Status and prospects of the first-row CKM unitarity test

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CKM unitarity

Benchmarks numbers for CKM tests from PDG "12. CKM Quark-Mixing Matrix"

first row:	$\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2 + \lvert V_{ub}\rvert^2 = 0.9985(5)$
second row:	$\lvert V_{cd}\rvert^2 + \lvert V_{cs}\rvert^2 + \lvert V_{cb}\rvert^2 = 1.025(22)$
first column:	$\lvert V_{ud}\rvert^2 + \lvert V_{cd}\rvert^2 + \lvert V_{td}\rvert^2 = 0.9970(18)$
second column:	$\lvert V_{us}\rvert^2 + \lvert V_{cs}\rvert^2 + \lvert V_{ts}\rvert^2 = 1.026(22)$

- **First-row unitarity test**
  - Testing consistency of V_{ud} and V_{us} at precision of a few times 10^{-4}
  - $\lvert V_{ub}\rvert^2 \simeq 1.5 \times 10^{-5}$
  - Deficit of $(2–3)\sigma$ (also deficit in first-column test, but less sensitive)
  - Second row/column more than an order of magnitude away; third row/column $\mathcal{O}(\lambda^4)$
- This talk:
  - Review inputs to first-row test, focus on uncertainties
  - Discuss prospects for improvements
  - Explain why we like pion β decay

Determination of V_{ud} from superallowed β decays

- Master formula Hardy, Towner 2018

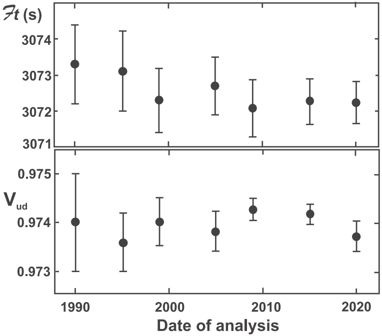
$$|V_{ud}|^2 = \frac{2984.432(3)\,\text{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

with (universal) radiative corrections Δ_R^V

- Value of V_{ud} crucially depends on Δ_R^V:

Ref.	Δ_R^V
Marciano, Sirlin 2006	0.02361(38)
Seng, Gorchtein, Patel, Ramsey-Musolf 2018	0.02467(22)
Czarnecki, Marciano, Sirlin 2019	0.02426(32)
Seng, Feng, Gorchtein, Jin 2020	0.02477(24)
Hayen 2020	0.02474(31)
Shiells, Blunden, Melnitchouk 2021	0.02472(18)
Cirigliano, Crivellin, MH, Moulson 2022	0.02467(27)

$\hookrightarrow$ main uncertainty from Regge region, lattice QCD to improve?



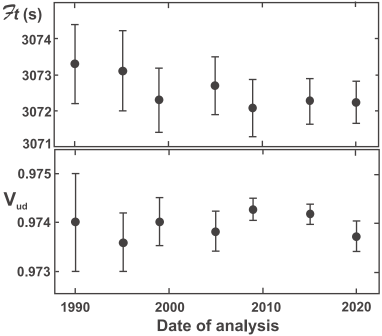
Hardy, Towner 2020

Determination of V_{ud} from superallowed β decays

- Further corrections
  - Isospin breaking Miller, Schwenk 2008, 2009, Condren, Miller 2022, Seng, Gorchtein 2022, Crawford, Miller 2022
  - Nuclear corrections Seng, Gorchtein, Ramsey-Musolf 2018, Gorchtein 2018
- Estimate from Gorchtein 2018 becomes dominant source of uncertainty

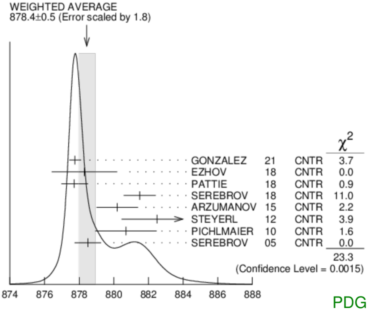
$$V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\text{exp}}(13)_{\Delta_V^R}(27)_{\text{NS}}[32]_{\text{total}}$$

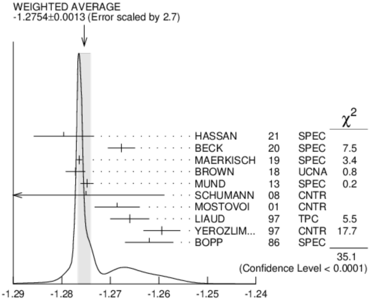
- Improvements from ab-initio nuclear structure? Martin, Stroberg, Holt, Leach 2021



Hardy, Towner 2020

Determination of V_{ud} from neutron decay





PDG 2022

- Master formula Czarnecki, Marciano, Sirlin 2018

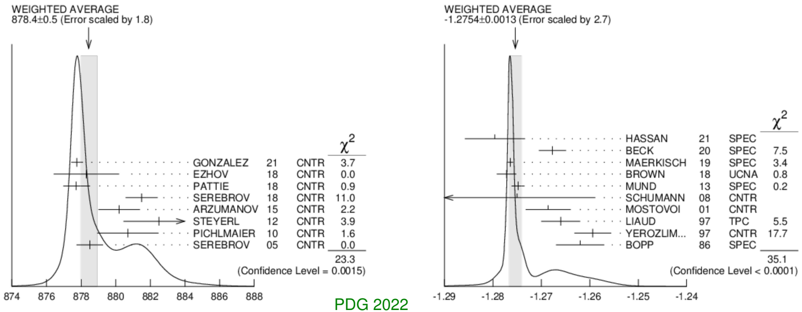
$$|V_{ud}|^2 \tau_n (1 + 3g_A^2)(1 + \Delta_{\text{RC}}) = 5099.3(3)\,\text{s}$$

  with radiative corrections Δ_{RC}

  $\hookrightarrow$ need lifetime τ_n and asymmetry $\lambda = g_A/g_V$

- PDG average especially for g_A includes large scale factors

Determination of V_{ud} from neutron decay



PDG 2022

- Results for V_{ud}

$$V_{ud}^{\text{n, PDG}} = 0.97441(3)_f(13)_{\Delta_R}(82)_\lambda(28)_{\tau_n}[88]_{\text{total}}$$

$$V_{ud}^{\text{n, best}} = 0.97413(3)_f(13)_{\Delta_R}(35)_\lambda(20)_{\tau_n}[43]_{\text{total}}$$

$\hookrightarrow$ average of $V_{ud}^{0^+\to 0^+}$ with $V_{ud}^{\text{n, best}}$ gives $V_{ud}^{\beta} = 0.97384(26)$

- Need improved measurements especially for g_A to make progress

Determination of V_{ud} from pion β decay

- Master formula Cirigliano, Knecht, Neufeld, Pichl 2003, Czarnecki, Marciano, Sirlin 2020, Feng et al. 2020

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu_e(\gamma)) = \frac{G_F^2 |V_{ud}|^2 M_{\pi^\pm}^5 |f_+^\pi(0)|^2}{64\pi^3}(1 + \Delta_{\text{RC}}^{\pi\ell}) I_{\pi\ell}$$

$\hookrightarrow$ need branching fraction and pion life time from experiment

- (Theory) inputs
  - Phase space $I_{\pi\ell} = 7.3766(43) \times 10^{-8}$
  - Form factor $f_+^\pi(0) = 1 - 7 \times 10^{-6}$

    $\hookrightarrow$ protected by $SU(2)$ Ademollo–Gatto theorem (Behrends–Sirlin)
  - Radiative corrections $\Delta_{\text{RC}}^{\pi\ell} = 0.0334(10)$ ChPT, Cirigliano et al., $\Delta_{\text{RC}}^{\pi\ell} = 0.0332(3)$ lattice QCD, Feng et al.

- Resulting V_{ud} extracted from PIBETA 2004

$$V_{ud}^{\pi,\text{ChPT}} = 0.97376(281)_{\text{BR}}(9)_{\tau_\pi}(47)_{\Delta_{\text{RC}}^{\pi\ell}}(28)_{I_{\pi\ell}}[287]_{\text{total}}$$

$$V_{ud}^{\pi,\text{lattice}} = 0.97386(281)_{\text{BR}}(9)_{\tau_\pi}(14)_{\Delta_{\text{RC}}^{\pi\ell}}(28)_{I_{\pi\ell}}[283]_{\text{total}}$$

$\hookrightarrow$ factor 10 possible before other errors creep in (same as for $R_{e/\mu}$)

Determination of V_{us}/V_{ud} from kaon decays: $K_{\ell 2}/\pi_{\ell 2}$

- **$K_{\ell 2}$ decays**: $K \to \ell \nu_\ell$

$$\frac{V_{us}}{V_{ud}}\frac{F_K}{F_\pi} = \left(\frac{\Gamma(K^+ \to \mu^+\nu_\mu(\gamma))M_\pi}{\Gamma(\pi^+ \to \mu^+\nu_\mu(\gamma))M_K}\right)^{1/2} \frac{1-\frac{m_\mu^2}{M_\pi^2}}{1-\frac{m_\mu^2}{M_K^2}} \Bigg(1 - \underbrace{\frac{\Delta_{\text{RC}}^K - \Delta_{\text{RC}}^\pi}{2}}_{\Delta_{\text{RC}}^{K\pi}/2}\Bigg)$$

- Consider the ratio over $\pi_{\mu 2}$ because
  - Only need ratio of decay constant
  - Certain structure-dependent radiative corrections cancel
- Need theory input for:
  - **Decay constants** in isospin limit: $F_K/F_\pi = 1.1978(22)$ HPQCD 2013, Fermilab/MILC 2017, CalLat 2020, ETMC 2021
  - **Isospin-breaking corrections**: $\Delta_{\text{RC}}^{K\pi} = -0.0112(21)$ ChPT, Cirigliano, Neufeld 2011, $\Delta_{\text{RC}}^{K\pi} = -0.0126(14)$ lattice, Di Carlo et al. 2019
- Result:

$$\frac{V_{us}}{V_{ud}}\bigg|_{K_{\ell 2}/\pi_{\ell 2}} = 0.23108(23)_{\text{exp}}(42)_{F_K/F_\pi}(16)_{\text{IB}}[51]_{\text{total}}$$

Determination of V_{us} from kaon decays: $K_{\ell 3}$

- **$K_{\ell 3}$ decays**: $K \to \pi \ell \nu_\ell$

$$\Gamma(K \to \pi \ell \nu_\ell(\gamma)) = \frac{C_K^2 G_F^2 |V_{us}|^2 M_K^5 |f_+^{K\pi}(0)|^2}{192\pi^3} \Big(1 + \underbrace{\Delta_{\text{RC}}^{K\ell}}_{\Delta_{\text{EM}}^{K\ell} + \Delta_{SU(2)}}\Big) I_{K\ell}$$

$\hookrightarrow$ $\ell = \mu$, e and two charge channels

- Need theory input for:
  - **Form factor**: $f_+^{K\pi}(0) = 0.9698(17)$ ETMC 2016, Fermilab/MILC 2019
  - **Radiative corrections**: $\Delta_{SU(2)} = 0.0252(11)$ Cirigliano et al. 2002, $\Delta_{\text{EM}}^{K^0 e} = 0.0116(3)$, $\Delta_{\text{EM}}^{K^+ e} = 0.0021(5)$, $\Delta_{\text{EM}}^{K^0 \mu} = 0.0154(4)$, $\Delta_{\text{EM}}^{K^+ \mu} = 0.0005(5)$ Seng et al. 2022
- Result:

$$V_{us}^{K_{\ell 3}} = 0.22330(35)_{\text{exp}}(39)_{f_+}(8)_{\text{IB}}[53]_{\text{total}}$$

- Global-fit point away from unitarity line
  ($\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 - 1$)

$$V_{ud} = 0.97378(26) \quad V_{us} = 0.22422(36)$$

$$\Delta_{\text{CKM}} = -1.48(53)\times 10^{-3} \quad [2.8\sigma]$$

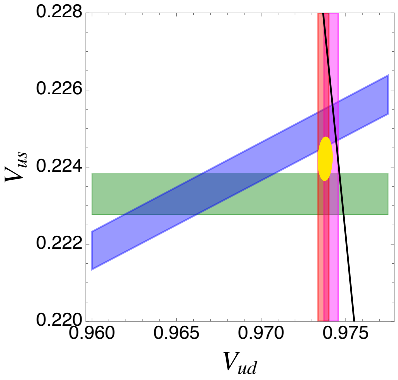
- Three possible measures of the CKM tension

$$\Delta^{(1)}_{\text{CKM}} = \big|V_{ud}^{\beta}\big|^2 + \big|V_{us}^{K_{\ell 3}}\big|^2 - 1 = -1.76(56)\times 10^{-3} \quad [3.1\sigma]$$

$$\Delta^{(2)}_{\text{CKM}} = \big|V_{ud}^{\beta}\big|^2 + \big|V_{us}^{K_{\ell 2}/\pi_{\ell 2},\,\beta}\big|^2 - 1 = -0.98(58)\times 10^{-3} \quad [1.7\sigma]$$

$$\Delta^{(3)}_{\text{CKM}} = \big|V_{ud}^{K_{\ell 2}/\pi_{\ell 2},\,K_{\ell 3}}\big|^2 + \big|V_{us}^{K_{\ell 3}}\big|^2 - 1 = -1.64(63)\times 10^{-2} \quad [2.6\sigma]$$

$\hookrightarrow$ already tension in kaon sector alone 2.6σ



Cirigliano, Crivellin, MH, Moulson 2022

What can we do to clarify the situation?

- Corroborating V_{ud}
  - Nuclear-structure corrections for superallowed β decays
  - Improved neutron-decay measurements (g_A, τ_n)
  - **Pion β decay**
- Corroborating V_{us}
  - Improved lattice calculations of F_K/F_π
  - **A new measurement of $K_{\mu3}/K_{\mu2}$**
  - τ and hyperon decays sensitive to V_{us}, but feasible at the relevant level of accuracy?

A new measurement of $K_{\mu 3}/K_{\mu 2}$, why?

	current fit	$K_{\mu 3}/K_{\mu 2}$ BR at 0.5%			$K_{\mu 3}/K_{\mu 2}$ BR at 0.2%		
		central	$+2\sigma$	-2σ	central	$+2\sigma$	-2σ
$\frac{V_{us}}{V_{ud}}\Big\rvert_{K_{\ell 2}/\pi_{\ell 2}}$	0.23108(51)	0.23108(50)	0.23085(51)	0.23133(51)	0.23108(49)	0.23071(51)	0.23147(52)
$V_{us}^{K_{\ell 3}}$	0.22330(53)	0.22337(51)	0.22360(52)	0.22309(54)	0.22342(49)	0.22386(52)	0.22287(52)
$10^2\Delta_{\text{CKM}}^{(3)}$	$-1.64(63)$	$-1.57(60)$	$-1.18(62)$	$-2.02(63)$	$-1.53(59)$	$-0.83(62)$	$-2.33(62)$
	-2.6σ	-2.6σ	-1.9σ	-3.2σ	-2.6σ	-1.4σ	-3.8σ

- Is the $K_{\ell 3}$ vs. $K_{\ell 2}$ tension real or an experimental problem?
  - $K_{\ell 2}$ data base completely dominated by KLOE 2006
  - Global fit to kaon data not great, p-value $\simeq 1\%$
- This can be clarified with **a new precision measurement of $K_{\mu 3}/K_{\mu 2}$**:
  - In case the tension were of experimental origin, there should be a positive shift compared to current fit

    $\hookrightarrow$ $\Delta_{\text{CKM}}^{(3)}$ would move from -2.6σ to -1.4σ for a $+2\sigma$ shift with a 0.2% measurement
  - In case the tension were of BSM origin, the current value would be confirmed (or move further in the other direction)

$\hookrightarrow$ **a single new precision measurement would have a huge impact!**

Back to pion β decay

- For $K_{\ell 2}$ and $\pi_{\ell 2}$ decays one uses the ratio

$$R_A = \frac{\Gamma(K^+ \to \mu^+\nu_\mu(\gamma))}{\Gamma(\pi^+ \to \mu^+\nu_\mu(\gamma))} = \left(\frac{V_{us}}{V_{ud}}\frac{F_K}{F_\pi}\right)^2 \frac{M_K}{M_\pi}\left(\frac{1-\frac{m_\mu^2}{M_K^2}}{1-\frac{m_\mu^2}{M_\pi^2}}\right)^2 \left(1+\Delta_{\text{RC}}^K - \Delta_{\text{RC}}^\pi\right)$$

  to cancel uncertainties and extract V_{us}/V_{ud}

- Can do the same for $K_{\ell 3}$ and $\pi_{\ell 3}$ Czarnecki, Marciano, Sirlin 2020

$$R_V = \frac{\Gamma(K \to \pi\ell\nu_\ell(\gamma))}{\Gamma(\pi^+ \to \pi^0 e^+\nu_e(\gamma))}$$

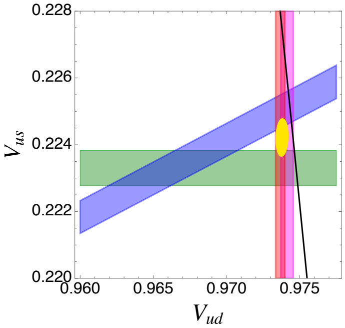
- Need a factor 2–3 to obtain a competitive value of V_{us}/V_{ud}, first goal for PIONEER

- Caveats: contrary to R_A no cancellation of structure-dependent radiative corrections nor gains in form-factor determination

  $\hookrightarrow$ **need factor 10 of Phase III to unleash full potential**

Conclusions

- Tensions among β decays and kaon decays point to the **apparent violation of CKM unitarity**
- Tension at the level of $(2\text{–}3)\sigma$
  $\hookrightarrow$ more work needed to corroborate or resolve
- **Pion β decay** clean, competitive probe of V_{ud} if branching fraction improved by a factor 10
- One order of magnitude to go before other errors become relevant, next one from M_{π^0}, not theory!
- **New precision measurement of $K_{\mu 3}/K_{\mu 2}$** to clarify situation in kaon sector
- Possible BSM interpretations of it all next talk by A. Crivellin



BSM searches with pion β decay

- Generalize master formula to include **effective operators** not present in SM

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu_e(\gamma)) = \frac{G_F^2 |V_{ud}|^2}{192\pi^3 M_\pi^3}(1+\Delta_{\text{RC}}^{\pi\ell}) \int_{m_e^2}^{(M_\pi - M_{\pi^0})^2} ds\, \lambda^{3/2}(s)\left(1+\frac{m_e^2}{2s}\right)\left(1-\frac{m_e^2}{s}\right)^2$$
$$\times \left[|V(s)|^2 + |A(s)|^2 + \frac{4(s-m_e^2)^2}{9sm_e^2}|T(s)|^2 + \frac{3m_e^2(M_\pi^2 - M_{\pi^0}^2)^2}{(2s+m_e^2)\lambda(s)}\left(|S(s)|^2 + |P(s)|^2\right)\right]$$

  with $V(s)$, $A(s)$, …depending on Wilson coefficients c_V, c_A, …

- **Tensor**: $T(s) = \frac{3s}{2s+m_e^2}\frac{m_e}{M_\pi} c_T B_T^\pi(s)$

  $\hookrightarrow$ suppressed by electron mass and tensor form factor

- **Scalar**: more competitive constraints, but still not at the same level as other β decays Falkowski, Gonzáles-Alonso, Naviliat-Cuncic 2020

V_{ud} tension as a sign for the violation of lepton flavor universality?

- Let us parameterize the ***W* couplings** as $\mathcal{L} = -i\frac{g_2}{\sqrt{2}}\bar{\ell}_i\gamma^\mu P_L \nu_j W_\mu \big(\delta_{ij} + \varepsilon_{ij}\big)$
- Modifies Fermi constant in **muon decay**

$$\frac{1}{\tau_\mu} = \frac{(G_F^{\mathcal{L}})^2 m_\mu^5}{192\pi^3}(1+\Delta q)(1+\varepsilon_{ee}+\varepsilon_{\mu\mu})^2$$

$\hookrightarrow$ measured Fermi constant $G_F = G_F^{\mathcal{L}}(1+\varepsilon_{ee}+\varepsilon_{\mu\mu})$

V_{ud} tension as a sign for the violation of lepton flavor universality?

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$\hookrightarrow$ measured Fermi constant $G_F = G_F^{\mathcal{L}}(1+\varepsilon_{ee}+\varepsilon_{\mu\mu})$

- All β-decay observables affected according to

$$V_{ud} \to V_{ud}^\beta = V_{ud}^{\mathcal{L}}(1-\varepsilon_{\mu\mu})$$

where $V_{ij}^{\mathcal{L}}$ fulfill CKM unitarity

- Construct ratio Crivellin, MH 2020

$$R(V_{us}) \equiv \frac{V_{us}^{K_{\mu 2}}}{V_{us}^\beta} \equiv \frac{V_{us}^{K_{\mu 2}}}{\sqrt{1-(V_{ud}^\beta)^2-|V_{ub}|^2}} = 1 - \bigg(\frac{V_{ud}}{V_{us}}\bigg)^2\varepsilon_{\mu\mu} + \mathcal{O}(\varepsilon^2)$$

$\hookrightarrow$ LFUV effect enhanced by $(V_{ud}/V_{us})^2 \sim 20$!

V_{ud} tension as a sign for the violation of lepton flavor universality?

Observable	Measurement	Constraint $\times 10^3$
$\frac{K\to\pi\mu\bar\nu}{K\to\pi e\bar\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	1.0010(25)	1.0(2.5)
$\frac{K\to\mu\nu}{K\to e\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	0.9978(18)	−2.2(1.8)
$\frac{\pi\to\mu\nu}{\pi\to e\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	1.0010(9)	1.0(9)
$\frac{\tau\to\mu\nu\bar\nu}{\tau\to e\nu\bar\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	1.0018(14)	1.8(1.4)
$\frac{W\to\mu\bar\nu}{W\to e\bar\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	0.9960(100)	−4(10)
$\frac{B\to D^{(*)}\mu\nu}{B\to D^{(*)}e\nu} \simeq 1+\varepsilon_{\mu\mu}-\varepsilon_{ee}$	0.9890(120)	−11(12)
$R(V_{us}) \simeq 1-\left(\frac{V_{ud}}{V_{us}}\right)^2\varepsilon_{\mu\mu}$	0.9891(33) (SGPR)	0.58(17)
	0.9927(39) (CMS)	0.39(21)

- Most stringent constraint on $\varepsilon_{\mu\mu}$ thanks to **CKM enhancement**
- Could explain tension between β decays and kaon decays, but not between $K_{\ell 2}$ and $K_{\ell 3}$ (right-handed currents?)
- Best constraint on $\varepsilon_{\mu\mu}-\varepsilon_{ee}$ from

$$R^{\pi}_{e/\mu} = \frac{\Gamma(\pi\to e\nu_e(\gamma))}{\Gamma(\pi\to\mu\nu_\mu(\gamma))}$$

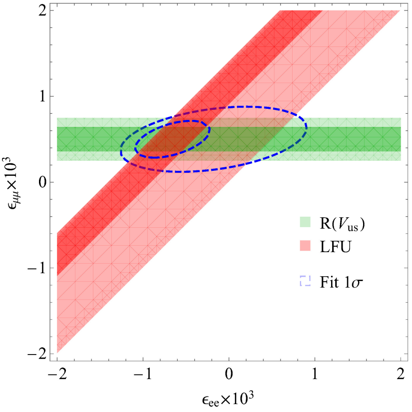
Projections in ε_{ee}–$\varepsilon_{\mu\mu}$ plane

- Potential improvements for $R(V_{us})$
  - Radiative corrections for β decays
  - Improved measurements of τ_n and g_A
  - New data on $K_{\ell 3}$ decays

$\hookrightarrow$ assumed $\sqrt{2}$ due to neutron decay

- Potential improvements for $\varepsilon_{\mu\mu} - \varepsilon_{ee}$
  - **Factor 3 from PEN/PiENu**
  - Factor 3 for τ decays from Belle II

$\hookrightarrow$ would probe ε_{ee} below $\mathcal{O}(10^{-3})$



Crivellin, MH 2020