc}	2.0
B_{peak}/E_{acc}	4.26 mT/(MV/m)
tuning range	± 300 kHz
$\Delta f/\Delta L$	315 kHz/mm
Lorentz force detuning at 25 MV/m	≈ 600 Hz
Q_{ext} of input coupler	$3 \cdot 10^6$
cavity bandwidth at $Q_{ext} = 3 \cdot 10^6$	430 Hz
RF pulse duration	1330 μs
repetition rate	5 Hz
fill time	530 μs
beam acceleration time	800 μs
RF power peak/average	208 kW/1.4 kW
number of HOM couplers	2
cavity longitudinal loss factor $k_\parallel$ for $\sigma_z = 0.7$ mm	10.2 V/pC
cavity transversal loss factor $k_\perp$ for $\sigma_z = 0.7$ mm	15.1 V/pC/m
parasitic modes with the highest impedance : type $\pi/9$ $(R/Q)/$ frequency $2\pi/9$ $(R/Q)/$ frequency	TM_{011} 80 Ω/2454 MHz 67 Ω/2443 MHz
bellows longitudinal loss factor $k_\parallel$ for $\sigma_z = 0.7$ mm	1.54 V/pC
bellows transversal loss factor $k_\perp$ for $\sigma_z = 0.7$ mm	1.97 V/pC/m

From design report

Studies & BLonD code

(Beam Longitudinal Dynamics code)

- BLonD: macro-particle tracking code, developed at CERN since 2014
- Links: documentation and github
- MuC-specific to multiple RF stations & muon decay
- Using the BLonD code to observe effects of
  - Short-range wakefields
  - Fundamental beam loading
  - Synchrotron tune Q_s between RF stations
- First studies with only one bunch, 2nd to follow



From BLonD documentation

Outline

- The TESLA cavity and RCS parameter
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  Short-range wakefield and beam loading compared to the cavity voltage
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Induced voltages: Short-range wakefields

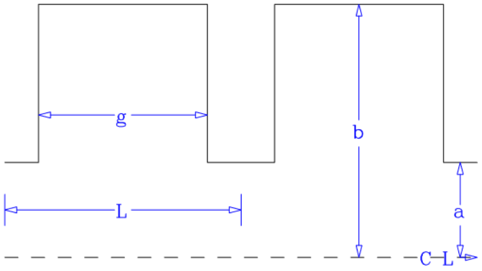
Based on K. Bane et al., '[Calculation of the short-range longitudinal wakefields in the NLC linac]', ICAP98, 1998

$$W_L \approx \frac{Z_0 c}{\pi a^2} \exp\left(-\frac{2\pi\alpha^2 L^2 s}{a^2 g}\right) \operatorname{erfc}\left(\frac{\alpha L}{a}\sqrt{\frac{2\pi s}{g}}\right) \quad [s \text{ small}] \qquad (3)$$

One can approximate this by a semi-analytically expression, valid for small *s* and *s/L* < 0.15:

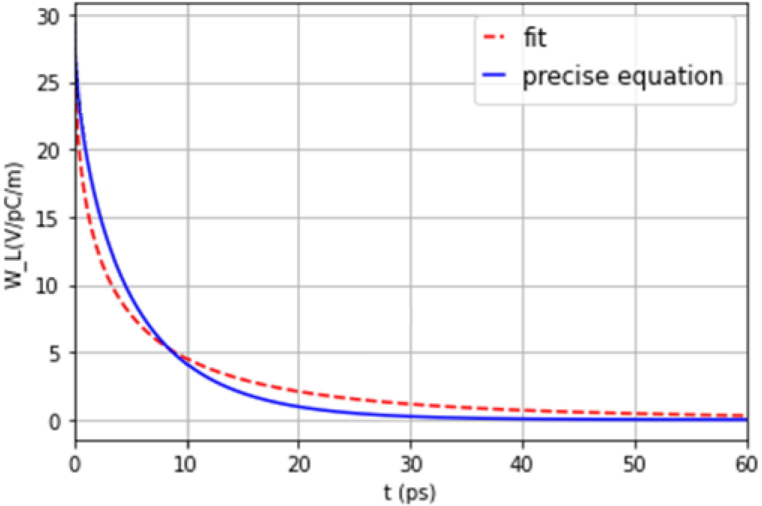
$$W_L = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{s/s_0}\right) \qquad s_0 = 0.41 \frac{a^{1.8} g^{1.6}}{L^{2.4}}$$

The parameters for the Tesla cavity[1] gives long. wake functions on the order of 30 V/pC/m:



L= 115.4 mm
g= 82 mm
a= 35 mm
b= 103.3 mm

Adjusting 'a' can be a powerful tool to mitigate wakefields!



The BLonD code convolutes the wake function with the beam profile to obtain the induced voltage

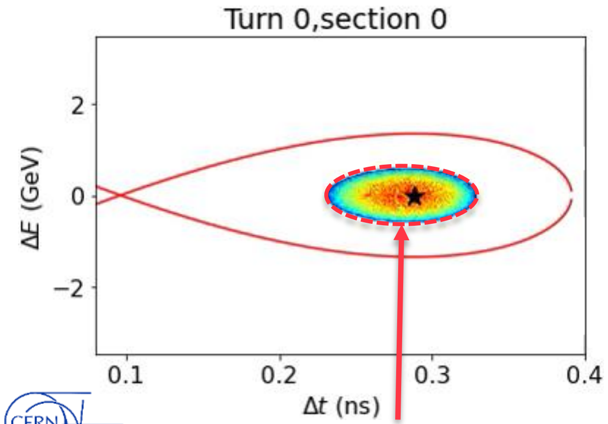
[1]Wakefield studies for the Tesla cavity shown in TESLA Report 2003-19

Short-range wakefields

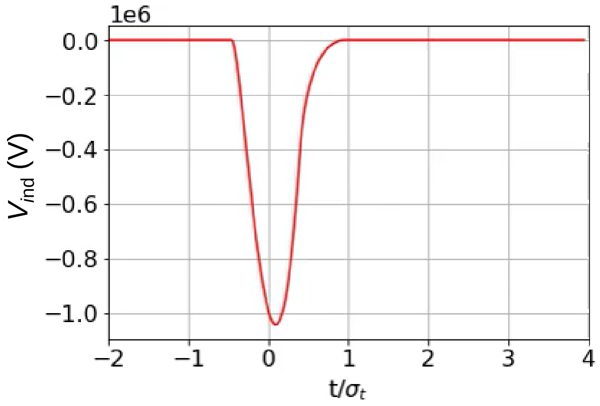
Simulations for RCS1 (63 → 314 GeV), n_{RF} = 48

- **Initial mismatch due to assumption of continuous synchrotron motion in matching routine**
  → Improves with a higher n_{RF} (see following slides)
- **Induced voltage from the short-range wakefield: 1.5 MV per cavity, i.e., also 1.5 MV/m**
- **Total induced voltage per turn is around 5% of V_{acc}**

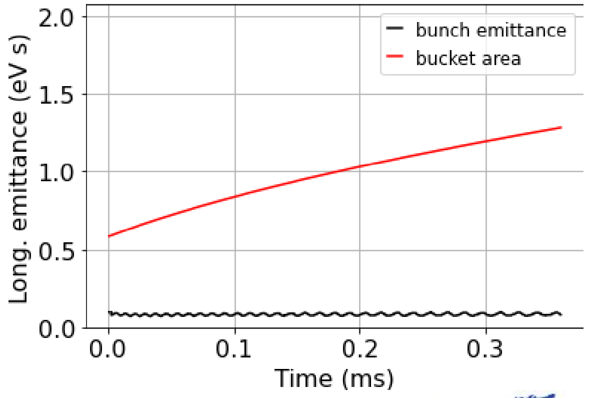
Phase space



Induced voltage per cavity



Longit. emittance



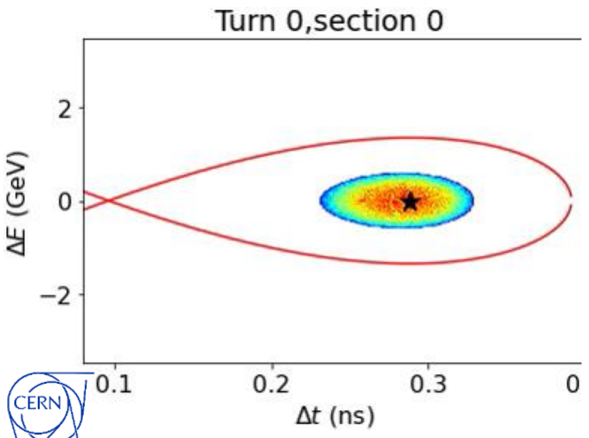
Emittance = area under Hamiltonian containing 99% of the bunch

Induced voltages: Fundamental Beam loading

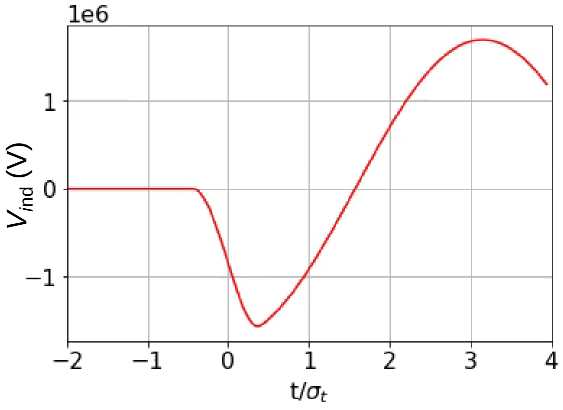
- Simulations for RCS1 (63 → 314 GeV), n_{RF} = 48, with induced voltage from fundamental mode beam loading, single turn, no short-range wakefields
- Induced voltage from beam loading for a single turn is similar, 1.5 MV per cavity (confirmed by simulations from A. Grudiev, see this presentation), with similar effects
- **Note**: So far for a single turn only, multi-turn implementation to follow

Reminder: resonator parameters :
- Gradient in cavity: 30 MV/m
- L=1.04 m
- R/Q = 518 Ω
- Δf = 320 Hz
- Q_L = 2.2e6

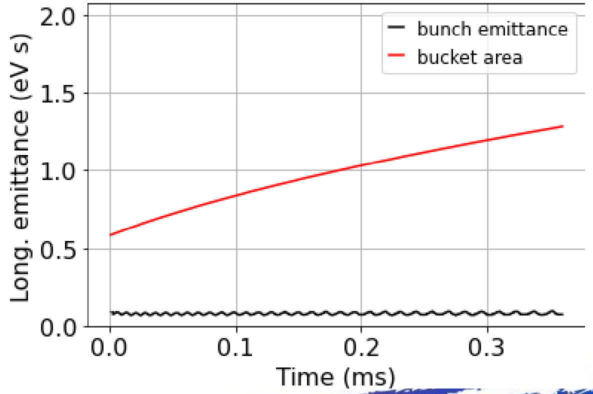
Phase space



Induced voltage per cavity



Longit. emittance



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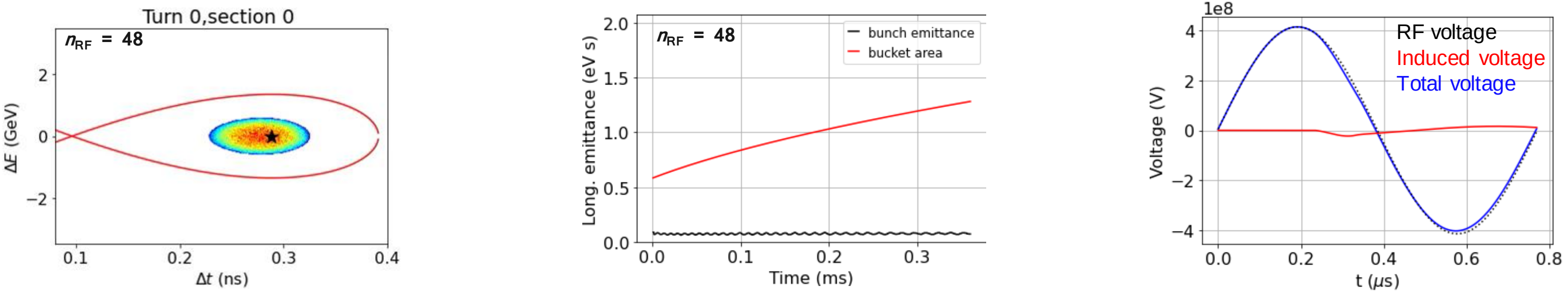
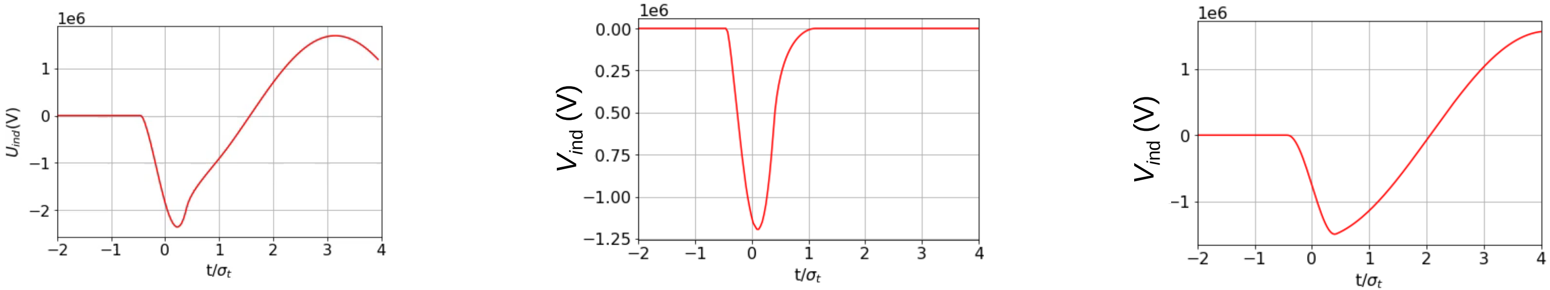
Induced voltages: both contributions

- **Both effects combined: total induced voltage in a cavity is around 2.2 MV per cavity / per meter, i.e. 10-11% of the RF voltage**



Total induced voltage per cavity = short-range wakefield + beam loading, fundamental



- **Intensity effects on bunch are mitigated by high RF voltage**

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Synchrotron tune and number of RF stations

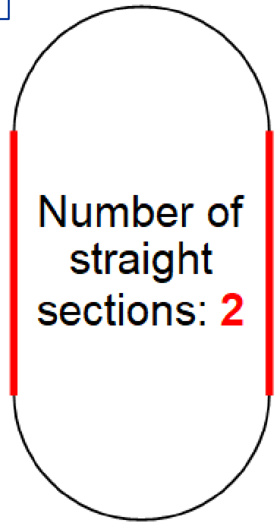
From: H. Damerau

- **Number of synchrotron oscillations per turn:**

$$Q_S = \frac{\omega_S}{\omega_0} = \sqrt{-\frac{h\eta e V_{RF}\cos\phi_S}{2\pi E\beta^2}} \propto \sqrt{V_{RF}\cos\phi_S}$$

LHC: Q_s=0.005

- **Stable synchrotron oscillations and phase focusing only for $Q_s \ll 1/\pi$** (T. Suzuki, KEK Report 96-10)
  - → **RCSs exceed this limit: 0.3 < Q_s < 1.5**
  - → **Several longitudinal kicks per turn for small Q_s between stations**
  - → **Distribute RF system over n_{RF} sections**
- **Advantageous also for counter-rotating beams!**



→ **n_{RF} is an important quantity to determine!**

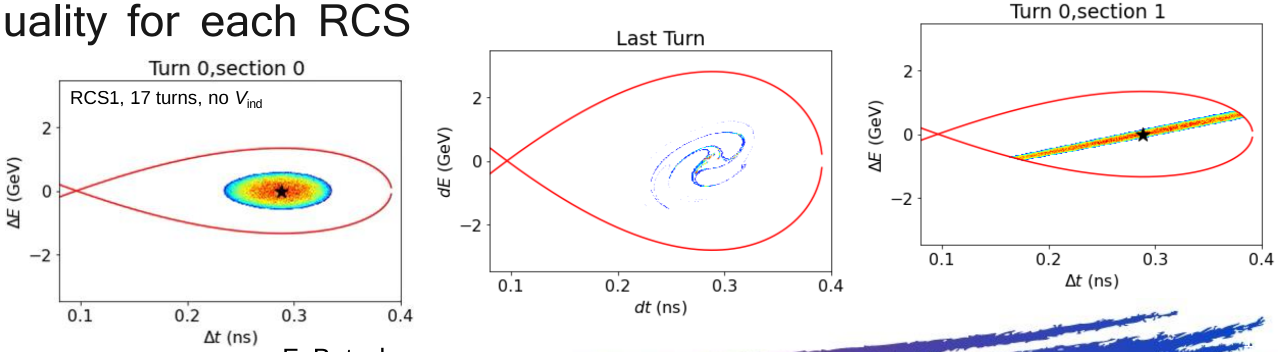
Synchrotron tune and number of RF stations

Why not choosing a high n_{RF} to fulfil $Q_s << 1/\pi$?

- High n_{RF} → smaller quadrupole-like oscillations caused by discrete energy steps and resulting mismatching
- **BUT:** higher n_{RF} results in higher construction / cooling / cryogenics and powering costs, even though the number of cavities is constant and defined by ΔE per turn

→ Determine emittance growth using BLonD, also as a function of n_{RF} as main criteria for beam quality for each RCS

Examples: n_{RF} = 4



High Q_s destroys bunch!

F. Batsch

Synchrotron tune and number of RF stations

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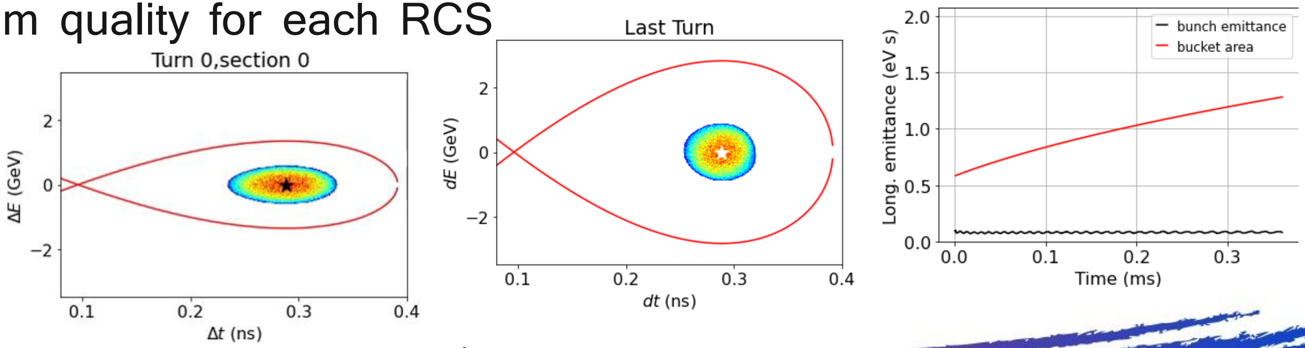
→ Determine emittance growth using BLonD, also as a function of n_{RF} as main criteria for beam quality for each RCS

Examples: n_{RF} = 48

Emittance growth: 4%



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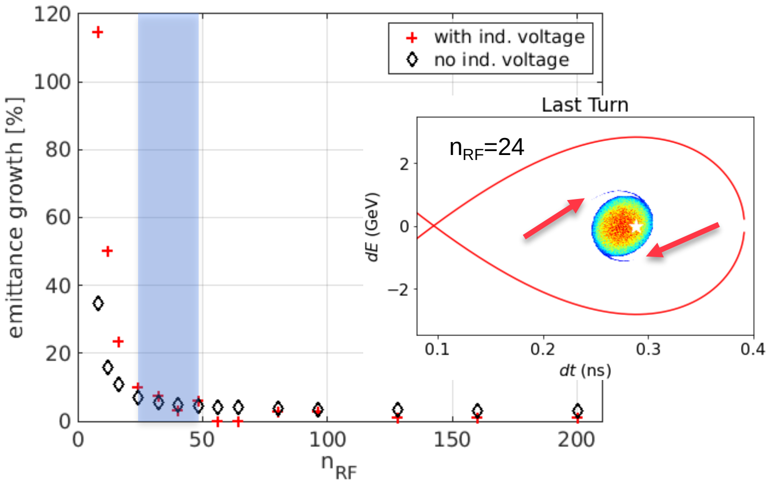
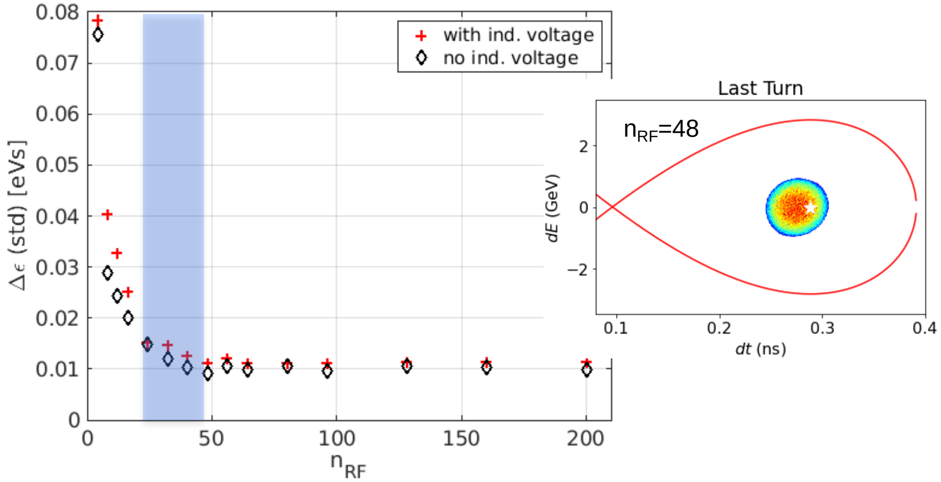
Synchrotron tune and number of RF stations

Results for each RCS, with and without induced voltage / intensity effects:

RCS1



Standard deviation measure for bunch oscillations:

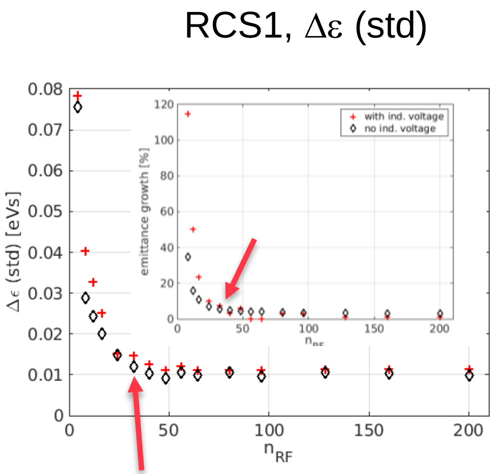


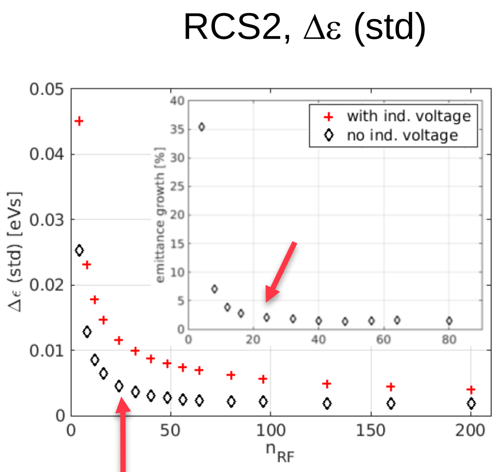
No improvement in emittance growth for n_{RF} >48

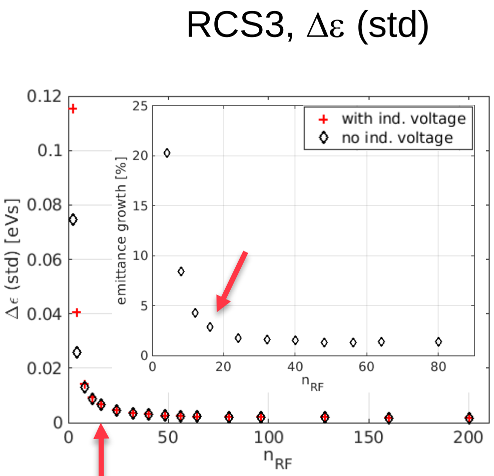
Minimum n_{RF}>24 RF stations for RCS1, e.g. 32.

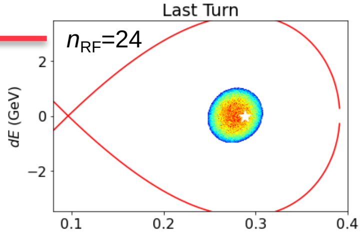
Synchrotron tune and number of RF stations

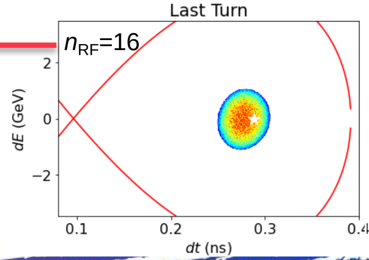
Results for each RCS and with and without induced voltage / intensity effects:











Similar behaviour for RCS 2&3, no change for n_{RF} > 48

Lower Q_s allows for slightly smaller number in RCS 2&3, e.g. n_{RF} = 24 and 16

➢ **Influence of small n_{RF} on two bunches to be investigated**

Outline

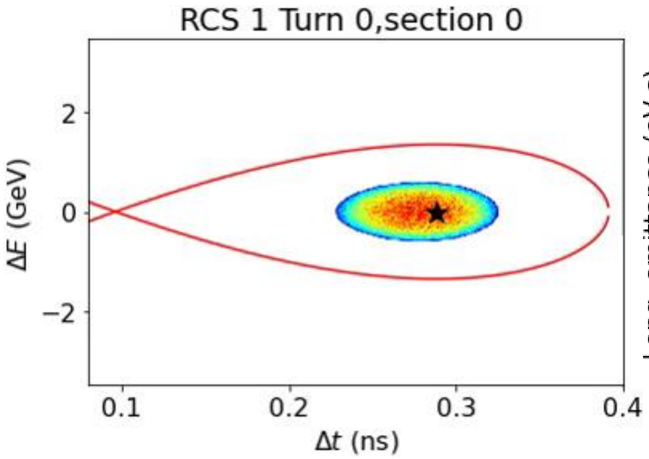
- The TESLA cavity and RCS parameter
- Intensity effects & longitudinal beam dynamics:

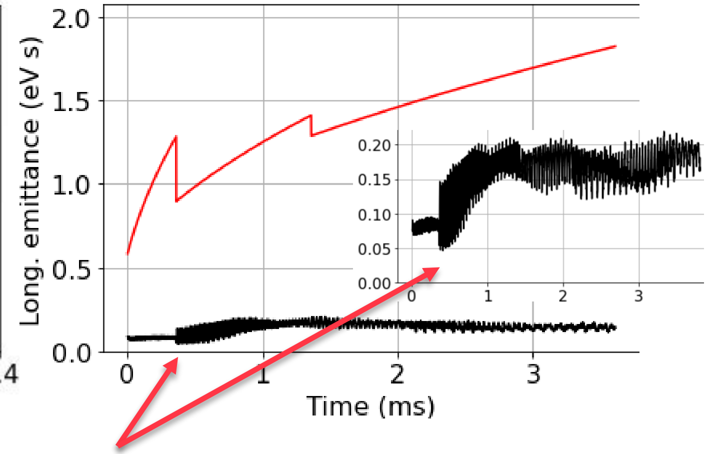
  Short-range wakefield and beam loading compared to the cavity voltage
- Synchrotron tune & Number of RF stations versus emittance
- Emittance evolution during the acceleration stages
- Outlook and Summary

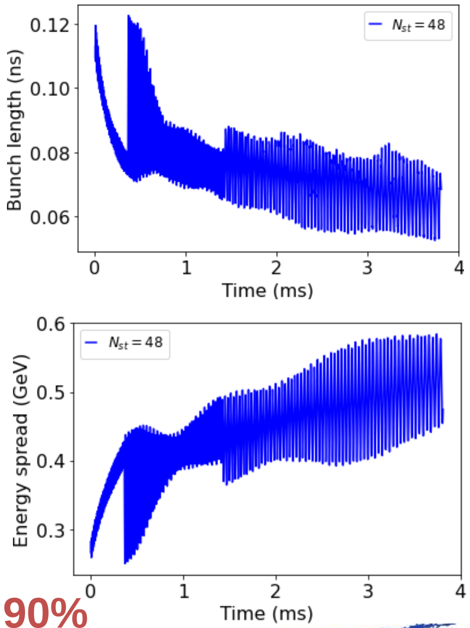
Beam transport though the RCS chain: Evolution of bunch parameter

How do the bunch length, energy spread and emittance evolve when propagating the bunch through not one, but all RCS?

All RCS with n_{RF}=48







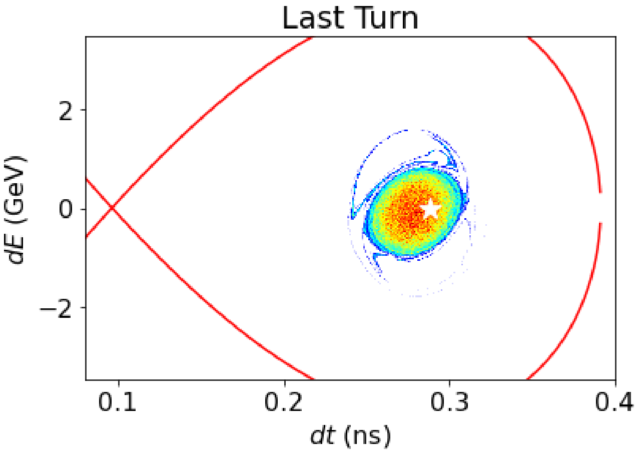
- **Unmatched bunch remains main problem, emittance growth 90%**

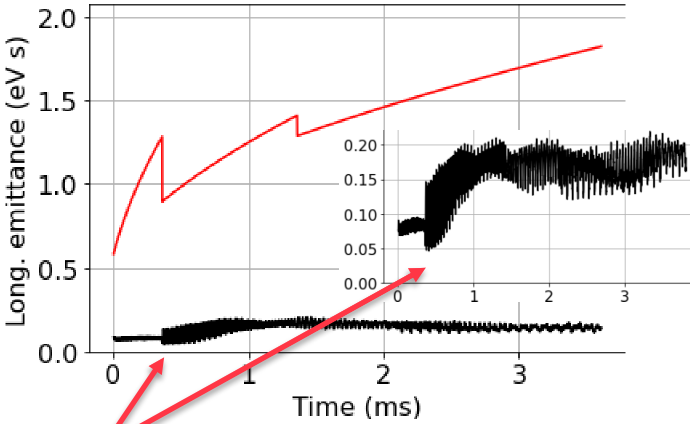
Beam transport though the RCS chain: Evolution of bunch parameter

International UON Collider Collaboration

How do the bunch length, energy spread and emittance evolve when propagating the bunch through not one, but all RCS?

All RCS with n_{RF} = 48





Final bunch parameter:

$4\pi\sigma_t\sigma_E < 0.19$ eVs (190MeVm)

$4\sigma_t = 0.08$ ns

$\sigma_E = 0.6$ GeV

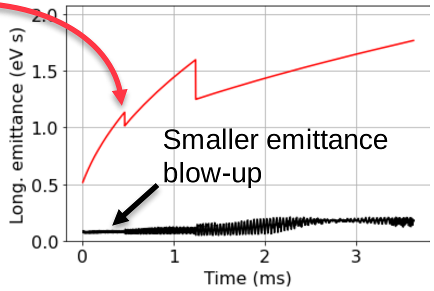
Factor 2-3 above nominal, but improvement with better matching

- **Unmatched beam main problem, emittance growth 90%**

→ Optimize voltages ≙ acc. time ≙ decay rate between RCS

Outlook – Follow ups

- **Parameter to be optimized for the construction point-of-view:**
  - **Decay rate:** RF voltage and ramp rates better distributed to e.g. lose more muons earlier and less later. **Matching improved?**
  - **Synchronous phase** ⇔ higher voltage ⇔ emittance budget
  - **Beam optics**, see e.g. talk by A. Chancé before
  - **Magnet's ramping functions:** See my talk tomorrow at 14:20



- **Multi-turn wakefields and resistive-wall impedance will have an effect, which must be included**

- **Wakefields of the counter-rotating bunch remains open question**

Summary

- **1.3 GHz TESLA Cavity suited for muon acceleration in the RCSs, but not all effects studied**
- **High intensities: Short-range wakefields and beam loading cause induced voltage ~2 MV/m per cavity, or 10% of V_{acc}, but do not harm beam transport due to the high Q_s and high V_{RF}**
- **Beam is transported with % level emittance growth in each RCS, if n_{RF} high enough**
- **A large number of RF stations on the order n_{RF} = 32 needed to ensure a sufficiently low synchrotron tune between the stations**

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