RCS 1 transverse impedance and stability

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Thanks to F. Batsch, F. Boattini, L. Bottura, C. Carli, A. Chancé, H. Damerau, A. Grudiev, I. Karpov, K. Skoufaris

Muon collider collaboration – Annual meeting
2022-10-12 / 14:40 - 14:55

Swiss Accelerator Research and Technology

This work was performed under the auspices and with support from the Swiss Accelerator Research and Technology (CHART) program (www.chart.ch).

Goal and scope of the study

- **O(700) RF cavities** will be needed to provide the large acceleration gradient

Details on RF system for the RCS in H. Damerau presentation

- These cavities will create **high-order resonant modes** whose properties (frequency, amplitude, bandwidth) depend on the cavity design

- First impedance and stability studies **focus on their impact**

Goal and scope of the study

- Obtain a general **limit for resonator shunt impedance, resonance frequency and quality factor** in the transverse plane

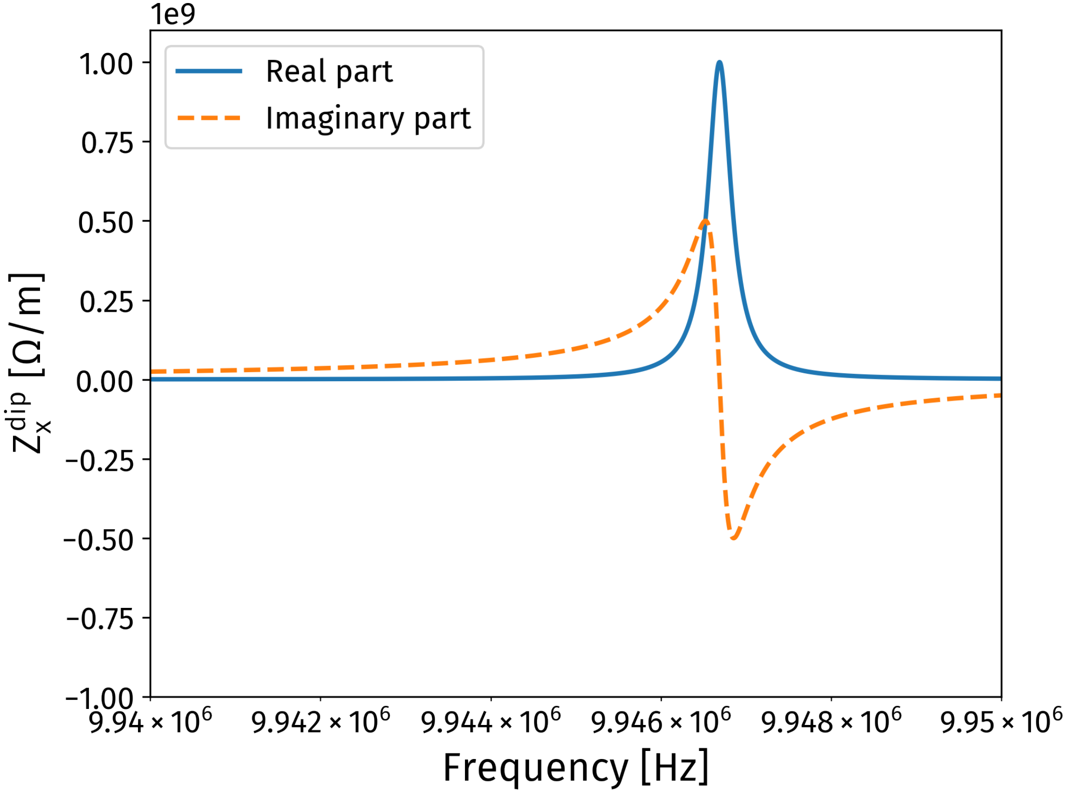
- **Check transverse beam stability** for a possible **RF cavity design**

- Studies are performed with a **single beam** at the moment

Resonator impedance and wakefield

Resonator impedance model

Resonator impedance
$f_{res} \approx 10$ MHz, $R_s = 1 G\Omega/m$, $Q = 30000$

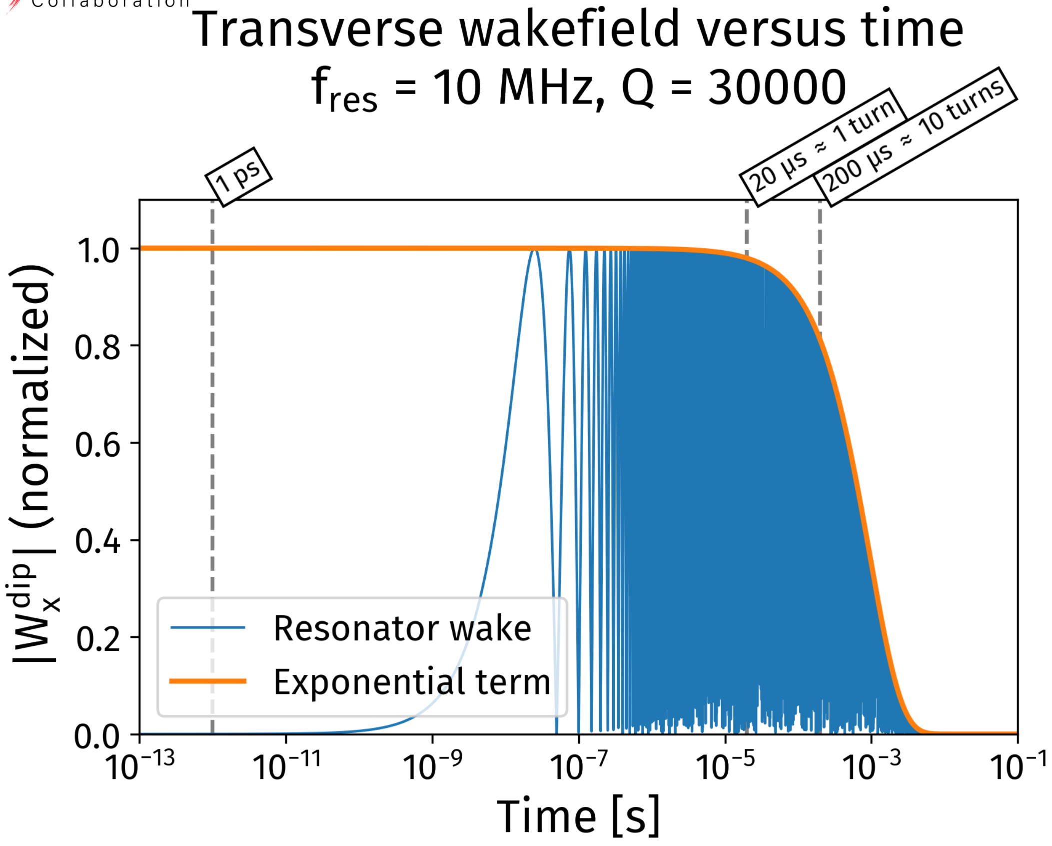


- Use a single horizontal dipolar resonator impedance/wakefield
- Scan its shunt impedance R_s, its resonance frequency f_{res} and its quality factor Q

Scan parameters

	Value
Resonator shunt impedance R_s	1 kΩ/m to 100 TΩ/m
Resonance frequency f_{res}	10 MHz to 1 THz
Quality factor Q	100, 300, 1000, 3000, 10000, 30000

Resonator impedance model



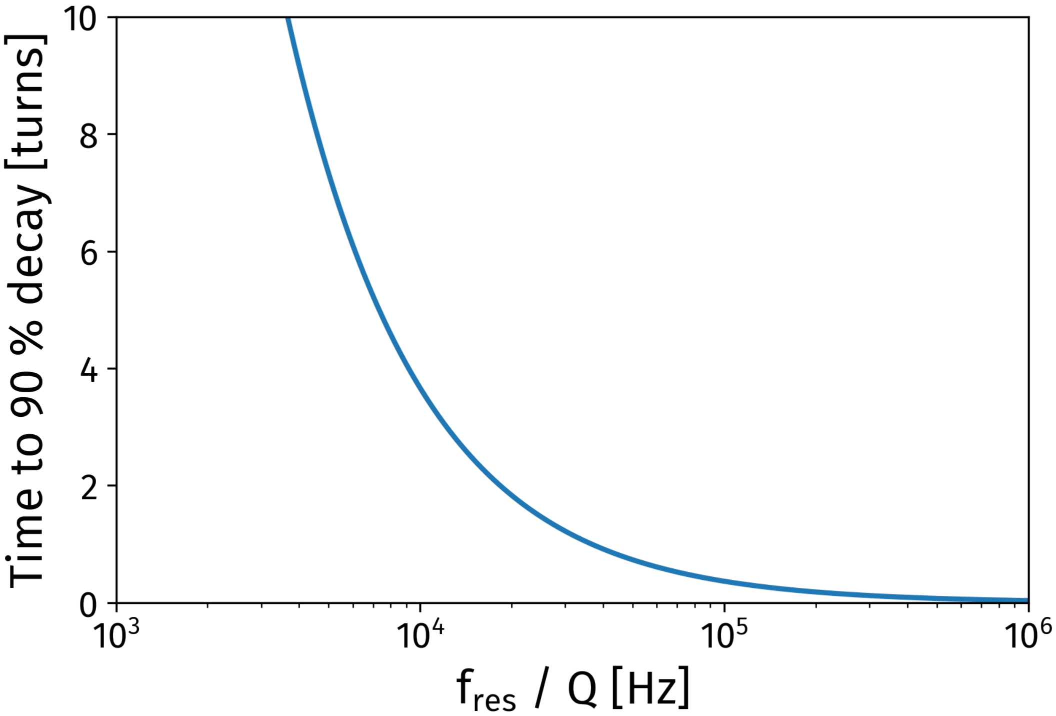
- For some (f_{res}, Q), the wakefield can extend well beyond one turn
- Example here with f_{res}=10 MHz.
- The wake can be written

$$W(t) = \frac{2\pi f_{res} R_s}{Q\sqrt{1-\frac{1}{4Q^2}}} \exp\left(-\frac{2\pi f_{res}}{2Q}t\right) \sin\left(2\pi f_{res}\sqrt{1-\frac{1}{4Q^2}}t\right)$$

- The exponential term (plotted in orange) dictates the wakefield decay

Resonator impedance model

Number of turns (1 turn = 20µs) to reach 90 % wakefield decay



- We can easily deduce the time t required to reach a 90 % wakefield decay:

$$t = -\frac{\ln(0.1)}{\pi}\left(\frac{f_{res}}{Q}\right)^{-1}$$

- With our machine parameters, **multiturn wakefield is required if $f_{res}/Q < \sim 10^5$**

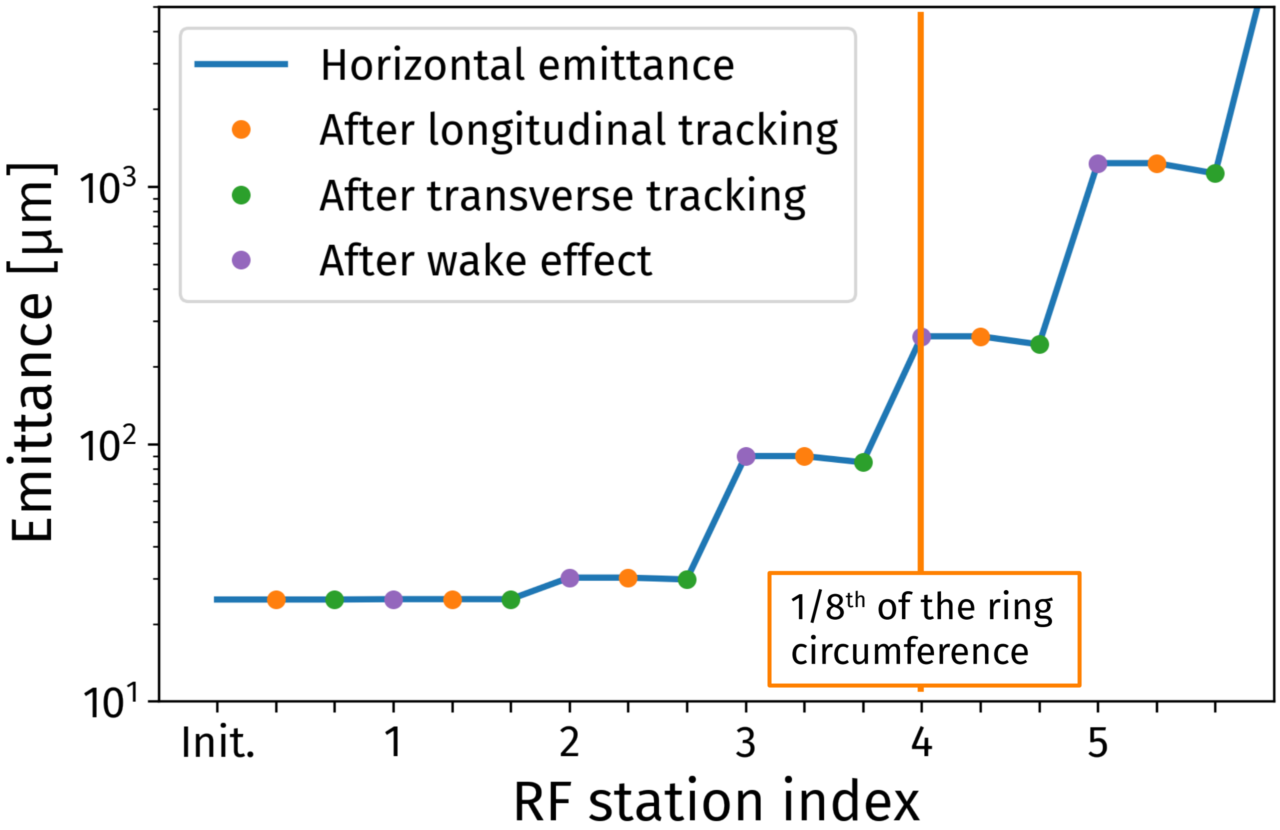
Transverse stability simulations parameters

Transverse stability simulation in the RCS 1

- Simulation including **longitudinal map + transverse map + transverse wakefield** (single turn or multi-turn)
  - **32 RF stations are used**, wakefield and transverse map also divided in 32 stations (for the number of stations, more info in F. Batsch presentation)
- Tracking simulations with macroparticle code PyHEADTAIL
  - Injection energy 63 GeV, momentum increment 14.2 GeV/c per turn (equally distributed in the 32 RF stations)
  - Only 17 turns are needed for the first acceleration stage (63 GeV to 313 GeV)
  - Chromaticity to 0 in both planes (**natural chromaticity is compensated**)
  - **No damper** (transverse feedback) / **No octupoles** for Landau damping

Instability growth can be very quick

- Between each RF station, there are three tracking elements: longitudinal, transverse and wakefield
- In this example the **instability appears** already **after the second RF station**



Transverse stability simulations results

Stability summary plots

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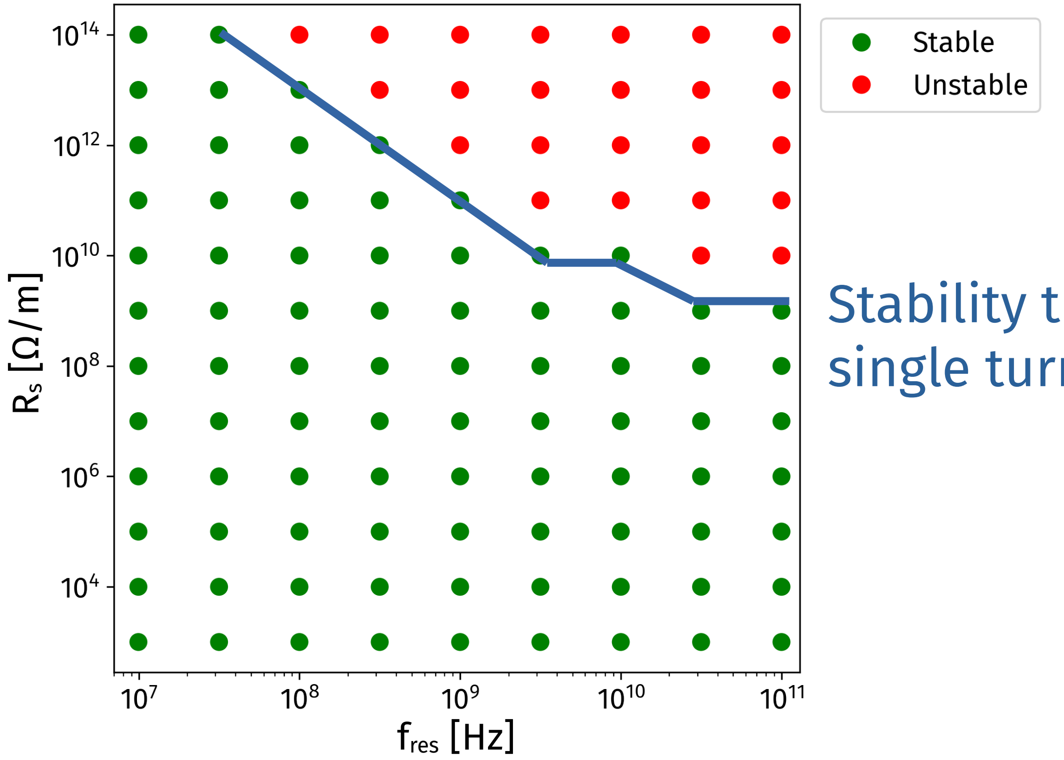
- Emittance is evaluated after 20 turns, slightly longer time than the 17 turns required for the acceleration in the RCS 1

- For each (R_s, f_{res}), indicate if there is **emittance growth (red dot) or not (green dot)**
  - Emittance growth = $\varepsilon_{turn\ 20}$ / $\varepsilon_{initial}$
  - Consider the beam unstable if **emittance growth > 20 %** (criterion to be refined)

- Focus on one high-Q case (Q=10 000, other cases reported in appendix)

Q = 10000, 1-turn wakefield

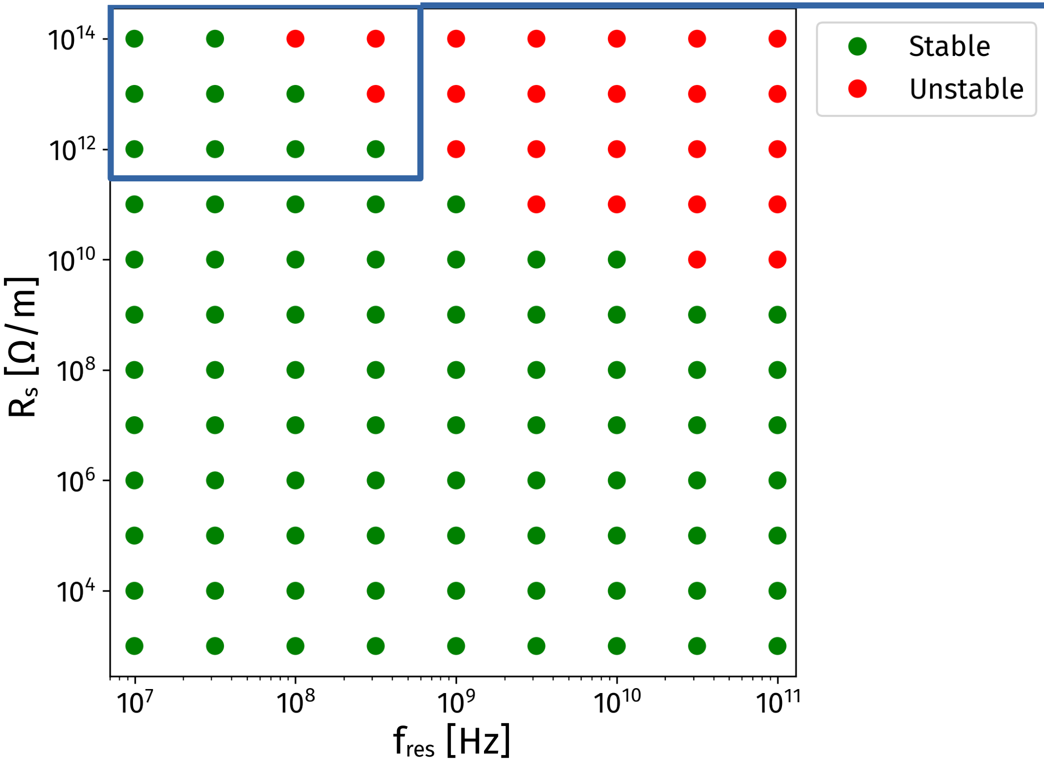
1-turn wakefield



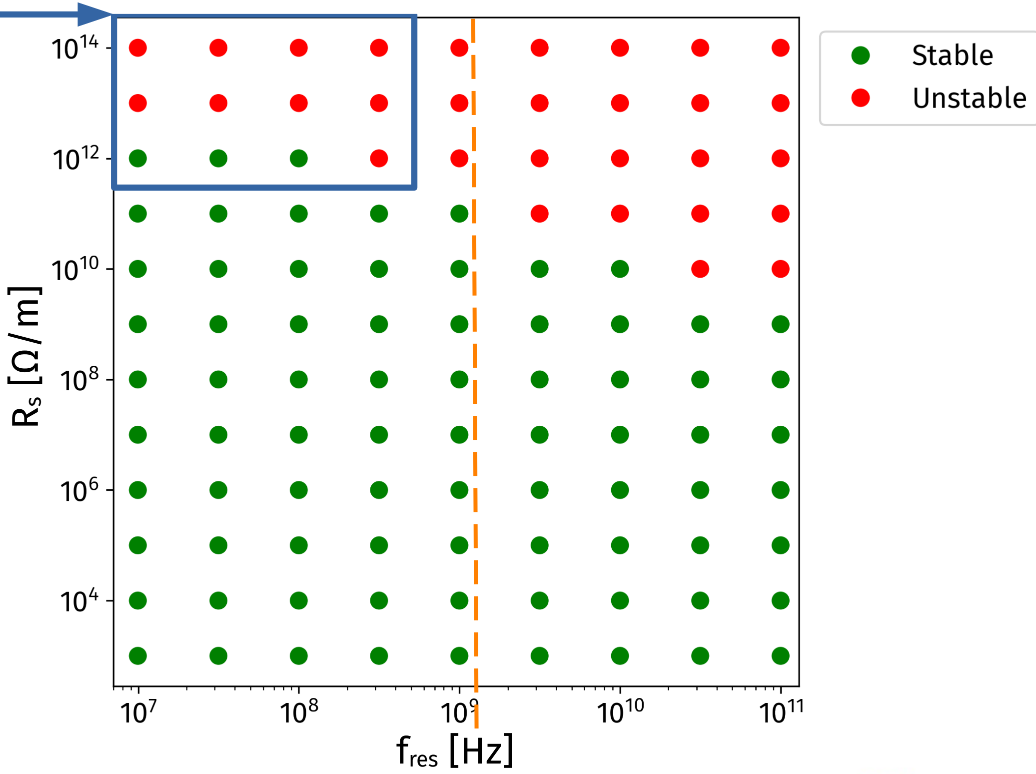
Stability threshold, single turn wakefield

Q = 10000, 50-turn wakefield

1-turn wakefield



50-turn wakefield

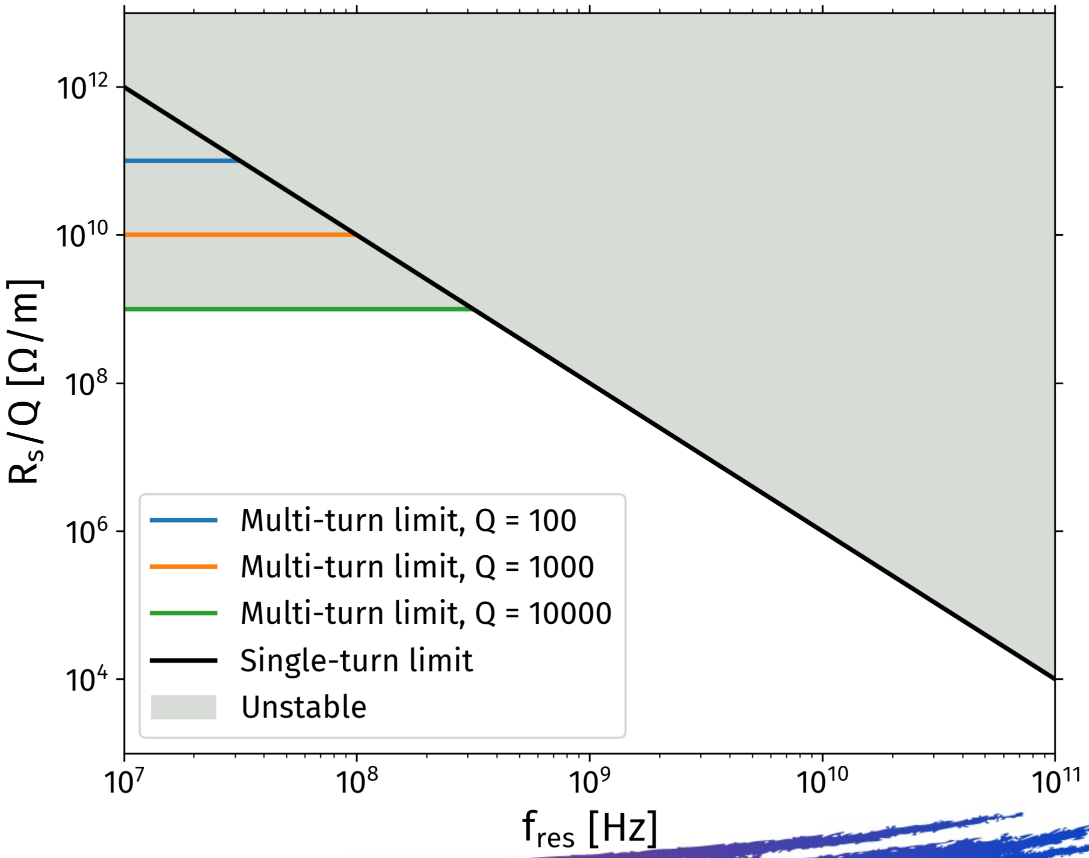


$f_{res}/Q < 10^5$

Summary of transverse stability simulation in the RCS 1

- Summarize on one plot the single-turn and multi-turn wakefield stability limits (divide y-axis by quality factor Q)
- The **single-turn limit** (black line) depends on the resonator frequency and the R_s/Q
  - **R_s < 100 [MΩ/m] * Q / f² [GHz²]**
- The **multi-turn limit** (color lines) depends on the resonator shunt impedance R_s
  - **$R_s < 10^{13}$ Ω/m**

Stability limit versus resonator parameters



Example with one type of cavity

Application of the stability criteria

- First proposition from A. Grudiev: ILC type cavities
- **RCS 1** requires **20 GV** accelerating voltage
- Using **Low Losses SRF cavity** described in Sekutowicz et al. “Design of a Low Loss SRF Cavity for the ILC”, 2005
  - Fundamental RF frequency: 1.3GHz
  - Assuming 30 MV/m and 1 m length per cavity
- **670 cavities** would be required

One HOM from LL ILC cavity and S-turn stability

- Some RF cavity parameters:
  - Active Acc. Gradient: 30 MV/m
  - Cavity length: L_cav~1m
  - V_cav: 30MV
  - N_cav = 20100/30 = **670**
- Max R/Q HOM: R/Q = 32 linacΩ/cm^2 => Rs/Q = R/Q/2*c/ω = **3.1 kΩ/m** per cavity
- For 670 cavities: Rs/Q = **2.1 MΩ/m**
- f = **2.45 GHz,**
- Ncav*Rs/Q*f^2 = **12.6 [MΩ/m*GHz^2] < 100,** Single turn stability limit from David
- **It is below stability limit by about factor 8 for one HOM.**
- In fact, all HOMs must be taken into account for S-turn stability calculation

A. Grudiev, *Transverse stability in RCS and TESLA cavity,* HEMAC meeting, 2022-10-04

Details on ILC type cavities in A. Yamamoto presentation

Application of the stability criteria

- Largest transverse High Order Mode for a single cavity
  - $\mathbf{f_{res} = 2.45\ GHz}$
  - $\mathbf{R_s/Q = 3.1\ k\Omega/m}$

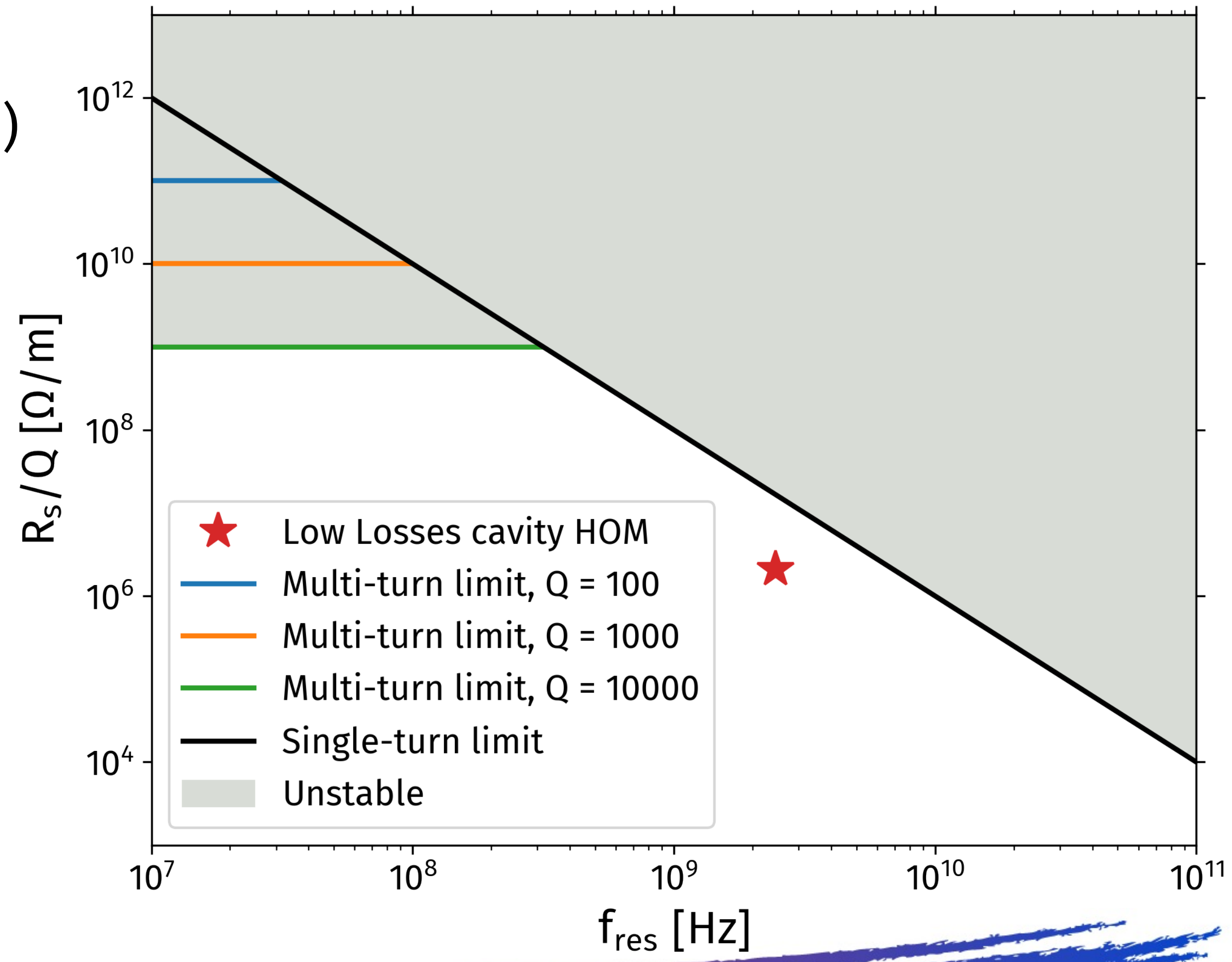
- Chose $\mathbf{Q = 10^4}$ to remain in single turn wakefield regime

- With 670 cavities
  - $\mathbf{[R_s/Q]_{total}} = 670 * 3.1\ k\Omega/m = \mathbf{2.1\ M\Omega/m}$
  - $\mathbf{R_{s,\ total}} = [R_s/Q]_{total} * Q = 2.1\ M\Omega/m * 10000 = 2.1 \cdot 10^{10}\ \Omega/m = \mathbf{21\ G\Omega/m}$

Mode stability prediction

- Stability threshold (single turn) $R_s/Q \sim 100\ [M\Omega/m]\ /\ f^2\ [GHz^2]$
- This **HOM** at **2.45 GHz:**
  - $\mathbf{[R_s/Q]_{threshold}} = 100/2.45^2 =$ **16.7MΩ/m**
  - $\mathbf{[R_s/Q]_{total}}$ **= 2.1 MΩ/m**
- HOM below the predicted stability limit by factor 8

Stability limit versus resonator parameters

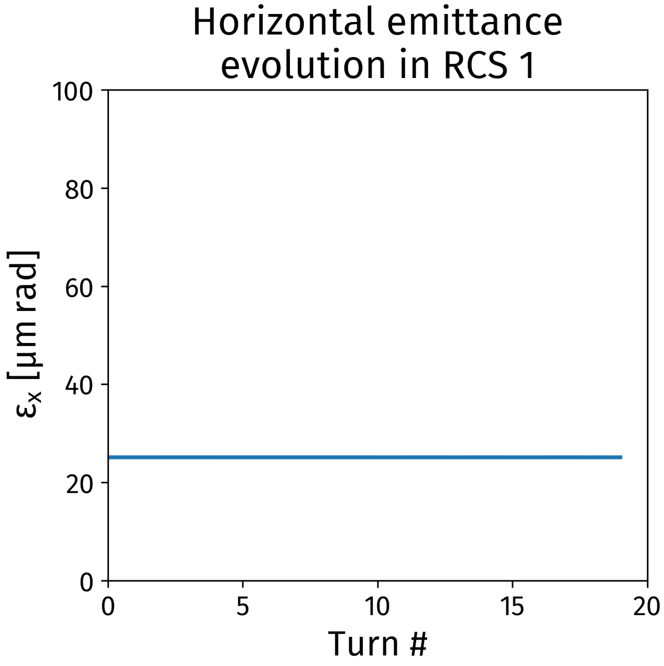


Stability simulations with most critical HOM

- Perform tracking simulations with this single most critical HOM
- To check the stability threshold, the HOM shunt impedance R_s is also multiplied by 6 and 8
- Note: stability simulation parameters are slightly different
  - Number of RF stations: 24 (was 32) following first optimization by F. Batsch
  - Number of macroparticles: 10 000 (was 5000)

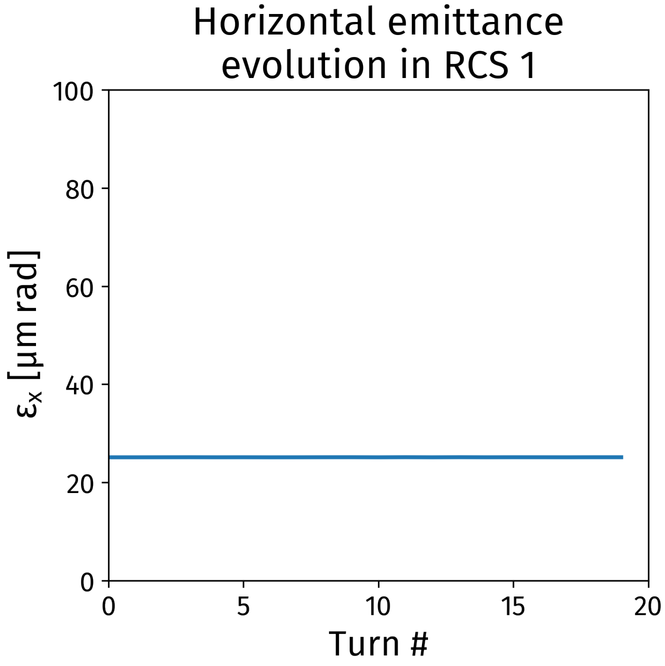
Stability simulations with most critical HOM

$[R_s/Q]_{total}$ = 2.1 MΩ/m
(factor 1)



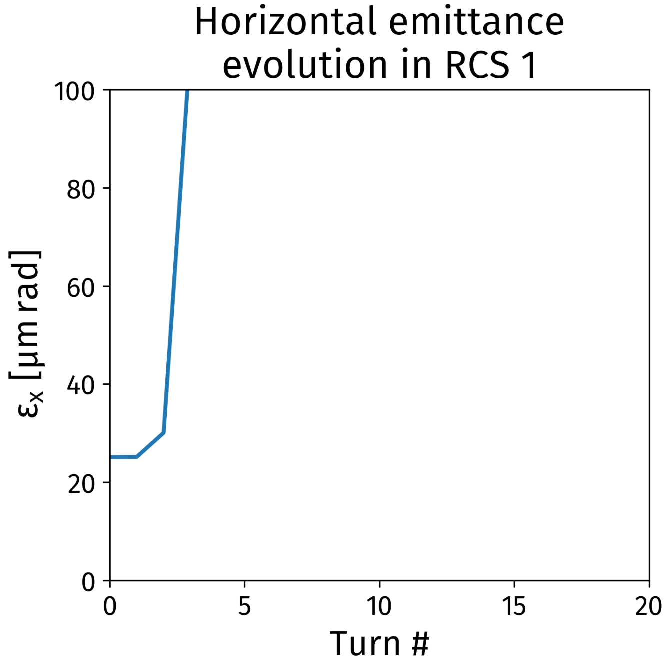
Stable

$[R_s/Q]_{total}$ = 12.6 MΩ/m
(factor 6)



Stable

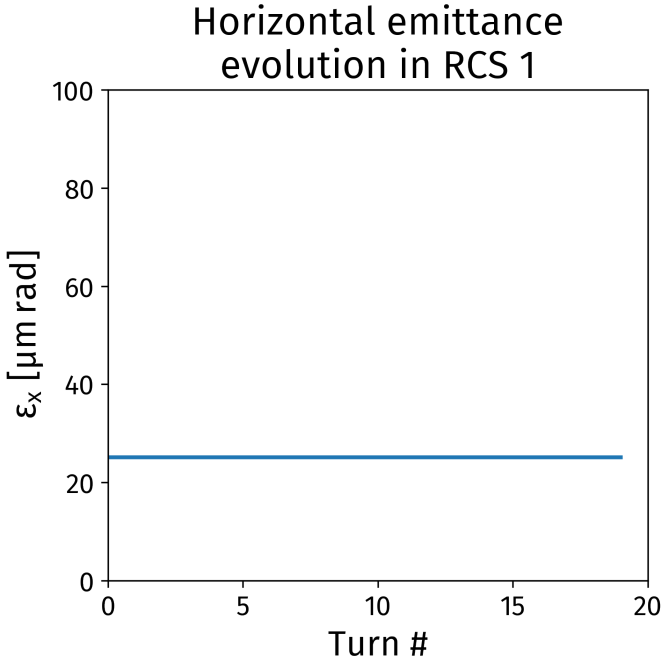
$[R_s/Q]_{total}$ = 16.8 MΩ/m
(factor 8)



Unstable

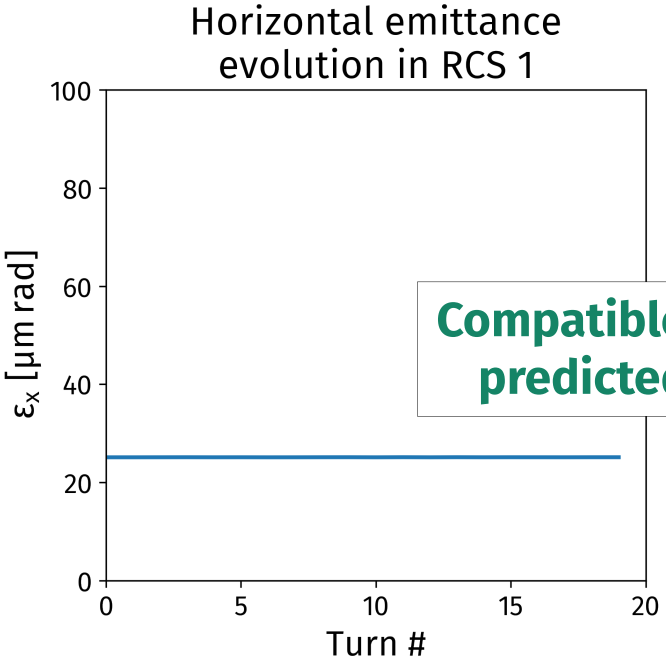
Stability simulations with most critical HOM

$[R_s/Q]_{total}$ = 2.1 MΩ/m
(factor 1)



Stable

$[R_s/Q]_{total}$ = 12.6 MΩ/m
(factor 6)



Stable

$[R_s/Q]_{total}$ = 16.8 MΩ/m
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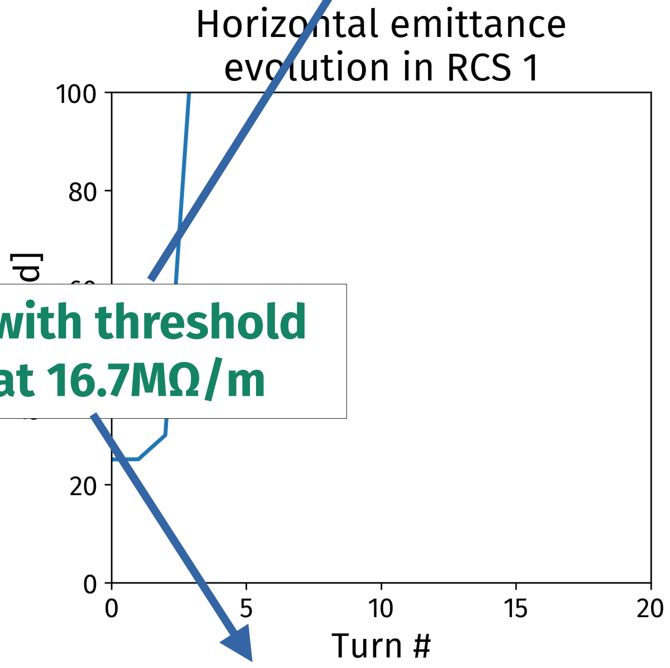


Unstable

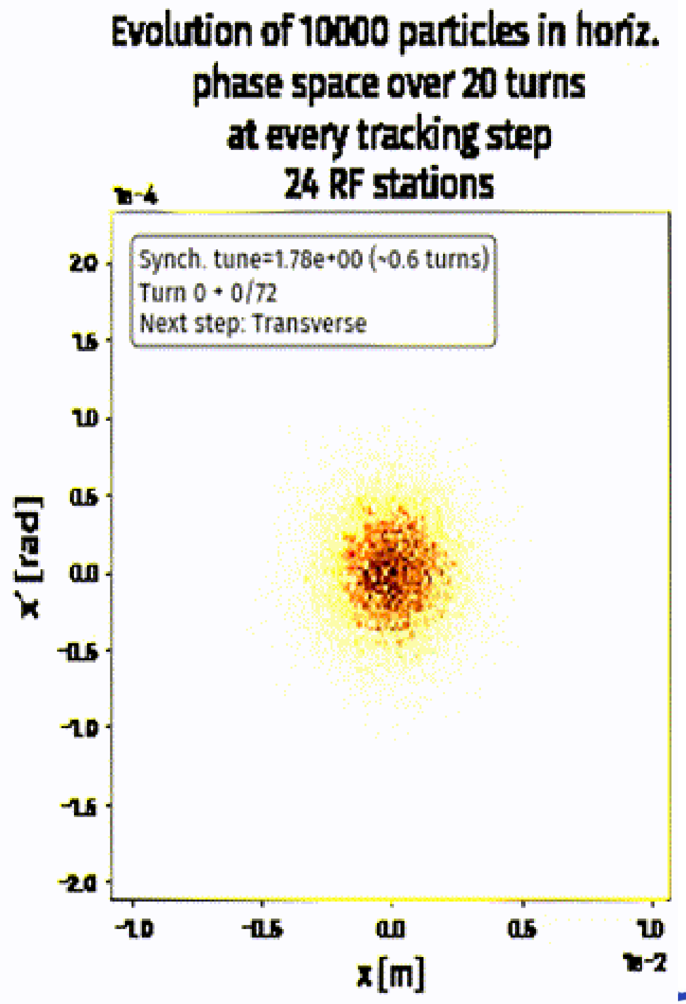
Compatible with threshold predicted at 16.7MΩ/m

Unstable case (factor 8 on shunt impedance)

Transverse phase space evolution

24 RF stations * 3 tracking elements = 72 tracking steps per turn

- The transverse instability is extremely quick, beam is lost in 2 turns



Conclusion

- General transverse stability criteria were derived for the RCS 1 RF cavities high-order modes

$f_{res}/Q < \sim 10^5$

Yes → **Multi-turn regime:**
$R_s < 10^{13}\ \Omega/m$

No → **Single turn regime:**
$R_s < 100\ [M\Omega/m] * Q\ /\ f^2\ [GHz^2]$

Conclusion

- One type of cavity (Low Losses SRF cavity described by Sekutowicz et al., as proposed by A. Grudiev in HEMAC meeting) was investigated from transverse stability side
  - The most critical HOM remains below the stability threshold, even with 670 cavities
  - Simulations show that there is a factor ~8 margin for this single mode shunt impedance

Next steps and possible further studies

- Include more detailed cavity models with all HOMs and/or detailed short range wakefield
- Perform similar impedance and stability studies for the RCS 2, 3 and 4
- Include the second, counter-rotating, beam effects
- Study the beam dynamic with natural (uncompensated) chromaticity
  - Check if sextupoles are needed at all in the machine for transverse beam stability
- Investigate mitigation measures if required: positive chromaticity, Landau octupoles, effect of α_p (i.e. γ_t)

Thanks for your attention

Appendix: effect of α_p/γ_t

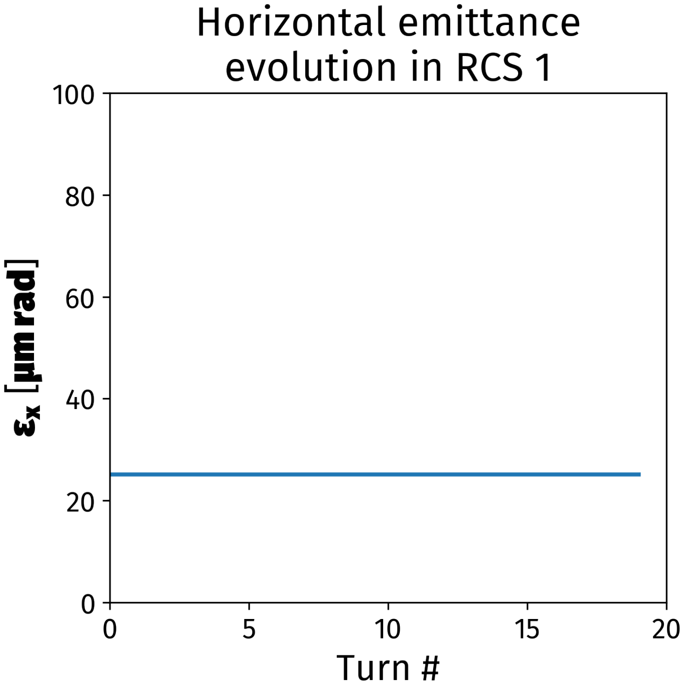
Resonator impedance model

- Use RCS 1 simulation with a single resonator
  - f_{res} = 2.45 GHz
  - R/Q=2.1 MΩ/m * 7 = 14.7 MΩ/m
  - $Q=10^4$
- Change α_p to scan the transition gamma value γ_t
- Nominal case is with γ_t=20
  - In this case, the beam is right on the transverse instability threshold

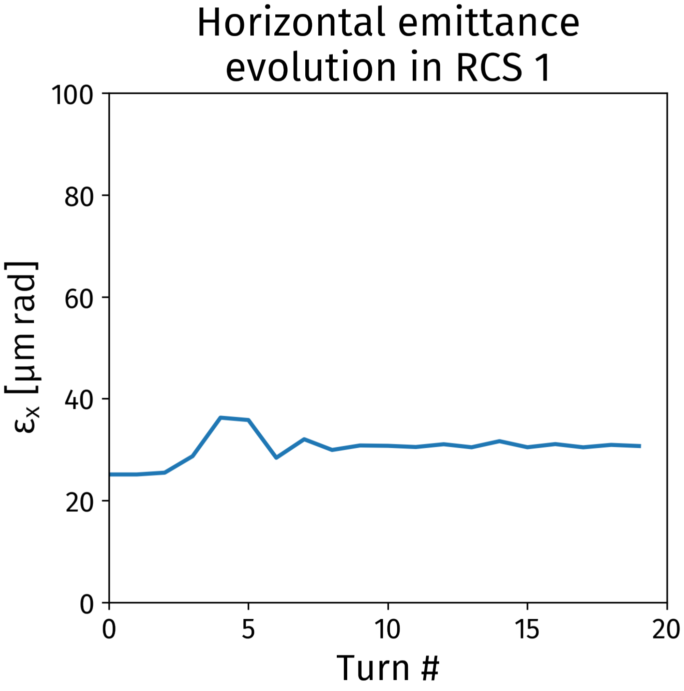
Resonator impedance model

Nominal case

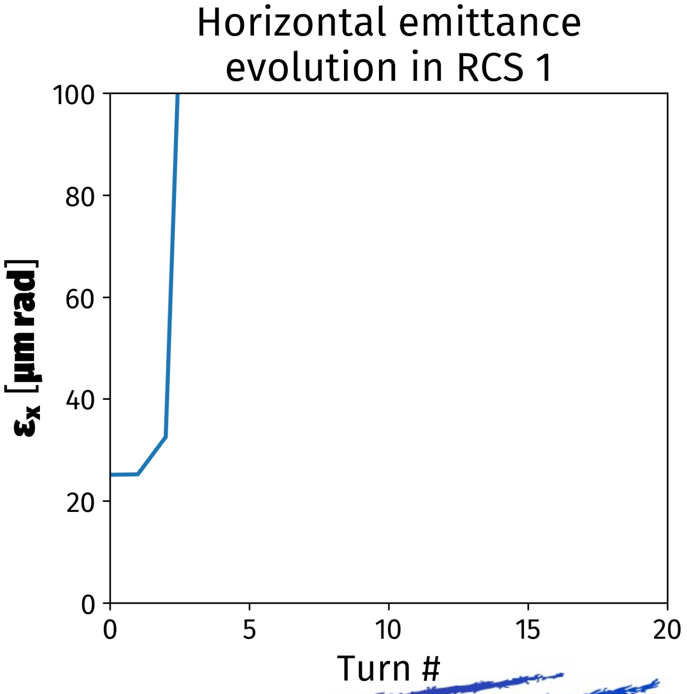
γ_t**=10** (α_p=0.01)



γ_t**=20** (α_p=0.0024)



γ_t**=40** (α_p=0.000625)



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Bunch intensity

Chromaticity, here = 0

Instability threshold criterion from A. Chao:

$$\Upsilon = \frac{\pi N r_0 W_0 c^2}{4\gamma C \omega_\beta \omega_s}\left(1 + i\frac{4\xi\omega_\beta \hat{z}}{\pi c \eta}\right).$$

Synchrotron tune

In our case, the chromaticity is corrected to $\xi = 0$, therefore

$$\Upsilon \propto \frac{N_b}{Q_\beta Q_s}$$

Synchrotron tune is proportional to √η

$\eta = 1/\gamma_t^2 - 1/\gamma^2 \sim 1/\gamma_t^2 \sim \alpha_p$

Synchrotron tune is proportional to $1/\gamma_t$

$$Q_s \propto \frac{1}{\gamma_t}$$

$$\Upsilon \propto \frac{N_b \gamma_t}{Q_\beta}$$

The instability occurs when $\Upsilon = 2$, therefore the instability criteria becomes

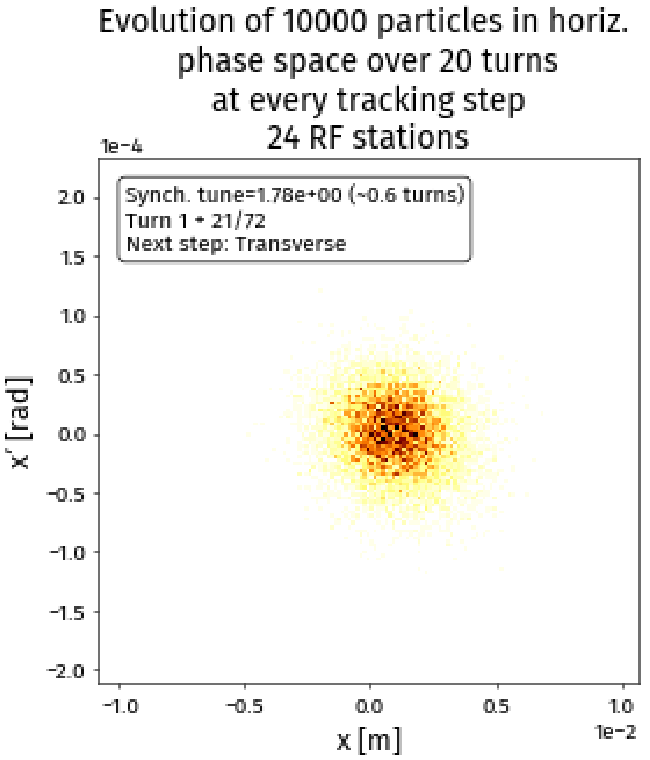
$$N_b \propto \frac{2Q_\beta}{\gamma_t} \quad \text{or} \quad \gamma_t \propto \frac{2Q_\beta}{N_b}$$

Appendix: transverse phase space during an instability

Unstable case (factor 8 on shunt impedance)

Transverse phase space during turn 2, at RF station #7

24 RF stations * 3 tracking elements = 72 tracking steps per turn



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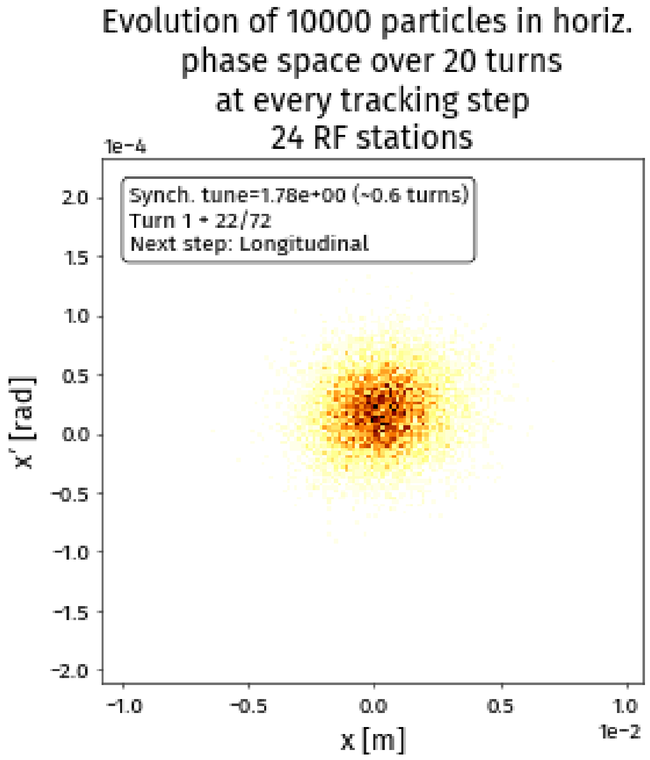

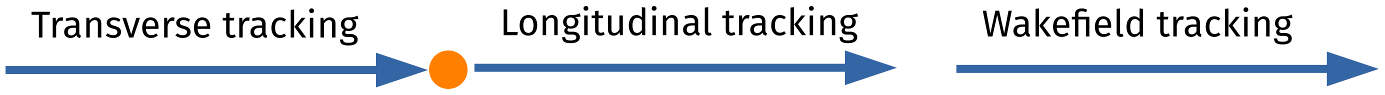

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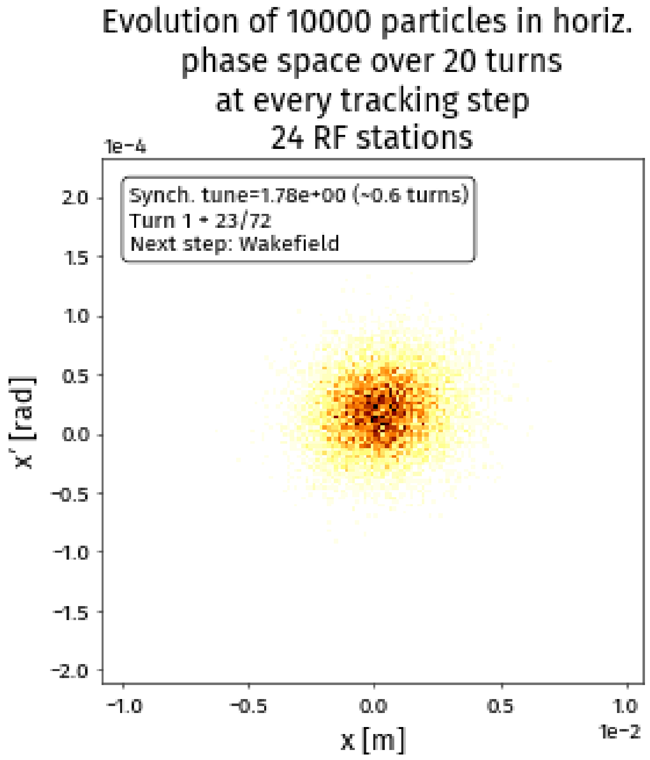
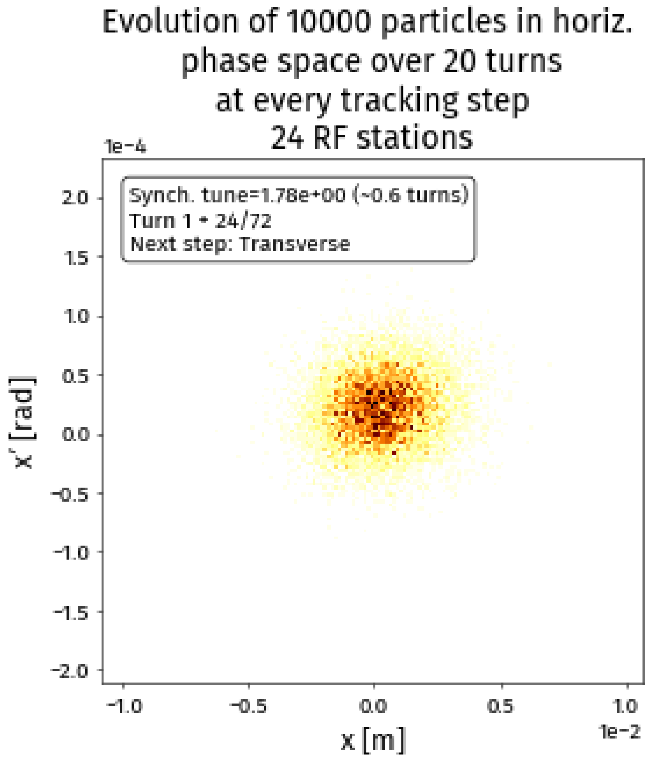


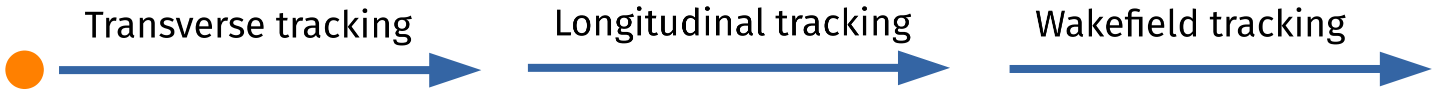
Transverse tracking → Longitudinal tracking → ● Wakefield tracking →

Unstable case (factor 8 on shunt impedance)

Transverse phase space during turn 2, at RF station #7

24 RF stations * 3 tracking elements = 72 tracking steps per turn





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Transverse phase space during turn 2, at RF station #7

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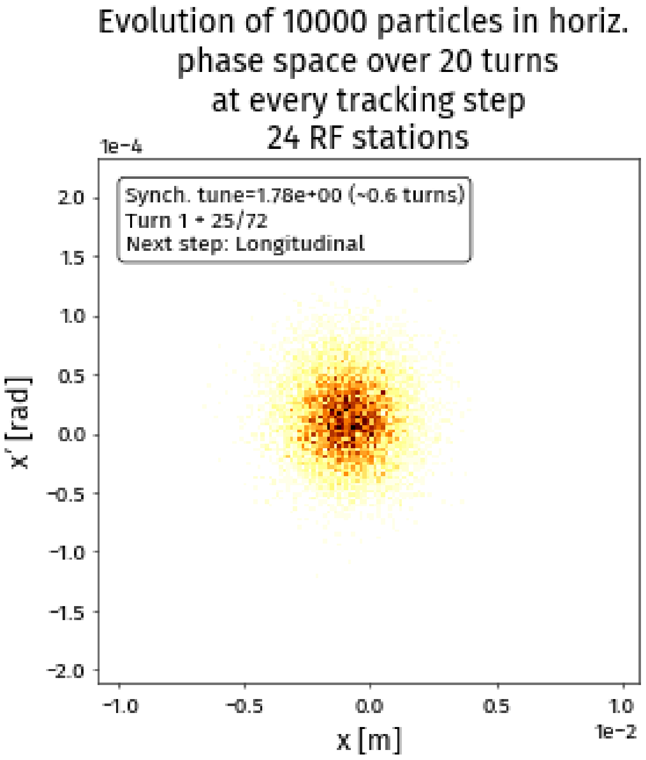
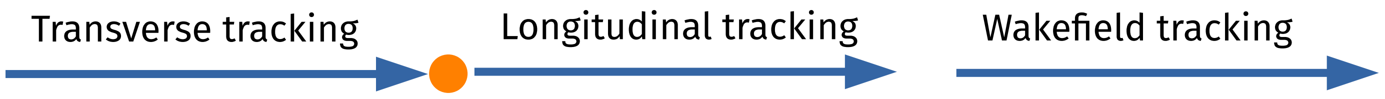
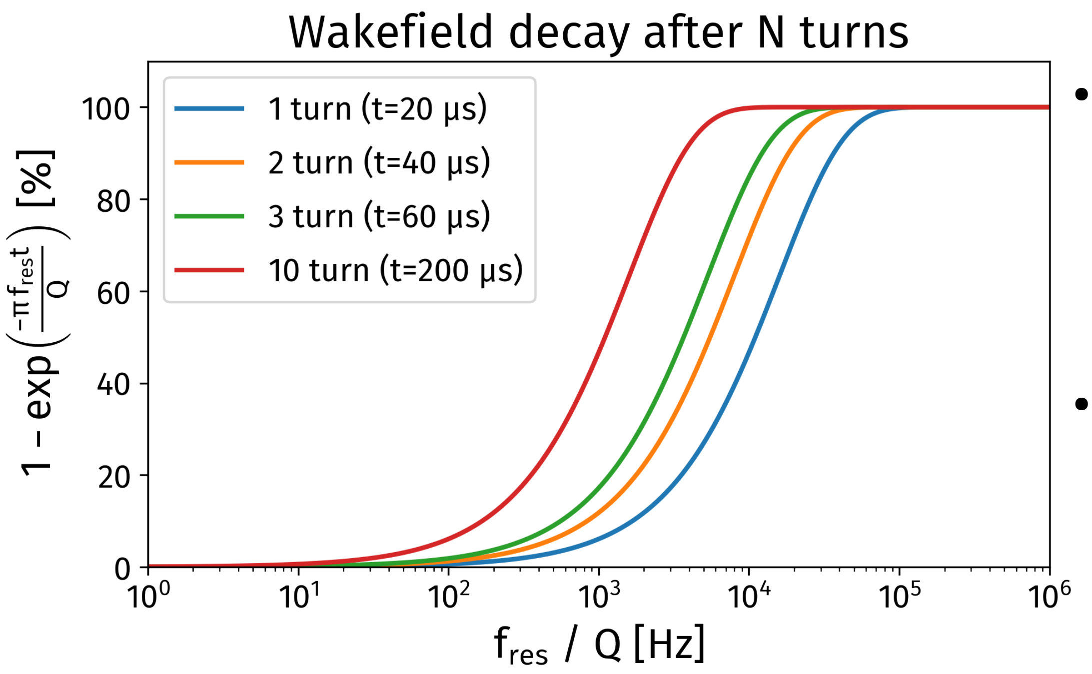




Appendix: resonator impedance and wakefield

Resonator impedance model

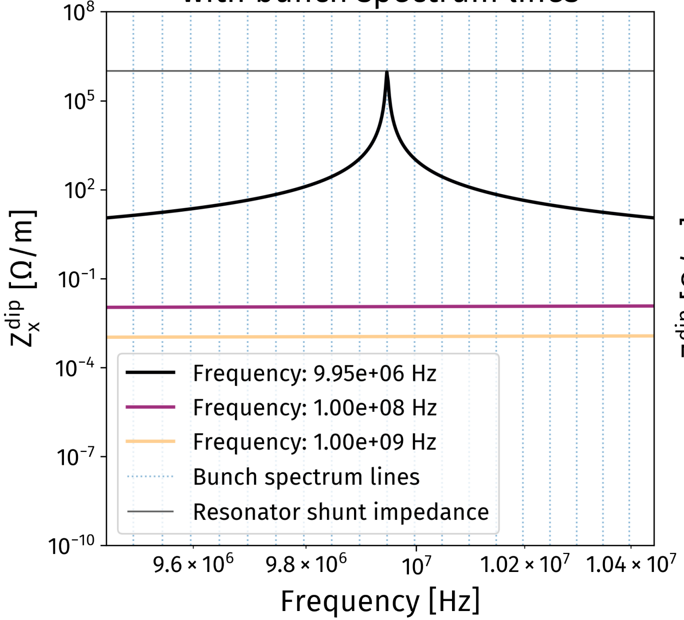


- We can plot the exponential term versus f_{res}/Q for a given time t (number of turns)

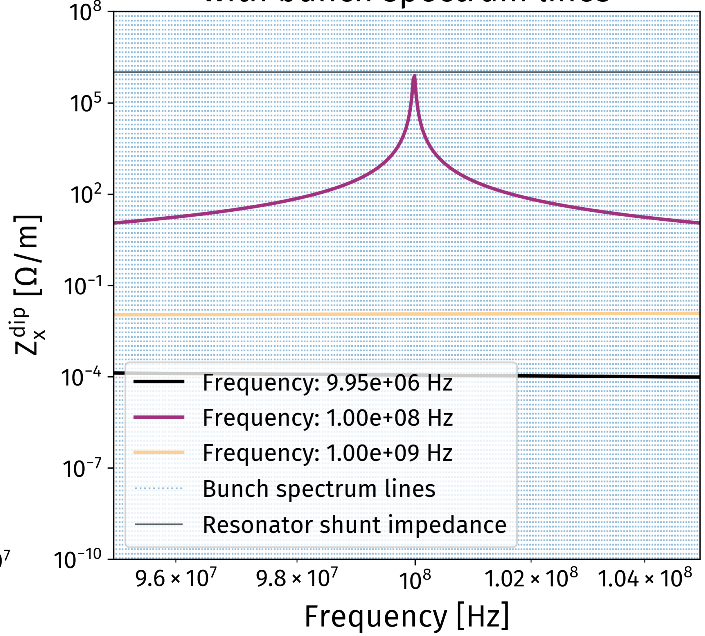
$$\exp\left(-\frac{2\pi f_{res}}{2Q} t\right)$$

- Shows by how much the wake decreased after N turns for a given (f_{res}, Q)

Resonator impedance model



f_{res} = 10 MHz
R_s = 1 MΩ/m
Q = 3000



f_{res} = 100 MHz
R_s = 1 MΩ/m
Q = 3000

- Resonator frequency is chosen to fall on a bunch spectrum line
- At high frequency (right plot), the resonance overlaps with many spectrum line
- Assumptions:
  - injection energy revolution frequency f_0
  - Q_x / Q_y = 0.26 / 0.26

Appendix: impedance and stability simulation parameters

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Stability simulation parameters

Machine parameters

	Unit	Value
Circumference	m	5990
Beam momentum at injection	GeV/c	63.1
Momentum increase per turn	GeV/c	14.212
Rev. frequency	kHz	50
RF frequency	MHz	1300
Harmonic number		25957
RF voltage	MV	20 100
α_p		2.4e-3
Avg. beta x/y	m	50 / 50
Chromaticity Q'_x/Q'_y		0 / 0
Detuning from octupoles x/y	m^{-1}	0 / 0

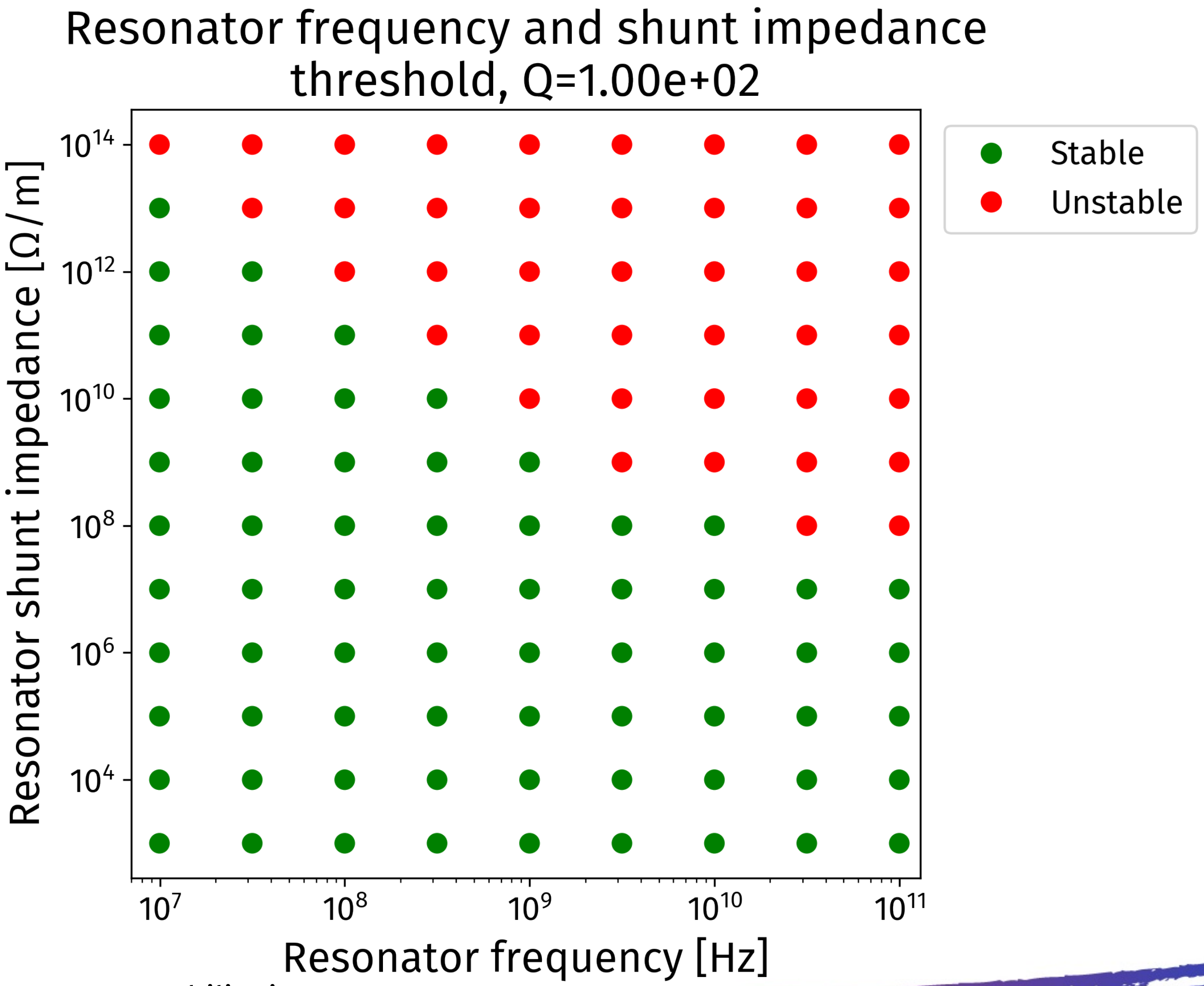
	Unit	Value
Synchrotron tune Q_s at injection		1.52
Synchrotron period	turns	0.66
Bunch length 1σ	**mm**	**25**
Bunch intensity	**Particles per bunch**	**2.6e12**
ε_x / ε_y	**µm rad**	**25**
# of macropaticles		5000

Scanned parameters

	Value
Resonator shunt impedance R_s	1 kΩ/m to 100 TΩ/m
Resonance frequency f_{res}	10 MHz to 1 THz
Quality factor Q	100, 300, 1000, 3000, 10 000, 30 000
Wakefield turns	1, 2, 3, 10, 50

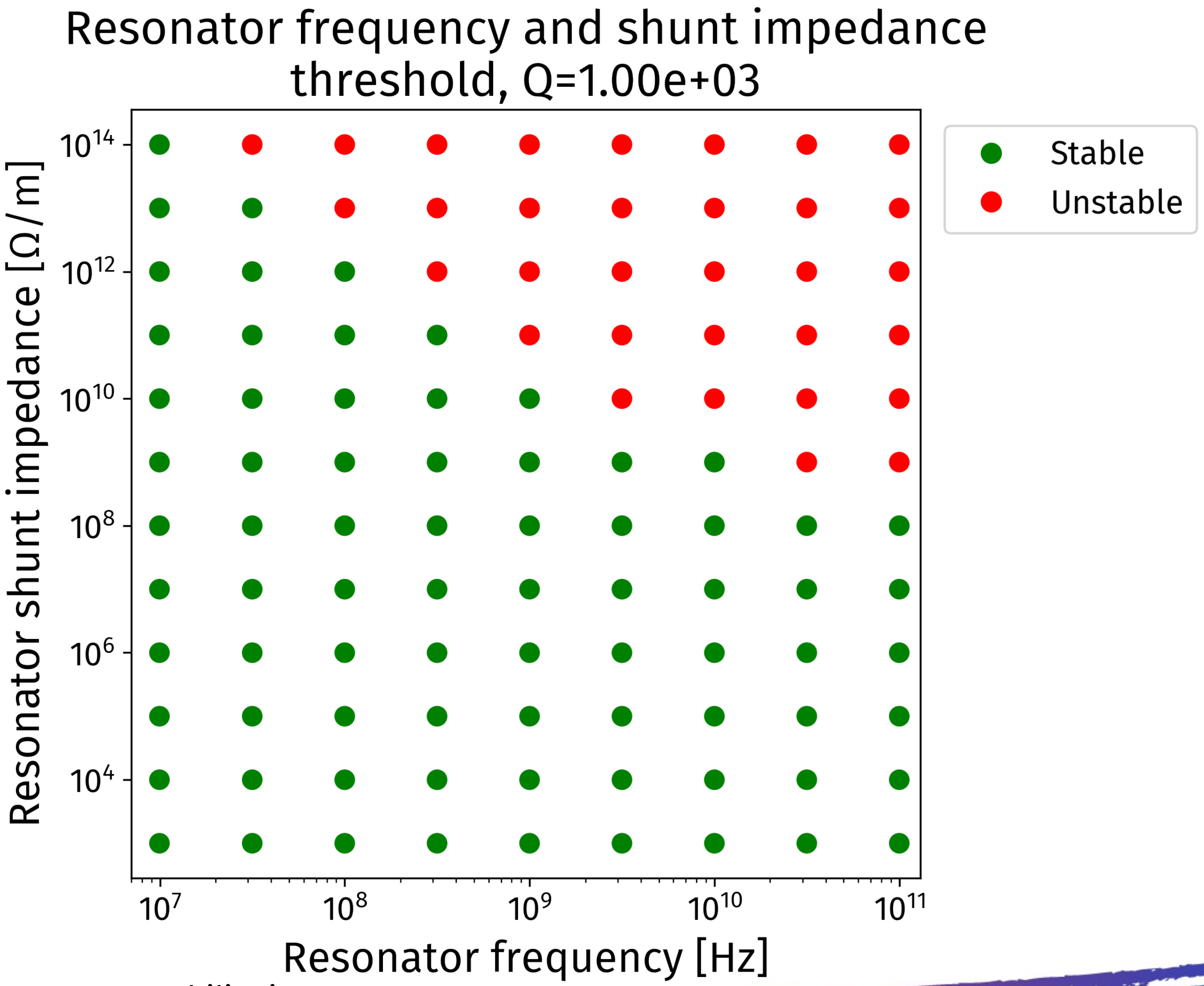
Appendix: stability results with single-turn wakefield

Q = 100



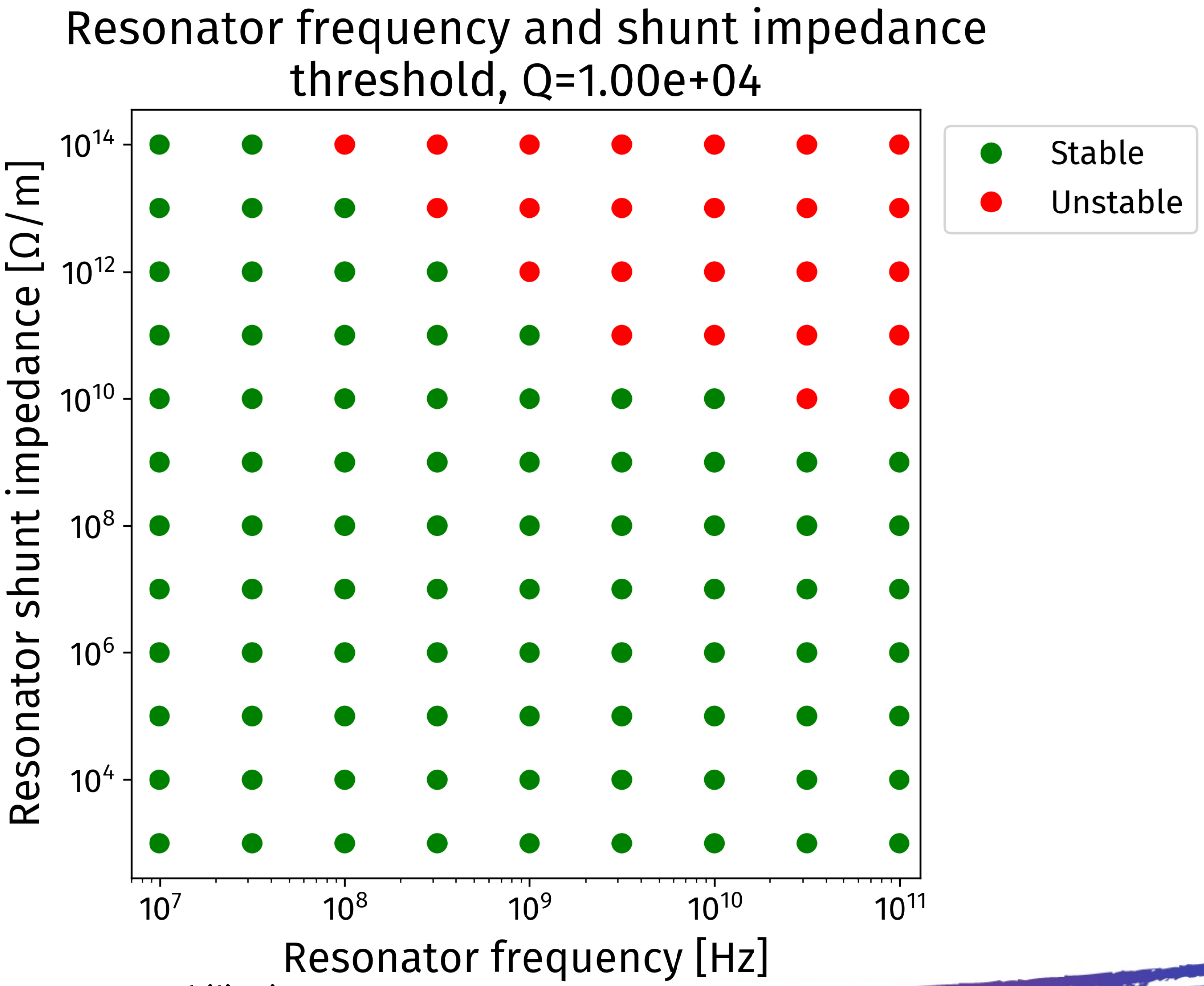
Resonator frequency and shunt impedance threshold, Q=1.00e+02
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
Stable
Unstable
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}

Q = 1000

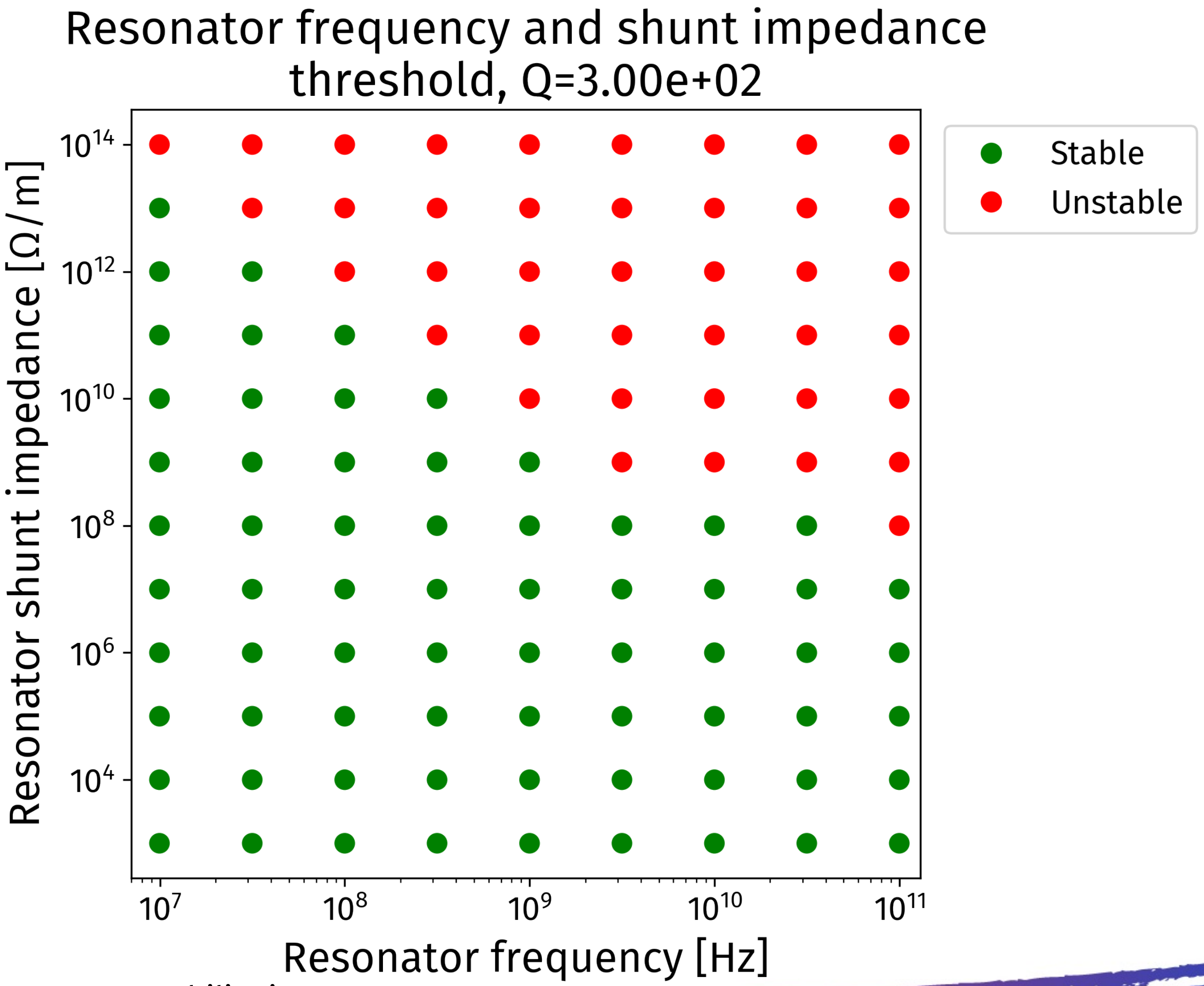


Resonator frequency and shunt impedance threshold, Q=1.00e+03
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
10^{4}
10^{6}
10^{8}
10^{10}
10^{12}
10^{14}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
Stable
Unstable

Q = 10000

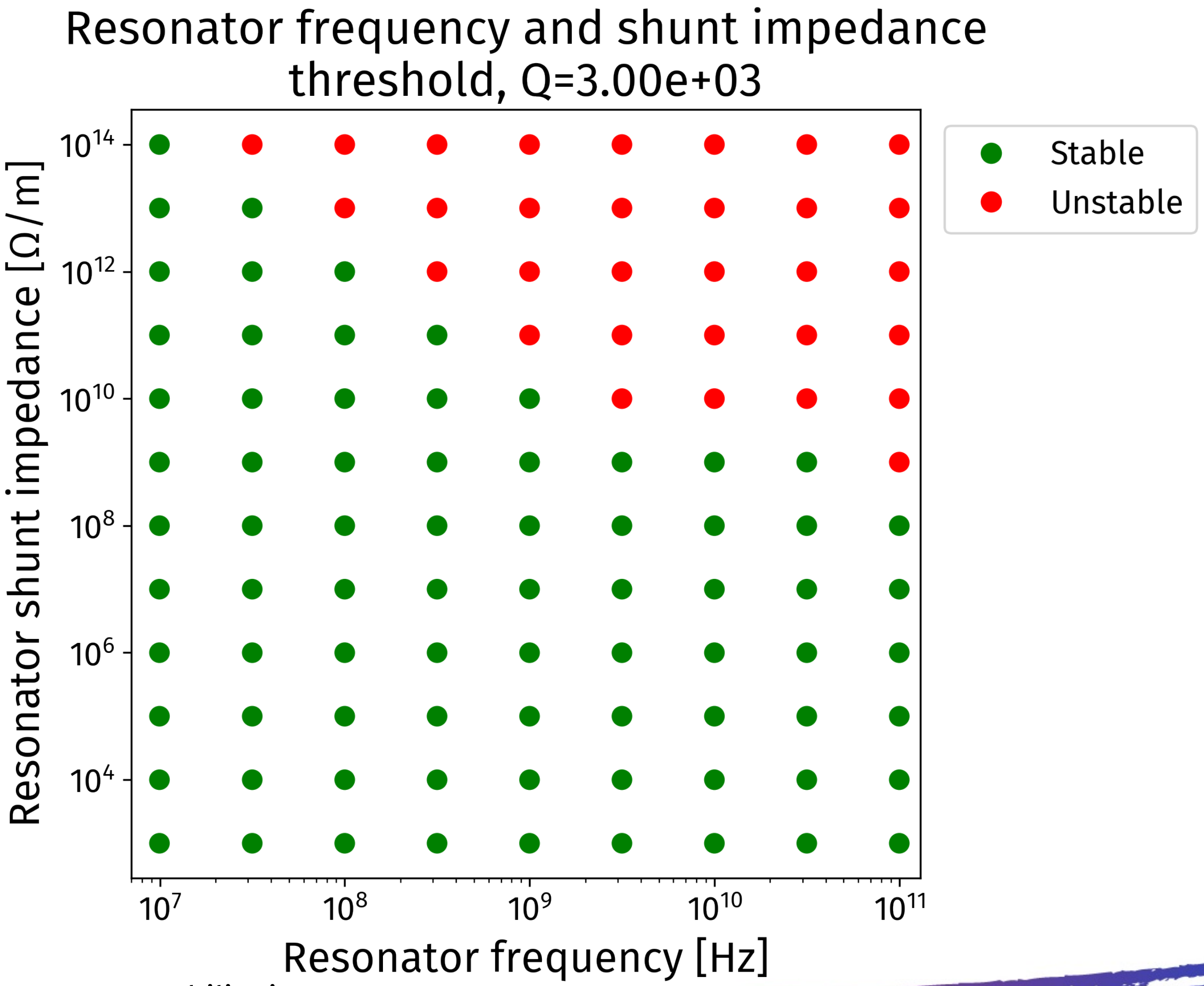

Resonator frequency and shunt impedance threshold, Q=1.00e+04
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
Stable
Unstable
10^14
10^12
10^10
10^8
10^6
10^4
10^7
10^8
10^9
10^10
10^11

Q = 300



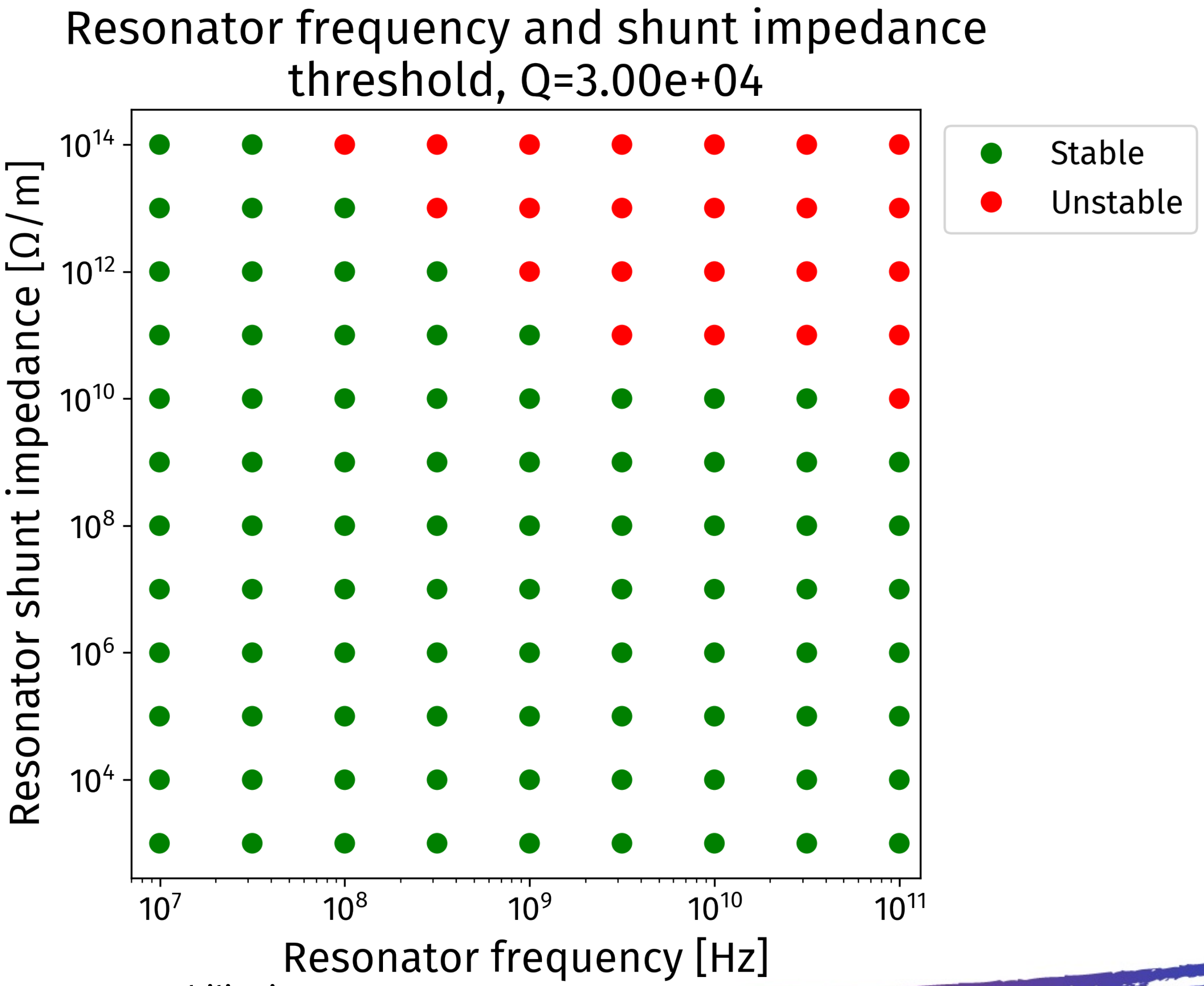
Resonator frequency and shunt impedance threshold, Q=3.00e+02
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
Stable
Unstable

Q = 3000



Resonator frequency and shunt impedance threshold, Q=3.00e+03
Resonator shunt impedance [Ω/m]
10^14
10^12
10^10
10^8
10^6
10^4
10^7
10^8
10^9
10^10
10^11
Resonator frequency [Hz]
Stable
Unstable

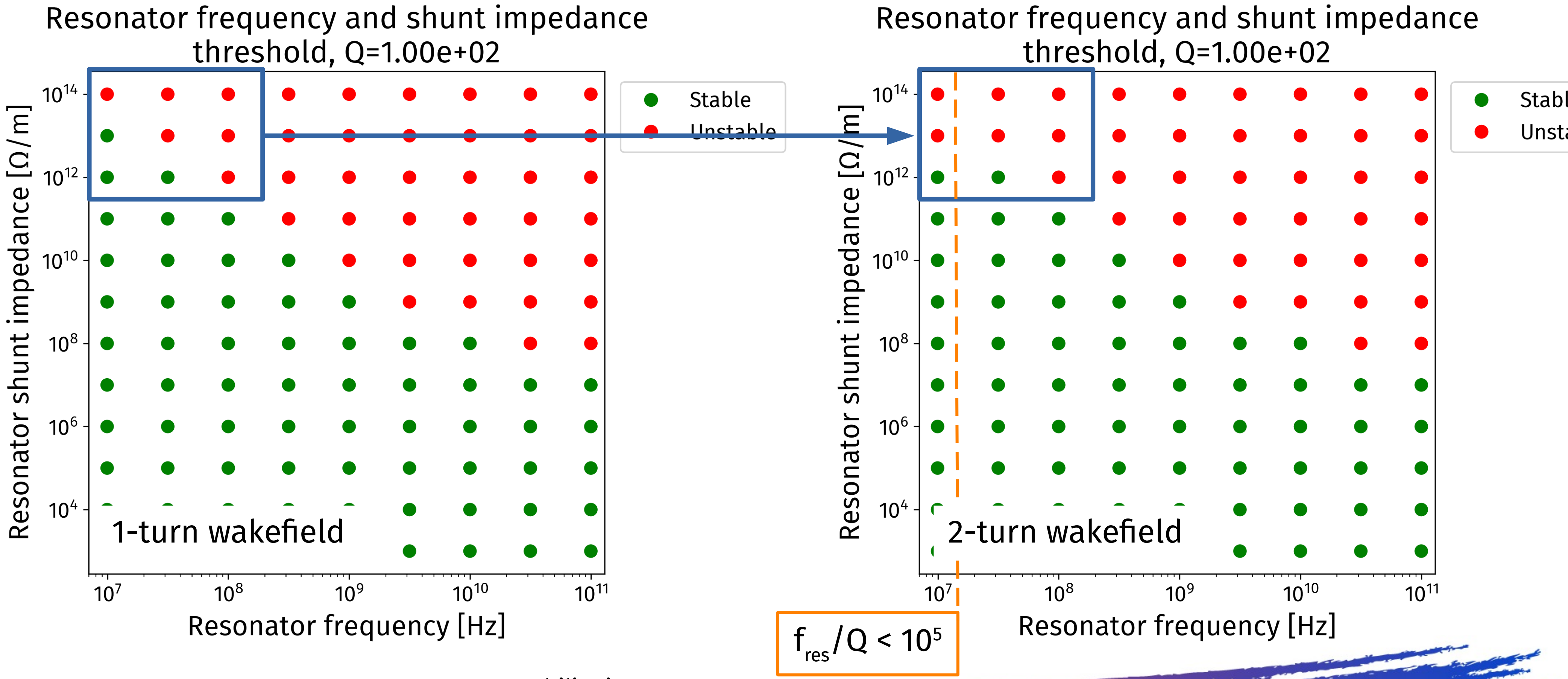
Q = 30000



Resonator frequency and shunt impedance
threshold, Q=3.00e+04
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
Stable
Unstable
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}

Appendix: stability results with multi-turn wakefield

Q = 100, 2-turn wakefield



Q = 100, 10-turn wakefield

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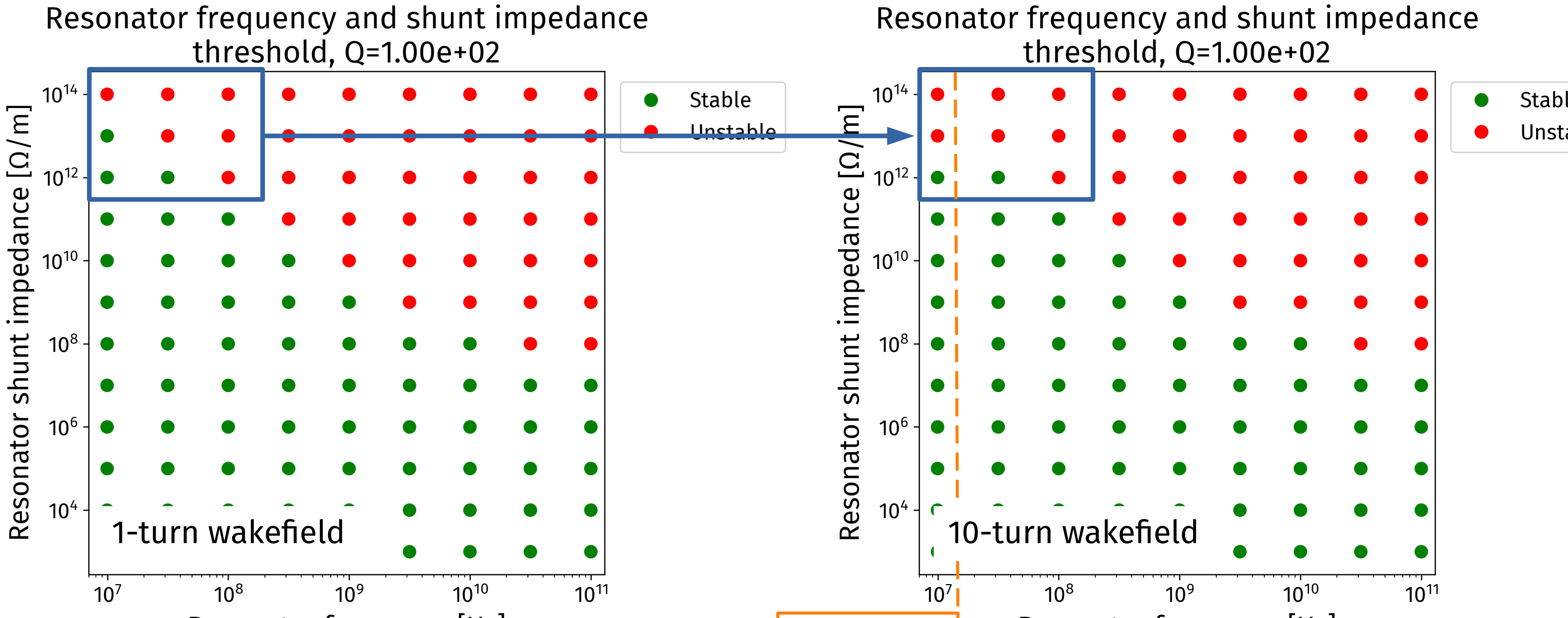
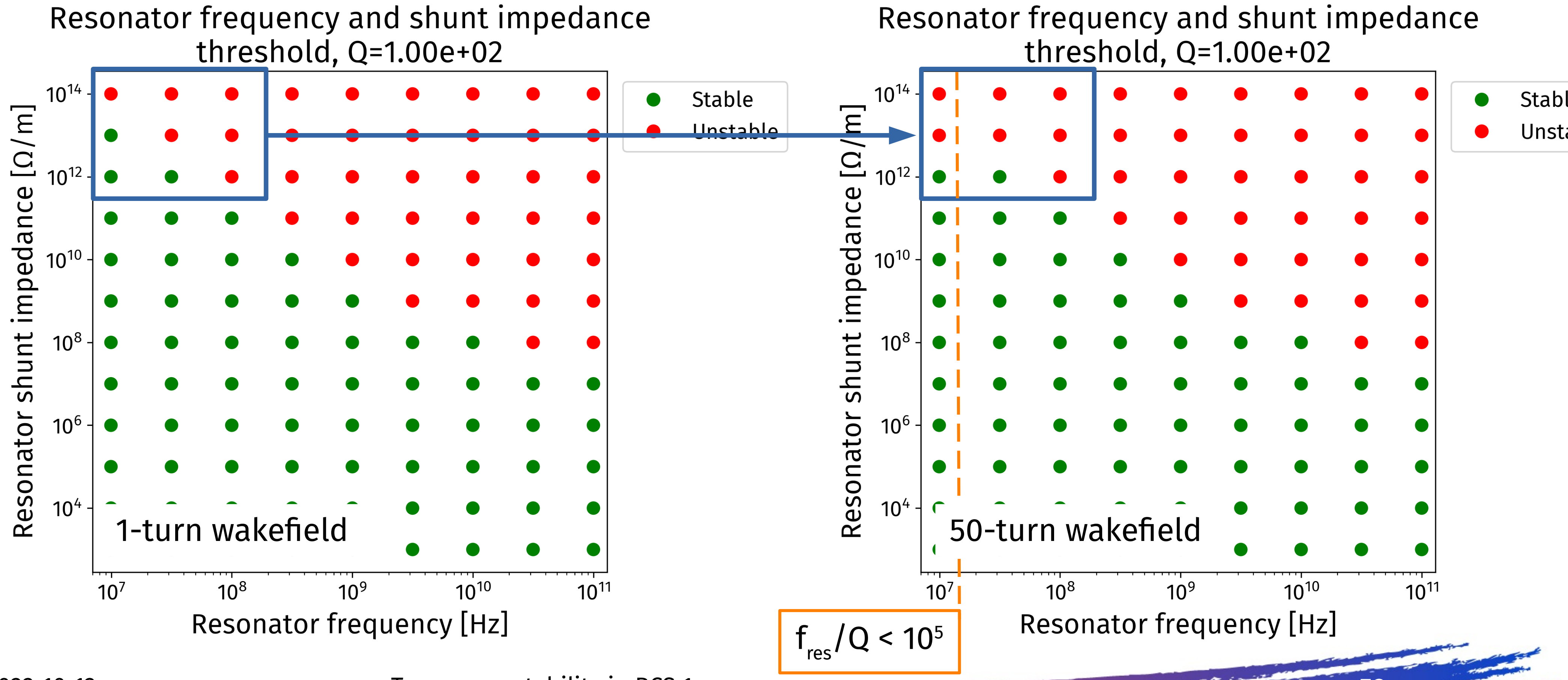
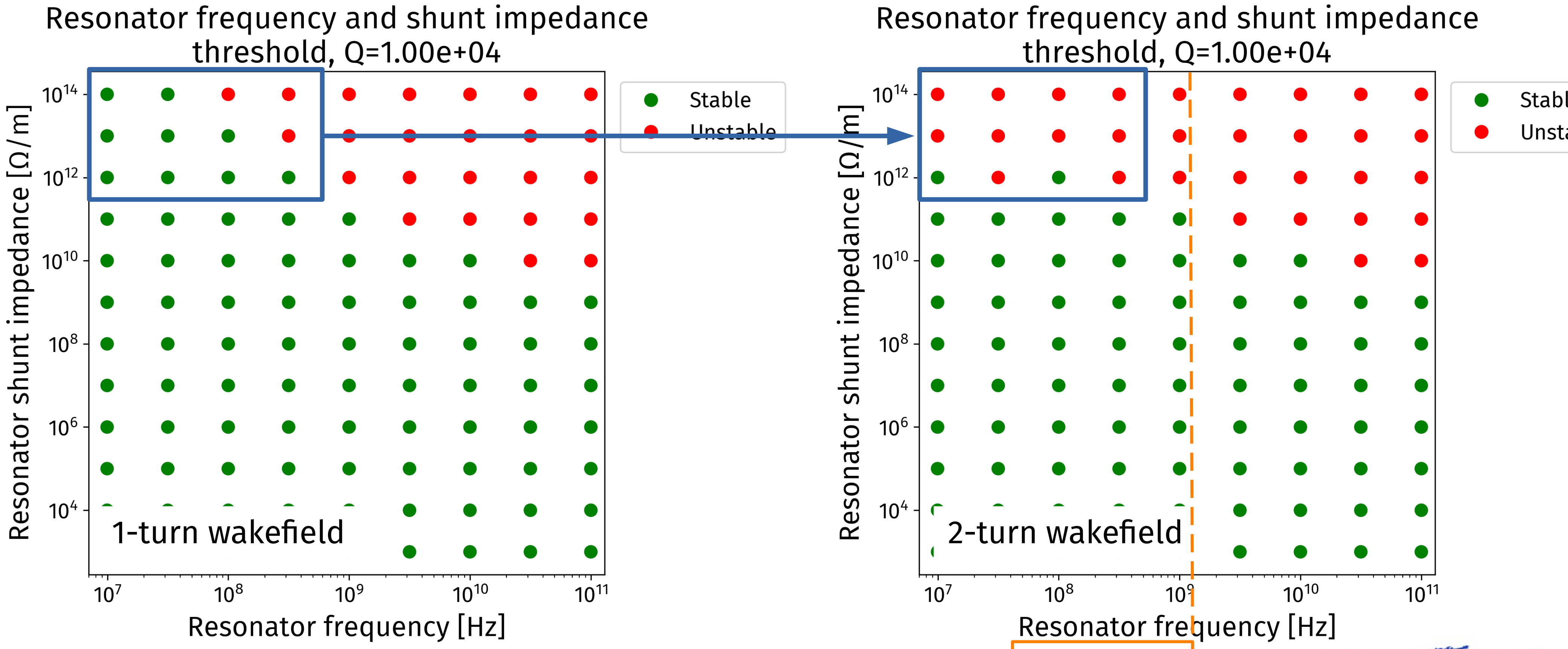


$f_{res}/Q < 10^5$

Q = 100, 50-turn wakefield

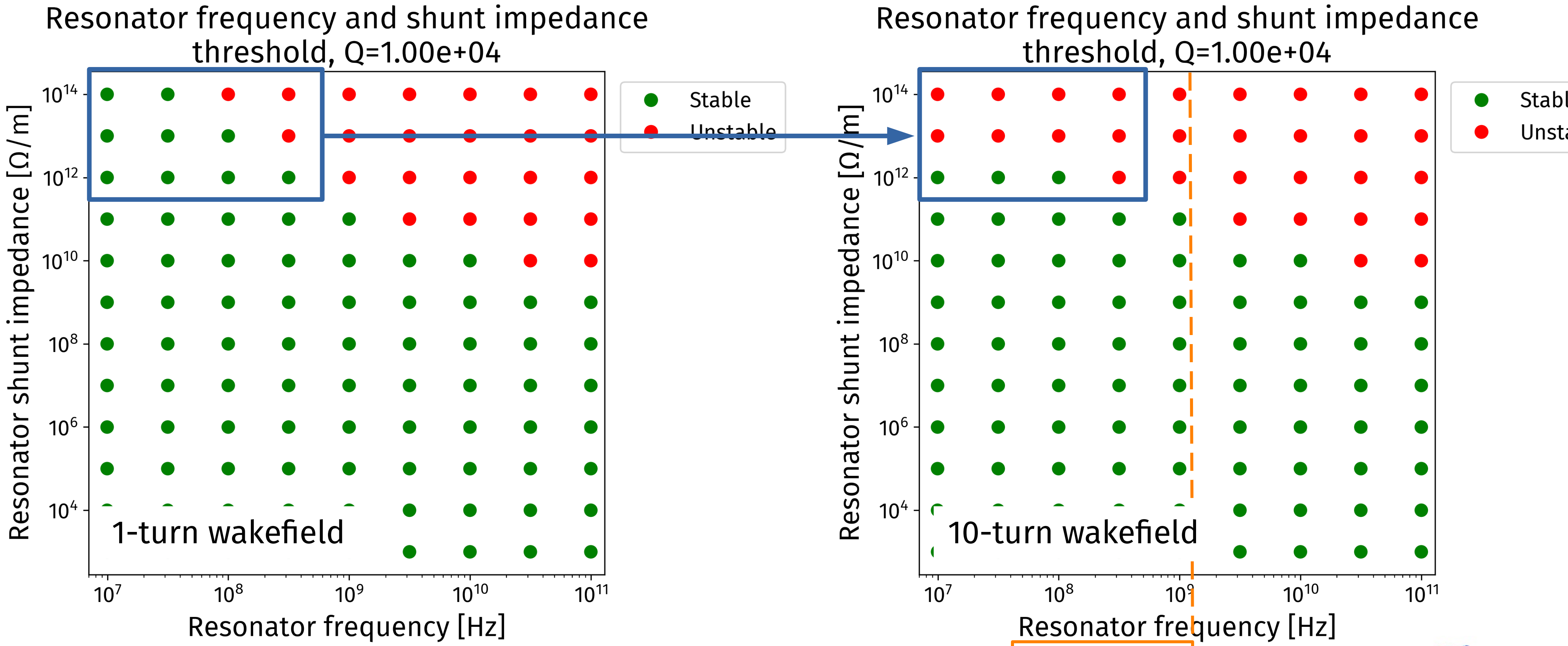


Q = 10000, 2-turn wakefield



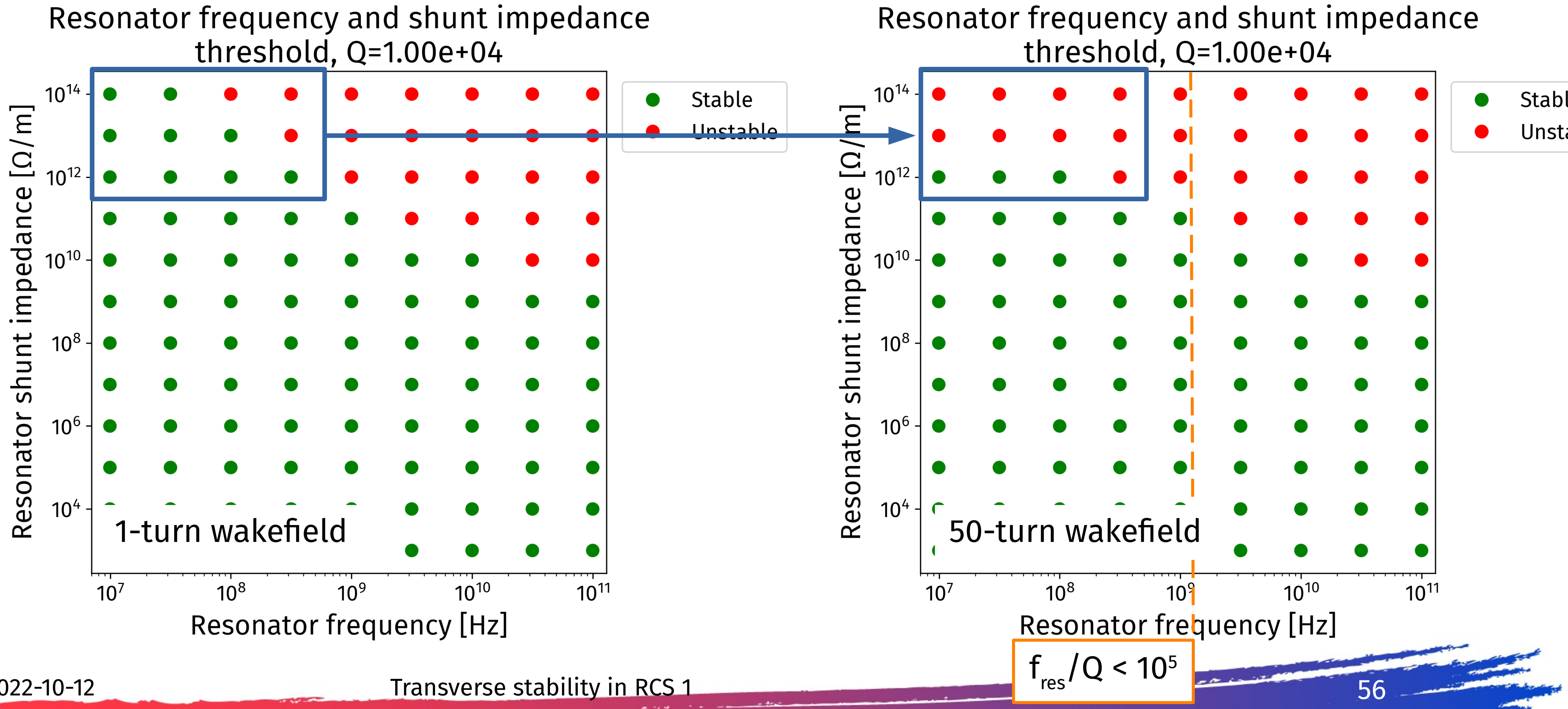
$f_{res}/Q < 10^5$

Q = 10000, 10-turn wakefield

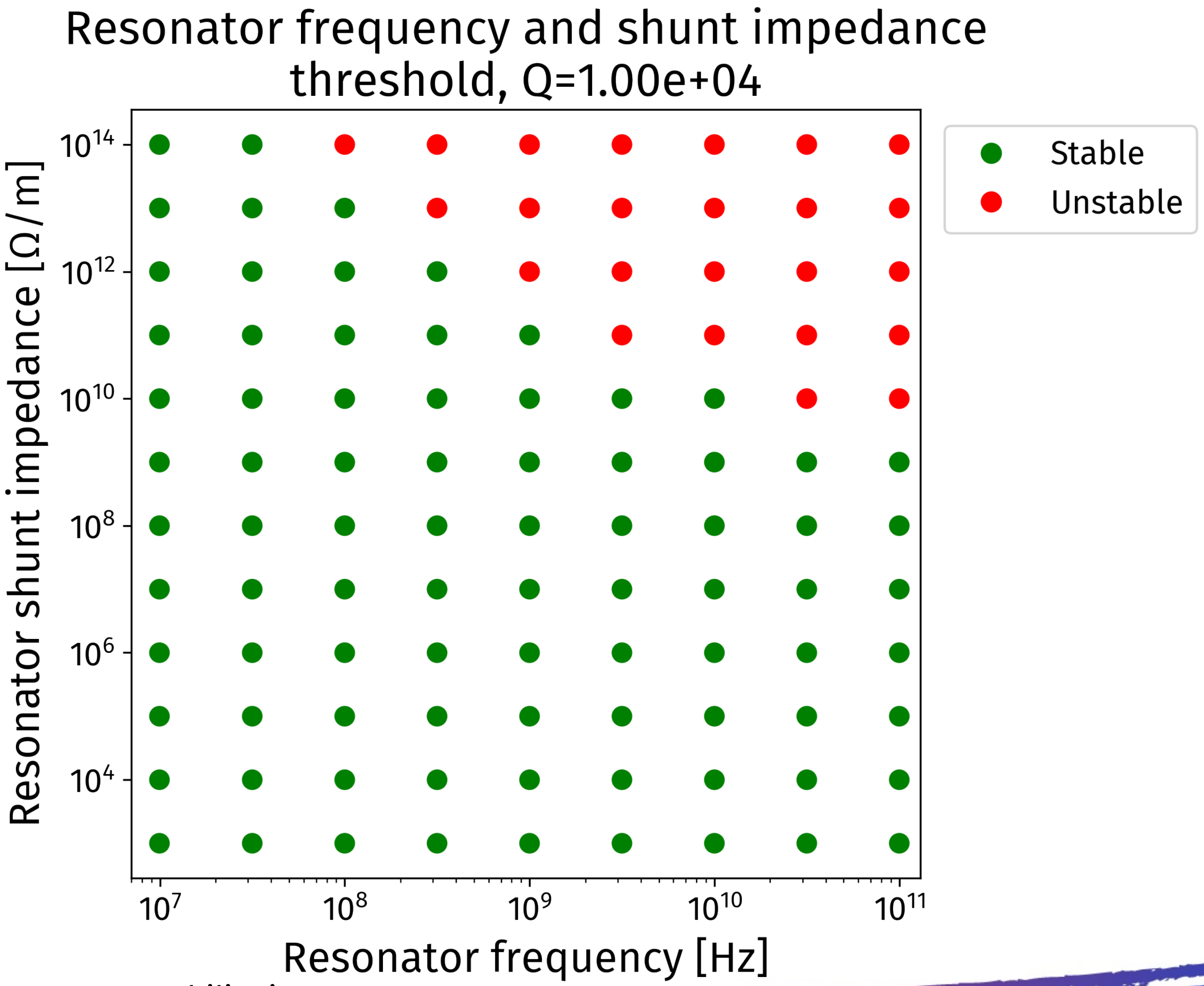


$f_{res}/Q < 10^5$

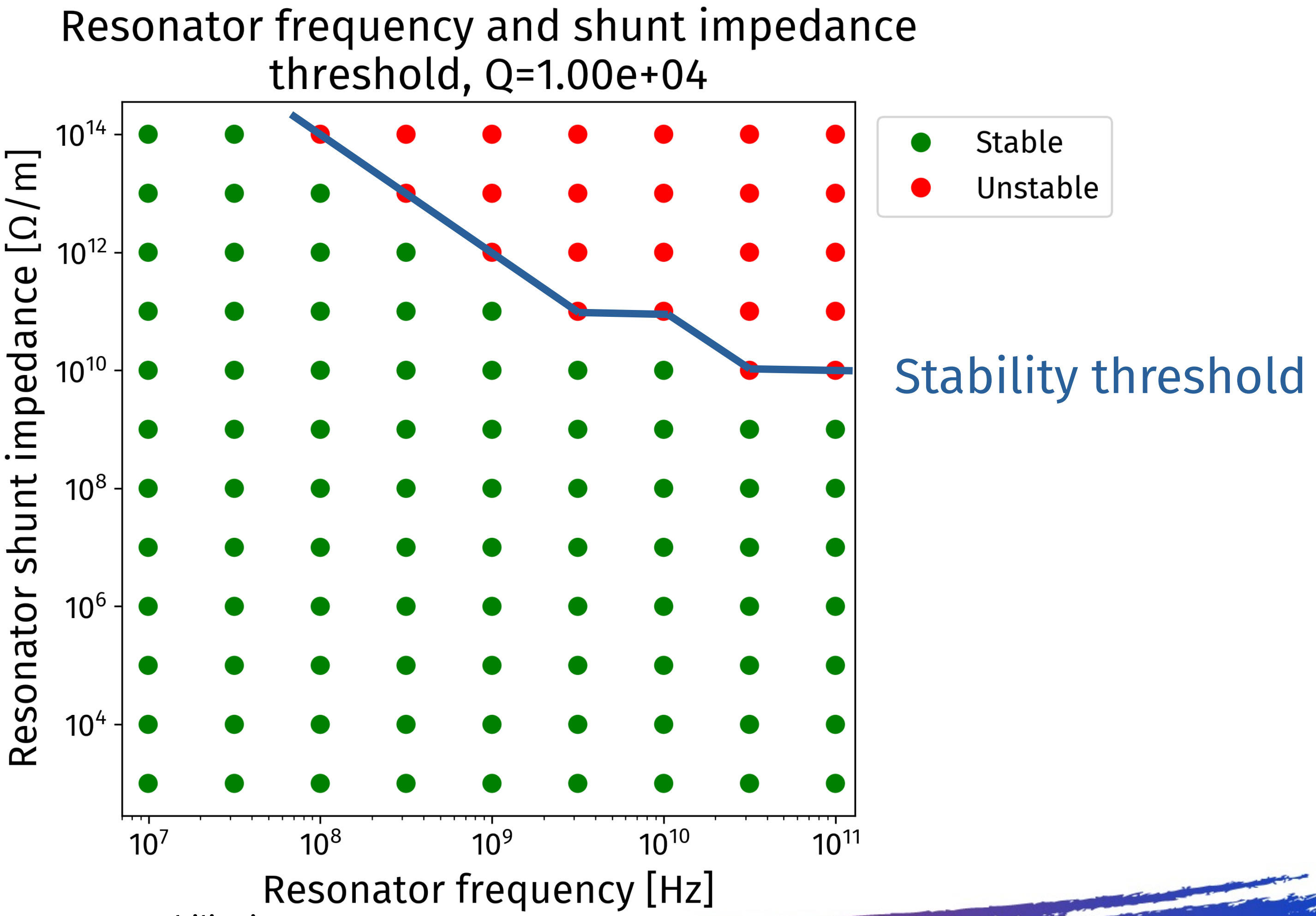
Q = 10000, 50-turn wakefield



Q = 10000


Resonator frequency and shunt impedance
threshold, Q=1.00e+04
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
Stable
Unstable

Q = 10000

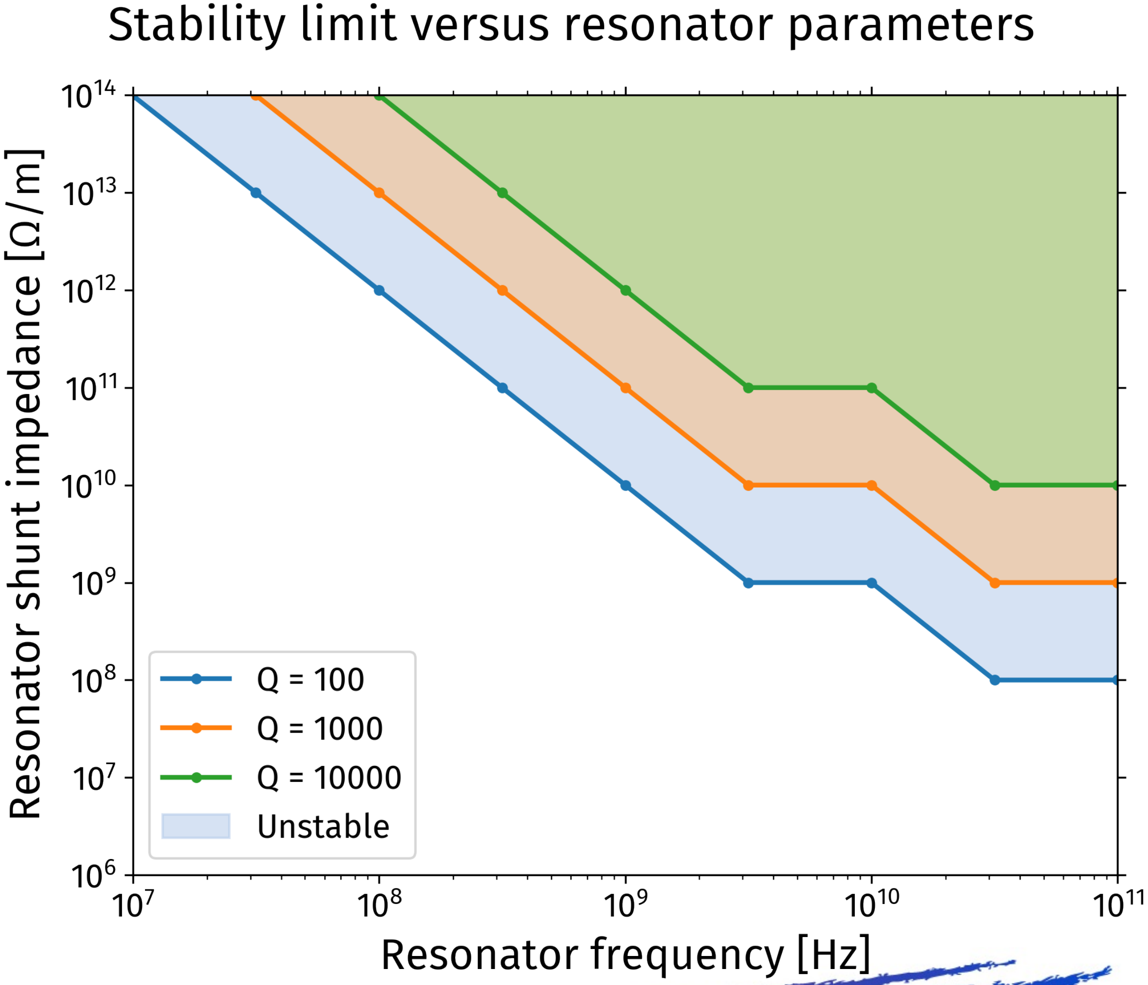


Resonator frequency and shunt impedance
threshold, Q=1.00e+04
Resonator shunt impedance [Ω/m]
Resonator frequency [Hz]
10^14
10^12
10^10
10^8
10^6
10^4
10^7
10^8
10^9
10^10
10^11
Stable
Unstable
Stability threshold

Appendix: summary plots

Summary plot for Q=100/1000/10000

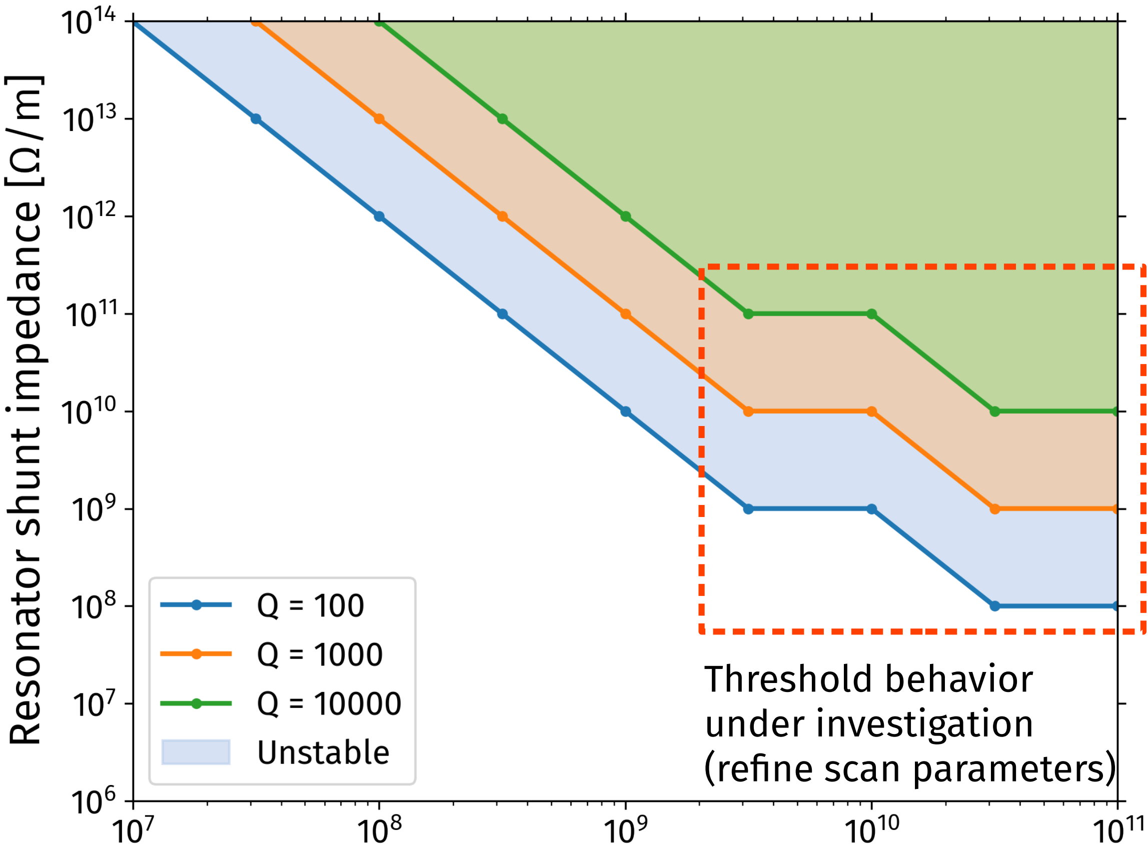
- Group the results for the different Q factor in one plot
- Line shows the first unstable simulation for a given Q factor, versus resonator shunt impedance and frequency
- Shaded area corresponds to the parameter space where the beam is unstable



Summary plot for Q=100/1000/10000

- Group the results for the different Q factor in one plot
- Line shows the first unstable simulation for a given Q factor, versus resonator shunt impedance and frequency
- Shaded area corresponds to the parameter space where the beam is unstable

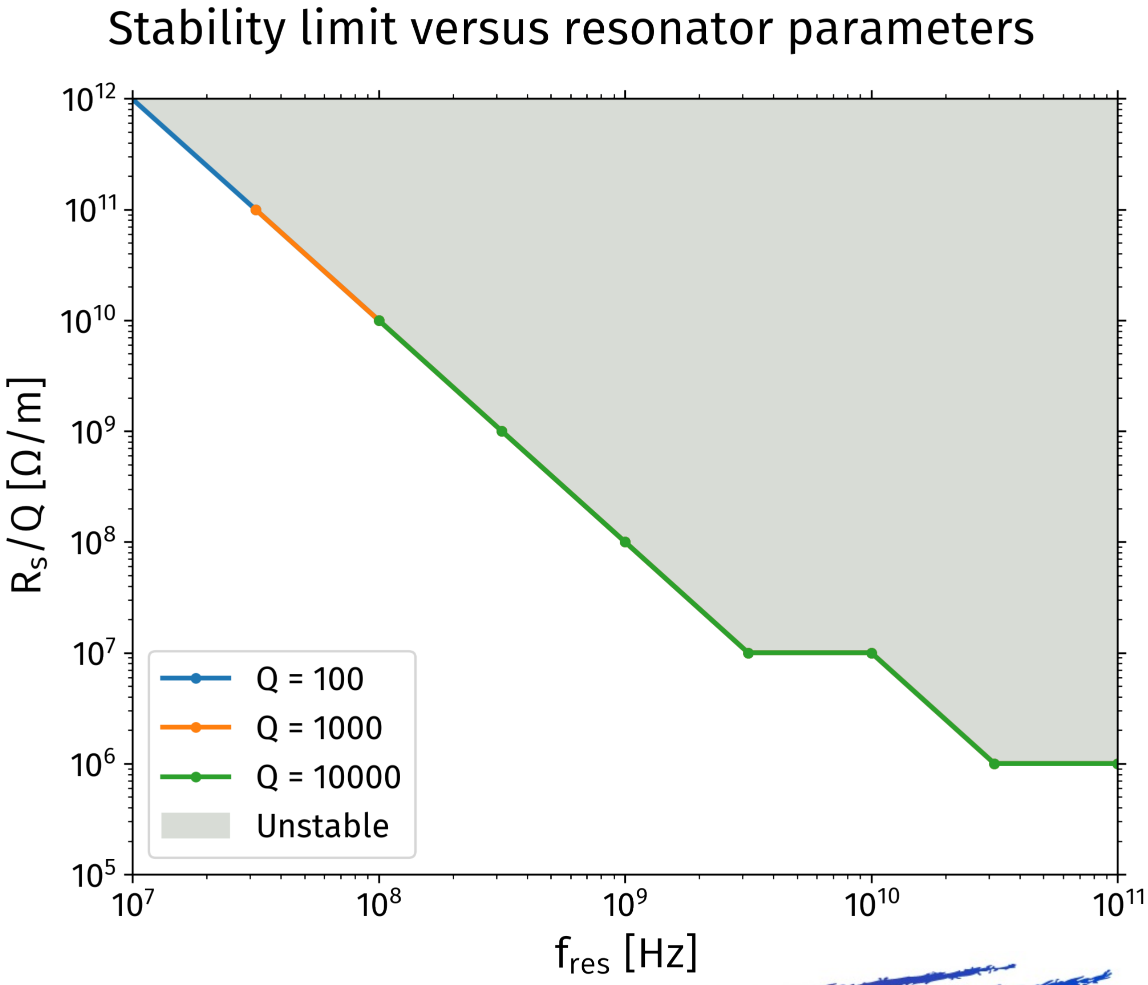
Stability limit versus resonator parameters



Summary plot for Q=100/1000/10000

- Now the **shunt impedance limit is divided by the resonator quality factor Q**
  - Provides a **limit on R_s/Q** for the whole ring

- This limit should be divided by the number of cavities to check if their design R_s/Q is within the limit
  - Example: at 100 MHz, R_s/Q = 10 GΩ/m. With 1000 cavities, R_s/Q limit is 10 MΩ/m per cavity



Stability summary plots, multi-turn wakefield

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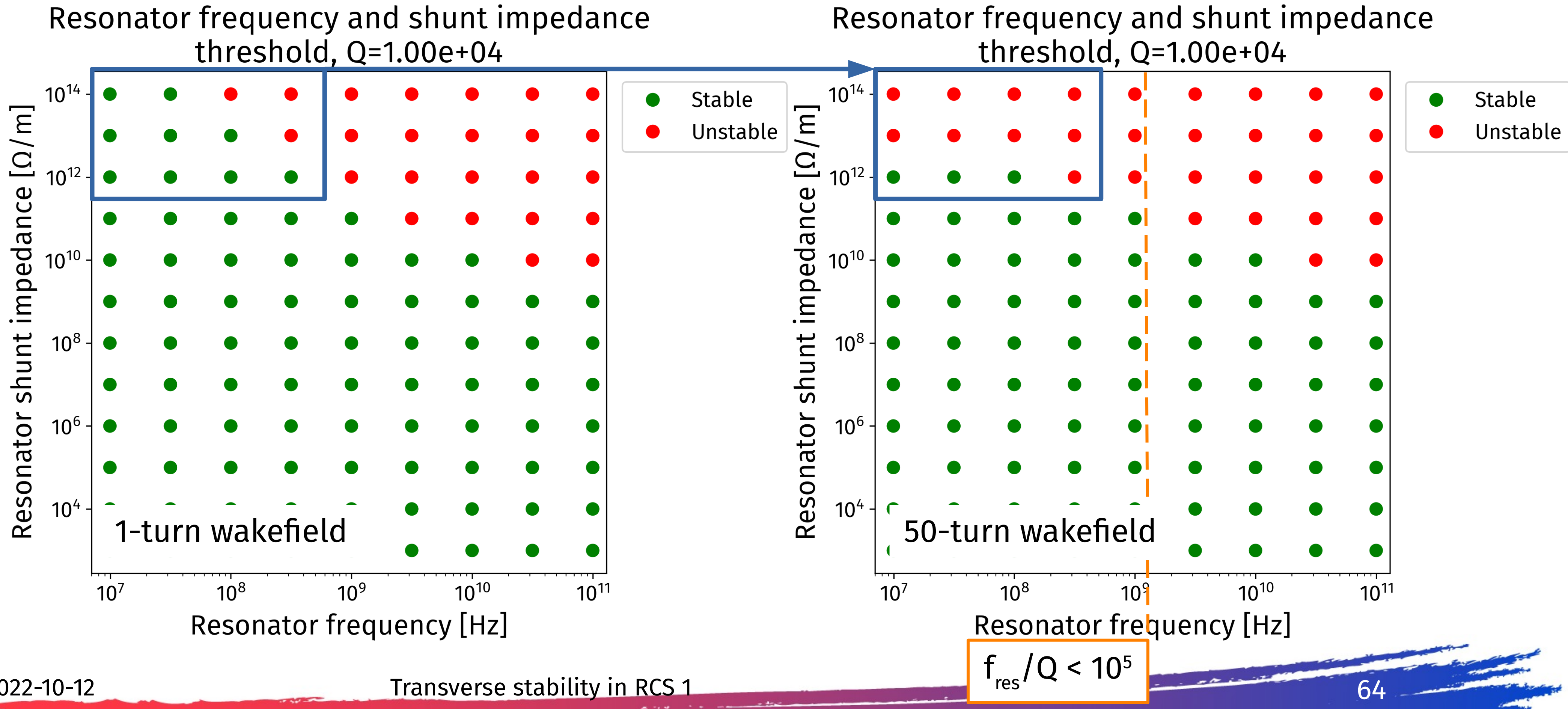
- Focus on one high-Q case (Q=10 000)
  - Results for different Q factor are reported in appendix
- Compare the 50-turn wakefield results to the single-turn wakefield case
  - Results for different number of wakefield turns and Q factor are reported in appendix
- In the plots, highlight the resonator frequency for which $f_{res}/Q < 10^5$
  - For Q=100, if $f_{res} < 10^7$ Hz, multi-turn wake should affect stability
  - For Q=10000, if $f_{res} < 10^9$ Hz, multi-turn wake should affect stability

Q = 10000, 50-turn wakefield



Summary of transverse stability simulation in the RCS 1

- Simulation including longitudinal map (**32 RF stations**) + transverse map + **transverse single-turn or multi-turn wakefield**
- Tracking over 100 turns, 5000 macroparticles with PyHEADTAIL
- In single-turn wakefield regime, the stability criterion depend on R_s /Q and f_{res}
- Multi-turn wakefield is required when $\mathbf{f_{res}/Q < 10^5}$ for the RCS1 case
- Effect is mostly visible for high-Q resonator (Q > 10000)
  - Below the $f_{res}/Q < 10^5$ criterion, simulations with high R_s become unstable
  - Above this criteron, we recover the single turn behavior studied previously

Summary of transverse stability simulation in the RCS 1, with multi-turn wakefield

- Summarize on one plot the single-turn and multi-turn wakefield stability limits
- The single-turn limit (black line) depends on the resonator frequency and the R_s/Q
- The multi-turn limit (color lines) depends on the resonator shunt impedance R_s

Stability limit versus resonator parameters