# Neutrino Radiation for a realistic Collider

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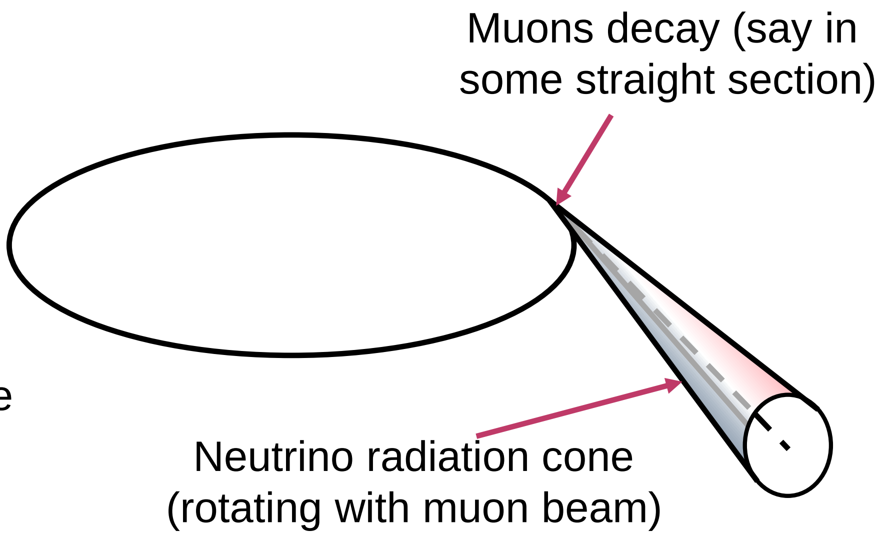
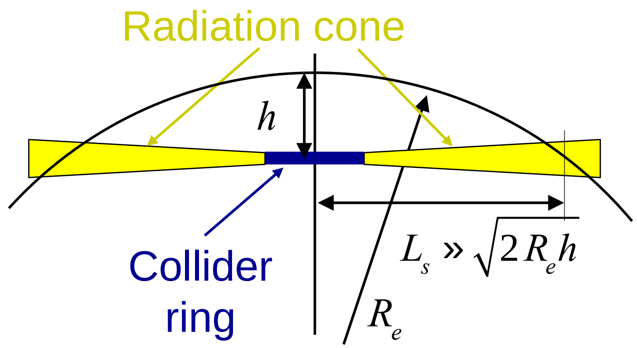
# Introduction
## Neutrino Radiation Issue

- Radiation due to neutrino beam reaching the earth surface
  - Narrow radiation “cone” for a short piece of the machine
  - Showers from neutrinos interacting close to earth surface generate dose seen at surface
  - Matter in front (“shielding”) does not help but makes situation even worse



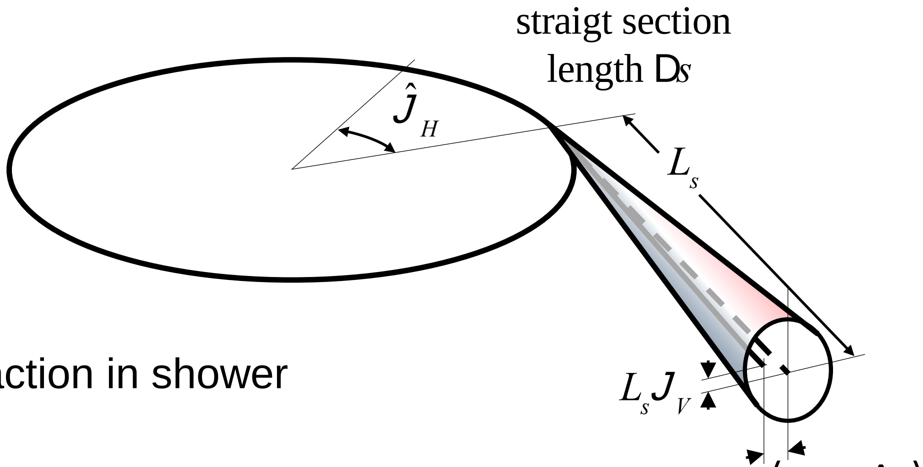

- Strong increase of maximum dose with muon energy
  - Cross sections about proportional to energy
  - Typical energy per interaction of neutrino with matter proportional to muon energy
  - Opening of radiation cone inversely proportional to muon energy



# Analytical estimates
## absorbed dose per decay

Following approach by B.J. King, (see, e.g., arXiv:physics/9908017)

- Consider say decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$
- In muon rest system: anisotropic neutrino distribution with known energy distribution
- Lorentz boost to obtain neutrino direction and energies in lab system (Lorentz factor $\gamma$)
- Sum over
  - Both neutrino types
  - cross sections of possible interactions times fraction of energy deposited per interaction in shower




$$d = L_s \Delta\theta_H = L_s\left(\theta_H - \hat{\theta}_H\right)$$

- Gives

$$\Delta D = \underbrace{\left(0.410\cdot 10^{-39}\,\frac{\mathrm{m}^2}{\mathrm{TeV}}\,\frac{E_{\mu r}^2}{4}\right)\frac{1}{m_u}}_{1.10\cdot 10^{-28}\,\mathrm{Gy\,m}^2}\frac{1}{L_s^2}\frac{\langle E_\nu^2\rangle}{\langle E_\nu^{i2}\rangle}\frac{1}{N}\frac{dN}{d\Omega} = \left(1.10\cdot 10^{-28}\ \mathrm{Gy\ m}^2\right)\frac{4\gamma^4}{\pi L_s^2}\frac{1}{\left(1+\gamma^2\left(d/L_s\right)^2\right)^4}$$

with $\langle E_\nu^2\rangle / \langle E_\nu^{i2}\rangle = 4\gamma^2 / \left(1+\gamma^2\theta^2\right)^2$ the ratio between the square of the neutrino energy in the lab and the rest system
and $N^{-1}\,dN/d\Omega = \left(\gamma^2/\pi\right)/\left(1+\gamma^2\theta^2\right)^2$ the angular distribution of neutrinos

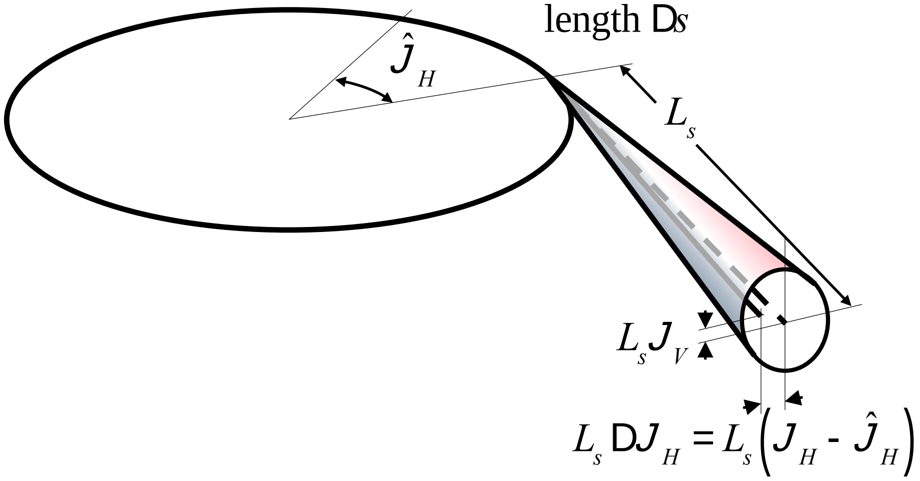
- The decay $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ gives (almost) identical final result despite different cross sections and fractions

# Analytical estimates
## approximation by Gaussian

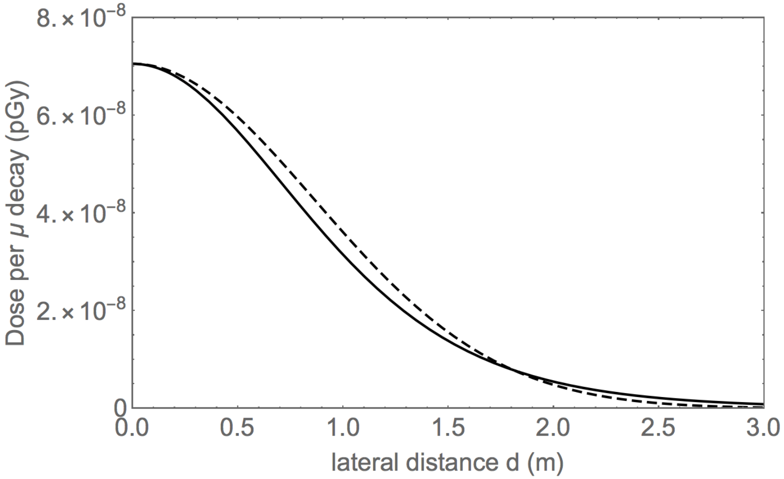
- Absorbed dose reasonably well described by Gaussian

$$\Delta D \approx \left(1.10\times10^{-28}\ \mathrm{Gy\ m^2}\right)\frac{4\gamma^4}{\pi L_s^2}\frac{1}{\left(1+\gamma^2\left(d/L_s\right)^2\right)^4}$$

$$\approx \left(1.10\cdot10^{-28}\ \mathrm{Gy\ m^2}\right)\frac{4\gamma^4}{\pi L_s^2}\,e^{-3\gamma^2\left(d/L_s\right)^2}$$




  - Rms opening angle of neutrino radiation cone of $\theta_{rms} = 1/\left(\sqrt{6}\,\gamma\right)$




Dose per muon decay at distance $L_s$ = 100 km and $\gamma$ = 47 322, i.e., 10 TeV com energy

- Assuming Gaussian for the beam divergence
  - Folding of divergence from muon decay process and beam divergence simple
- Almost suitable as source term to estimate doses generated by neutrino interactions

# FLUKA simulations of doses due to neutrino interactions

- Motivation: improvement and check of analytical derivations
  - Possible widening of neutrino radiation cone due to lateral extension of shower
  - Effective dose instead of absorbed dose
- Ansatz

$$\Delta H \approx \Delta\hat{H}\, e^{-d^2/2\sigma^2_{\Delta H}}$$

$$= w_{R,eff} \cdot \left(1.104 \cdot 10^{-28}\ \mathrm{Gy\ m^2}\right) \frac{4\gamma^4}{\pi\left(L_s^2 + 6\gamma^2\sigma_s^2\right)} \exp\left(-\frac{d^2}{2\left(L_s^2/6\gamma^2 + \sigma_s^2\right)}\right)$$

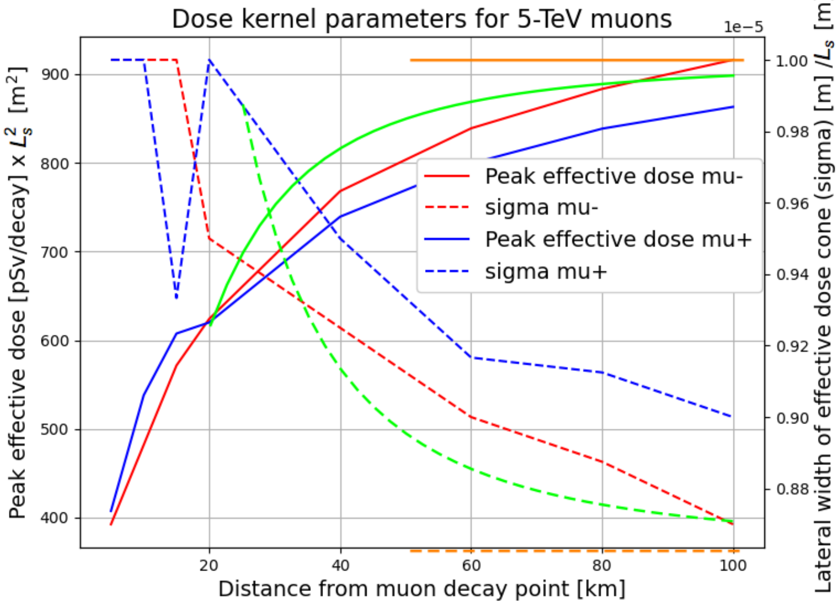
$$\Delta\hat{H} \approx w_{R,eff} \cdot \left(1.104 \cdot 10^{-28}\right) 4\gamma^4 \Big/ \left(\pi\left(L_s^2 + 6\gamma^2\sigma_s^2\right)\right)$$

$$\sigma_{\Delta H} \approx \sqrt{L_s^2/\left(6\gamma^2\right) + \sigma_s^2}$$

$$\Delta\hat{H}\sigma^2_{\Delta H} \approx w_{R,eff} \cdot \left(1.104 \cdot 10^{-28}\right) 2\gamma^2 / \left(3\pi\right)$$

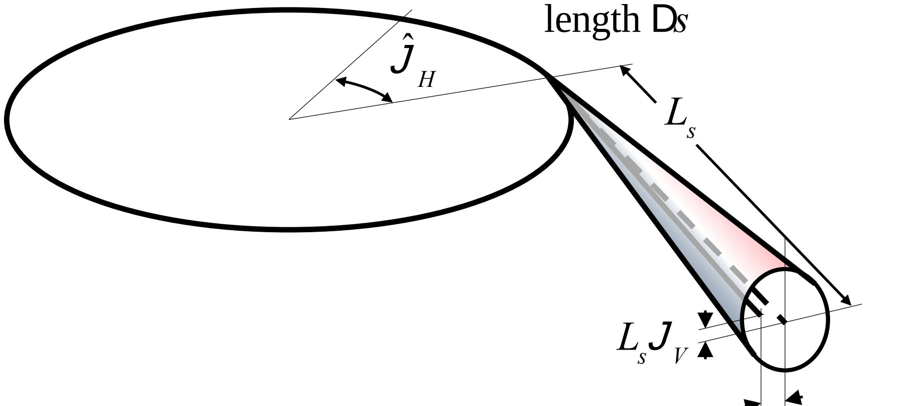
  - with $w_{R,eff}$ to convert absorbed dose $D$ to dose equivalent $H = w_{R,eff} D$
  - and $\sigma_s$ the rms radial extension of the shower (assumed as well to be Gaussian?!)
- Result of fitting "by hand" to FLUKA results for 5 TeV muons (green lines in plot)
  - $w_{R,eff} = 1.3\ \mathrm{Sv/Gy}$ and $\sigma_s = 0.12\ \mathrm{m}$
  - Results may be different for other energies
  - Shower extension $\sigma_s$ neglected for further studies (slightly pessimistic)

FLUKA studies by G. Lerner et al.



# Extrapolation to a collider lattice

- Taking details of lattice into account:
  - Twiss gamma functions $\gamma_H(s)$ and $\gamma_V(s)$ and derivative of dispersion $D'(s)$
  - Physical rms emittances $\varepsilon_H = \varepsilon_V = 25\,\mu\text{m}/\gamma \approx 0.528\,\text{nm}$
  - Rel. momentum spread $\sigma_p/p = 10^{-3}$




$$d = L_s \Delta\theta_H = L_s\left(\theta_H - \hat{\theta}_H\right)$$

- Extrapolation of argument of exponential

$$-3\gamma^2\left(\theta_V^2+\theta_H^2\right) = \frac{-\theta_V^2}{2\times\frac{1}{6\gamma^2}} + \frac{-\theta_H^2}{2\times\frac{1}{6\gamma^2}} \Rightarrow -\frac{\theta_V^2}{2\sigma_{\theta_V}^2} - \frac{\left(\theta_H - \hat{\theta}(s)\right)^2}{2\sigma_{\theta_H}^2}$$

$$\sigma_{\theta_H}^2 = \frac{1}{6\gamma^2} + \varepsilon_H\gamma_H(s) + \left(\frac{\sigma_p}{p}\cdot D'(s)\right)^2$$

$$\sigma_{\theta_V}^2 = \frac{1}{6\gamma^2} + \varepsilon_V\gamma_V(s)$$

  - Without mitigation measures gives

$$\frac{dH}{dt} = \left(1.104\cdot 10^{-28}\ \text{Gy m}^2\right) w_{R,eff} \frac{4\gamma^4 f_r N_\mu}{\pi L_s^2 C} \int ds \frac{1/\left(6\gamma^2\right)}{\sigma_{\theta_H}\cdot\sigma_{\theta_H}} \exp\left[-\frac{\left(\theta_H - \hat{\theta}(s)\right)^2}{2\sigma_{\theta_H}^2} - \frac{\theta_V^2}{2\sigma_{\theta_V}^2}\right]$$

with $N_\mu$ the number if muons per bunch, $f_r$ the repetition rate and $C$ the circumference

# Numerical evaluations
## Simple cases

- Divergence of muon beam neglected, peak dose rate
  - Straight section $\Delta s$ , using $C = 2\pi\gamma E_{\mu}/\left(c\,e\,\bar{B}\right)$ and $L_s \approx 2\sqrt{R_e h}$ with $R_e \approx 6.38\times10^6\ \mathrm{m}$ the earth radius

$$\frac{dH_S}{dt} = \left(6.85\cdot10^{-22}\ \frac{\mathrm{Sv}}{\mathrm{T}}\right) f_r N_{\mu} \left(\frac{E}{5\mathrm{TeV}}\right)^3 \frac{\Delta s\,\bar{B}}{h}$$

for beam energies around $E \approx 5\mathrm{TeV}$

  - Bending magnet – integration w.r.t $s$ using $J_H - \hat{J}(s) = \left(eB/\left(\gamma E_{\mu}/c\right)\right)s$

$$\frac{dH_B}{dt} = \left(6.85\cdot10^{-22}\ \frac{\mathrm{Sv}}{\mathrm{T}}\right) f_r N_{\mu} \left(\frac{E}{5\mathrm{TeV}}\right)^3 \frac{\bar{B}}{h} \int ds\, \exp\left(-3\left(\frac{eBc}{E_{\mu}}s\right)^2\right) = \left(2.47\cdot10^{-22}\ \mathrm{Sv\ m}\right) f_r N_{\mu} \left(\frac{E}{5\mathrm{TeV}}\right)^3 \frac{\bar{B}}{h\,B}$$

  - Integrated peak equivalent dose per muon beam for one year operation (5000 h = 18 $10^6$ s) without mitigation measures

Contribution only from of the two muon beams

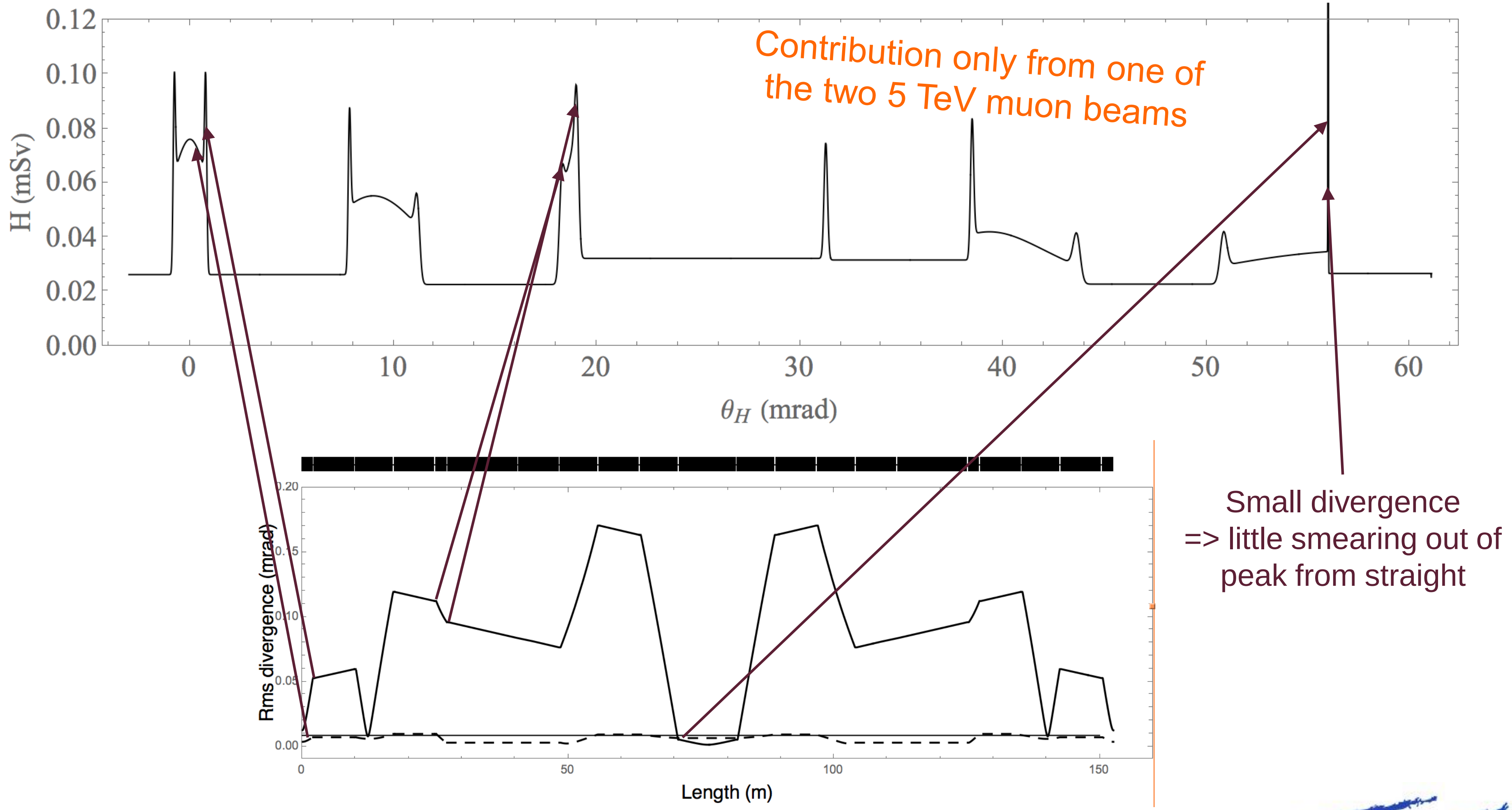
Mitigation mandatory!!

$N_{\mu} = 1.8\times10^{12}$ muons per bunch,
$f_r = 5\mathrm{Hz}$ repetition,
$E \approx 5\mathrm{TeV}$ beam energy,
$C = 10000\,\mathrm{km}$ circumference and
$\bar{B} = 10.42\,\mathrm{T}$ average field

| h (m) | $L_s$ (km) | $H_s$ (mSv) for $\Delta s$ = 0.3 m | $H_B$ (mSv) for B = 8 T |
|---|---|---|---|
| 100 | 35.7 | 3.5 | 0.52 |
| 200 | 50.5 | 1.75 | 0.26 |
| 500 | 79.7 | 0.70 | 0.105 |
| 784 | 100 | 0.45 | 0.067 |

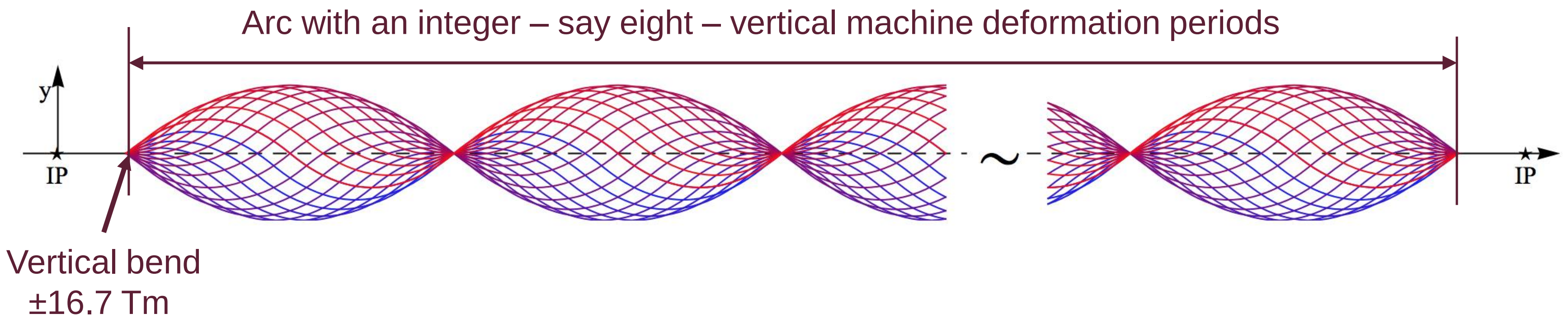
# Numerical evaluations
## equivalent dose from arc cell at 100 km

- Integrals evaluated for present (work in progress by K. Skoufaris) 10 TeV collider arc half cell
  - In collider mid-plane as function of $J_H$ (i.e., $J_V = 0$) for one year (5000 h operation)



# Mitigation by “Wobbling”

- Wobbling of machine in vertical direction – part of MAP proposal?
  - Time-dependent mechanical deformation of ring around arc (including chromatic compensation, matching section and FMC arc cells
  - High precision movement system
  - Impact on optics?
- For 10 TeV com collider with 10 km circumference and say 4.8 km arcs




  - Combination of pieces of parabola – two pieces with opposite curvature one period
  - Say 8 periods 660 m long periods generating angles between -1 mrad and + 1 mrad
  - Magnetic field (average) bending in vertical ±0.11 T
  - Excursion (maximum total) ±150 mm
  - Replaces vertical Gaussian angle distribution with rms opening of ≈0.0086 mrad by about rectangular distribution within ±1 mrad

=> About two order of magnitude reduction of peak dose rates

# Summary and Outlook

- Dose generated at the earth surface due to showers generated by neutrino interactions is a serious issue
  - Muon decays generate narrow cone swept by bendings in bending plane
  - In particular high doses in direction of (short) straights sections
  - Numerical estimates based on “source term” from FLUKA simulations (=> Comparison with MARS results to be done?)

- Time-dependent vertical deformation of the whole arc proposed as mitigation measure
  - For a 10 TeV com collider angle variations (linear up and down) in a range ±1 mrad would allow to gain about two orders of magnitude
  - To be combination with other mitigation measures
    - Collider installed deep underground with suitable positioning
    - Appropriate orientation towards uncritical areas (possibly owned by facility)

- Feasibility to be studied
  - High precision movement system
  - Impact on beam dynamics in particular in chromatic compensation and matching sections (vertical dispersion, orbit stability …)