

*“Insights” on the Rare Pion Decay Program*  
*“Pioneer”*

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## A Homage To Alberto Sirlin (1930-2022) “Pioneer” and Master of EW Radiative Corrections



Important original contributions to SM Renormalization, Phenomenology, Precision Measurements & “New Physics” searches including: Muon decay  $\rightarrow G_F$ , Nuclear Beta Decays,  $V_{ud}$ , W & Z Masses, Neutrino Scattering, Atomic physics, Tau decays ... **Rare Pion Decays...**

### Pioneer Experiment Goals (Test Lepton & CKM Universality)

- 1)  $R_{e/\mu} = BR(\pi^+ \rightarrow e^+ \nu(\gamma))_{\text{exp}} / BR(\pi^+ \rightarrow \mu^+ \nu(\gamma))_{\text{exp}}$       *10-15 x improvement Phase I*
- 2)  $BR(\pi^+ \rightarrow \pi^0 e^+ \nu(\gamma))_{\text{exp}} \rightarrow V_{ud}$  Pristine Theory (Unique).      *3-10 x better! Phase II & III*

## Popular Experimental Hints of New Physics

Anomalous Magnetic Moments  $\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = \underline{251(41)_{exp}(43) \times 10^{-11}}_{th}$  **4.2 $\sigma$ !** **e<sup>+</sup>e<sup>-</sup> data vs Lattice QCD 1.5 $\sigma$ ?**

$$\Delta a_e = a_e^{exp} - a_e^{SM} = \underline{-87(28)_{exp}(23) \alpha(2)_{th}} \times 10^{-14}$$

$\Delta a_e$  (-2.4 $\sigma$  or +1.6 $\sigma$ ) depends on  $\alpha$  from Cs vs Rb differ by 5.5 $\sigma$

**\*Recent Result:** Fan, Myers, Sukra, Gabrielse improvement of  $a_e^{exp}$  by factor of 2.2 **agreement! with 2008 exp**

B  $\rightarrow$  K<sup>(\*)</sup> l<sup>+</sup> l<sup>-</sup> decays & Possible electron-muon **universality** breakdown **3 $\sigma$ ?**

**\* CDF II  $m_W = 80.433(9)$  GeV vs Global EW Fit (without  $M_W^{exp}$ )  $\rightarrow m_W = 80.359(3)$  GeV **7 sigma difference!!****

First Row CKM Unitarity Violation? (see **C.-Y. Seng**, Mod. Phys. Lett. (2022) for a recent review)

Current  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)?$   **$\sim 2.5\sigma$  effect**  $V_{ud} = 0.97373(31)$  from beta decays (Nuclear Unc?)

$V_{us} = 0.2243(5)$  from  $K_{\mu 2}$  &  $K_{l 3}$  ave.

$V_{ub} \sim 4 \times 10^{-3}$  negligible effect

Axial Current  $|V_{us}|/|V_{ud}| = \underline{0.23131(51)}$  from  $K_{\mu 2}/\pi_{\mu 2}$  + unitarity  $\rightarrow V_{ud} = \underline{0.97428(11)}$   $V_{us} = \underline{0.22536(47)}$

Vector Current  $|V_{us}|/|V_{ud}| = \underline{0.22910(77)}$  from  $K_{l 3}/\pi_{e 3}$  + unitarity  $\rightarrow V_{ud} = 0.97475(17)$   $V_{us} = 0.22331(72)$

**2.4 $\sigma$  difference**

BACK TO ARTICLE

High-precision measurement of the W boson mass with the CDF II detector  
Fig. 5

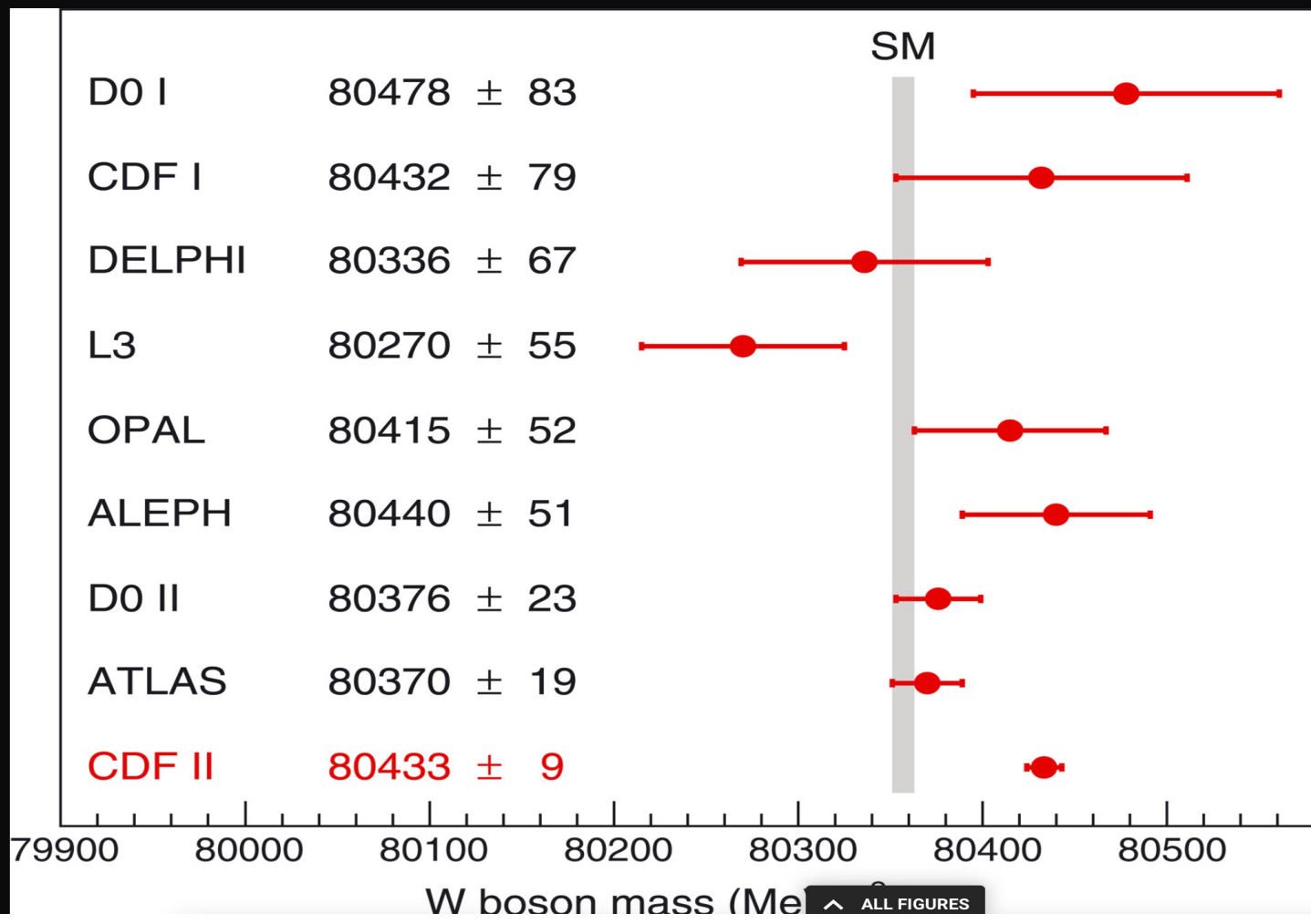


Fig. 5. and pa expect The lat the unc order c uncert measu such a

# Radiative Inclusive Charged Pion Decays

(Current Status PDG 2022).

$$\Gamma(\pi \rightarrow l \bar{\nu}_l(\gamma)) = \frac{G_\mu^2 |V_{ud}|^2}{8\pi} f_\pi^2 m_\pi m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2 \left[ \right]$$

- $BR(\pi^+ \rightarrow \mu^+ \nu(\gamma))_{\text{exp}} = 0.9998770(4)$

**Determines**  $|f_\pi V_{ud}| = 127.13(0.02)(0.13)_{\text{RC}} \text{MeV}$

Lattice FLAG Ave.  $f_\pi = 130.2(0.8) \text{MeV} \rightarrow V_{ud} = 0.9764(60)(2)$  not competitive by factor 20

$V_{ud} = 0.9740(27) \rightarrow f_\pi = 130.5(0.4) \text{MeV}$

- $BR(\pi^+ \rightarrow e^+ \nu(\gamma))_{\text{exp}} = 1.2325(23) \times 10^{-4}$  **vs**  $BR(\pi^+ \rightarrow e^+ \nu(\gamma))_{\text{SM}} = 1.2350(1) \times 10^{-4}$

(Includes  $BR(\pi^+ \rightarrow e^+ e^- e^+ \nu)_{\text{exp}} = 3.2(5) \times 10^{-9}$  part of radiative inclusive).

- $BR(\pi^+ \rightarrow \pi^0 e^+ \nu(\gamma))_{\text{exp}} = 1.036(6) \times 10^{-8}$  **PIBETA Exp.** Determines  $|f_+^\pi(0) V_{ud}| = 0.9740(27)$

$f_+^\pi(0) = 1 - O(10^{-5})$  Behrends – Sirlin Theorem (1960)

make certain BRs are Radiative Inclusive!

$\pi^\pm$  Lifetime :  $\tau_\pi = 2.6033(5) \times 10^{-8} \text{sec}$  **New Measurement Warranted?**

$$R_{e/\mu} = \Gamma(\pi^+ \rightarrow e^+ \nu(\gamma)) / \Gamma(\pi^+ \rightarrow \mu^+ \nu(\gamma))$$

Test of Electron-Muon Universality

**1959) T. Kinoshita:** QED radiative corrections reduce  $R_{e/\mu}$   
 In a V-A theory by 3.9% for point-like pions  
 to  $1.2326 \times 10^{-4}$  (ignoring strong interactions)  
 for radiative inclusive decay rates.

$$R_0 = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.28 \times 10^{-4},$$

Main effect  $-3\frac{\alpha}{\pi} \ln\left(\frac{m_\mu}{m_e}\right) = -16\frac{\alpha}{\pi} = -3.72\%$

**1976) WJM & A. Sirlin** Theorem: Strong interaction effects on mass singularities ( $\ln(M/m_l)$  terms) present in partial decay rates but **cancel for radiative inclusive decay rates (infrared safe)**.

**1993) WJM & A. Sirlin:** Radiative Corrections to  $\pi_{l2}$  Decays (generic SM formula)

$$\Gamma(\pi \rightarrow l \bar{\nu}_l(\gamma)) = \frac{G_\mu^2 |V_{ud}|^2}{8\pi} f_\pi^2 m_\pi m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2 \left[1 + \frac{2\alpha}{\pi} \ln\left(\frac{m_Z}{m_\rho}\right)\right]$$

$$\times \left[1 - \frac{\alpha}{\pi} \left\{ \frac{3}{2} \ln\left(\frac{m_\rho}{m_\pi}\right) + C_1 + C_2 \frac{m_l^2}{m_\rho^2} \ln \frac{m_\rho^2}{m_l^2} + C_3 \frac{m_l^2}{m_\rho^2} + \dots \right\}\right] \left[1 + \frac{\alpha}{\pi} F(x)\right],$$

+ Leading Logs Summation,  $C_2$  calculated and expect  $C_1$  to largely cancel in ratio, we used  $C_3 = 0 \pm 10$  (conservative)  $\rightarrow R_{e/\mu} = 1.2352(5) \times 10^{-4}$  1993 SM Prediction

2007 Cirigliano and Rosell evaluate  $R_{e/\mu} = 1.2352(1) \times 10^{-4}$  in SM using Chiral Pert. Theory. Small uncertainty (Pioneer uses  $\pm 1.5 \times 10^{-8}$ ). Could be slightly further improved using RGE to include NLO corrections.

### First Row CKM Unitarity

A. Sirlin: (1973-1978) Standard Model Semi-Leptonic Radiative Corrections Finite & Calculable **2.32%** *universal* normalization using  $G_\mu = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$  based on

$\tau_\mu = 2.1969803(22) \times 10^{-6} \text{ sec}$  (today's value from MuLAN 2010)

3 Generation CKM mixing via unitary matrix  $V_{\text{CKM}}$

$|V_{ud}^0|^2 + |V_{us}^0|^2 + |V_{ub}^0|^2 = 1$  **First Row Bare Natural Relation**

EW Radiative Corrections Finite and calculable relative to  $\tau_\mu$ . Different weak  $U(1)_Y$  hypercharges lead to different RC.

$(ud)_L$   $Y = \frac{1}{3}$  **and**  $(ve)_L$   $Y = -1$  doublets  $\rightarrow$  divergences become  $\ln(m_Z/m_p)$  corrections

20 sigma quantum loop confirmation of SM radiative corrections!

## SIRLIN'S UNIVERSAL $\Delta_R^V$ TO NUCLEAR & NEUTRON BETA DECAYS

$$\text{Universal } \Delta_R^V = \alpha/2\pi[3\ln(m_Z/m_p) + \ln(m_Z/m_A) + 2C + A_{\text{QCD}}]$$

$3\alpha/2\pi\ln(m_Z/m_p)$  short-distance **not** renormalized by strong int.

$[\alpha/2\pi[\ln(m_Z/m_A) + 2C + A_{\text{QCD}}]]$  Induced by axial-current loop

$m_A = 1.2\text{GeV}$  long/short distance matching scale

$C = 0.8g_A(\mu_N + \mu_p) = 0.891$  (long distance  $\gamma W$  Box diagram)

$A_{\text{QCD}} = -\alpha_s/\pi(\ln(m_Z/m_A) + \text{cons}) = -0.34$  QCD Correction

$[\alpha/\pi\ln(m_Z/m)]^n$  leading logs summed via renormalization group  $\simeq +0.001$

Next to leading short distance logs  $\sim -0.0001$ , small?

and  $-\alpha^2\ln(m_p/m_e) = -0.00043$  estimated (for neutron decay)

Czarnecki, WJM, Sirlin (CMS)(2004)

2006 matching short and long distance  $\gamma W$  (VA) Box Diagram 1+ RC = **1.0390(4)** **WJM & Sirlin**

\*2018 **DR(dispersion Relation) give 1+RC = 1.0399(2)** **Seng, Gorchtein, Patel, Ramsey-Musolf (PRL2018)**

$$\Delta_R^V \rightarrow \mathbf{0.02467(22)} \rightarrow V_{ud} = \mathbf{0.97370(14)} \quad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = \mathbf{0.9985(5)}$$

**3 sigma tension with Unitarity! Quit while you are ahead.**



## Status of First Row CKM Unitarity (The Cabibbo Anomaly)

- $|V_{ud}| = 0.97373(14)(27)_{NS}$  **Superaligned (15 Nucl.  $\beta$  Decays)** Hardy & Towner. Unitarity  $\rightarrow 0.9742$
- $|V_{ud}| = 0.97330(30)_{\tau n}(30)_{gA}(10)_{RC}$  neutron PDG world ave.
- $|V_{ud}| = 0.97390(270)_{exp}(10)_{th.}$   $\pi_{e3}$   $\pi^+ \rightarrow \pi^0 e \nu(\gamma)$  **Very Clean Theory**  $\pm 0.01\%$  but BR  $\sim 10^{-8}$ !
- $|V_{us}| = 0.2243(6)_{ave}$  **1.6 PDG Scale factor** from average of  $K_{l3} \rightarrow 0.2234(6)$  &  $K_{\mu 2} \rightarrow 0.2252(6)$  tension

$|V_{ub}| = 4 \times 10^{-3}$ . negligible effect on unitarity sum

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5) \text{ 3 sigma deviation (Cabibbo Anomaly)}$$

- **Recent shorter neutron lifetime and larger  $g_A$  favors larger  $V_{ud}$  + unitarity restored?**  
Current degree of Unitarity violation in the eye of the beholder.

### PIONEER GOALS

Potential improvement of  $\frac{\Gamma(\pi \rightarrow e \nu(\gamma))}{\Gamma(\pi \rightarrow \mu \nu(\gamma))}$ ,  $f_\pi$ ,  $V_{ud}$  &  $R_V \equiv \frac{\Gamma(K \rightarrow \pi e \nu(\gamma))}{\Gamma(\pi \rightarrow \pi^0 e \nu(\gamma))} \rightarrow \frac{V_{us}}{V_{ud}}$

**SM:**  $f_\pi V_{ud} = 127.13(0.02)(0.13)_{RC} \text{MeV} \rightarrow f_\pi = 130.49(13) \text{MeV} \pm 0.1\%$  vs lattice  $130.2(8) \text{MeV} \pm 0.6\%$

$$\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu(\gamma)) = \frac{G_\mu^2 |V_{ud}|^2 m_{\pi^+}^5 |f_+^\pi(0)|^2}{64\pi^3} (1 + RC_\pi) I_\pi,$$

$$RC_\pi = 0.0332(3)$$

$$I_\pi = 7.376 \times 10^{-8} \text{ phase space suppression}$$

Combined with Kl3 decay rates leads to  $R_V = V_{us}/V_{ud} = 0.22908(66)_\pi(41)_K(20)_{lat}(2)_{\tau\pi}(1)_{RC}$

Recall  $R_A \rightarrow \frac{V_{us}}{V_{ud}} = 0.23131(45)$  from  $K_{\mu 2}/\pi_{\mu 2}$  2.3 sigma difference currently

$$R_A \equiv \frac{\Gamma(K \rightarrow \mu \nu(\gamma))}{\Gamma(\pi \rightarrow \mu \nu(\gamma))} \text{ vs } R_V \equiv \frac{\Gamma(K \rightarrow \pi e \nu(\gamma))}{\Gamma(\pi \rightarrow \pi^0 e \nu(\gamma))}$$

Axial current CKM  $\frac{V_{us}}{V_{ud}} = 0.23131(45)$ .

Vector Current CKM  $\frac{V_{us}}{V_{ud}} = 0.22908(66)_{\pi}(41)_K(20)_{\text{lat}}(2)_{\tau\pi}(1)_{\text{RC}}$

Reduction of pion beta decay by about a factor of 3 would make them comparable

Reduction by a factor of 10 would be unique  $V_{ud}$  determination

Current Difference: Lattice  $f_+^K(0)$  problem?

Status of SM Charged Pion & Kaon Radiative Inclusive Decay Rates  
& Future Improvements expected from the PIONEER Exp. at PSI

1)  $BR(\pi \rightarrow \mu\nu(\gamma))_{\text{exp}} = 0.9998770(4)$        $\rightarrow \Gamma(\pi \rightarrow \mu\nu(\gamma)) = 2.5280(5) \times 10^{-14} \text{ MeV} \rightarrow f_{\pi} |V_{ud}| = 127.13(1)_{\text{exp}}(17)_{\text{RC}} \text{ MeV}$   
 $\pi^{\pm}$  Lifetime :  $\tau_{\pi} = 2.6033(5) \times 10^{-8} \text{ sec}$        $\Gamma(K \rightarrow \mu\nu(\gamma)) = 3.379(7) \times 10^{-14} \text{ MeV.} \rightarrow f_K |V_{us}| = 35.09(4)_{\text{exp}}(4)_{\text{RC}} \text{ MeV}$   
 $\Gamma(\text{all}) = 2.5284(5) \times 10^{-14} \text{ MeV (unc. } \pm 0.02\%)$        $R_A \equiv \frac{\Gamma(K \rightarrow \mu\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} \rightarrow \frac{V_{us}}{V_{ud}} = 0.23131(45)$   
 **$\tau_{\pi}$  Improvement by order of magnitude?**       $\frac{f_K |V_{us}|}{f_{\pi} |V_{ud}|} = 0.27602(29)_{\text{exp}}(24)_{\text{RC}} + \text{Lattice } f_K/f_{\pi} = 1.1932(19) + \text{unitarity \& } |V_{ub}|^2 \approx 0$   
 useful but not required       $V_{ud} = 0.97426(10)$      $V_{us} = 0.22543(43)$     SM Prediction    Deviation  $\rightarrow$  "New Physics"

2) Electron Decay Mode:  $\frac{\Gamma(\pi \rightarrow e\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} = 1.2352(1) \times 10^{-4}$  very precise (radiative inclusive) SM theory prediction. **Universality Test**  
 $1.2327(28) \times 10^{-4}$  Exp.  $\rightarrow$  Pioneer goal factor 10-15 improvement  
 Constrains many examples of BSM new physics see (Bryman et al. Ann. Rev. N&P)

3) Pion Beta Decay: **Current**  $BR(\pi \rightarrow \pi^0 e\nu(\gamma))_{\text{exp}} = 1.038(6) \times 10^{-8} \rightarrow V_{ud} = 0.9740(28)_{\text{exp}}(1)_{\text{th}}$  very clean precise theory  
 $BR(\pi \rightarrow \pi^0 e\nu(\gamma)) = \frac{\Gamma(\pi \rightarrow \pi^0 e\nu(\gamma))}{1.00012327\Gamma(\pi \rightarrow \mu\nu(\gamma))} = \frac{\Gamma(\pi \rightarrow e\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} \times \frac{\Gamma(\pi \rightarrow \pi^0 e\nu(\gamma))}{\Gamma(\pi \rightarrow e\nu(\gamma))} |_{\text{exp}}$        $R_V \equiv \frac{\Gamma(K \rightarrow \pi e\nu(\gamma))}{\Gamma(\pi \rightarrow \pi^0 e\nu(\gamma))} \rightarrow \frac{V_{us}}{V_{ud}} = 0.22908(66)_{\pi}(41)_{\text{K}}(20)_{\text{Lat.}}$   
 $\Gamma(\pi \rightarrow \pi^0 e\nu(\gamma)) = BR(\pi \rightarrow \pi^0 e\nu(\gamma)) / \tau_{\pi}$

Pioneer (Phase 2): factor of 3 improvement in  $BR(\pi \rightarrow \pi^0 e\nu(\gamma))_{\text{exp}}$  + improvement in  $K_{l3}$  & Lattice  $\rightarrow R_V$  factor 3 improvement  
 Pioneer (Phase 3) Overall factor 10 improvement?  $\rightarrow$  Potentially best determination of  $V_{ud}$  but difficult systematics

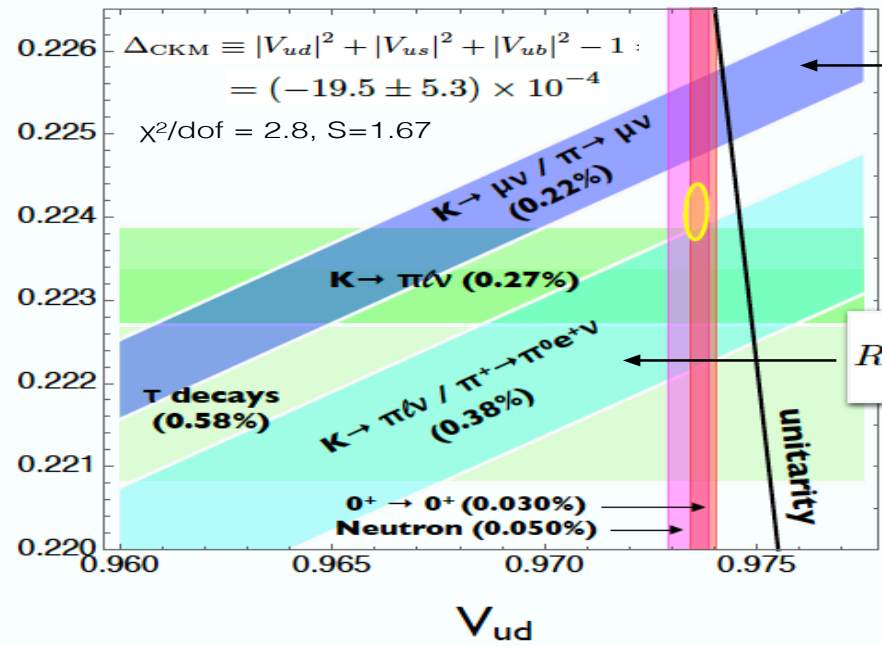
Slide from V. Cirigliano's Pioneer talk

# Cabibbo universality and $BR(\pi^\pm \rightarrow \pi^0 e^\pm \nu(\gamma))$

- Status of  $V_{ud}$  and  $V_{us}$  determinations: tension with SM hypothesis

$R_V$  is nearly competitive with  $R_A$ .  
 3x improvement in PIBETA BR, would lead to competitive  $V_{us}/V_{ud}$  @ 0.2%.  
 Realistic short term goal

Bryman, VC, Crivellin, Inguglia 2111.05338



$$R_A = \frac{\Gamma(K \rightarrow \mu \nu(\gamma))}{\Gamma(\pi \rightarrow \mu \nu(\gamma))}$$

Marciano, hep-ph/0402299

$$R_V = \frac{\Gamma(K_L \rightarrow \pi^\pm e^\mp \nu(\gamma))}{\Gamma(\pi^+ \rightarrow \pi^0 e^+ \nu(\gamma))}$$

Czarnecki, Marciano, Sirlin 1911.04685

## Summary of Charged Pion Properties & PIONEER Goals

$\tau_\pi = 2.6033(5) \times 10^{-8}$  sec. Known to  $\pm 0.02\%$ . Defines Potential Sensitivity Goals  
3 (radiative inclusive) Decay Modes 2 Axial & 1 Vector Current Primary Amplitude

1)  $BR(\pi \rightarrow \mu\nu(\gamma)) = 0.9998770(4)$   $\Gamma(\pi \rightarrow \mu\nu(\gamma)) = 2.5280(5) \times 10^{-14}$  MeV

SM prediction =  $1.5642(3) \times 10^{-18}/\text{MeV} \times |V_{ud} f_\pi|^2$

Lattice FLAG Ave.  $f_\pi = 130.2(0.8)$   $V_{ud} = 0.9764(60)(2)$

$V_{ud} = 0.97373(30) \rightarrow f_\pi = 130.56(4)$

1<sup>st</sup> row unitarity &  $V_{us}/V_{ud} = 0.23131(45) \rightarrow V_{ud} = 0.97426(10) \rightarrow f_\pi = 130.48(1)\text{MeV}$

2)  $\Gamma(\pi \rightarrow e\nu(\gamma))/\Gamma(\pi \rightarrow \mu\nu(\gamma)) = 1.2352(1) \times 10^{-4}$  SM prediction

$1.2327(28) \times 10^{-4}$  Current Experimental determination

3)  $BR(\pi \rightarrow \pi^0 e\nu(\gamma)) = \frac{\Gamma(\pi \rightarrow \pi^0 e\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} = \frac{\Gamma(\pi \rightarrow e\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} \times \frac{\Gamma(\pi \rightarrow \pi^0 e\nu(\gamma))}{\Gamma(\pi \rightarrow e\nu(\gamma))}|_{\text{exp}}$

$R_V \equiv \frac{\Gamma(K \rightarrow \pi e\nu(\gamma))}{\Gamma(\pi \rightarrow \pi^0 e\nu(\gamma))} \rightarrow \frac{V_{us}}{V_{ud}} = 0.22908(66)_{\pi(41)K(20)}_{\text{Lat.}}$  Reduce  $\pi_{e3}$  &  $K_{l3}$  uncertainties

Pioneer: expect phase 2 factor of 3 improvement in  $BR(\pi \rightarrow \pi^0 e\nu(\gamma))_{\text{exp}}$  &  $V_{ud} + K_{l3}$ ?, Lattice?  $\rightarrow R_V$

Phase 3 Long term goal: factor of 10 improvement in  $BR(\pi \rightarrow \pi^0 e\nu(\gamma))$  &  $V_{ud}$ ! Unique Capability