Phenomenology of DDVCS in the era of new experiments

Víctor Martínez-Fernández
PhD student at the National Centre for Nuclear Research
(NCBJ, Warsaw, Poland)

Work in collaboration with:

- **Katarzyna Deja** (NCBJ)
- **Bernard Pire** (CPHT)
- **Paweł Sznajder** (NCBJ)
- **Jakub Wagner** (NCBJ)

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Outline

- Starting point: **GPD**
- Double deeply virtual Compton scattering **(DDVCS)**
  - Goal & motivation
  - Formulation *à la* Kleiss & Stirling
  - Tests of our KS-based formulation
- Summary

Starting point: GPD

- **GPD** = Generalized Parton Distribution ≈ "3D version of a PDF (Parton Distribution Function)." With x the fraction of the hadron's longitudinal momentum carried by a quark:

$$\mathrm{GPD}(x,\eta,t)=\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ix\bar{p}^+z^-}\langle P_2|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathcal{W}[-z/2,z/2]\mathfrak{q}_f(z/2)|P_1\rangle\Big|_{z_\perp=z^+=0}$$

$$t=\Delta^2=(p_2-p_1)^2,\quad \eta=-\frac{q\Delta}{pq},\qquad q=\frac{q_1+q_2}{2},\quad p=p_1+p_2$$

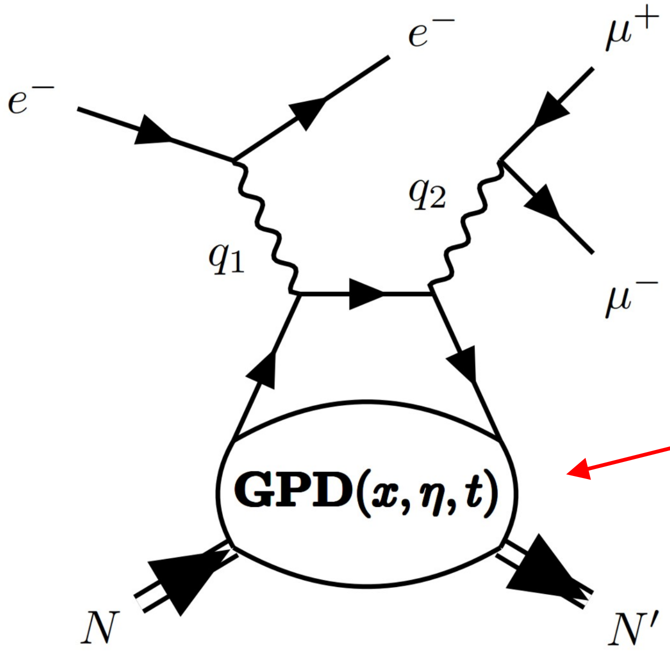
- **Importance:**
  - Connected to QCD energy-momentum tensor, and so to spin. GPDs are a way to address the hadron's **spin puzzle**
  - **Tomography:** distribution of longitudinal momentum on the transverse (to hadron's motion) plane

$$q(x,\mathbf{b}_\perp)=\int\frac{\mathrm{d}^2\boldsymbol{\Delta}}{4\pi^2}e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}}\underline{H^q(x,0,t=-\boldsymbol{\Delta}^2)}$$

A particular GPD

Our goal

- **Goal:** assessment of feasibility of double deeply virtual Compton scattering (DDVCS) at JLab12, EIC and JLab20+. Phenomenology: xsec, asymmetries, etc
- **What is DDVCS?** *Electroproduction of a lepton pair*



$$t = \Delta^2 = (p_2 - p_1)^2, \quad \eta = -\frac{q\Delta}{pq},$$

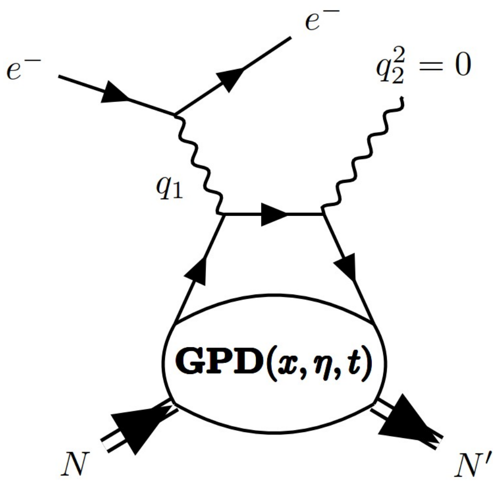
$$q = \frac{q_1 + q_2}{2}, \quad p = p_1 + p_2$$

GPD = Generalized Parton Distribution.
Factorized amplitude into GPD and hard contribution (perturbative term)

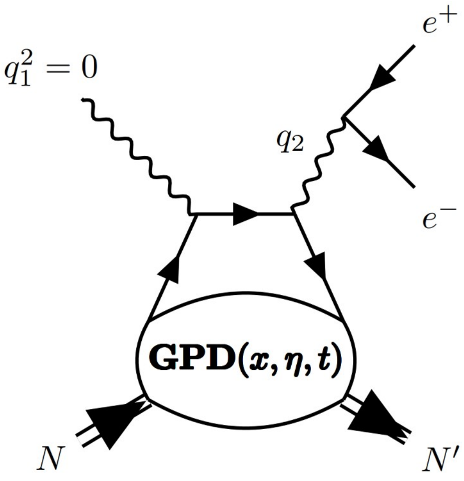
Why DDVCS?

- **Goal:** assessment of feasibility of double deeply virtual Compton scattering (DDVCS) at JLab12, EIC and JLab20+
- **Problem:** currently, GPDs are accessible experimentally via deeply virtual Compton scattering (DVCS), timelike Compton scattering (TCS) and deeply virtual meson production (DVMP) only

DVCS



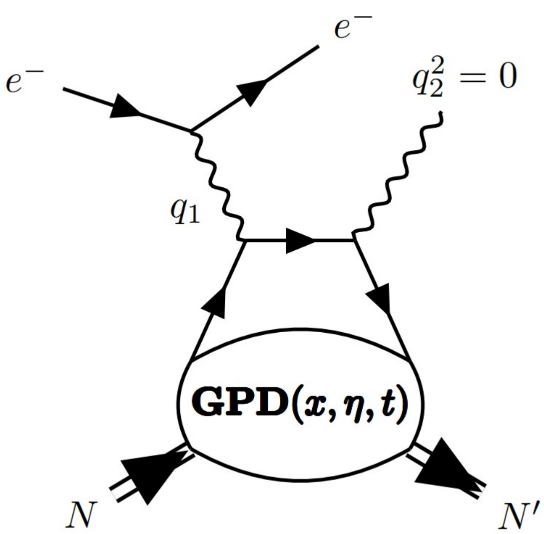
TCS



Why DDVCS?

- **Goal:** assessment of feasibility of double deeply virtual Compton scattering (DDVCS) at JLab12, EIC and JLab20+
- **Problem:** DVCS amplitude allows for measurement of the GPD with restriction to x = η. Similar situation happens with TCS for x = -η

DVCS



Sketch of DVCS amplitude (LO)

$$\mathcal{A}_{\rm DVCS} \sim \int_{-1}^{1} dx \, \frac{1}{x-\eta+i0} \mathrm{GPD}(x,\eta,t) + \cdots$$
$$= \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\eta} \mathrm{GPD}(x,\eta,t)\right) - \int_{-1}^{1} dx \; i\pi\delta(x-\eta)\mathrm{GPD}(x,\eta,t) + \cdots$$

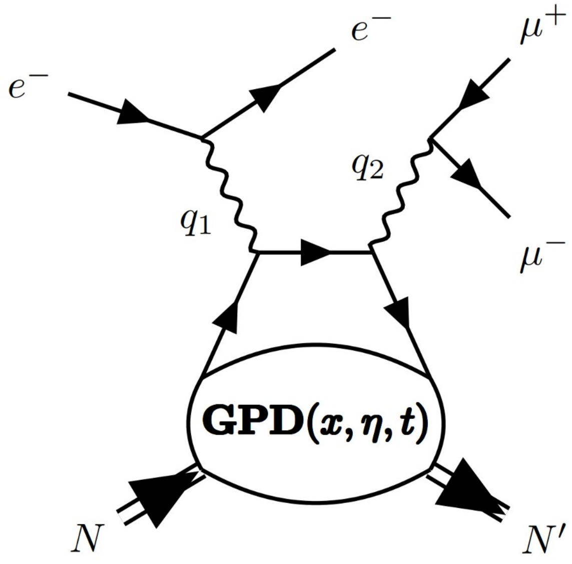
So we can measure GPDs at $x = \eta$ only, i.e., we can access $\mathrm{GPD}(\eta,\eta,t)$

Real part may be expressed by the imaginary part by means of dispersion relations

Why DDVCS?

- **Problem:** DVCS amplitude allows for measurement of the GPD with restriction to x = η. Similar situation happens with TCS for x = -η
- **Solution by DDVCS:** the extra virtuality allows for the introduction of a new (generalized) Bjorken variable ξ so that we can access GPDs for x = ξ ≠ η

DDVCS



Sketch of DDVCS amplitude (LO)

$$\mathcal{A}_{\mathrm{DDVCS}} \sim \int_{-1}^{1} dx \, \frac{1}{x-\xi+i0} \mathrm{GPD}(x,\eta,t) + \cdots$$

$$= \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\xi} \mathrm{GPD}(x,\eta,t)\right) - \int_{-1}^{1} dx \; i\pi\delta(x-\xi)\mathrm{GPD}(x,\eta,t) + \cdots$$

$$\xi = \frac{-q^2}{pq}$$

Formulation *à la* Kleiss & Stirling

- 1st proposed by Belitsky, Mueller, Guidal and Vanderhaeghen in:
  - ***Exclusive Electroproduction of Lepton Pairs as a Probe of Nucleon Structure***, PRL 90, 022001 (2003)
  - ***Double Deeply Virtual Compton Scattering off the Nucleon***, PRL 90, 012001 (2003)
- Xsec by Belitsky and Mueller in ***Probing generalized parton distributions with electroproduction of lepton pairs off the nucleon***, Phys. Rev. D 68, 116005 (2003)
- That work seems to present some typos or mismatches because we cannot reproduce appropriate limits with it: taking a virtuality of DDVCS to be a reality you get either DVCS or TCS
- Consequently, we have performed a **rederivation of DDVCS' formulae** via Kleiss & Stirling's methods

Kleiss & Stirling’s technique (KS): the basics

- **The idea of KS**: compute amplitudes, not the modulus squared of them
- Transform spinor products into new scalars *s* and *t* **(prevents the use of traces of Dirac gamma matrices)**:

$$s(p_1, p_2) := \bar{u}_+(p_1)u_-(p_2) = -s(p_2, p_1)$$

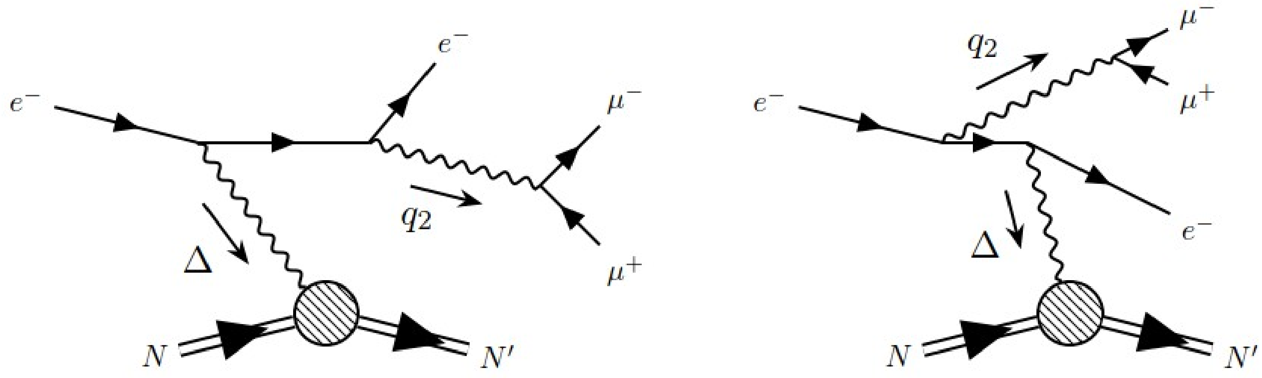
$$t(p_1, p_2) := \bar{u}_-(p_1)u_+(p_2) = [s(p_2, p_1)]^*$$

$$s(p_1, p_2) = (p_1^y + ip_1^z)\sqrt{\frac{p_2^0 - p_2^x}{p_1^0 - p_1^x}} - (p_2 \leftrightarrow p_1)$$

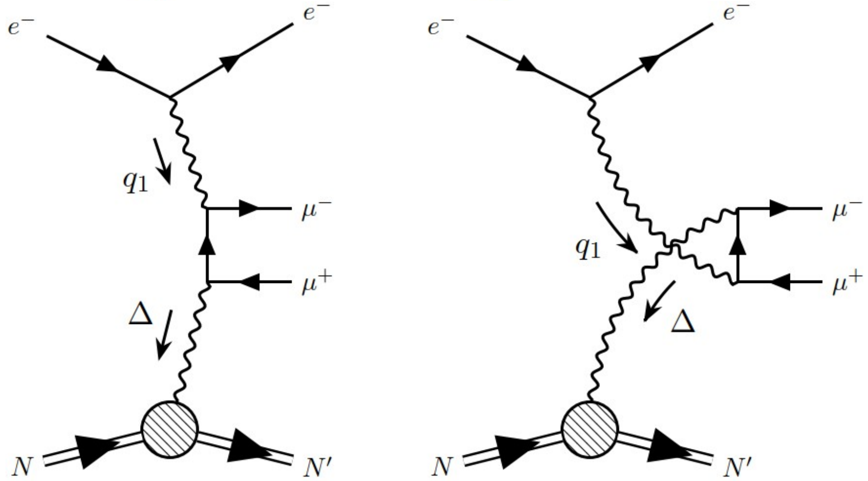
KS’ paper: ***Spinor Techniques for Calculating p anti p → W+-/Z0 +Jets***. Nuclear Physics B262 (1985) 235-262

Not only VCS but also **BH (pure QED)**

BH1 + BH1-crossed =



BH2 + BH2-crossed =



Example: BH1 *à la* KS

- KS application to BH1 diagram of DDVCS:

amplitude

$$i\widetilde{\mathcal{M}}_{\mathrm{BH1}} = \left(\frac{ie^4}{(q_2^2+i0)(\Delta^2+i0)((k-\Delta)^2+i0)}\right)^{-1} i\mathcal{M}_{\mathrm{BH1}}$$

$$\begin{aligned} i\widetilde{\mathcal{M}}_{\mathrm{BH1}} =&(F_1+F_2)\sum_L \Big(Y_{s_2 s_1} f(s_\ell,\ell_-,\ell_+;s,k',L) f(s,L,k;+,r'_{s_2},r_{s_1})+ \\ & Z_{s_2 s_1} f(s_\ell,\ell_-,\ell_+;s,k',L) f(s,L,k;-,r'_{-s_2},r_{-s_1})\Big) - \frac{F_2}{2M} J^{(2)}_{s_2,s_1} \sum_{L,R} f(s_\ell,\ell_-,\ell_+;s,k',L) g(s,L,R,k) \end{aligned}$$

- For example, *f* function is defined as

$$f(s,k_0,k_1;s',k_2,k_3) = u_s(k_0)\gamma^\mu u_s(k_1) u_{s'}(k_2)\gamma_\mu u_{s'}(k_3)$$

that can be expressed by means of *s* and *t* KS scalars

Dedicated softwares

➔ PARTONS platform: open-source C++ program

- Contains several GPD models
- Leading twist… but higher twist corrections will be included in near future
- Useful for theorists and experimentalists
- Provides xsecs, Compton Form Factors, etc
- DVCS, TCS and DVMP are already included

PARtonic Tomography Of Nucleon Software

To download and for tutorials:
http://partons.cea.fr

Description of architecture:
Eur. Phys. J. C78 (2018), 478

Dedicated softwares

➔ EpIC Monte Carlo event generator in C++

- Uses PARTONS framework
- Includes radiative corrections
- Generates the kinematic configurations following the probability distributions given by PARTONS
- DVCS, TCS and DVMP are already included

Access EpIC via GitHub:
https://github.com/pawelsznajder/epic

Detail description and architecture:
arXiv:2205.01762 [hep-ph]

DVCS & TCS limits of DDVCS

- **DDVCS to DVCS**

$$\int d\Omega_\ell \underbrace{\frac{d^7\sigma}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_\ell}}_{\text{DDVCS}} \xrightarrow{Q'^2 \to 0} \underbrace{\left(\frac{d^4\sigma}{dx_B dQ^2 d|t| d\phi}\right)}_{\text{DVCS}} \frac{N}{Q'^2}, \qquad N = \frac{\alpha_{em}}{3\pi}$$

- **DDVCS to TCS**

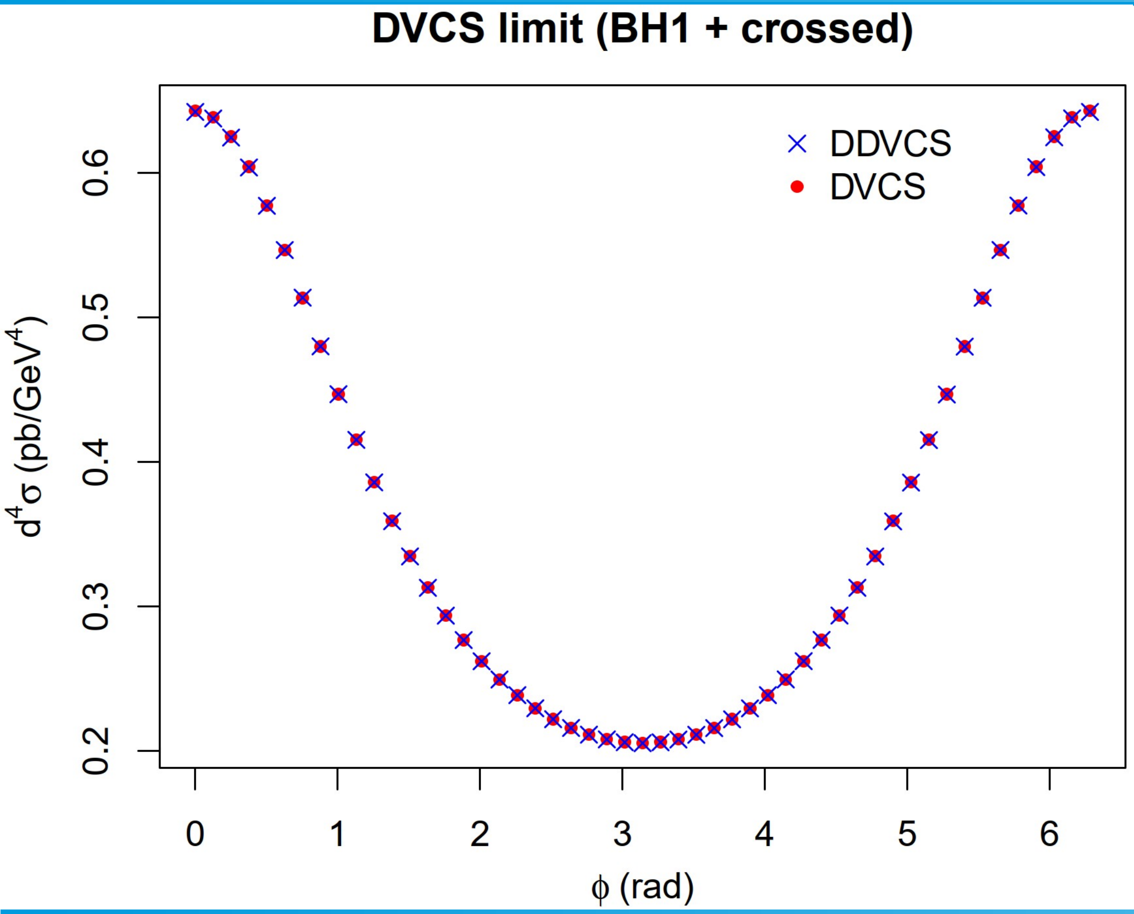
Evaluate energy of incoming virtual photon to be used as energy of TCS photon beam

$$\nu = \frac{Q^2}{2Mx_B}$$

Divide by flux Γ and get rid of x_B and Q^2 differentiation

$$\Gamma = \frac{\alpha_{em}}{2\pi Q^2}\left(1 + \frac{(1-y)^2}{y} - \frac{2(1-y)Q^2_{\text{min}}}{yQ^2}\right)\frac{\nu}{Ex_B}, \qquad Q^2_{\text{min}} = \frac{(ym_e)^2}{1-y}$$

DVCS & TCS limits of DDVCS



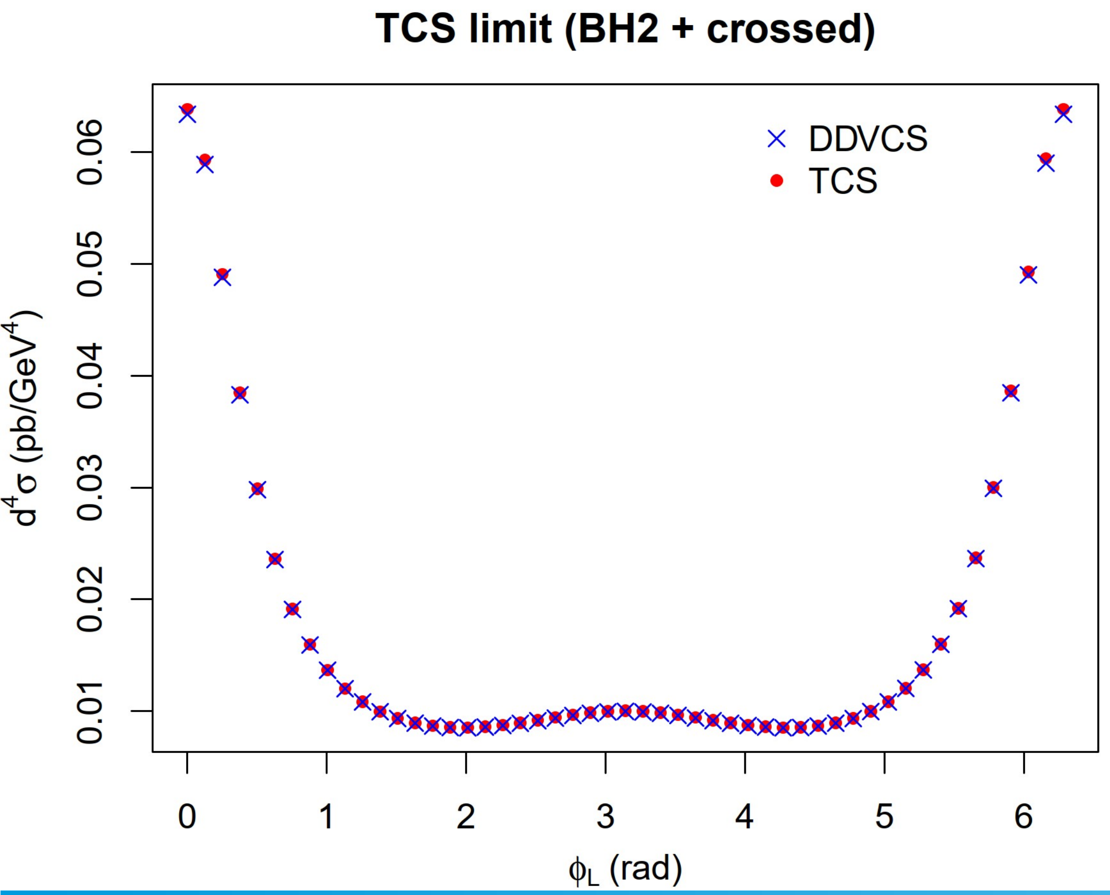
x_B = 0.04, t = -0.1 GeV^2, $-q_1^2$ = 10 GeV^2, $q_2^2 \approx$ 0.001 GeV^2, E_{beam} = 160 GeV

DVCS formulae:

Belitsky et al., *Theory of deeply virtual Compton scattering on the nucleon*, Nuclear Physics B629 (2002)

Belitsky et al., *Compton scattering: from deeply virtual to quasi-real*, Nuclear Physics B878 (2014)

DVCS & TCS limits of DDVCS

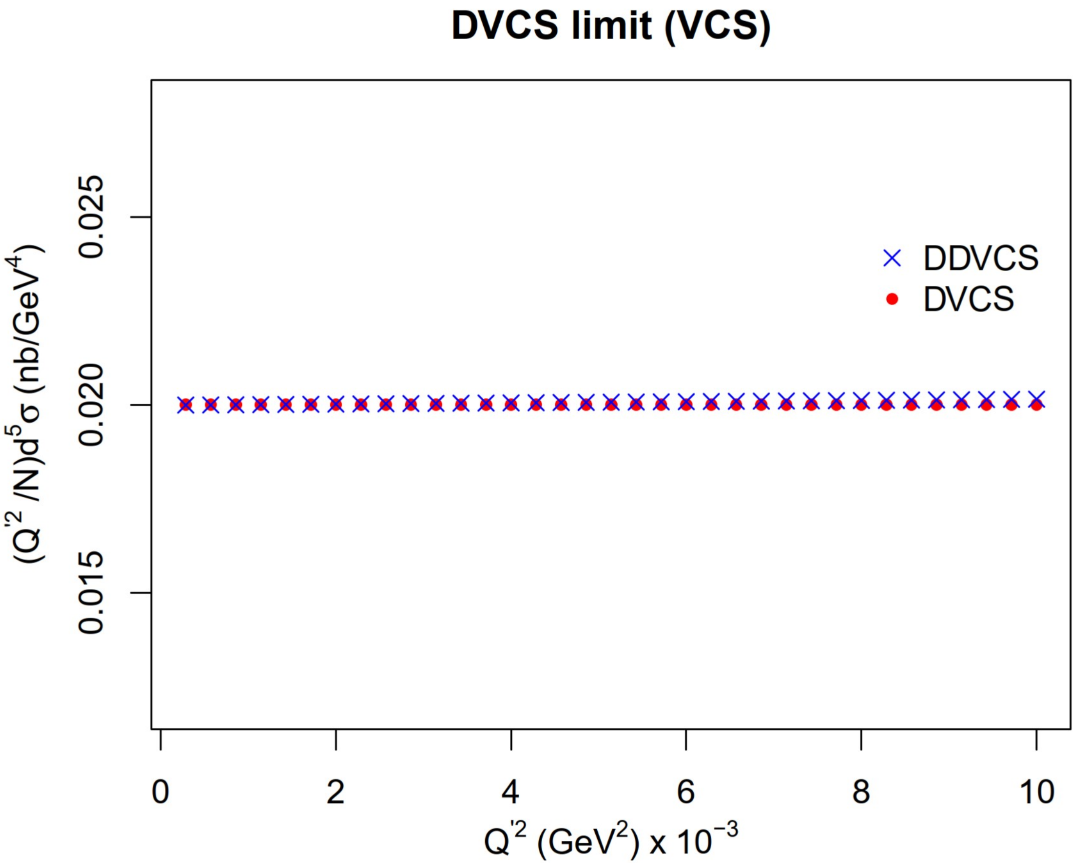


x_B = 2▪10^{-4}, t = -0.5 GeV^2, $-q_1^2$ = 2▪10^{-3} GeV^2, q_2^2 = 1 GeV^2, E_{beam} = 12 GeV

TCS formulae:

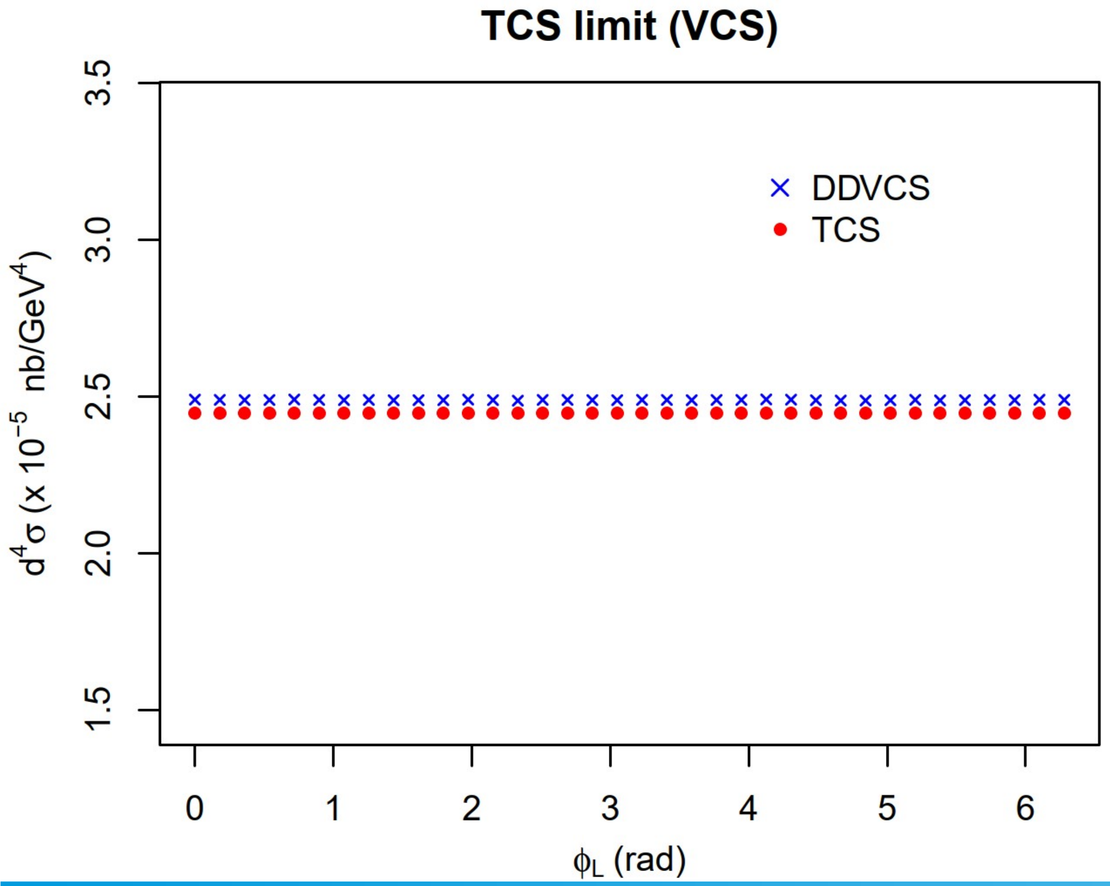
Berger et al., *Timelike Compton scattering: exclusive photoproduction of lepton pairs*, The European Physics Journal C 23 (2002)

DVCS & TCS limits of DDVCS



$x_B = 0.04$, $t = -0.01$ GeV^2, $-q_1^2 = 10$ GeV^2, $E_{beam} = 160$ GeV, $\phi = 0$

DVCS & TCS limits of DDVCS



$x_B = 0.5 \cdot 10^{-5}$, $t = -0.01$ GeV^2, $-q_1^2 = 0.5 \cdot 10^{-3}$ GeV^2, $q_2^2 = 10$ GeV^2, $E_{beam} = 60$ GeV, $\theta_L = \pi/2$

Relative errors in y-axis:

(DDVCS-TCS)/TCS ~ 2%

DVCS & TCS limits of DDVCS

- Xsec goes as $1/Q^2$ meaning that there is a delicate cancellation of such a dependence on the amplitude

- As shown, these limits are not trivial and require a careful analysis

Summary

- New analytical formulae has been derived
- DDVCS is already implemented in PARTONS (Lo and LT)
- We are interested in observables such as the beam spin asymmetry proportional to Im(BH x VCS*)
- Code will be included in EpIC MC generator to study feasibility of DDVCS
- We are in contact with experimentalists that will use it and we invite everybody to it