Lepton Flavour Universality tests across energy scales

Antonio Pich

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Standard Model of the Fundamental Interactions

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

1. **Interactions determined by gauge symmetries.** **Flavour Universality**
2. **Gauge symmetries require all elementary particles to be massless**
3. **Masses generated through the interaction with the Higgs doublet**

$$\mathcal{L}_Y = -\sum_{jk} \left\{ \left(\bar{u}_j, \bar{d}'_j\right)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)*} \\ -\phi^{(-)} \end{pmatrix} u_{kR} \right] + \left(\bar{\nu}_j, \bar{\ell}'_j\right)_L c^{(l)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} \ell'_{kR} \right\}$$

Mass is the only difference among the three fermion families

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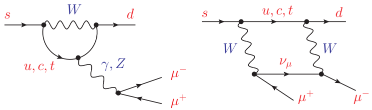
Mass is the only difference among the three fermion families

4. **Fermion mass eigenstates $\neq$ weak eigenstates**

→ **Flavour Mixing:** $d'_i = V_{ij}\, d_j \quad , \quad V^\dagger V = V\, V^\dagger = I$

CP violation (if $N_G \geq 3$)

Successful Description of Flavour & CP



Rare Decays

$\mathrm{Br}(K_L^0 \to \mu^+\mu^-) = 6.8 \times 10^{-9}$

$\mathrm{Br}(B_s^0 \to \mu^+\mu^-) = 3.0 \times 10^{-9}$

$\mathrm{Br}(\bar{b} \to \bar{s}\gamma) = 3.1 \times 10^{-4}$

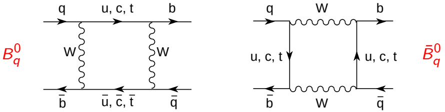
Meson-Antimeson Mixing

$\Delta M_{K^0}/M_{K^0} = 7.0 \times 10^{-15}$ ➡ m_c

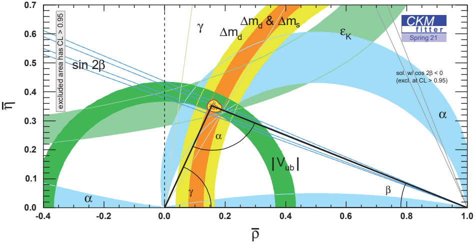
$\Delta M_{B_d^0}/M_{B_d^0} = 6.3 \times 10^{-14}$ ➡ m_t

$\Delta M_{B_s^0}/M_{B_s^0} = 2.2 \times 10^{-12}$ ➡ m_t

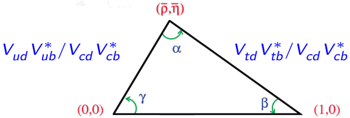
Sensitivity to (virtual) heavy scales



$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

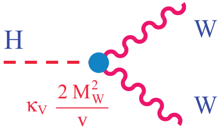


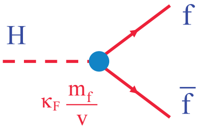
CKM Unitarity

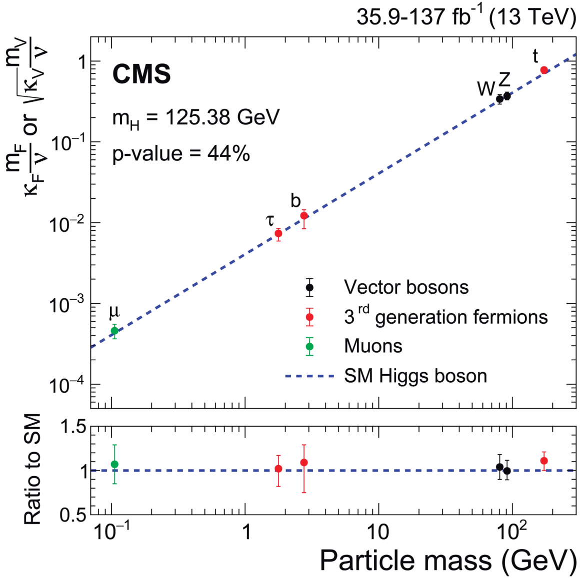


A Higgs field indeed

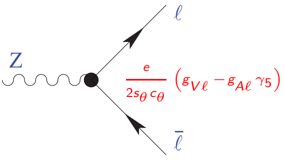
Interaction proportional to mass







Lepton Flavour Universality in Z Decays



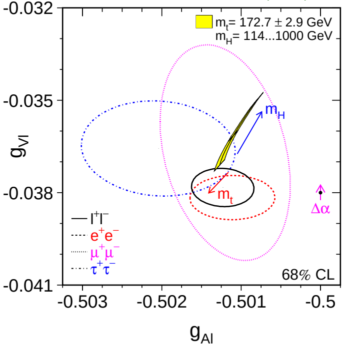
$$\frac{\Gamma(Z \to \mu^+\mu^-)}{\Gamma(Z \to e^+e^-)} = 1.0001 \pm 0.0024$$

$$\frac{\Gamma(Z \to \tau^+\tau^-)}{\Gamma(Z \to e^+e^-)} = 1.0020 \pm 0.0032$$

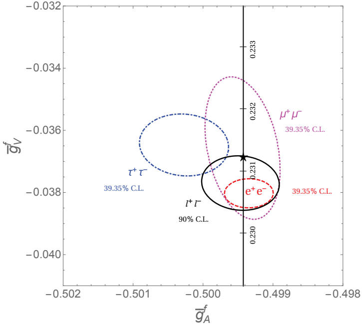
$$\frac{\Gamma(Z \to \tau^+\tau^-)}{\Gamma(Z \to \mu^+\mu^-)} = 1.0010 \pm 0.0026$$

PDG 2022

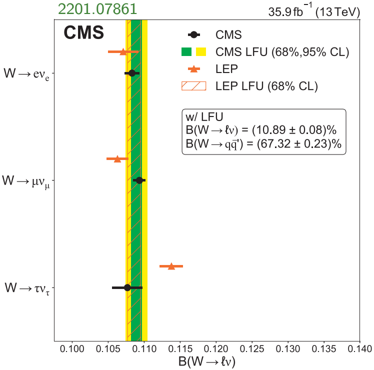
LEP Phys. Rept. 427 (2006) 257

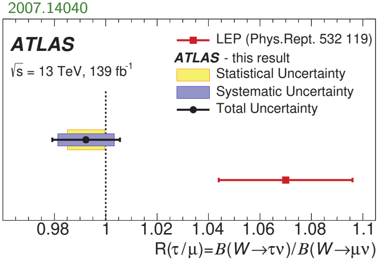


PDG 2022 J. Erler, A. Freitas



Lepton Flavour Universality in W Decays



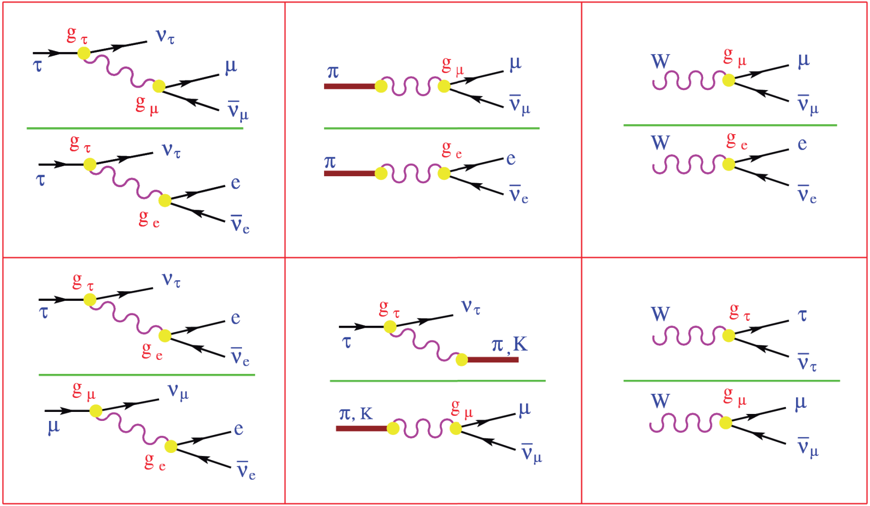


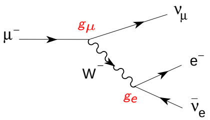
	CMS	LEP	ATLAS	LHCb	CDF	D0
$R_{\mu/e}$	1.009 ± 0.009	0.993 ± 0.019	1.003 ± 0.010	0.980 ± 0.012	0.991 ± 0.012	0.886 ± 0.121
$R_{\tau/e}$	0.994 ± 0.021	1.063 ± 0.027	—	—	—	—
$R_{\tau/\mu}$	0.985 ± 0.020	1.070 ± 0.026	0.992 ± 0.013	—	—	—
$R_{\tau/\ell}$	1.002 ± 0.019	1.066 ± 0.025	—	—	—	—

LEPTON UNIVERSALITY

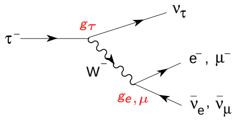
$\frac{g_\mu}{g_e}$

$\frac{g_\tau}{g_\mu}$





Leptonic Decays



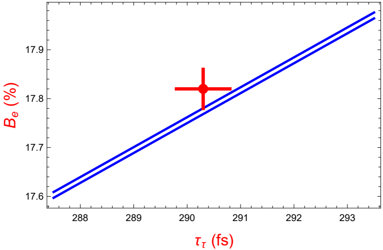
$$\Gamma(\tau \to \nu_\tau \ell \bar{\nu}_\ell) = \frac{G_F^2 m_\tau^5}{192\pi^3} f(m_\ell^2/m_\tau^2) (1 + \delta_{\rm RC})$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

G_F, m_i ➡

$$B_{\tau\to e} = \frac{B_{\tau\to\mu}}{0.972561 \pm 0.000004} = \frac{\tau_\tau}{(1632.7 \pm 0.5) \cdot 10^{-15}\ \rm s}$$

τ_τ (Belle), m_τ (BESIII), B_e (LEP)



$$(B_{\tau\to\mu}/B_{\tau\to e})_{\rm exp} = 0.9762 \pm 0.0028$$

Non-BF:	0.9725 ± 0.0039
BaBar'10:	0.9796 ± 0.0039

➡ $$B_{\tau\to e}^{\rm univ} = (17.815 \pm 0.023)\%$$

CHARGED CURRENT UNIVERSALITY

A. Pich, arXiv:2012.07099 (updated)

$|g_\mu / g_e|$

$B_{\tau\to\mu}/B_{\tau\to e}$	1.0017 ± 0.0016
$B_{\pi\to\mu}/B_{\pi\to e}$	
$B_{K\to\mu}/B_{K\to e}$	
$B_{K\to\pi\mu}/B_{K\to\pi e}$	
$B_{W\to\mu}/B_{W\to e}$	1.001 ± 0.003

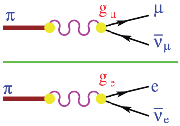
$|g_\tau / g_\mu|$

$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	1.0011 ± 0.0014
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	
$B_{W\to\tau}/B_{W\to\mu}$	1.001 ± 0.010

$|g_\tau / g_e|$

$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0028 ± 0.0015
$B_{W\to\tau}/B_{W\to e}$	1.008 ± 0.012

Leptonic Meson Decays



$$R_{P\to e/\mu} \equiv \frac{\Gamma[P\to e\bar{\nu}_e(\gamma)]}{\Gamma[P\to \mu\bar{\nu}_\mu(\gamma)]} = \left|\frac{g_e}{g_\mu}\right|^2 \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2-m_e^2}{m_P^2-m_\mu^2}\right)^2 \left(1+\delta R_{P\to e/\mu}\right)$$

Helicity suppression

QED/ChPT: $\delta R_{P\to e/\mu} = \Delta^P_{e^2Q^0} + \Delta^P_{e^2Q^2} + \Delta^P_{e^2Q^4} + \cdots + \Delta^P_{e^4Q^0} + \cdots = \begin{cases} -0.03748\ (12) & \pi \\ -0.0360\ \ (5) & K \end{cases}$

Cirigliano-Rosell

$R^{\rm SM}_{\pi\to e/\mu} = (1.23524 \pm 0.00015)\cdot 10^{-4}$, $R^{\rm SM}_{K\to e/\mu} = (2.477 \pm 0.001)\cdot 10^{-5}$

$R^{\rm exp}_{\pi\to e/\mu} = (1.2327 \pm 0.0023)\cdot 10^{-4}$ → $\left|\frac{g_\mu}{g_e}\right| = 1.0010 \pm 0.0009$

PIENU...

$R^{\rm exp}_{K\to e/\mu} = (2.488 \pm 0.009)\cdot 10^{-5}$ → $\left|\frac{g_\mu}{g_e}\right| = 0.9978 \pm 0.0018$

NA62, KLOE...

$K \to \pi \ell \bar{\nu}_\ell$ Decays

$$\Gamma(K_{\ell 3}) = \frac{G_F^2 m_K^5}{192\pi^3}\, C_K^2\, S_{\mathrm{EW}} \left[|V_{us}|\, f_+^{K^0\pi^-}(0) \right]^2 I_{K\ell} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi} \right)^2$$

$$C_{K^0} = 1\,, \qquad C_{K^\pm} = \frac{1}{\sqrt{2}}\,, \qquad S_{\mathrm{EW}} = 1.0232\,(3)\,, \qquad \delta_{\mathrm{SU(2)}}^{K^0\pi^-} = 0\,, \qquad \delta_{\mathrm{SU(2)}}^{K^+\pi^0} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1$$

$$I_{K\ell} = \int dt\, \frac{\lambda^{3/2}}{m_K^8} \left(1 + \frac{m_\ell^2}{2t} \right) \left(1 - \frac{m_\ell^2}{2t} \right)^2 \left[\bar{f}_+^2(t) + \frac{3 m_\ell^2 (m_K^2 - m_\pi^2)^2}{(2t + m_\ell^2)\,\lambda}\, \bar{f}_0^2(t) \right] , \qquad \lambda \equiv \lambda(t, m_K^2, m_\pi^2)\,, \qquad \bar{f}_{+,0}(t) \equiv \frac{f_{+,0}^{K\ell}(t)}{f_+^{K\ell}(0)}$$

$$\longrightarrow \qquad \boxed{\left| \frac{g_\mu}{g_e} \right|^2 = \frac{\Gamma(K_{\mu 3})}{\Gamma(K_{e 3})}\, \frac{I_{Ke}}{I_{K\mu}}\, \frac{\left(1 + 2\,\delta_{\mathrm{EM}}^{Ke}\right)}{\left(1 + 2\,\delta_{\mathrm{EM}}^{K\mu}\right)}}$$

Data: Flavianet 2010, Moulson 2017

$\delta_{\mathrm{EM}}^{K\ell}$: Cirigliano et al 2008, Seng et al 2021

$$\left| \frac{g_\mu}{g_e} \right| = \begin{cases} 1.0022\,(24) & (K_L) \\ 0.9995\,(26) & (K^\pm) \end{cases} \qquad \longrightarrow \qquad \boxed{\left| \frac{g_\mu}{g_e} \right| = 1.0009 \pm 0.0018}$$

Bryman-Cirigliano-Crivellin-Inguglia, 2111.05338

CHARGED CURRENT UNIVERSALITY

A. Pich, arXiv:2012.07099
(updated)

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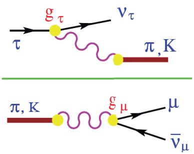
$\tau^- \to \nu_\tau P^-$ Decays



$$R_{\tau/P} \equiv \frac{\Gamma[\tau^- \to \nu_\tau P^-(\gamma)]}{\Gamma[P^- \to \mu^- \bar{\nu}_\mu(\gamma)]} = \left|\frac{g_\tau}{g_\mu}\right|^2 \frac{m_\tau^3}{2 m_P m_\mu^2} \left(\frac{1 - m_P^2/m_\tau^2}{1 - m_\mu^2/m_P^2}\right)^2 (1 + \delta R_{\tau/P})$$

$$\delta R_{\tau/\pi} = \begin{cases} (0.16 \pm 0.14)\% \\ (0.18 \pm 0.57)\% \end{cases} \quad , \quad \delta R_{\tau/K} = \begin{cases} (0.90 \pm 0.22)\% \\ (0.97 \pm 0.58)\% \end{cases}$$

Decker-Finkemeier, 1995

Arroyo-Ureña et al, 2107.04603

$$\left|\frac{g_\tau}{g_\mu}\right|_\pi = 0.9964 \pm 0.0038 \quad , \quad \left|\frac{g_\tau}{g_\mu}\right|_K = 0.9857 \pm 0.0078$$

CHARGED CURRENT UNIVERSALITY

A. Pich, arXiv:2012.07099 (updated)

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$|g_\tau / g_\mu|$

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$|g_\tau / g_e|$

$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0028 ± 0.0015
$B_{W\to\tau}/B_{W\to e}$	1.008 ± 0.012

Four-Fermion Operators

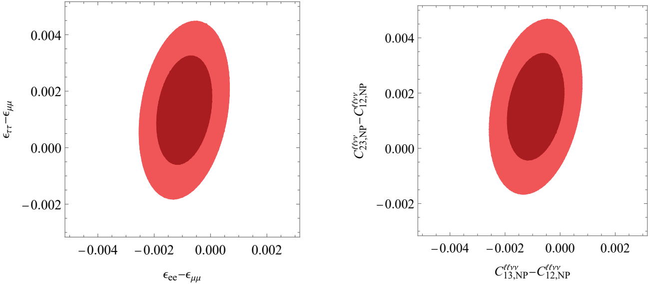
Low-energy EFT:

Cirigliano et al, 0908.1754, 2112.02087

$$\mathcal{L}_{\rm EFT} = -\frac{g_2^2}{2m_W^2}\left(1 + C^{\ell\ell\nu\nu}_{fi,\rm NP}\right)(\bar\ell_f\gamma_\mu P_L\ell_i)(\bar\nu_i\gamma^\mu P_L\nu_f) + \cdots$$

Bryman et al
2111.05338

$g_\ell \equiv g_2\,(1+\epsilon_{\ell\ell})$



➡ Constraints on **SMEFT** operators

➡ Generic constraints on New Physics

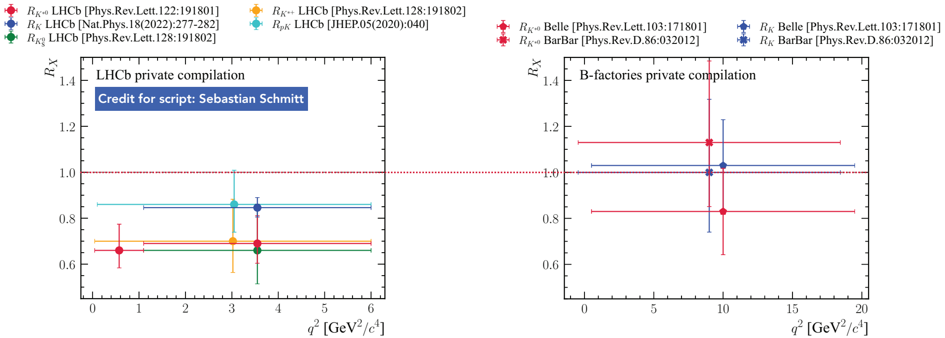
Violations of Lepton Flavour Universality

$$R_H \equiv \frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\Gamma(B\to H\,\mu^+\mu^-)}{dq^2}\, dq^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} \frac{d\Gamma(B\to H\,e^+e^-)}{dq^2}\, dq^2} \stackrel{\rm SM}{=} 1 \pm \mathcal{O}(10^{-2})$$

QED corrections

$$b \to s\,\ell^+\ell^-$$

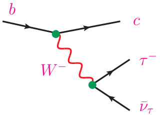
V. Gligorov, ICHEP 2022



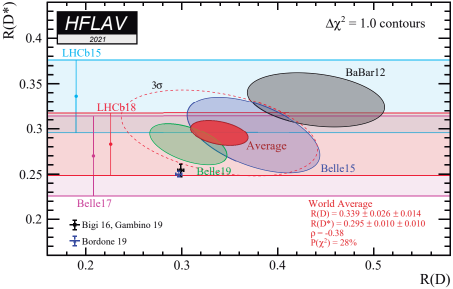
Precision dominated by LHCb, Belle 2 will be able to independently verify with ~10ab^{-1}. Will be interesting to see the eventual impact of the parked CMS dataset.

$$\mathcal{R}_{D^{(*)}} \equiv \frac{\mathcal{B}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell)}$$

Tree-level process



$b \to c\,\ell\nu$



3.4 σ discrepancy

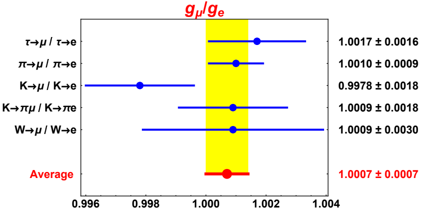
LHCb, 1711.05623: $\mathcal{R}_{J/\psi} \equiv \frac{\mathcal{B}(B_c \to J/\psi\tau\bar{\nu}_\tau)}{\mathcal{B}(B_c \to J/\psi\mu\bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18$ $(1.7\,\sigma)$ $\mathcal{R}^{\mathrm{SM}}_{J/\psi} \approx 0.26 - 0.28$

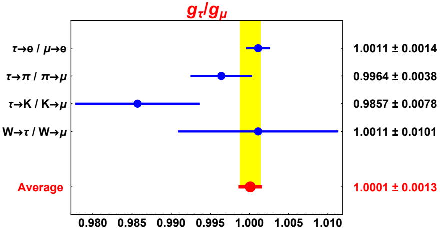
LHCb, 2201.03497: $\mathcal{R}_{\Lambda_b^0 \to \Lambda_c^+} = 0.242 \pm 0.026 \pm 0.040 \pm 0.059$ $\mathcal{R}^{\mathrm{SM}}_{\Lambda_b^0 \to \Lambda_c^+} \approx 0.324 \pm 0.004$

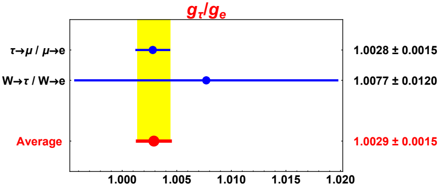
BUT $\frac{\Gamma(B \to D^{(*)}\mu\nu)}{\Gamma(B \to D^{(*)}e\nu)}$ ➡ $\left|\frac{g_\mu}{g_e}\right| = 0.989 \pm 0.012$

Jung-Straub 1801.01112

Lepton-Universality Averages







Complementary tests: sensitive to different new-physics contributions