GPD studies with hard exclusive processes

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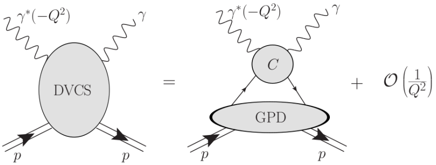
Outline

Accessing GPDs

- exclusive processes such as DVCS and DVMP
- at leading order four complex Compton form factors $\mathcal{H}\left(\xi, t, Q^2\right), \mathcal{E}\left(\xi, t, Q^2\right), \widetilde{\mathcal{H}}\left(\xi, t, Q^2\right), \widetilde{\mathcal{E}}\left(\xi, t, Q^2\right)$
- factorization theorem [Collins et al. '98]



- CFFs are a convolution [Müller '92, et al. '94, Ji, Radyushkin '96]

$$
{}^a\mathcal{H}\left(\xi, t, Q^2\right) = \int \mathrm{d}x\, C^a\left(x, \xi, \frac{Q^2}{Q_0^2}\right) \underbrace{H^a\left(x, \eta = \xi, t, Q_0^2\right)}_{\text{GPD}}, \quad a = q, G
$$

Types of models

1. "Physical" GPD (and CFF) model
2. Neural network parametrization of CFFs

Modelling GPDs

GPD evolution

- evolution in x space complicated, we introduce conformal moments

$$F_n(\eta, t) = \int_{-1}^{1} dx c_n(x, \eta) F(x, \eta, t)$$

$$c_n(x, \eta) = \eta^n \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(1+n)}{2^n\Gamma\left(\frac{3}{2}+n\right)} C_n^{\frac{3}{2}}\left(\frac{x}{\eta}\right)$$

- $C_n^{3/2}$ Gegenbauer polynomials
- analytic continuation $n \to j \in \mathbb{C}$
- evolution diagonal in j space at LO

$$\mu\frac{d}{d\mu}F_j^q\left(\eta, t, \mu^2\right) = -\frac{\alpha_s(\mu)}{2\pi}\gamma_j^{(0)}F_j^q\left(\eta, t^2, \mu^2\right)$$

Valence quark GPDs

- valence quarks modelled in x space ($q = u, d$) at crossover line $x = \eta$ (no Q^2 evolution)

$$\mathfrak{Im}\mathcal{H}(\xi, t) \overset{LO}{=} \pi \left[\frac{4}{9} H^{u_{\rm val}}(\xi, \xi, t) + \frac{1}{9} H^{d_{\rm val}}(\xi, \xi, t) + \frac{2}{9} H^{\textsf{sea}}(\xi, \xi, t) \right]$$

$$H(x, x, t) = n r 2^{\alpha} \left(\frac{2x}{1+x} \right)^{-\alpha(t)} \left(\frac{1-x}{1+x} \right)^{b} \frac{1}{\left(1 - \frac{1-x}{1+x} \frac{t}{M^2}\right)^{p}} .$$

$$\alpha_v(t) = 0.43 + 0.85 t/\mathrm{GeV}^2$$

- fixed parameters: n from PDFs, $\alpha(t)$ Regge trajectory, p counting rules

Sea quark and gluon GPDs

- sea quarks modelled in j space
- $SO(3)$ partial waves expansion

$$F_j(\eta,t) = \sum_{\substack{J=J_{\rm min}\\ \text{even}}}^{j+1} F_j^J(t)\eta^{j+1-J}\hat{d}^J_{\alpha,\beta}(\eta), \quad J = j{+}1, j{-}1, j{-}3, \ldots$$

- leading contribution

$$H_j^a(\eta=0,t) = N^a \frac{\mathrm{B}\left(1-\alpha^a+j, \beta^a+1\right)}{\mathrm{B}\left(2-\alpha^a, \beta^a+1\right)} \frac{\beta(t)}{1-\dfrac{t}{\left(m_j^a\right)^2}},$$

$$\left(m_j^a\right)^2 = \frac{1+j-\alpha^a}{\alpha'^a}, \quad \beta(t) = \left(1-\frac{t}{M^2}\right)^{-p}, \quad a=\{s,g\}$$

- full NLO QCD Q^2 evolution

- partial wave expansion implemented simply in Mellin-Barnes integral

$$\left\{ \begin{array}{c} {}^{s}\mathcal{H} \\ {}^{s}\mathcal{E} \end{array} \right\} = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \xi^{-j-1} \left[i + \tan\left(\frac{\pi j}{2} \right) \right] \times$$

$$\times \left[[\mathbb{C} \otimes \mathbb{E}]_j + [\mathbb{C} \otimes \mathbb{E}]_{j+2} \boldsymbol{S} + [\mathbb{C} \otimes \mathbb{E}]_{j+4} \boldsymbol{T} \right] \left\{ \begin{array}{c} \boldsymbol{H}_j^{(l)} \\ \boldsymbol{E}_j^{(l)} \end{array} \right\}$$

- 10-15 parameters

Cross sections

- DVCS

$$\frac{d\sigma^{\gamma^* N \to \gamma N}}{d\Delta^2} \approx \frac{\pi \alpha_{em}^2}{(W^2 + \mathcal{Q}^2)^2} \left[|\mathcal{H}|^2 + |\widetilde{\mathcal{H}}|^2 - \frac{\Delta^2}{4M^2} |\mathcal{E}|^2 \right]$$

- DVMP

$$\frac{d\sigma^{\gamma^* N \to VN}}{d\Delta^2} \approx \frac{4\pi^2 \alpha_{em} x_B^2}{\mathcal{Q}^4} \left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M^2} |\mathcal{E}|^2 \right]$$

→ for $|\Delta^2| < 1$ GeV^2 CFF $\mathcal{E}$ suppressed by

$$-\frac{\langle \Delta^2 \rangle}{4M^2} \approx 5 \times 10^{-2}$$

→ for $\widetilde{\mathcal{H}}$ Regge intercept $\alpha(0) \approx 1/2$, for $\mathcal{H}$ $\alpha(0) \approx 1$, $\widetilde{\mathcal{H}}$ also suppressed for low x

Dispersion relations

- CFFs constrained by dispersion relations

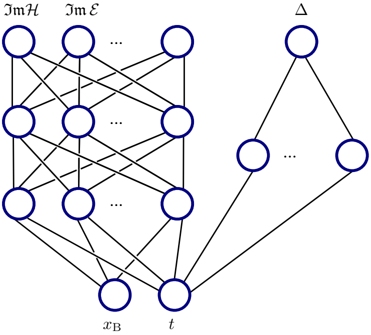
$$\mathfrak{Re}\,\mathcal{H}(\xi,t) \overset{LO}{=} \Delta(t) + \frac{1}{\pi}\mathrm{P.V.}\int_0^1 \mathrm{d}x \left(\frac{1}{\xi - x} - \frac{1}{\xi + x}\right) \mathfrak{Im}\,\mathcal{H}(x,t)$$

- subtraction constant model

$$\Delta(t) = \frac{C}{\left(1 - \frac{t}{M_C^2}\right)^2}$$

- $\Delta_{\mathcal{H}}(t) = -\Delta_{\mathcal{E}}(t)$, $\Delta_{\tilde{\mathcal{H}}}(t) = \Delta_{\tilde{\mathcal{E}}}(t) = 0$
- only imaginary part of CFFs and one subtraction constant $\Delta(t)$ are modelled

Neural networks constrained by dispersion relations

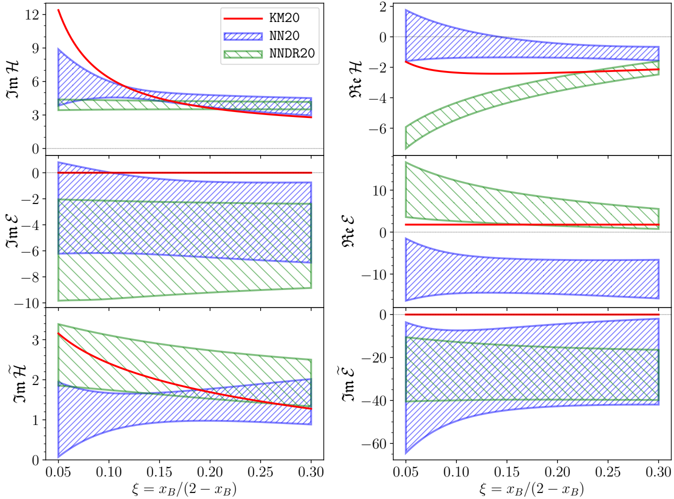


- Only imaginary part of CFFs and one subtraction constant $\Delta(t)$ are parametrized by neural nets

Results

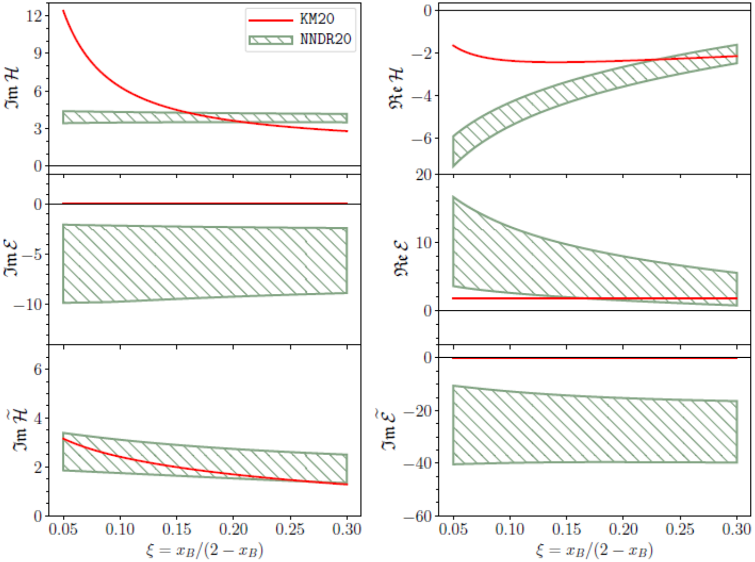
Extraction of 6 CFFs

[M. Č., K. Kumerički, A. Schäfer, '20], from JLab Hall A data



$Q^2 = 4$ GeV^2
$t = -0.2$ GeV^2

Proton DVCS

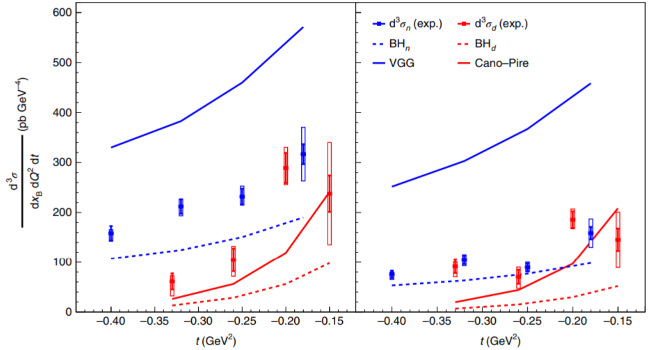


$\mathcal{Q}^2 = 4$ GeV^2

$t = -0.2$ GeV^2

Neutron DVCS

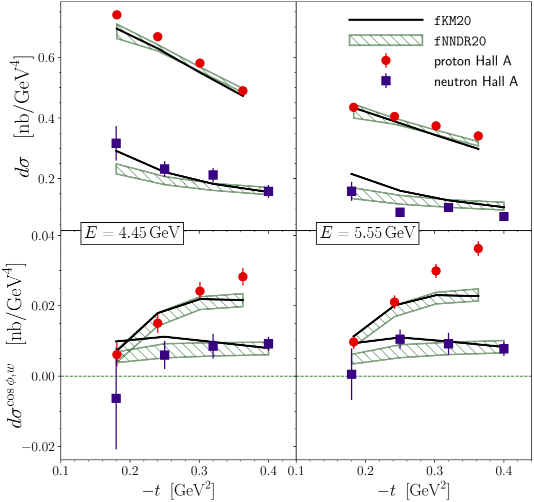
[Benali et al. '20], DVCS off a deuterium target



Using isospin symmetry (e.g. $H^{\text{val}}_{u,\text{proton}} = H^{\text{val}}_{d,\text{neutron}}$) we combine proton and neutron DVCS data to separate up and down quark contributions to CFFs.

Flavor separation

- separate model for each flavor CFF: $\mathcal{H}_u$, $\mathcal{H}_d$
- fKM20 "physical" flavored model, fNNDR neural nets and dispersion relations

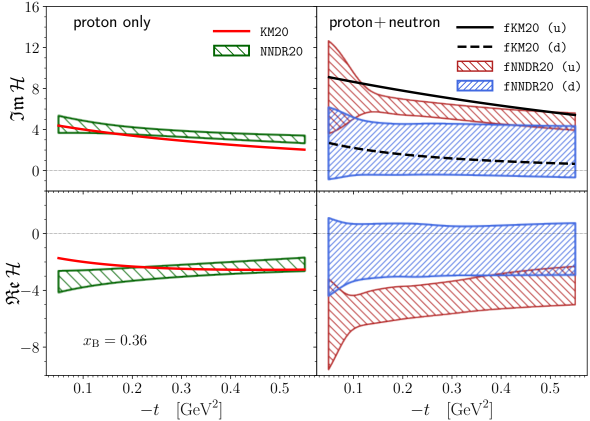


$x_B = 0.36$
$\mathcal{Q}^2 = 1.75$ GeV^2

Flavor CFFs

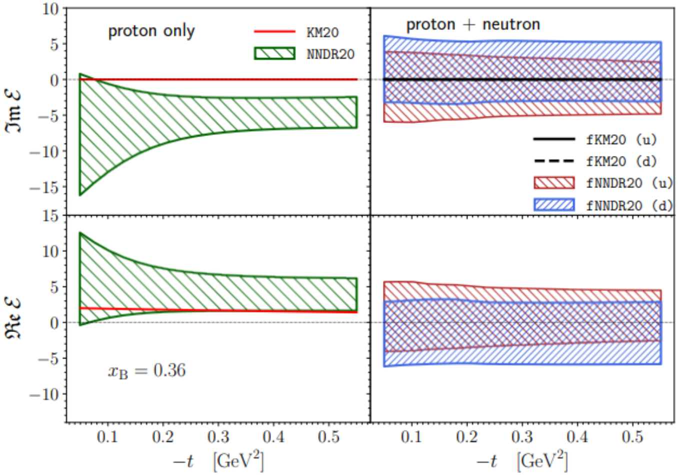
- up and down contributions to CFF $\mathcal{H}$ cleanly separated

$$\mathcal{H} = \frac{4}{9}\mathcal{H}_u + \frac{1}{9}\mathcal{H}_d$$



$x_B = 0.36$
$Q^2 = 4$ GeV^2

- $\mathcal{E}$ cannot be separated



CLAS 12 GeV predictions

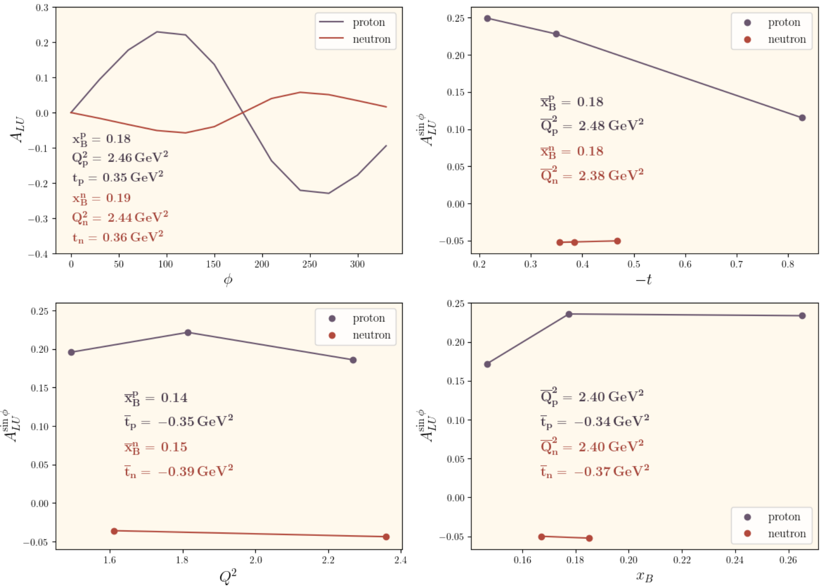
- proton and neutron beam spin asymmetry

$$A_{LU} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

- not a flavored model, assumes only isospin rotation

$$\mathcal{H}_n^{\rm val} = \frac{2e_d^2 + e_u^2}{2e_u^2 + e_d^2}\mathcal{H}^{\rm val} = \frac{2}{3}\mathcal{H}^{\rm val} \,, \quad \mathcal{H}_n^{\rm sea} = \mathcal{H}^{\rm sea}$$

CLAS predictions

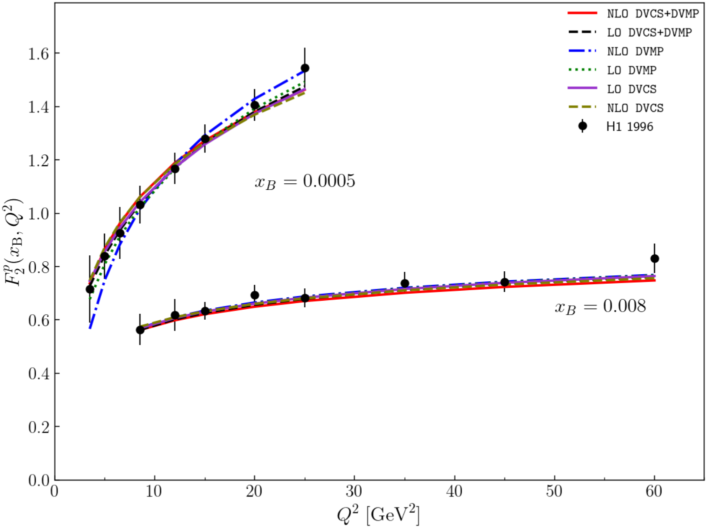


NLO DIS+DVCS+DVMP small-x global fit

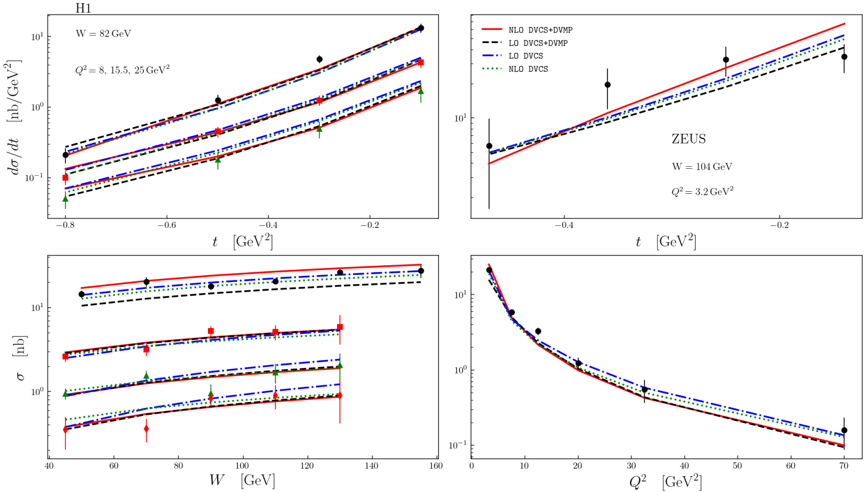
- First global fits to DIS+DVCS+DVMP HERA collider data [Lautenschlager, Müller, Schäfer, '13, unpublished!]
- hard scattering amplitude corrected in the meantime [Duplančić, Müller, Passek-Kumerički '17]

- [M. Č. et al., '22] preliminary results for NLO DIS+DVCS+DVMP small-x global fit
- only considered sea quarks and gluons, full NLO Q^2 evolution
- ρ^0 and ϕ DVMP
- fit to HERA collider data (excluding t-dependent DVMP data): $\chi^2/n_{\text{d.o.f.}} = 205.41/203 \approx 1.01$
- we also studied LO fits, fits to DIS+DVCS and fits to DIS+DVMP
- what are the effects of NLO corrections?
- can we get universal GPDs regardless of DVCS and DVMP data?

DIS F_2 data description

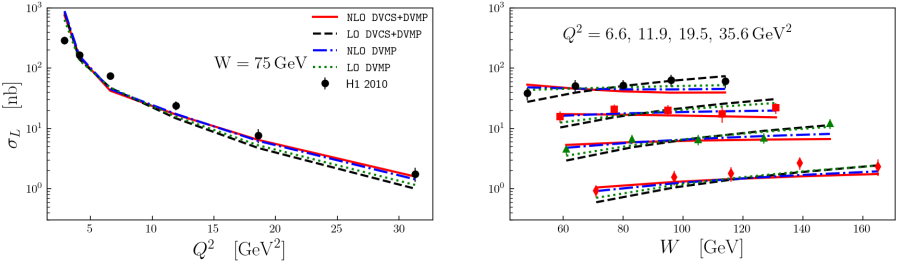


DVCS data description



DVMP data description

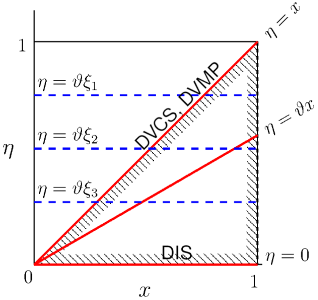
H1 DVMP



Skewness

- skewness: ratio of GPD to corresponding PDF

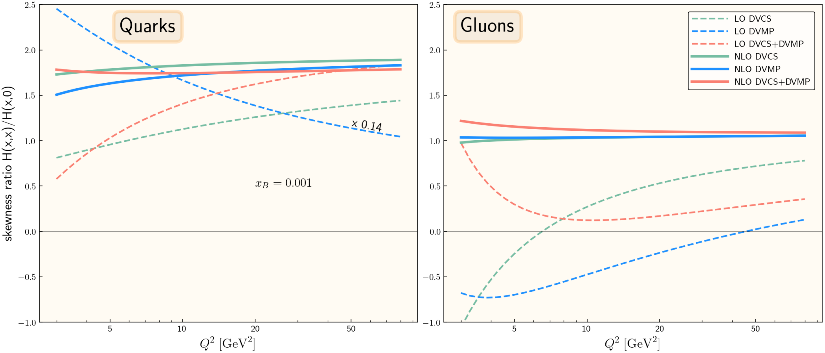
$$r = \frac{H(x, \eta = x)}{q(x)}$$



- conformal (Shuvaev) values, PDFs completely specify GPDs:

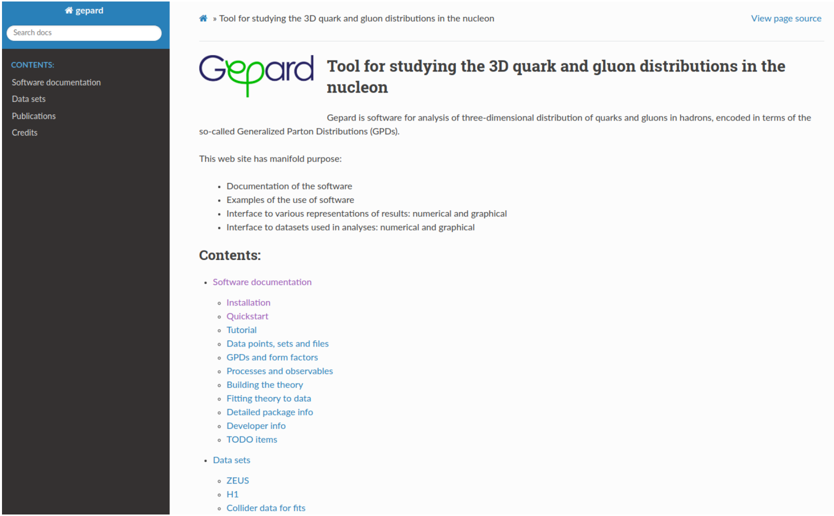
$$r^q \approx 1.65, \quad r^G \approx 1$$

Skewness at LO and NLO



Universal GPD structure emerges at NLO!

Open-source software Gepard https://gepard.phy.hr



Tool for studying the 3D quark and gluon distributions in the nucleon

Gepard is software for analysis of three-dimensional distribution of quarks and gluons in hadrons, encoded in terms of the so-called Generalized Parton Distributions (GPDs).

This web site has manifold purpose:

- Documentation of the software
- Examples of the use of software
- Interface to various representations of results: numerical and graphical
- Interface to datasets used in analyses: numerical and graphical

Contents:

- Software documentation
  - Installation
  - Quickstart
  - Tutorial
  - Data points, sets and files
  - GPDs and form factors
  - Processes and observables
  - Building the theory
  - Fitting theory to data
  - Detailed package info
  - Developer info
  - TODO items
- Data sets
  - ZEUS
  - H1
  - Collider data for fits

Future plans

- improve model so that all parts are in j space
- implement pseudoscalar mesons
- study EIC kinematics