GPDs in symmetric and asymmetric frames

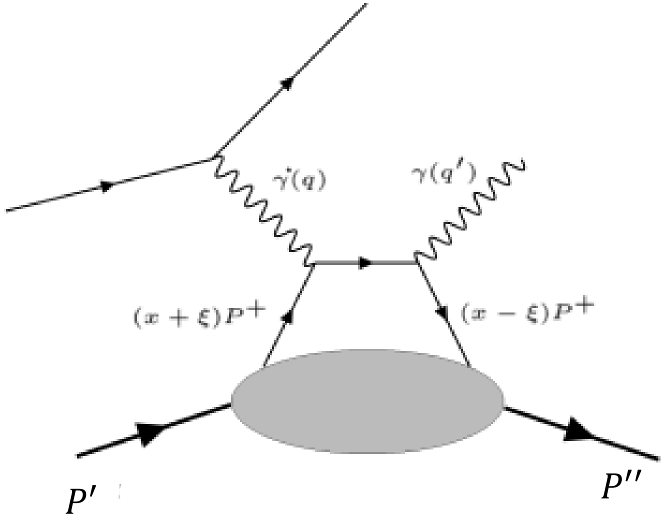
Fernanda Steffens

University of Bonn

With thanks to my collaborators for the works presented here:

C. Alexandrou, S. Bhattacharya, K. Cichy, M. Constantinou, J. Dodson, X. Gao, K. Hadjiyiannakou, K. Jansen, A. Metz, S. Mukherjee, A. Scapellato, Y. Zhao

Generalised PDFs (GPDs)

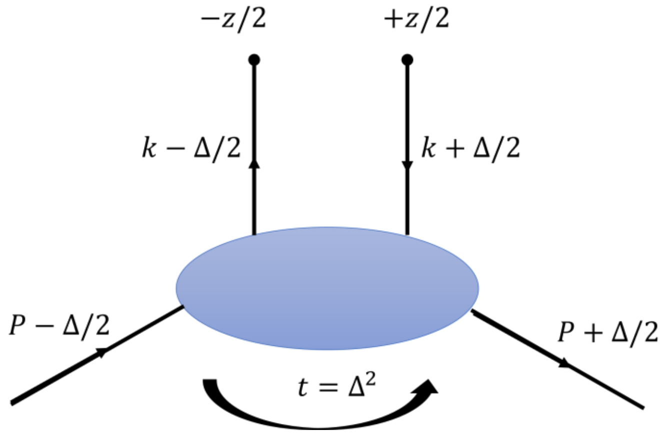


A virtual photon is exchanged, with a real photon measured in the final state

Momentum transfer: $\Delta \equiv P'' - P', \qquad t \equiv \Delta^2,$

Fraction of the momentum transfer: $\xi \equiv -\frac{P''^{+} - P'^{+}}{P''^{+} + P'^{+}} = -\frac{2\Delta^{+}}{P^{+}}$, ξ is called skewness

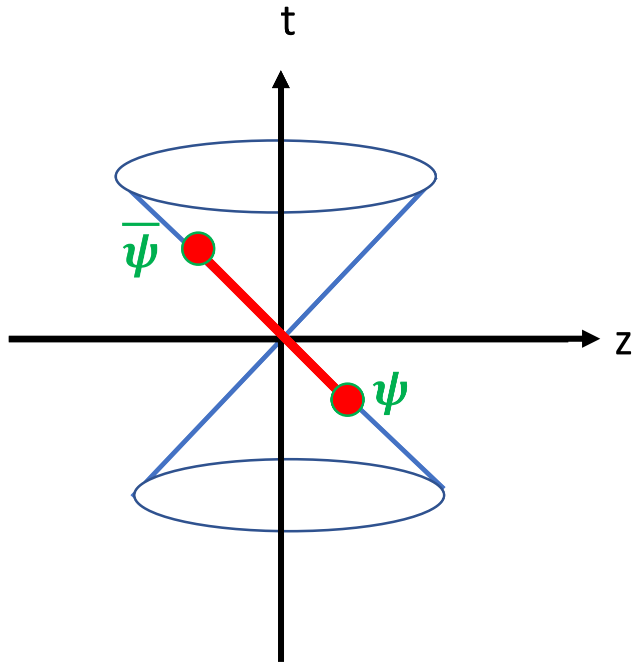
GPDs are multidimensional objects, depending on x, t, ξ



$$\xi = -\frac{2\Delta^+}{P^+}$$

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik.z} \left\langle p';\lambda' \middle| \bar{\psi}\left(-\frac{z}{2}\right)\Gamma \mathcal{W}\left(-\frac{z}{2},\frac{z}{2}\right)\psi\left(\frac{z}{2}\right) \middle| p;\lambda \right\rangle \Big|_{z^+=0,\, z_\perp=0}$$

$$F^{[\gamma^+]}(x,\xi,t) = \bar{u}(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2m}E(x,\xi,t)\right]u(p,\lambda)$$



$f_1^q(x) = H^q(x,0,0)$ PDFs (deep inelastic and semi-inclusive scattering)

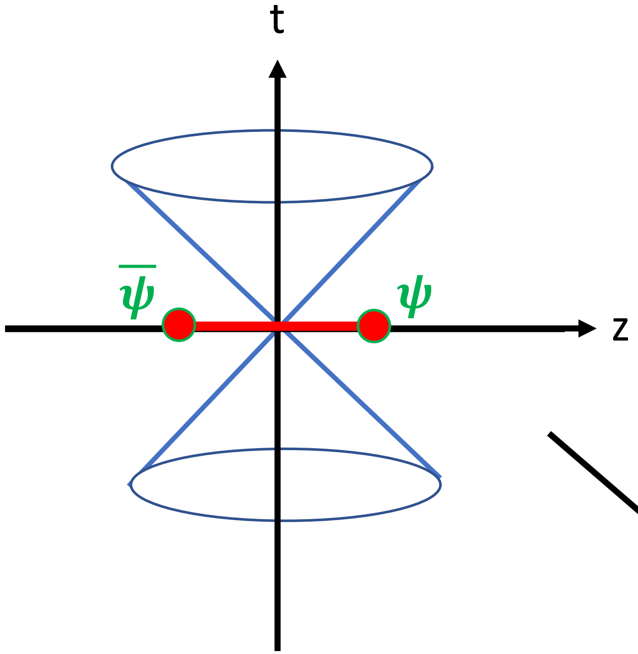
$F_1^q(t) = \int_{-1}^{+1} dx\, H^q(x,t,\xi)$

s (elastic scattering)

ngular momentum

- Light-cone correlation
- Can not be computed on the lattice
- It can be computed in the quasi-PDF approach

GPDs unify momentum, spin, and spatial structure of hadrons

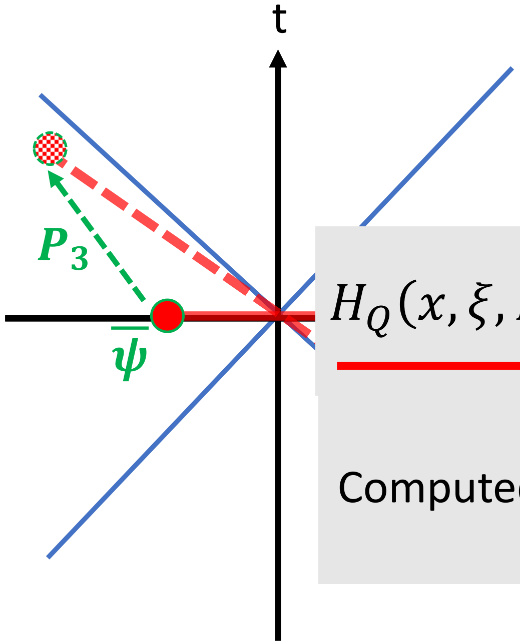


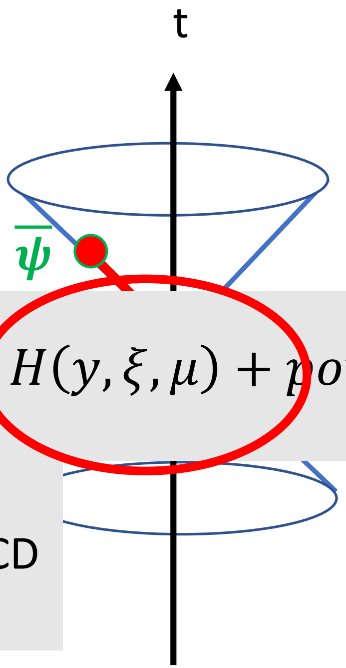
$$\frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik^3 z^3} \left\langle P';\lambda' \middle| \bar{\psi}\left(-\frac{z^3}{2}\right) \Gamma \mathcal{W}\left(-\frac{z^3}{2},\frac{z^3}{2}\right) \psi\left(\frac{z^3}{2}\right) \middle| P;\lambda \right\rangle$$

Purely spatial correlation

X. Ji, PRL 110 (2013) 262002.

We want to go from a purely spatial correlation to a light-front correlation





$$H_Q(x,\xi,P^3,\mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{yP^3}\right) H(y,\xi,\mu) + \textit{power corrections}$$

Computed in LQCD

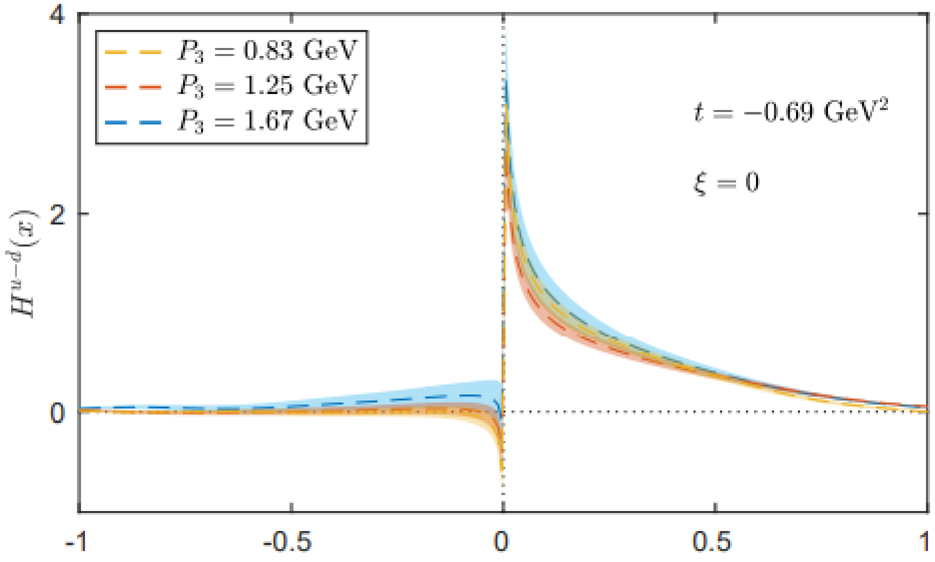
Computed in pQCD

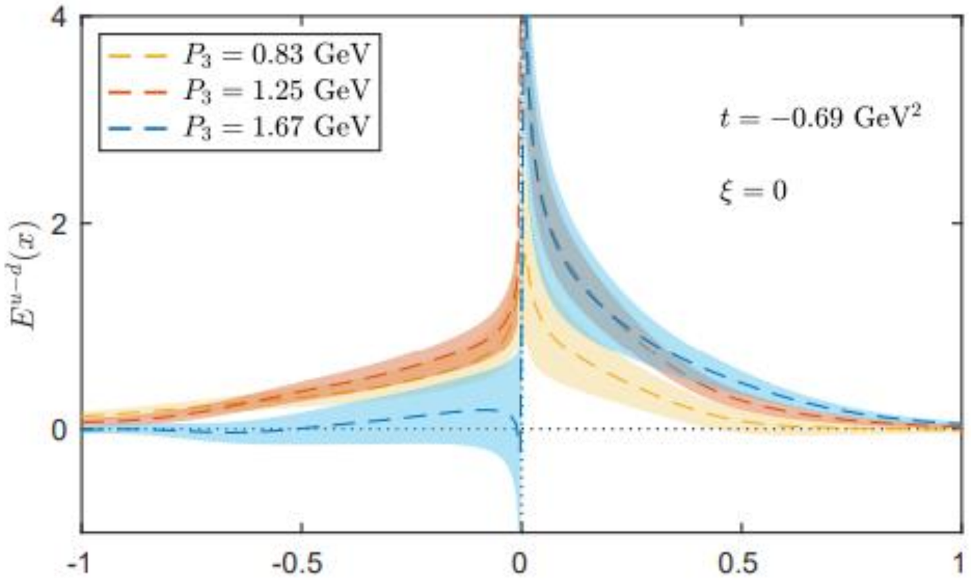
Latt

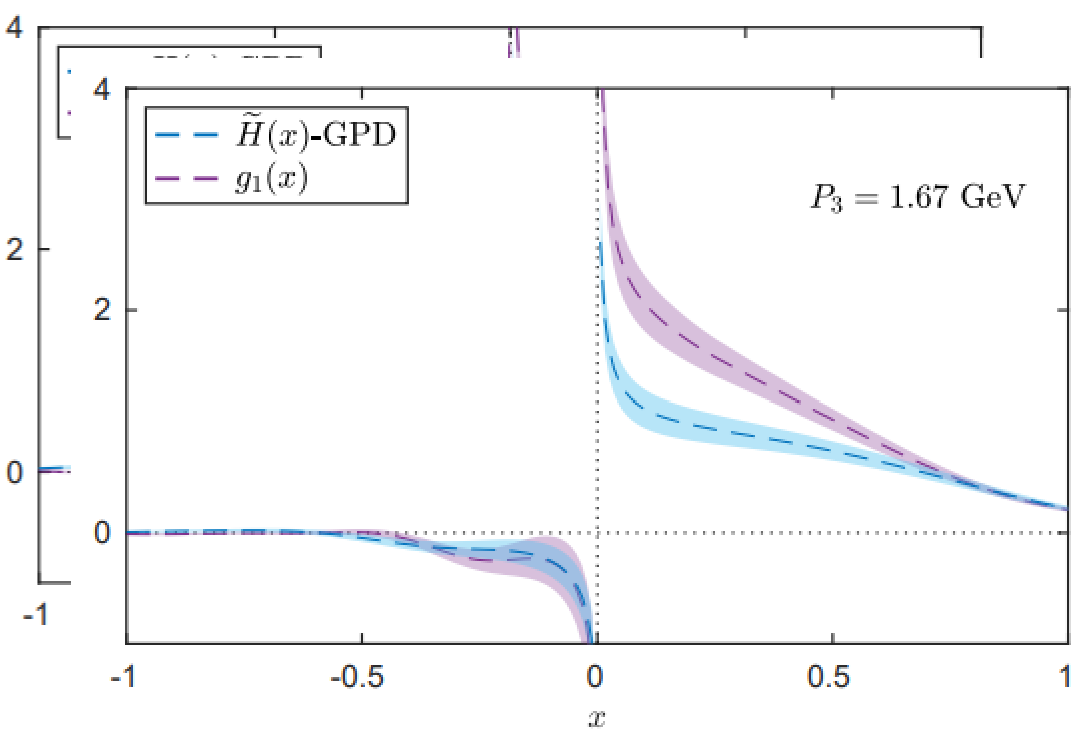
- f
- g
- g
- l
- $32^3 \times 64$ lattice, $L \approx 3$ fm
-

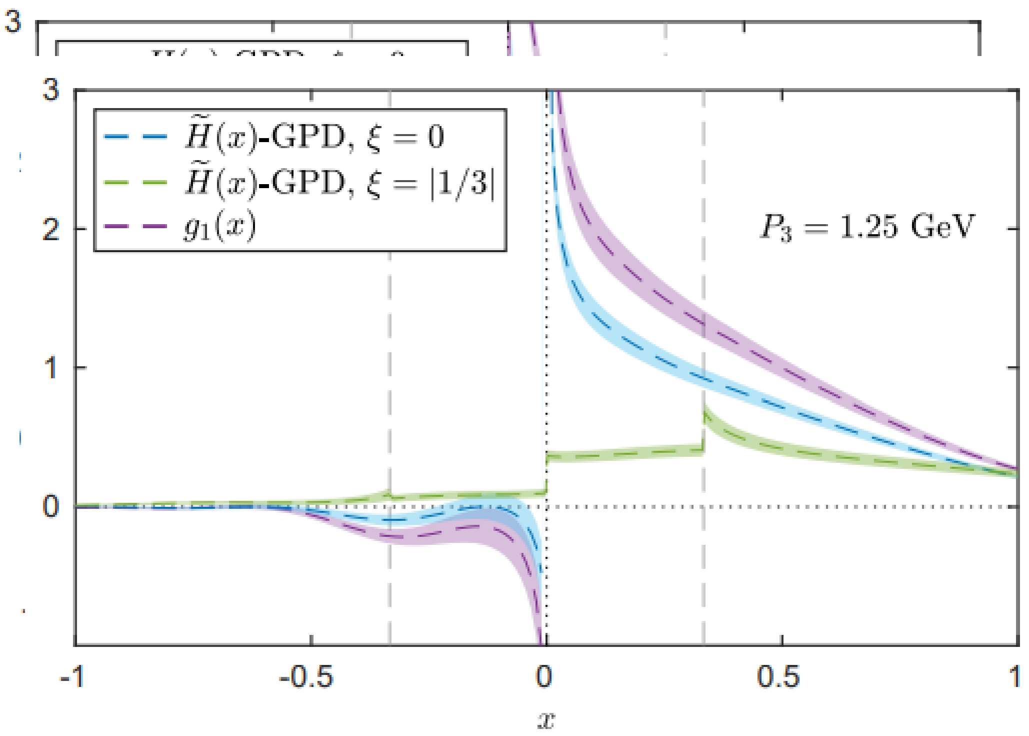
GeV

V^2

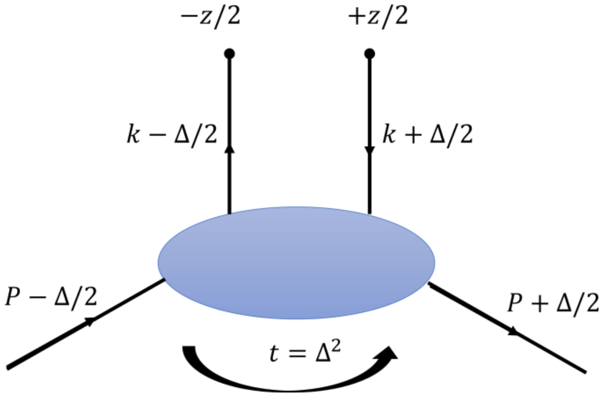


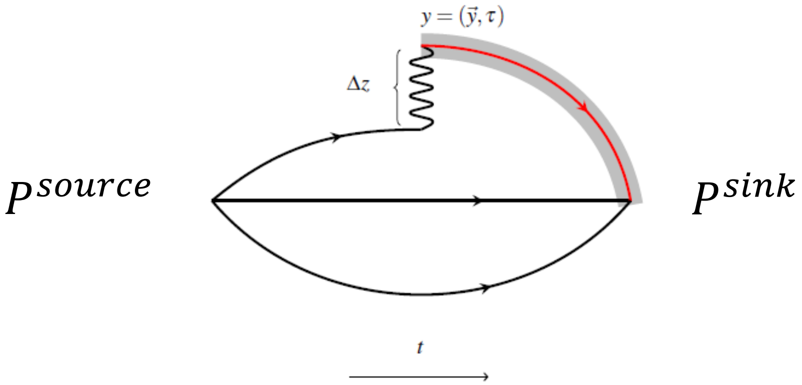






Problem with the current approach: not efficient



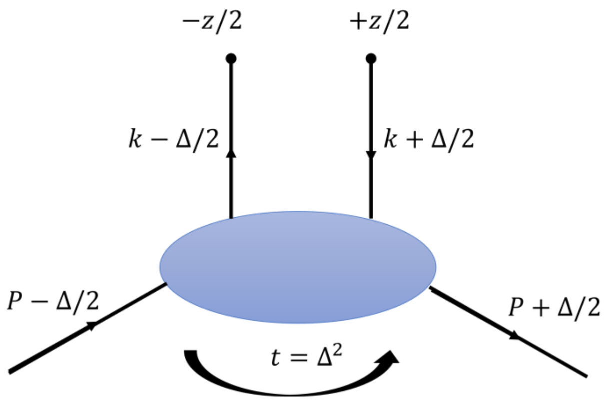


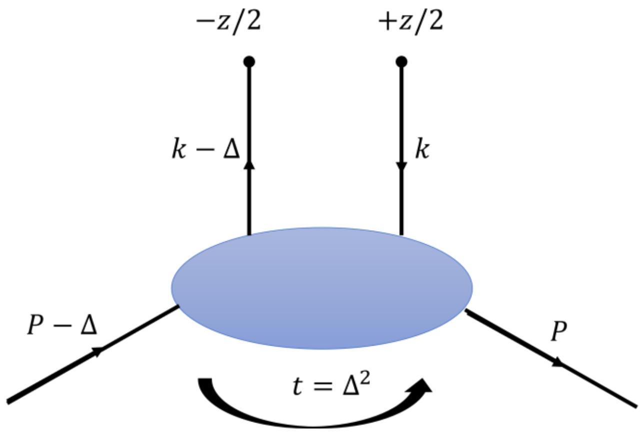
$$P^{source} = \left(-\frac{\Delta_x}{2}, -\frac{\Delta_y}{2}, P_3\right) \qquad P^{sink} = \left(-\frac{\Delta_x}{2}, -\frac{\Delta_y}{2}, P_3\right)$$

- Separate calculation for each momentum transfer: $P^{sink} = \left(-\frac{\Delta_x}{2}, -\frac{\Delta_y}{2}, P_3\right)$
- Much more efficient if $P^{sink} = (0,0,P_3)$

Symmetric and asymmetric frames

S. Bhattacharya et al., arXiv: 2209.05373





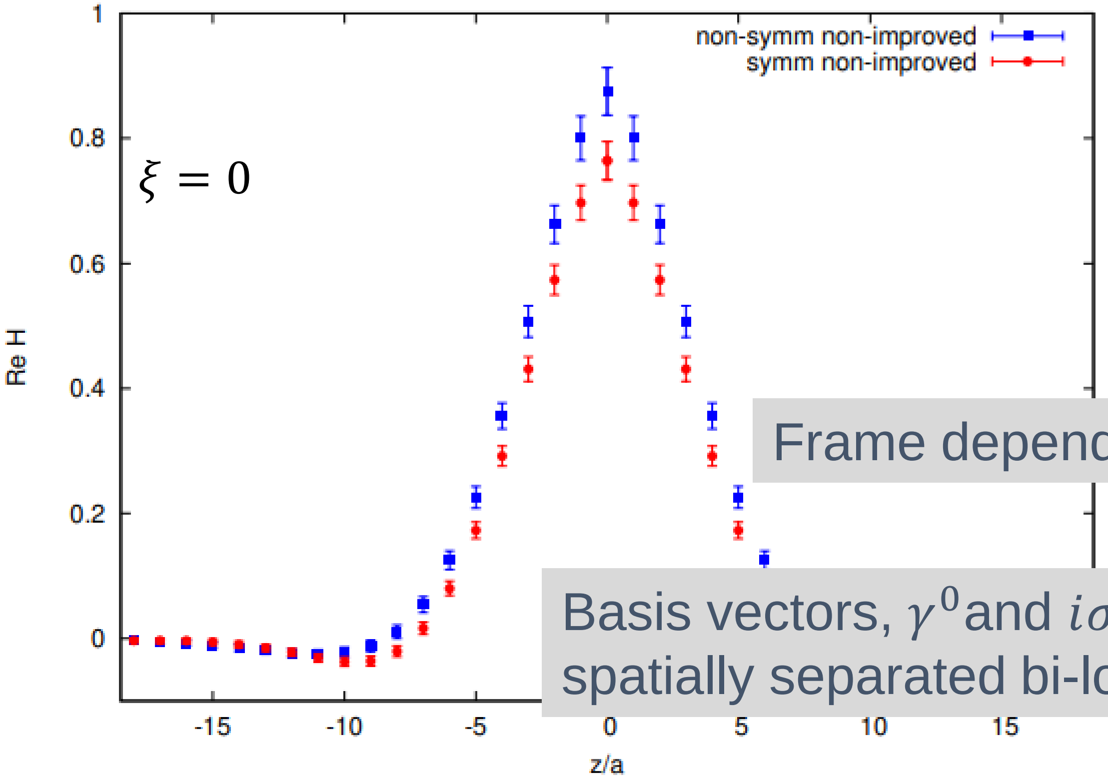
$$\begin{pmatrix} E_{i,s} \\ p^1_{i,s} \\ p^2_{i,s} \\ p^3_{i,s} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_{i,a} \\ -\Delta^1_a \\ 0 \\ P^3 \end{pmatrix}$$

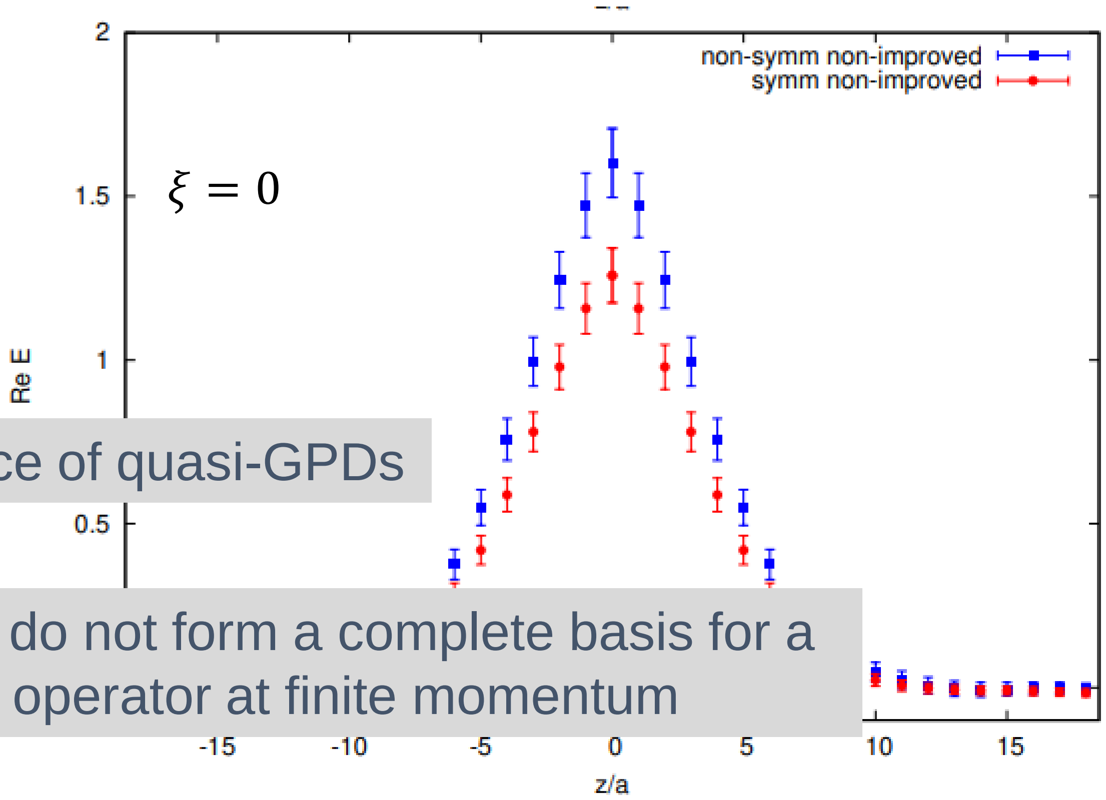
Transverse boost

$$\Longrightarrow \quad \langle \bar{\psi}\gamma^0\psi \rangle^s = \gamma \langle \bar{\psi}\gamma^0\psi \rangle^a - \gamma\beta \langle \bar{\psi}\gamma^1\psi \rangle^a$$

Historical definitions of quasi-GPD

$$F^0(z,P,\Delta) = \left\langle p';\lambda' \middle| \bar{\psi}\left(-\frac{z^3}{2}\right)\gamma^0 \mathcal{W}\left(-\frac{z^3}{2},\frac{z^3}{2}\right)\psi\left(\frac{z^3}{2}\right) \middle| p;\lambda \right\rangle$$





Frame dependence of quasi-GPDs

Basis vectors, $\gamma^0$and $i\sigma^{0\mu}$, do not form a complete basis for a spatially separated bi-local operator at finite momentum

New parametrization of position-space matrix elements

$$F^{\mu}(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}A_1 + mz^{\mu}A_2 + \frac{\Delta^{\mu}}{m}A_3 + im\sigma^{\mu\nu}z_{\nu}A_4 + \frac{i\sigma^{\mu\nu}\Delta_{\nu}}{m}A_5 + \frac{P^{\mu}i\sigma^{\mu\nu}z_{\mu}\Delta_{\nu}}{m}A_6 + mz^{\mu}i\sigma^{\mu\nu}z_{\mu}\Delta_{\nu}A_7 + \frac{\Delta^{\mu}i\sigma^{\mu\nu}z_{\mu}\Delta_{\nu}}{m}A_8\right]u(p_i,\lambda)$$

- General structure of matrix elements based on constraints from Parity
- 8 linearly independent Dirac structures
- 8 Lorentz invariant amplitudes (Form Factors): $A_i = A_i(z\cdot P, z\cdot\Delta, t, z^2)$

Light cone case

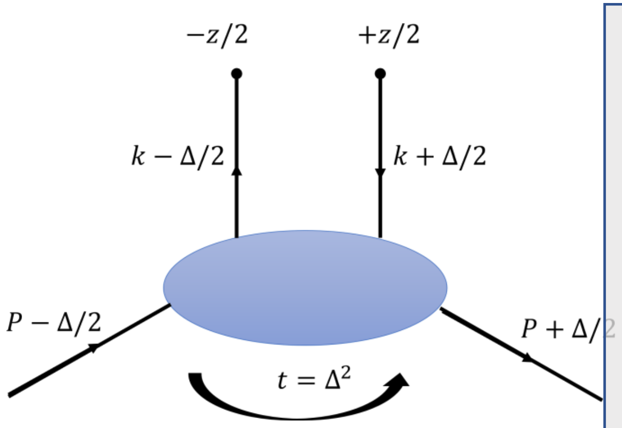
$$F^{+}(z,P,\Delta) = \bar{u}^{s/a}\left(p_f^{s/a},\lambda'\right)\left[\gamma^{+}H\left(z,P^{s/a},\Delta^{s/a}\right) + \frac{i\sigma^{+\nu}\Delta_{\nu}}{m}E\left(z,P^{s/a},\Delta^{s/a}\right)\right]u^{s/a}(p_i^{s/a},\lambda)$$

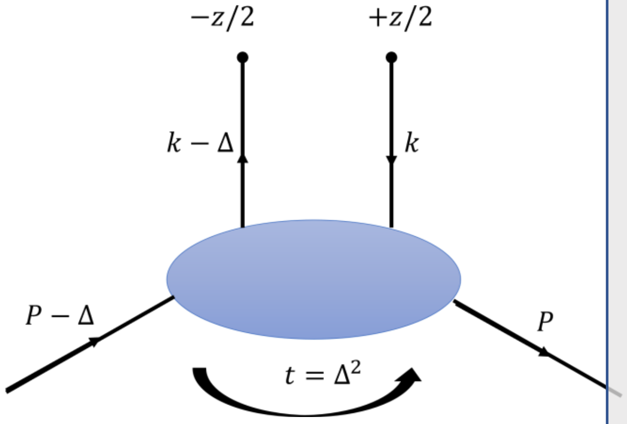
$$H\left(z,P^{s/a},\Delta^{s/a}\right) = A_1 + \frac{\Delta^{+,s/a}}{P^{+,s/a}}A_3$$

$$H\left(z\cdot P_{s/a}, z\cdot\Delta_{s/a}, t_{s/a}, z^2\right) = A_1 + \frac{z\cdot\Delta^{s/a}}{z\cdot P^{s/a}}A_3$$

Lorentz invariant

Quasi case:





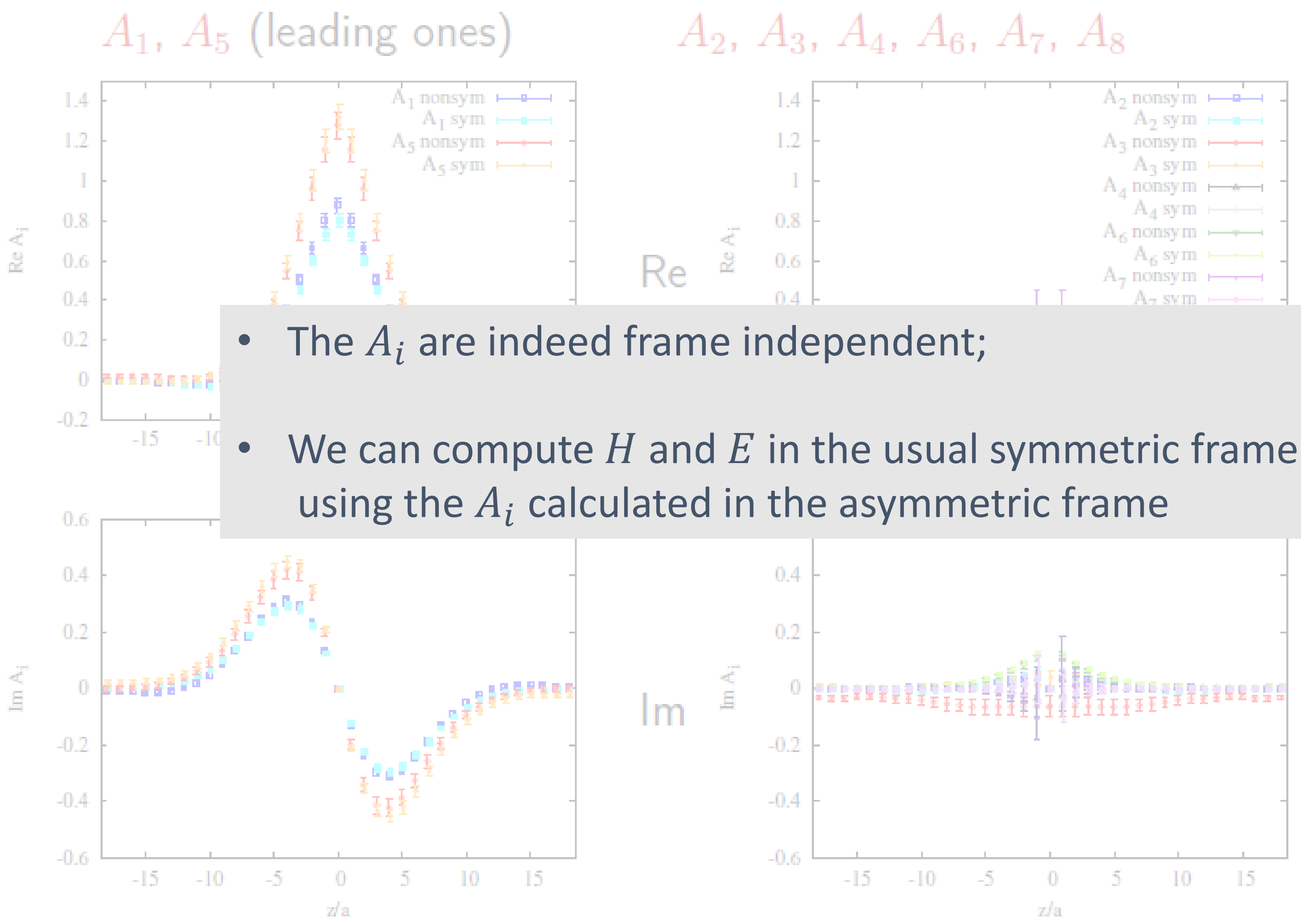
$$\mathcal{H}_0(z,P_s,\Delta_s)\big|_s = A_1 + \frac{\Delta_s^0}{P_s^0}A_3 - \frac{\Delta_s^0 z^3}{2P_s^0P_s^3}A_4 + \left(\frac{(\Delta_s^0)^2 z^3}{2M^2P_s^3} - \frac{\Delta_s^0\Delta_s^3 z^3 P_s^0}{2M^2(P_s^3)^2} - \frac{z^3\Delta_\perp^2}{2M^2P_s^3}\right)A_6 + \left(\frac{(\Delta_s^0)^3 z^3}{2M^2P_s^0P_s^3} - \frac{(\Delta_s^0)^2\Delta_s^3 z^3}{2M^2(P_s^3)^2} - \frac{\Delta_s^0 z^3\Delta_\perp^2}{2M^2P_s^0P_s^3}\right)A_8$$

$$\mathcal{H}_0\left(z,P^{s/a},\Delta^{s/a}\right) \to A_1 + \frac{\Delta^0_{s/a}}{P^0_{s/a}}A_3 \quad \text{in the } P_3 \to \infty \text{ limit}$$

Reduces to the LC result in the IMF limit

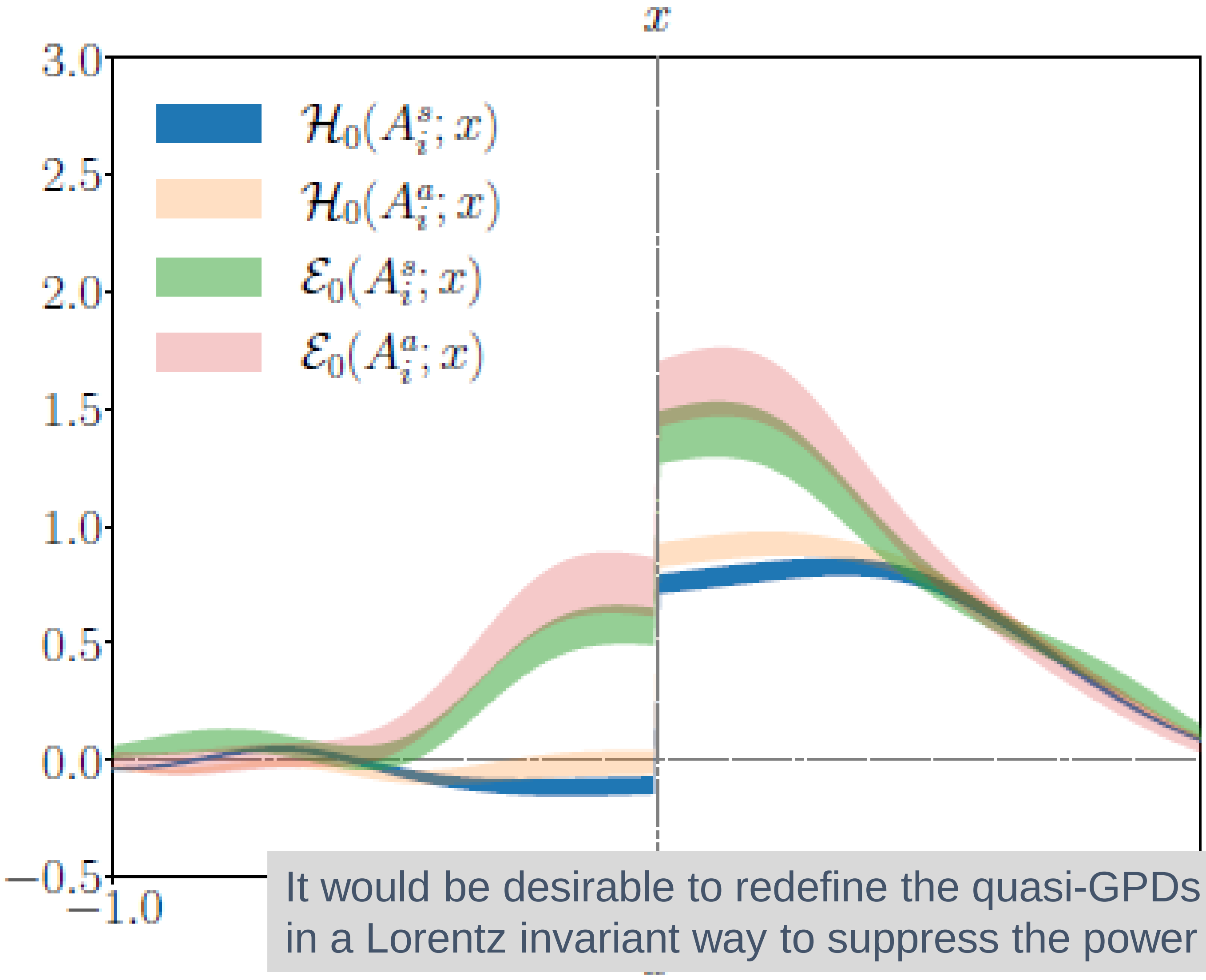
$$\mathcal{H}_0(z,P_s,\Delta_s) = A_1 + \frac{\Delta_a^0}{P^0_{avg,a}}A_3 - \left(\frac{\Delta_a^0 z^3}{2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{\Delta_a^0\Delta_a^3 z^3}{4P^0_{avg,a}(P^3_{avg,a})^2}\right)A_4$$
$$+\left(\frac{(\Delta_a^0)^2 z^3}{2M^2P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{4M^2(P^3_{avg,a})^2} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{P^0_{avg,a}\Delta_a^0\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2}{2M^2P^3_{avg,a}}\right)A_6$$
$$+\left(\frac{(\Delta_a^0)^3 z^3}{2M^2P^0_{avg,a}P^3_{avg,a}} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{(\Delta_a^0)^3\Delta_a^3 z^3}{4M^2P^0_{avg,a}(P^3_{avg,a})^2} - \frac{1}{\left(1+\frac{\Delta_a^3}{2P^3_{avg,a}}\right)}\frac{(\Delta_a^0)^2\Delta_a^3 z^3}{2M^2(P^3_{avg,a})^2} - \frac{z^3\Delta_\perp^2\Delta_a^0}{2M^2P^0_{avg,a}P^3_{avg,a}}\right)A_8$$

Extraction of the A_i in different frames



- The A_i are indeed frame independent;
- We can compute H and E in the usual symmetric frame using the A_i calculated in the asymmetric frame

Computing $\mathcal{H}_0$ and $\mathcal{E}_0$ in the two frames, with $\xi = 0$



It would be desirable to redefine the quasi-GPDs
in a Lorentz invariant way to suppress the power corrections

The Light-cone Lorentz Invariant definitions:

$$H(z\cdot P_{s/a}, z\cdot\Delta_{s/a}, t_{s/a}, z^2) = A_1 + \frac{z\cdot\Delta_{s/a}}{z\cdot P_{s/a}}A_3 \xrightarrow[\xi\,=\,0]{} A_1$$

$$A_i \equiv A_i(z^2 = 0)$$

$$E(z\cdot P_{s/a}, z\cdot\Delta_{s/a}, t_{s/a}, z^2) = -A_1 - \frac{z\cdot\Delta_{s/a}}{z\cdot P_{s/a}}A_3 + 2A_5 + 2\mathrm{z}\cdot P_{s/a}A_6 + 2\mathrm{z}\cdot\Delta_{s/a}A_8 \xrightarrow[\xi\,=\,0]{} -A_1 + 2A_5 + 2\mathrm{z}\cdot P_{s/a}A_6$$

Lorentz Invariant definitions for quasi $(z^2 \neq 0)$:

$$\mathcal{H}(z\cdot P_{s/a}, z\cdot\Delta_{s/a}, t_{s/a}, z^2) = A_1 + \frac{z\cdot\Delta_{s/a}}{z\cdot P_{s/a}}A_3 \to A_1$$

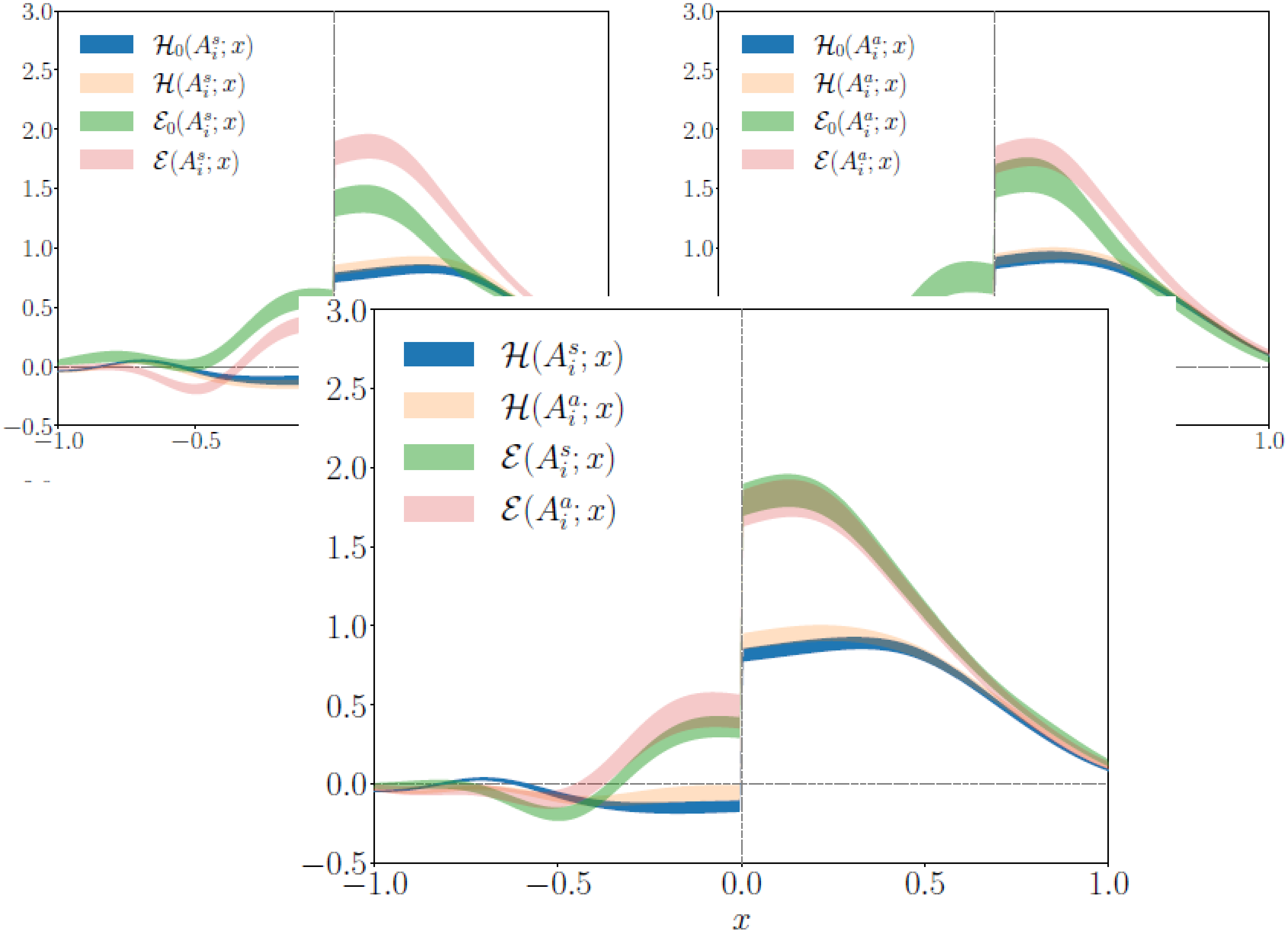
$$\mathcal{E}(z\cdot P_{s/a}, z\cdot\Delta_{s/a}, t_{s/a}, z^2) = -A_1 - \frac{z\cdot\Delta_{s/a}}{z\cdot P_{s/a}}A_3 + 2A_5 + 2\mathrm{z}\cdot P_{s/a}A_6 + 2\mathrm{z}\cdot\Delta_{s/a}A_8 \to -A_1 + 2A_5 + 2\mathrm{z}\cdot P_{s/a}A_6$$

Equivalent to adding extra structures:

$$\mathcal{H}_0 \to c_0\langle\bar{\psi}\gamma^0\psi\rangle$$

$$\mathcal{H} \to c_0\langle\bar{\psi}\gamma^0\psi\rangle + c_1\langle\bar{\psi}\gamma^1\psi\rangle + c_2\langle\bar{\psi}\gamma^2\psi\rangle$$

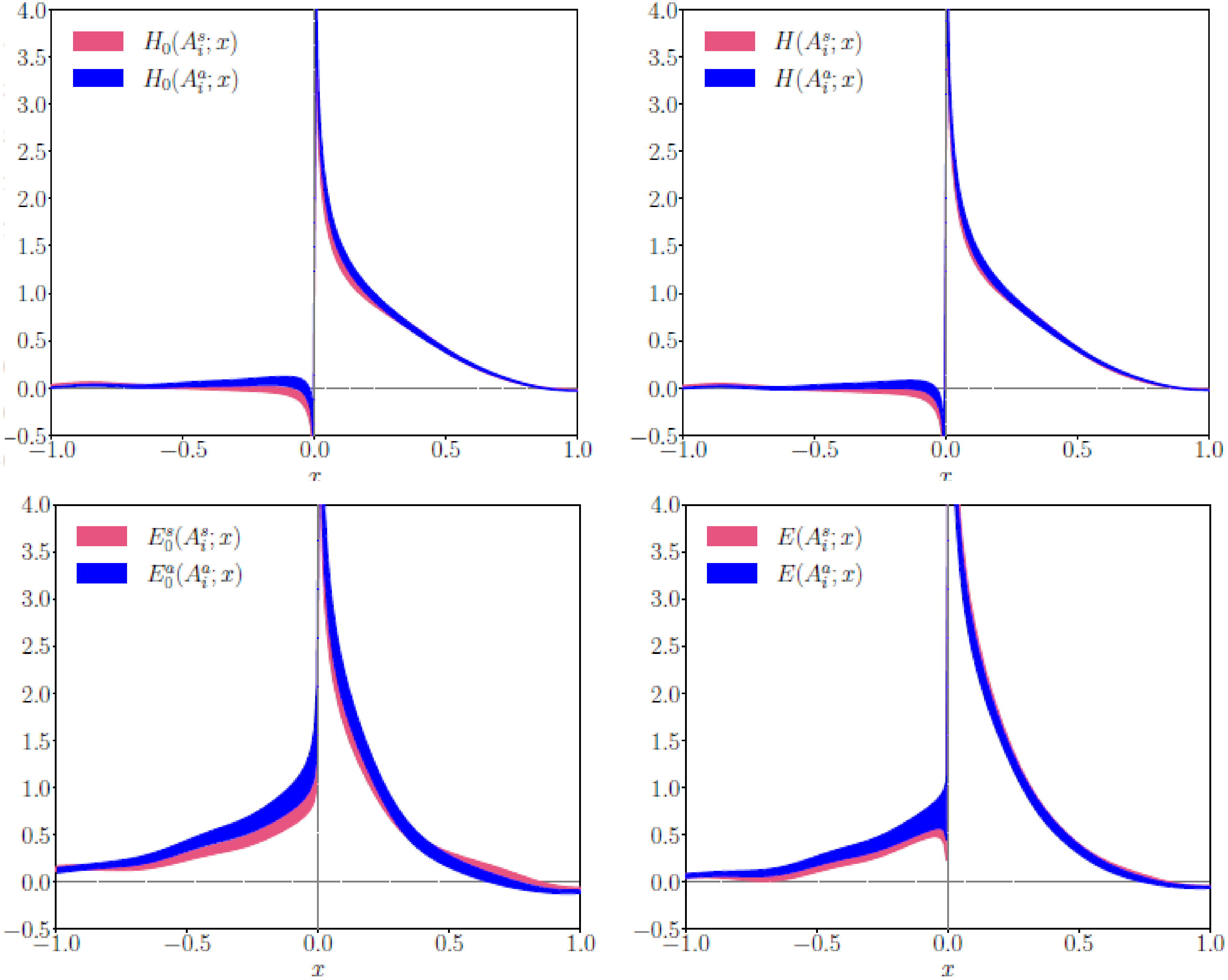
Using the LI definitions



Matching to the LC GPDs

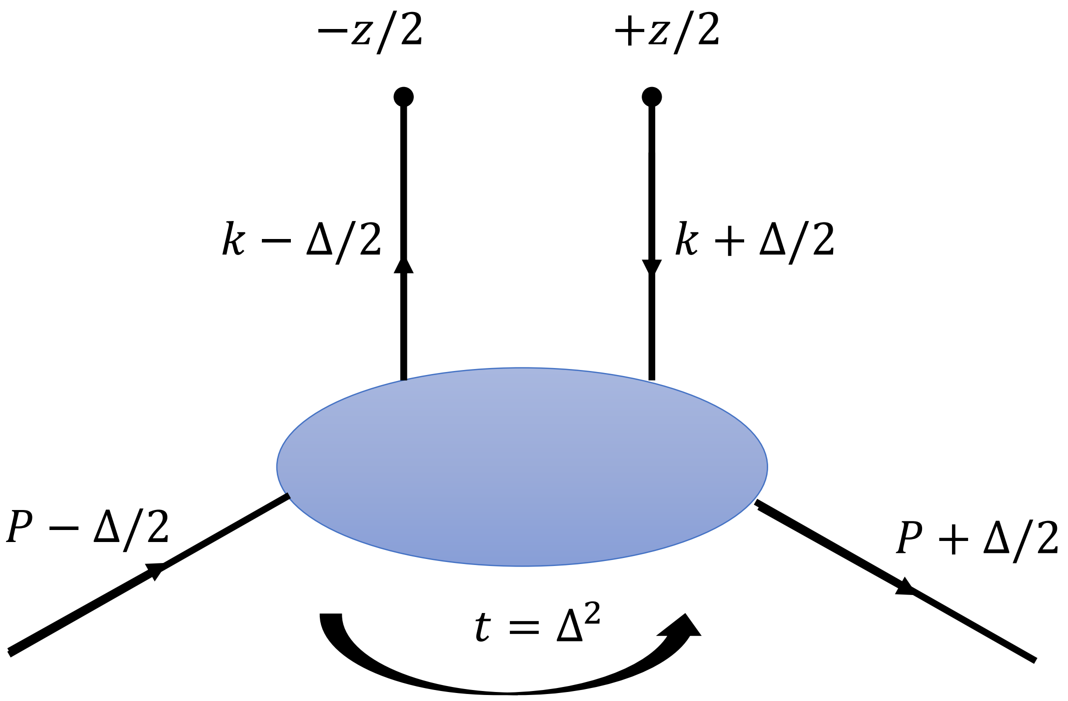
We use the $RI \to \overline{MS}$ matching as computed in

Y.-S. Liu et al., Phys. Rev D 100, 034006 (2019), arXiv:1902.00307


$H_0(A_i^s;x)$
$H_0(A_i^a;x)$
$H(A_i^s;x)$
$H(A_i^a;x)$
$E_0^s(A_i^s;x)$
$E_0^a(A_i^a;x)$
$E(A_i^s;x)$
$E(A_i^a;x)$
x

Summary

- Huge developments on first principles GPDs calculations
- Perform calculations in the asymmetric frame is more efficient
- Can we calculate quasi-GPDs in the symmetric frame through the asymmetric frame?
- Historic definitions of quasi $\mathcal{H}_0$ and $\mathcal{E}_0$ are not manifestly Lorentz invariant
- Lorentz invariant definition of quasi-GPDs may allow for faster convergence to LC GPDs


$-z/2$
$+z/2$
$k - \Delta/2$
$k + \Delta/2$
$P - \Delta/2$
$P + \Delta/2$
$t = \Delta^2$