Nucleon GPDs and Valence Light Front Wave Functions

Michael Joseph Riberdy

Université Paris Saclay, DPhN, IRFU, CEA
Under the Supervision of Cédric Mezrag, Hervé Moutarde

October 28 2022

Working on the Lightfront with Lightcone Coordinates

$$v^{\pm} \equiv \frac{v^0 \pm v^3}{\sqrt{2}}, \quad (1)$$

$$\vec{v}_{\perp} = (v^1, v^2) \quad (2)$$

such that Minkowski 4-vectors become

$$v = (v^+, \vec{v}_{\perp}, v^-) \quad (3)$$

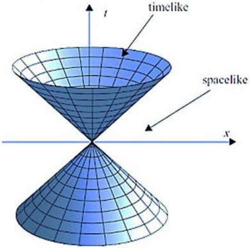
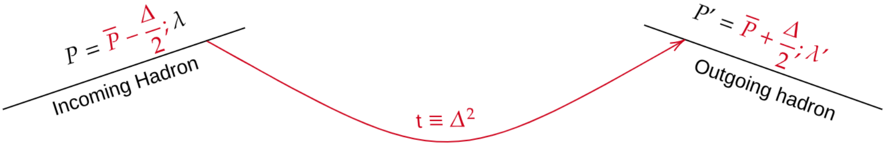


Figure: Light Cone

Defining GPDs

[Ji, 1997a] [D. Müller et al., 1994] [Radyushkin, 1997]

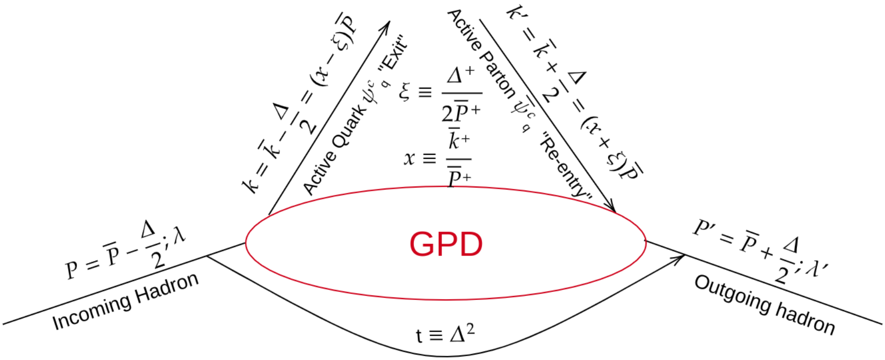
$$\overline{P} \equiv \frac{P' + P}{2}, \quad \Delta = P' - P \rightarrow t = \Delta^2$$



$$\equiv \mathcal{H}^q_{\lambda'\lambda} = \langle P'; \lambda' || P; \lambda \rangle \tag{4}$$

Defining GPDs

[Ji, 1997a] [D. Müller et al., 1994] [Radyushkin, 1997]



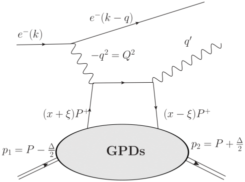
$$\equiv \mathcal{H}^q_{\lambda'\lambda} = \frac{\sum_c \int \frac{dz^-}{2\pi} e^{i\bar{k}^+ z^-}}{2\sqrt{1-\xi^2}} \langle P'; \lambda' | \bar{\psi}^c_q(-\frac{\bar{z}}{2}) \gamma^+ \psi^c_q(\frac{\bar{z}}{2}) | P; \lambda \rangle \quad (4)$$

Motivation for GPDs

- GPDs are Universal Objects

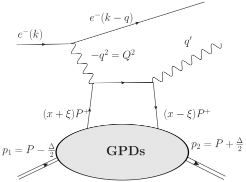
Motivation for GPDs

- GPDs are Universal Objects
- Probed in exclusive processes (DVCS, etc.)



Motivation for GPDs

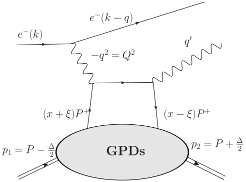
- GPDs are Universal Objects
- Probed in exclusive processes (DVCS, etc.)



- Multidimensional picture of the nucleon (off-forward generalization of PDFs)

Motivation for GPDs

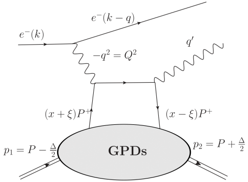
- GPDs are Universal Objects
- Probed in exclusive processes (DVCS, etc.)



- Multidimensional picture of the nucleon (off-forward generalization of PDFs)
  - Available for quarks and gluons $\rightarrow$ quark focus here

Motivation for GPDs

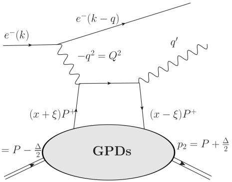
- GPDs are Universal Objects
- Probed in exclusive processes (DVCS, etc.)



- Multidimensional picture of the nucleon (off-forward generalization of PDFs)
  - Available for quarks and gluons $\rightarrow$ quark focus here
- Related to the energy momentum tensor

Motivation for GPDs

- GPDs are Universal Objects
- Probed in exclusive processes (DVCS, etc.)



- Multidimensional picture of the nucleon (off-forward generalization of PDFs)
  - Available for quarks and gluons $\rightarrow$ quark focus here
- Related to the energy momentum tensor
- Access quark and gluon contributions to the total angular momentum of the nucleon [Ji, 1997b]

GPD Modeling: Difficulties

Simultaneous satisfaction of all theoretical constraints is hard!

GPD Modeling: Difficulties

Simultaneous satisfaction of all theoretical constraints is hard!

- The literature now includes simple algebraic models [Mezrag et al., 2015] and advanced computations [Raya et al., 2022]

GPD Modeling: Difficulties

Simultaneous satisfaction of all theoretical constraints is hard!

- The literature now includes simple algebraic models [Mezrag et al., 2015] and advanced computations [Raya et al., 2022]
- Quark GPD Positivity: $|H_\pi^q(x,\xi,t)| \leq \sqrt{q_\pi(x_{\text{in}})q_\pi(x_{\text{out}})}$ (with q the corresponding quark PDF)

GPD Modeling: Difficulties

Simultaneous satisfaction of all theoretical constraints is hard!

- The literature now includes simple algebraic models [Mezrag et al., 2015] and advanced computations [Raya et al., 2022]
- Quark GPD Positivity: $|H_\pi^q(x,\xi,t)| \leq \sqrt{q_\pi(x_{\text{in}})q_\pi(x_{\text{out}})}$ (with q the corresponding quark PDF)
- Quark GPD Polynomality: $\int dx x^n H^q(x,\xi) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor}(2\xi)^{2i} A^q_{n+1,2i} + \text{mod}(2,n)(2\xi)^{n+1} C^q_{n+1}$

GPD Modeling: Overcoming Difficulties in the Meson Case

- Positivity: Use of Light Front Wave Functions

GPD Modeling: Overcoming Difficulties in the Meson Case

- Positivity: Use of Light Front Wave Functions
- Polynomality: By Radon Transform [Chouika et al., 2017]

GPD Modeling: Overcoming Difficulties in the Meson Case

- Positivity: Use of Light Front Wave Functions
- Polynomality: By Radon Transform [Chouika et al., 2017]

DVCS Meson Sector Prediction: Publication for phenomenological studies [Chávez et al., 2022]

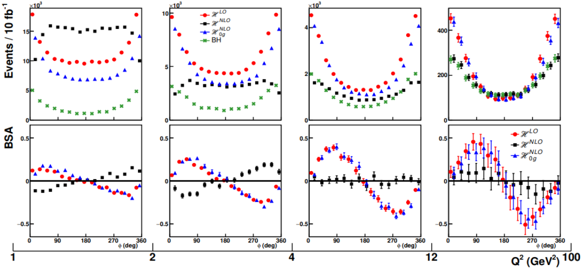


FIG. 3. Number of DVCS events (upper charts) and expected beam-spin asymmetries (lower chart) as a function of Q^2 for $x_B^\pi \in [10^{-3}; 10^{-2}]$. Red circles: LO evaluation of the CFF; blue triangles: NLO evalution but without taking gluon GPDs into account; black circles: full NLO results. The BH event rates is as well displayed by the green crosses.

GPD Modeling: Bottom Line

- The meson case is interesting and will be probed eventually

GPD Modeling: Bottom Line

- The meson case is interesting and will be probed eventually
- Extension from the meson to the nucleon will serve as a good test of the methods described due to existing and forthcoming data (JLab 6, JLab 12, etc.)

Light Front Wave Functions (LFWFs) as Fock Coefficients

Matrix element of ultimate interest:

$$\langle P'; \lambda' | \bar{\psi}_q^c(-\bar{z}/2) \, \gamma^+ \, \psi_q^c(\bar{z}/2) \, | P; \lambda \rangle \tag{5}$$

Light Front Wave Functions (LFWFs) as Fock Coefficients

Matrix element of ultimate interest:

$$\langle P'; \lambda' |\, \bar{\psi}_q^c(-\bar{z}/2)\, \gamma^+\, \psi_q^c(\bar{z}/2)\, | P; \lambda \rangle \tag{5}$$

For further investigation a basis for the incoming and outgoing nucleon states is required:

$$| P; \lambda_{\mathrm{N}} \rangle = \sum_{\mathrm{Fock}} \Psi_{\lambda_{\mathrm{N}}}^{\mathrm{Fock}} | \mathrm{Fock} \rangle \tag{6}$$

where the Ψ are the LFWFs (which admit a probabilistic interpretation).

Light Front Wave Functions (LFWFs) as Fock Coefficients

Matrix element of ultimate interest:

$$\langle P'; \lambda' | \, \bar{\psi}_q^c(-\bar{z}/2) \, \gamma^+ \, \psi_q^c(\bar{z}/2) \, | P; \lambda \rangle \tag{5}$$

For further investigation a basis for the incoming and outgoing nucleon states is required:

$$|P; \lambda_\mathrm{N}\rangle = \sum_{\mathrm{Fock}} \Psi_{\lambda_\mathrm{N}}^{\mathrm{Fock}} |\mathrm{Fock}\rangle \tag{6}$$

where the Ψ are the LFWFs (which admit a probabilistic interpretation).

But why choose a Fock expansion?

Parametrizing Hadronic States: Why Choose a Fock Expansion?

$$|P; \lambda_{\mathrm{N}}\rangle = \sum_{\mathrm{Fock}} \Psi_{\lambda_{\mathrm{N}}}^{\mathrm{Fock}} |\mathrm{Fock}\rangle \tag{7}$$

Parametrizing Hadronic States: Why Choose a Fock Expansion?

$$|P; \lambda_{\mathrm{N}}\rangle = \sum_{\alpha} \Psi^{\alpha}_{\lambda_{\mathrm{N}}} |\alpha\rangle \tag{7}$$

- Out of the box with a defined set of quantum numbers α

Parametrizing Hadronic States: Why Choose a Fock Expansion?

$$|P;\lambda_{\mathrm{N}}\rangle = \sum_{\alpha} \Psi^{\alpha}_{\lambda_{\mathrm{N}}} |\alpha\rangle \tag{7}$$

- Out of the box with a defined set of quantum numbers α
  - $\rightarrow$ Relative contributions of hadronic states $|\alpha\rangle$ can be assessed in a systematic way as LFWFs are the associated amplitudes

Parametrizing Hadronic States: Why Choose a Fock Expansion?

$$|P;\lambda_{\mathrm{N}}\rangle = \sum_{\alpha'}\sum_{\lambda_q} \Psi^{\alpha'}_{\lambda_{\mathrm{N}},\lambda_q}|\alpha',\lambda_q\rangle \tag{7}$$

- Out of the box with a defined set of quantum numbers α
  - $\rightarrow$ Relative contributions of hadronic states can be assessed in a systematic way as LFWFs are the associated amplitudes
- $\rightarrow$ Each valence LFWF corresponds to a particular set of quark helicities (with sum λ_q)

Parametrizing Hadronic States: Why Choose a Fock Expansion?

$$|P; \lambda_N\rangle = \sum_{\alpha'} \sum_{\lambda_q} \Psi^{\alpha'}_{\lambda_N, \lambda_q} |\alpha', \lambda_q\rangle \tag{7}$$

- Out of the box with a defined set of quantum numbers α
  - $\rightarrow$ Relative contributions of hadronic states can be assessed in a systematic way as LFWFs are the associated amplitudes
- $\rightarrow$ Each valence LFWF corresponds to a particular set of quark helicities (with sum λ_q)
  - $\rightarrow$ Due to conservation of angular momentum each LFWF corresponds to a definite quark orbital angular momentum (qOAM $= \lambda_N - \lambda_q$)

λ_N	$1/2$	$1/2$	$1/2$	$1/2$	$-1/2$	$-1/2$	$-1/2$	$-1/2$
λ_q	$3/2$	$1/2$	$-1/2$	$-3/2$	$3/2$	$1/2$	$-1/2$	$-3/2$
qOAM	-1	0	1	2	-2	-1	0	1

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

We define $$N_\sigma^+ \Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0 | (\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i)) | P; \lambda_\mathsf{N} \rangle \quad (8)$$

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

$$\text{We define} \quad N_\sigma^+ \Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0|(\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i))|P;\lambda_\mathrm{N}\rangle \tag{8}$$

where the quark annhilation operator q_i

- has been projected onto its '+' lightcone component $q_i^+ \equiv \frac{1}{2}\gamma^-\gamma^+ q_i$ to restrict the analysis to leading twist

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

$$\text{We define} \quad N_\sigma^+ \Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0 | (\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i)) | P; \lambda_\mathrm{N} \rangle \tag{8}$$

where the quark annhilation operator q_i

- has been projected onto its '+' lightcone component $q_i^+ \equiv \frac{1}{2}\gamma^-\gamma^+ q_i$
- is of flavor f_i

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

We define $$N_\sigma^+ \Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0 | (\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i)) | P; \lambda_N \rangle \quad (8)$$

where the quark annhilation operator q_i

- has been projected onto its '+' lightcone component $q_i^+ \equiv \frac{1}{2}\gamma^-\gamma^+ q_i$
- is of flavor f_i
- carries the Dirac index α_i

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

We define $$N_\sigma^+ \Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0 | (\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i)) | P; \lambda_N \rangle \qquad (8)$$

where the quark annhilation operator q_i

- has been projected onto its '+' lightcone component $q_i^+ \equiv \frac{1}{2}\gamma^-\gamma^+ q_i$
- is of flavor f_i
- carries the Dirac index α_i
- is a function of the momentum set κ_i containing the longitudinal momentum fraction x_i and transverse momenta $\vec{k}_{i\perp}$.

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

We define $$N_{\sigma}^{+}\Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0|(\prod_{i=1}^{3} q^{+}_{i,f_i,\alpha_i}(\kappa_i))|P;\lambda_{\mathrm{N}}\rangle \tag{8}$$

- has been projected onto its '$+$' lightcone component $q_i^+ \equiv \frac{1}{2}\gamma^-\gamma^+ q_i$
- is of flavor f_i
- carries the Dirac index α_i
- is a function of the momentum set κ_i containing the longitudinal momentum fraction x_i and transverse momenta $\vec{k}_{i\perp}$.

and where N_{σ}^{+} is the nucleon spinor with Dirac index σ

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

We define
$$N_\sigma^+ \Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0 | (\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i)) | P; \lambda_\mathrm{N} \rangle \tag{8}$$

We parametrize at leading twist
$$N_\sigma^+ \Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta}\phi_j(\kappa)) N_\beta^+ \tag{9}$$

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

We define
$$N_\sigma^+ \Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0 | (\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i)) | P; \lambda_{\mathrm{N}} \rangle \tag{8}$$

We parametrize at leading twist
$$N_\sigma^+ \Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta} \phi_j(\kappa)) N_\beta^+ \tag{9}$$

by using Lorentz covariance as a constraint on the tensorial structures $T_{j,\beta}$ [Ji et al., 2003][Braun et al., 2000]

Accessing LFWFs from Hadronic Matrix Elements

How can we actually access them?

We define $$N_\sigma^+\Omega_{f,\alpha,\sigma}(\kappa) \equiv \langle 0|(\prod_{i=1}^{3} q^+_{i,f_i,\alpha_i}(\kappa_i))|P;\lambda_{\mathrm{N}}\rangle \quad (8)$$

We parametrize at leading twist $$N_\sigma^+\Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta}\phi_j(\kappa))N_\beta^+ \quad (9)$$

by using Lorentz covariance as a constraint on the tensorial structures $T_{j,\beta}$ [Ji et al., 2003][Braun et al., 2000]
The $\phi_j(\kappa)$ exhibit various symmetry properties due to u-quark symmetry and futher due to imposition of isospin symmetry

Comments on Choice of Basis ϕ_j

- A given parametrization $N_\sigma^+ \Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta}\phi_j(\kappa))N_\beta^+$ is NOT unique.

Comments on Choice of Basis ϕ_j

- A given parametrization $N_\sigma^+ \Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta}\phi_j(\kappa))N_\beta^+$ is NOT unique.
- $\rightarrow$ We are free to choose a basis of $T_{j,\beta}\phi_j$ which is proportional to the set of $\Psi_{\lambda_{\mathrm{N}},\lambda_q}$

Comments on Choice of Basis ϕ_j

- A given parametrization $N_\sigma^+ \Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta}\phi_j(\kappa))N_\beta^+$ is NOT unique.
- $\rightarrow$ We are free to choose a basis of $T_{j,\beta}\phi_j$ which is proportional to the set of $\Psi_{\lambda_\mathrm{N},\lambda_q}$
  - $\rightarrow$ We choose to parametrize $\Omega_{f,\alpha,\sigma}$ in terms of the LFWFs $\Psi_{\lambda_\mathrm{N},\lambda_q}$

Comments on Choice of Basis ϕ_j

- A given parametrization $N_\sigma^+ \Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta}\phi_j(\kappa))N_\beta^+$ is NOT unique.
- $\rightarrow$ We are free to choose a basis of $T_{j,\beta}\phi_j$ which is proportional to the set of $\Psi_{\lambda_{\mathrm{N}},\lambda_q}$
  - $\rightarrow$ We choose to parametrize $\Omega_{f,\alpha,\sigma}$ in terms of the LFWFs $\Psi_{\lambda_{\mathrm{N}},\lambda_q}$
    - ★ Not all of the functions $\Psi_{\lambda_{\mathrm{N}},\lambda_q}$ are independent as some are related by the Jacobi-Wick transformation [Ji et al., 2003]

Comments on Choice of Basis ϕ_j

- A given parametrization $N_\sigma^+ \Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta}\phi_j(\kappa))N_\beta^+$ is NOT unique.
- $\rightarrow$ We are free to choose a basis of $T_{j,\beta}\phi_j$ which is proportional to the set of $\Psi_{\lambda_N,\lambda_q}$
  - $\rightarrow$ We choose to parametrize $\Omega_{f,\alpha,\sigma}$ in terms of the LFWFs $\Psi_{\lambda_N,\lambda_q}$
    - ★ Not all of the functions $\Psi_{\lambda_N,\lambda_q}$ are independent as some are related by the Jacobi-Wick transformation [Ji et al., 2003]
    - ★ There are 6 independent functions $\Psi_{\lambda_N,\lambda_q}$

Comments on Choice of Basis ϕ_j

- A given parametrization $N_\sigma^+ \Omega_{f,\alpha,\sigma} = (\sum_j T_{j,\beta}\phi_j(\kappa))N_\beta^+$ is NOT unique.
- $\rightarrow$ We are free to choose a basis of $T_{j,\beta}\phi_j$ which is proportional to the set of $\Psi_{\lambda_N,\lambda_q}$
  - $\rightarrow$ We choose to parametrize $\Omega_{f,\alpha,\sigma}$ in terms of the LFWFs $\Psi_{\lambda_N,\lambda_q}$
    - Not all of the functions $\Psi_{\lambda_N,\lambda_q}$ are independent as some are related by the Jacobi-Wick transformation [Ji et al., 2003]
    - There are 6 independent functions $\Psi_{\lambda_N,\lambda_q}$
    - By projecting our general matrix element onto carefully selected Dirac structures we may isolate the $\Psi_{\lambda_N,\lambda_q}$ directly

Recap

Recap

- Goal: Model GPDs

Recap

- Goal: Model GPDs
- Subgoal: Express individual qOAM contributions to GPDs

Recap

- Goal: Model GPDs
- Subgoal: Express individual qOAM contributions to GPDs
- Hurdle: Decomposition of the nucleon states $|P; \lambda_N\rangle$ is necessary

Recap

- Goal: Model GPDs
- Subgoal: Express individual qOAM contributions to GPDs
- Hurdle: Decomposition of the nucleon states $|P; \lambda_N\rangle$ is necessary
  - Leap: We choose to parametrize the matrix element characterizing the contribution of various Fock states, $\Omega_{f,\alpha,\sigma}$, to the state $|P; \lambda_N\rangle$, in terms of the $\Psi_{\lambda_N,\lambda_q}$, a Fock basis which makes manifest contributions of distinct qOAM

Recap

- Goal: Model GPDs
- Subgoal: Express individual qOAM contributions to GPDs
- Hurdle: Decomposition of the nucleon states $|P;\lambda_N\rangle$ is necessary
  - Leap: We choose to parametrize the matrix element characterizing the contribution of various Fock states, $\Omega_{f,\alpha,\sigma}$, to the state $|P;\lambda_N\rangle$, in terms of the $\Psi_{\lambda_\mathrm{N},\lambda_q}$, a Fock basis which makes manifest contributions of distinct qOAM
    - ★ $\rightarrow$ After using your favorite way of calculating $\Omega_{f,\alpha,\sigma}$ you may directly calculate the LFWFs

Recap

- Goal: Model GPDs
- Subgoal: Express individual qOAM contributions to GPDs
- Hurdle: Decomposition of the nucleon states $|P; \lambda_N\rangle$ is necessary
  - Leap: We choose to parametrize the matrix element characterizing the contribution of various Fock states, $\Omega_{f,\alpha,\sigma}$, to the state $|P; \lambda_N\rangle$, in terms of the $\Psi_{\lambda_\mathrm{N},\lambda_q}$, a Fock basis which makes manifest contributions of distinct qOAM
    - $\rightarrow$ After using your favorite way of calculating $\Omega_{f,\alpha,\sigma}$ you may directly calculate the LFWFs
- Let's calculate GPDs using our convenient new basis!

Overlap Representation of GPDs

According to [Diehl et al., 2001] one may calculate GPDs as sums of overlaps of LFWFs.

Original Matrix Element

$$\mathcal{H}^{q}_{\lambda'_N \lambda_N} \equiv \frac{1}{2\sqrt{1-\xi^2}} \sum_c \int \frac{dz^- e^{ik^+ z^-}}{2\pi} \langle P'; \lambda' | \bar{\psi}^c_q(-\bar{z}/2) \gamma^+ \psi^c_q(\bar{z}/2) | P; \lambda \rangle$$

Leading Fock $\rightarrow$

$$= \sqrt{1-\xi}\sqrt{1+\xi} \sum_{\lambda_q = \lambda'_q} \sum_j \delta_{s_j q} \int [d\bar{x}]_N [d^2 \bar{\mathbf{k}}_\perp]_N \delta(\bar{x} - \bar{x}_j)$$

$$\Psi^*_{\lambda'_N, \lambda'_q} \Psi_{\lambda_N, \lambda_q}$$

Original Matrix Element

$$\mathcal{H}^q_{\lambda'_N\lambda_N} \equiv \frac{1}{2\sqrt{1-\xi^2}} \sum_c \int \frac{dz^- e^{ik^+z^-}}{2\pi} \langle P';\lambda'|\bar{\psi}^c_q(-\bar{z}/2)\gamma^+\psi^c_q(\bar{z}/2)|P;\lambda\rangle$$

Leading Fock $\rightarrow$

$$= \sqrt{1-\xi}\sqrt{1+\xi} \sum_{\lambda_q=\lambda'_q} \sum_j \delta_{s_j q} \int [d\bar{x}]_N [d^2\bar{\mathbf{k}}_\perp]_N \delta(\bar{x}-\bar{x}_j)$$

$$\Psi^*_{\lambda'_N,\lambda'_q}\Psi_{\lambda_N,\lambda_q}$$

$$\equiv \frac{\sum_{\lambda_q=\lambda'_q}}{2\sqrt{1-\xi^2}} \mathcal{O}^q(\hat{\Psi}'_{\lambda'_N,\lambda_q},\hat{\Psi}_{\lambda_N,\lambda_q})$$

Original Matrix Element

$$\mathcal{H}^q_{\lambda'_N \lambda_N} \equiv \frac{1}{2\sqrt{1-\xi^2}} \sum_c \int \frac{dz^- e^{ik^+ z^-}}{2\pi} \langle P'; \lambda' | \bar{\psi}^c_q(-\bar{z}/2)\gamma^+ \psi^c_q(\bar{z}/2) | P; \lambda \rangle$$

Leading Fock $\rightarrow$

$$= \sqrt{1-\xi}\sqrt{1+\xi} \sum_{\lambda_q = \lambda'_q} \sum_j \delta_{s_j q} \int [d\bar{x}]_N [d^2 \bar{\mathbf{k}}_\perp]_N \delta(\bar{x} - \bar{x}_j)$$

$$\Psi^*_{\lambda'_N, \lambda'_q} \Psi_{\lambda_N, \lambda_q}$$

$$\equiv \frac{\sum_{\lambda_q = \lambda'_q}}{2\sqrt{1-\xi^2}} \mathcal{O}^q(\hat{\Psi}'_{\lambda'_N, \lambda_q}, \hat{\Psi}_{\lambda_N, \lambda_q})$$

$$\mathcal{O}^q(\hat{\Psi}'_{\lambda'_N, \lambda_q}, \hat{\Psi}_{\lambda_N, \lambda_q})$$

$$\equiv \int \mathcal{D} \sum_{\sigma \in S_3} \delta^{c,f,h}_{\sigma(c'),\sigma(f'),\sigma(h')} \sum_{l=1}^{3} \delta_{f_l,q} \Psi^*_{\lambda'_N, \lambda'_q}(\sigma(\kappa))|_l \Psi_{\lambda_N, \lambda_q}$$

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q}, \Psi_{\lambda_N,\lambda_q})$$

$$\equiv$$

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q},\Psi_{\lambda_N,\lambda_q}) \equiv \int \mathcal{D}$$

$$\mathcal{D} \equiv \frac{1}{2}\prod_{l=1}^{3}\left(\frac{dx_l d^2\vec{k}_{lT}}{(2\pi)^2\sqrt{x_l}}\right)\delta(1-\sum_{l=1}^{3}x_l)\delta^{(2)}(\sum_{l=1}^{3}\vec{k}_{lT})$$

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q},\Psi_{\lambda_N,\lambda_q}) \equiv \int \mathcal{D} \sum_{\sigma\in S_3}$$

$$\mathcal{D} \equiv \frac{1}{2}\prod_{l=1}^{3}\left(\frac{dx_l d^2\vec{k}_{lT}}{(2\pi)^2\sqrt{x_l}}\right)\delta(1-\sum_{l=1}^{3}x_l)\delta^{(2)}(\sum_{l=1}^{3}\vec{k}_{lT})$$

Sum over all possible permutations of:

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q}, \Psi_{\lambda_N,\lambda_q})$$

$$\equiv \int \mathcal{D} \sum_{\sigma \in S_3} \delta^{c}_{\sigma(c')}$$

$$\mathcal{D} \equiv \frac{1}{2}\prod_{l=1}^{3}(\frac{dx_l d^2\vec{k}_{lT}}{(2\pi)^2\sqrt{x_l}})\delta(1-\sum_{l=1}^{3}x_l)\delta^{(2)}(\sum_{l=1}^{3}\vec{k}_{lT})$$

Sum over all possible permutations of:
-Color

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q}, \Psi_{\lambda_N,\lambda_q}) \equiv \int \mathcal{D} \sum_{\sigma \in S_3} \delta^{c,f}_{\sigma(c'),\sigma(f')}$$

$$\mathcal{D} \equiv \frac{1}{2}\prod_{l=1}^{3}\left(\frac{dx_l d^2\vec{k}_{lT}}{(2\pi)^2\sqrt{x_l}}\right)\delta(1-\sum_{l=1}^{3}x_l)\delta^{(2)}(\sum_{l=1}^{3}\vec{k}_{lT})$$

Sum over all possible permutations of:

-Color

-Flavor

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q},\Psi_{\lambda_N,\lambda_q})$$
$$\equiv \int \mathcal{D} \sum_{\sigma\in S_3} \delta^{c,f,h}_{\sigma(c'),\sigma(f'),\sigma(h')}$$

$$\mathcal{D} \equiv \frac{1}{2}\prod_{l=1}^{3}\left(\frac{dx_l d^2\vec{k}_{lT}}{(2\pi)^2\sqrt{x_l}}\right)\delta(1-\sum_{l=1}^{3}x_l)\delta^{(2)}(\sum_{l=1}^{3}\vec{k}_{lT})$$

Sum over all possible permutations of:

-Color

-Flavor

-Quark helicities

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q},\Psi_{\lambda_N,\lambda_q})$$

$$\equiv \quad \int \mathcal{D} \sum_{\sigma\in S_3} \delta^{c,f,h}_{\sigma(c'),\sigma(f'),\sigma(h')} \sum_{l=1}^{3}$$

$$\mathcal{D} \equiv \frac{1}{2}\prod_{l=1}^{3}\left(\frac{dx_l d^2\vec{k}_{lT}}{(2\pi)^2\sqrt{x_l}}\right)\delta(1-\sum_{l=1}^{3}x_l)\delta^{(2)}(\sum_{l=1}^{3}\vec{k}_{lT})$$

Sum over all possible permutations of:

-Color

-Flavor

-Quark helicities

Sum over all active quarks

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^{q}(\Psi^{*}_{\lambda'_N,\lambda'_q},\Psi_{\lambda_N,\lambda_q})$$

$$\equiv \int \mathcal{D} \sum_{\sigma\in S_3} \delta^{c,f,h}_{\sigma(c'),\sigma(f'),\sigma(h')} \sum_{l=1}^{3} \delta_{f_l,q}$$

$$\mathcal{D} \equiv \frac{1}{2}\prod_{l=1}^{3}\left(\frac{dx_l d^2\vec{k}_{lT}}{(2\pi)^2\sqrt{x_l}}\right)\delta(1-\sum_{l=1}^{3}x_l)\delta^{(2)}(\sum_{l=1}^{3}\vec{k}_{lT})$$

Sum over all possible permutations of:

-Color

-Flavor

-Quark helicities

Sum over all active quarks (of the correct flavor)

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\hat{\Psi}^*_{\lambda'_N,\lambda'_q}(\sigma(\kappa)), \Psi_{\lambda_N,\lambda_q})$$

$$\equiv \int \mathcal{D} \sum_{\sigma\in S_3} \delta^{c,f,h}_{\sigma(c'),\sigma(f'),\sigma(h')} \sum_{l=1}^{3} \delta_{f_l,q} \Psi^*_{\lambda'_N,\lambda'_q}(\sigma(\kappa))|_l \Psi_{\lambda_N,\lambda_q}$$

$$\mathcal{D} \equiv \frac{1}{2} \prod_{l=1}^{3} (\frac{dx_l d^2\vec{k}_{lT}}{(2\pi)^2\sqrt{x_l}}) \delta(1 - \sum_{l=1}^{3} x_l) \delta^{(2)}(\sum_{l=1}^{3} \vec{k}_{lT})$$

Sum over all possible permutations of:
-Color
-Flavor
-Quark helicities
Sum over all active quarks (of the correct flavor)
LFWFs with the corresponding permutation of momenta

Overlap ($\mathcal{O}$) Notation

$$\mathcal{O}^q(\Psi^*_{\lambda'_N,\lambda'_q}, \Psi_{\lambda_N,\lambda_q})$$

$$\equiv \int \mathcal{D} \sum_{\sigma \in S_3} \delta^{c,f,h}_{\sigma(c'),\sigma(f'),\sigma(h')} \sum_{l=1}^{3} \delta_{f_l,q} \Psi^*_{\lambda'_N,\lambda'_q}(\sigma(\kappa))|_l \Psi_{\lambda_N,\lambda_q}$$

Sum over all possible permutations of:

-Color

-Flavor

-Quark helicities

Sum over all active quarks (of the correct flavor)

LFWFs with the corresponding permutation of momenta

$$\Psi'^*|_l\Psi \equiv \Psi'^*|^{\text{Outgoing variables}}_{l\text{th quark active}} \Psi|^{\text{Incoming variables}}_{l\text{th quark active}} \delta(\bar{x} - x_l)$$

The lth quark is active.

Expressing GPDs

Quark orbital angular momentum: 0, 1, 2.

$$\mathcal{H}^q_{++} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^q(\hat{\Psi}_{\frac{1}{2},\frac{1}{2}},\hat{\Psi}_{\frac{1}{2},\frac{1}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{1}{2},\frac{-1}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-1}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \quad (10)$$

$$\mathcal{H}^q_{-+} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \quad (11)$$

$$H^q = \mathcal{H}^q_{++} + \frac{\xi^2 2m|\vec{\Delta}_\perp|}{(\Delta_1 + i\Delta_2)\sqrt{1-\xi^2}\sqrt{\frac{4\xi^2 m^2}{\xi^2-1} - t}}\mathcal{H}^q_{-+} \quad (12)$$

$$E^q = \frac{2m|\vec{\Delta}_\perp|\sqrt{1-\xi^2}}{(\Delta_1 + i\Delta_2)\sqrt{\frac{4\xi^2 m^2}{\xi^2-1} - t}}\mathcal{H}^q_{-+} \quad (13)$$

$$(14)$$

All Contributions to E Are Off-Diagonal in Nucleon Helicity

$$\mathcal{H}^q_{-+} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \quad (15)$$

All Contributions to E Are Off-Diagonal in Nucleon Helicity

$$\mathcal{H}^q_{-+} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \quad (15)$$

No OAM$=0$ contributions

- Included as all quark helicites interfere constructively:

$$\mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) \quad (16)$$

All Contributions to E Are Off-Diagonal in Nucleon Helicity

$$\mathcal{H}^q_{-+} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \quad (15)$$

No OAM$=0$ contributions

- Included as all quark helicites interfere constructively:

$$\mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) \quad (16)$$

- Excluded as some quark helicities interfere destructively:

$$\hat{\Psi}_{\frac{-1}{2},\frac{1}{2}}, \quad \hat{\Psi}_{\frac{1}{2},\frac{-1}{2}} \quad (17)$$

All Contributions to E Are Off-Diagonal in Nucleon Helicity

$$\mathcal{H}^q_{-+} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \quad (15)$$

No OAM$=0$ contributions

- Included as all quark helicites interfere constructively:

$$\mathcal{O}^q(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) \quad (16)$$

- Excluded as some quark helicities interfere destructively:

$$\hat{\Psi}_{\frac{-1}{2},\frac{1}{2}}, \quad \hat{\Psi}_{\frac{1}{2},\frac{-1}{2}} \quad (17)$$

→ Non-zero quark OAM states are therefore expected to contribute to the energy momentum tensor through the GPD E.

- This is consistent with existing computations in the literature [Ji et al., 2003]

- This is consistent with existing computations in the literature [Ji et al., 2003]
- has been extended to the polarized GPDs $\tilde{H}$ and $\tilde{E}$

Recap Two

- Goal: Model GPDs
- Subgoal: Express individual qOAM contributions to GPDs
- Hurdle: Decomposition of the nucleon states $|P;\lambda_N\rangle$ is necessary
  - Leap: We choose to parametrize the matrix element characterizing the contribution of various Fock states, $\Omega_{f,\alpha,\sigma}$, to the state $|P;\lambda_N\rangle$, in terms of the $\Psi_{\lambda_N,\lambda_q}$, a Fock basis which makes manifest contributions of distinct qOAM
    - ★ $\rightarrow$ After using your favorite way of calculating $\Omega_{f,\alpha,\sigma}$ you may directly calculate the LFWFs
- Distinct qOAM contributions to GPDs H, E, H̃, Ẽ and are calculable from the LFWF basis

Conclusions and Future Perspectives

- Anticipated experimental access to GPDs provides ample motivation to investigate their modeling

Conclusions and Future Perspectives

- Anticipated experimental access to GPDs provides ample motivation to investigate their modeling
- Techniques developed for analogous modeling in the meson sector are reaching the maturity necessary to be applied to the nucleon

Conclusions and Future Perspectives

- Anticipated experimental access to GPDs provides ample motivation to investigate their modeling
- Techniques developed for analogous modeling in the meson sector are reaching the maturity necessary to be applied to the nucleon
- LFWFs arise naturally as amplitudes characterizing individual Fock contributions to low-energy QCD bound states

Conclusions and Future Perspectives

- Anticipated experimental access to GPDs provides ample motivation to investigate their modeling
- Techniques developed for analogous modeling in the meson sector are reaching the maturity necessary to be applied to the nucleon
- LFWFs arise naturally as amplitudes characterizing individual Fock contributions to low-energy QCD bound states
- One may choose a minimal basis of 6 LFWFs whose overlaps may be used to model individual qOAM to nucleon GPDs

Conclusions and Future Perspectives

- Anticipated experimental access to GPDs provides ample motivation to investigate their modeling
- Techniques developed for analogous modeling in the meson sector are reaching the maturity necessary to be applied to the nucleon
- LFWFs arise naturally as amplitudes characterizing individual Fock contributions to low-energy QCD bound states
- One may choose a minimal basis of 6 LFWFs whose overlaps may be used to model individual qOAM to nucleon GPDs
- Future work will include calculating valence nucleon LFWFs from projected nucleon Fadeev amplitudes in a quark-diquark framework

Conclusions and Future Perspectives

- Anticipated experimental access to GPDs provides ample motivation to investigate their modeling
- Techniques developed for analogous modeling in the meson sector are reaching the maturity necessary to be applied to the nucleon
- LFWFs arise naturally as amplitudes characterizing individual Fock contributions to low-energy QCD bound states
- One may choose a minimal basis of 6 LFWFs whose overlaps may be used to model individual qOAM to nucleon GPDs
- Future work will include calculating valence nucleon LFWFs from projected nucleon Fadeev amplitudes in a quark-diquark framework
  - Relative contributions of distinct values of qOAM to GPDs, PDFs, FFs, and the electric radius of the nucleus will be assessed

Thank you

Thank you!

Projection of Ω

$$\begin{aligned}
&\epsilon^{c_1,c_2,c_3}\langle 0|q^{+,c_1}_{\alpha_1,f_1}(z_1^-,z_{\perp 1})q^{+,c_2}_{\alpha_2,f_2}(z_2^-,z_{\perp 2})q^{+,c_3}_{\alpha_3,f_3}(z_3^-,z_{\perp 3})|P,\lambda\rangle|_{z^+=0} \\
= \quad & \frac{1}{4}f_N N_\sigma(P,\lambda)\int\left[\prod_{j=1}^{3}\mathrm{d}k_j^+\mathrm{d}^{(2)}k_{\perp j}\right]e^{-i(k_j^+z_j^- - k_{\perp j}z_{\perp j})}\delta(P^+ - \sum_j k_j^+) \\
\times \quad & \delta^{(2)}(P_\perp - \sum_j k_{\perp j})\Omega_{\alpha_1\alpha_2\alpha_3;\sigma} \qquad (18)
\end{aligned}$$

Framed Coordinates

For the incoming frame:

$$x'_i \equiv \frac{\bar{x}_i}{1+\xi}, \qquad \vec{k}'_{iT} \equiv \vec{\bar{k}}_{iT} + \frac{\bar{x}_i}{1+\xi}\frac{\vec{\Delta}_T}{2}$$
$$x'_j \equiv \frac{\bar{x}_j+\xi}{1+\xi}, \qquad \vec{k}'_{jT} \equiv \vec{\bar{k}}_{jT} - \frac{1-\bar{x}_j}{1+\xi}\frac{\vec{\Delta}_T}{2} \tag{19}$$

and for the outgoing frame

$$x_i \equiv \frac{\bar{x}_i}{1-\xi}, \qquad \vec{k}_{iT} \equiv \vec{\bar{k}}_{iT} - \frac{\bar{x}_i}{1-\xi}\frac{\vec{\Delta}_T}{2}$$
$$x_j \equiv \frac{\bar{x}_j-\xi}{1-\xi}, \qquad \vec{k}_{jT} \equiv \vec{\bar{k}}_{jT} + \frac{1-\bar{x}_j}{1-\xi}\frac{\vec{\Delta}_T}{2} \tag{20}$$

Proton PDFs and FFs

$$f^{p,q} = H^{p,q}|_{t=\xi=0} = \Big(\mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{1}{2}}, \hat{\Psi}_{1,\frac{1}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-1}{2}}, \hat{\Psi}_{1,\frac{-1}{2}}) \quad (21)$$

$$+ \quad \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{3}{2}}, \hat{\Psi}_{1,\frac{3}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-3}{2}}, \hat{\Psi}_{1,\frac{-3}{2}})\Big)|_{t=\xi=0}$$

Proton PDFs and FFs

$$f^{p,q} = H^{p,q}|_{t=\xi=0} = \Big(\mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{1}{2}}, \hat{\Psi}_{1,\frac{1}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-1}{2}}, \hat{\Psi}_{1,\frac{-1}{2}}) \quad (21)$$

$$+ \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{3}{2}}, \hat{\Psi}_{1,\frac{3}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-3}{2}}, \hat{\Psi}_{1,\frac{-3}{2}})\Big)|_{t=\xi=0}$$

$$F_i^{p;P}(t) = \frac{4}{3}F_i^{p,u}(t) - \frac{1}{3}F_i^{p,d}(t) \quad (22)$$

Proton PDFs and FFs

$$f^{p,q} = H^{p,q}|_{t=\xi=0} = \Big(\mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{1}{2}}, \hat{\Psi}_{1,\frac{1}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-1}{2}}, \hat{\Psi}_{1,\frac{-1}{2}}) \quad (21)$$
$$+ \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{3}{2}}, \hat{\Psi}_{1,\frac{3}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-3}{2}}, \hat{\Psi}_{1,\frac{-3}{2}})\Big)|_{t=\xi=0}$$

$$F_i^{p;P}(t) = \frac{4}{3}F_i^{p,u}(t) - \frac{1}{3}F_i^{p,d}(t) \quad (22)$$

$$F_1^{p,q}(t) \equiv \int_{-1}^{1} dx H^{p,q}(x,0,t) = \int_{-1}^{1} dx \Big(\mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{1}{2}}, \hat{\Psi}_{1,\frac{1}{2}})$$
$$+ \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-1}{2}}, \hat{\Psi}_{1,\frac{-1}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{3}{2}}, \hat{\Psi}_{1,\frac{3}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{1,\frac{-3}{2}}, \hat{\Psi}_{1,\frac{-3}{2}})\Big)|_{\xi=0}$$

$$F_2^{p,q}(t) \equiv \int_{-1}^{1} dx E^{p,q}(x,0,t) = \int_{-1}^{1} dx \frac{(-1)^p 2M_N|\vec{\Delta}_\perp|}{(\Delta_1 + i\Delta_2)\sqrt{-t}}$$
$$\times \Big(\mathcal{O}^{p,q}(\hat{\Psi}_{-1,\frac{-3}{2}}, \hat{\Psi}_{1,\frac{3}{2}}) + \mathcal{O}^{p,q}(\hat{\Psi}_{-1,\frac{3}{2}}, \hat{\Psi}_{1,\frac{-3}{2}})\Big)|_{\xi=0}$$

Proton Electric Radius

$$\langle (r_E^P)^2 \rangle \;=\; 6\hbar^2 \partial_t \Big(F_1^P(t) - \frac{t}{4M_N^2} F_2^P(t) \Big)\Big|_{t=0} \qquad (23)$$

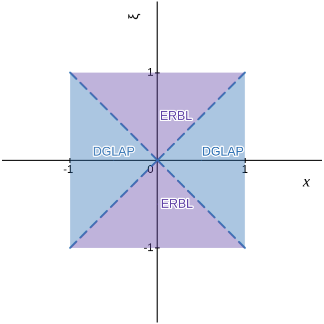
M_N represents the nucleon mass.

Proton Electric Radius

$$\langle (r_E^P)^2 \rangle \;=\; 6\hbar^2 \partial_t \Big(F_1^P(t) - \frac{t}{4M_N^2} F_2^P(t) \Big)|_{t=0} \tag{23}$$

M_N represents the nucleon mass.
This object relies on the $t = 0$ behaviour of the LFWFs, which will be used in future work to constrain modeling assumptions, with specific regard given to Nakanishi weight function based models.

DGLAP & ERBL



- LFWFs calculated in the DGLAP region feature incoming and outgoing states with identical numbers of partons, whereas the ERBL region requires an unequal number of partons in each state.
- By Radon transforming and subsequently inverse Radon transforming expressions for GPDs in the DGLAP region, one finds ERBL GPDs satisfying important modeling assumptions (i.e. polynomality is conserved)

Expressing Polarized GPDs

Quark orbital angular momentum: 0, 1, 2.
($p = 0 \leftrightarrow$ unpolarized; $p = 1 \leftrightarrow$ polarized)

$$\mathcal{H}^{q,p}_{++} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^{q,p}(\hat{\Psi}_{\frac{1}{2},\frac{1}{2}},\hat{\Psi}_{\frac{1}{2},\frac{1}{2}}) + \mathcal{O}^{q,p}(\hat{\Psi}_{\frac{1}{2},\frac{-1}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-1}{2}}) + \mathcal{O}^{q,p}(\hat{\Psi}_{\frac{1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^{q,p}(\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \tag{24}$$

$$\mathcal{H}^{q,p}_{-+} = (1-\xi^2)^{-1/2}\Big(\mathcal{O}^{q,p}(\hat{\Psi}_{\frac{-1}{2},\frac{-3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{3}{2}}) + \mathcal{O}^{q,p}(\hat{\Psi}_{\frac{-1}{2},\frac{3}{2}},\hat{\Psi}_{\frac{1}{2},\frac{-3}{2}})\Big) \tag{25}$$

$$H^{q,p} = \mathcal{H}^{q,p}_{++} + \frac{\xi^2 2m|\vec{\Delta}_\perp|}{(\Delta_1 + i\Delta_2)\sqrt{1-\xi^2}\sqrt{\frac{4\xi^2 m^2}{\xi^2-1} - t}}\mathcal{H}^{q,p}_{-+} \tag{26}$$

$$E^{q,p} = \frac{2m|\vec{\Delta}_\perp|\sqrt{1-\xi^2}}{(\Delta_1 + i\Delta_2)\sqrt{\frac{4\xi^2 m^2}{\xi^2-1} - t}}\mathcal{H}^{q,p}_{-+} \tag{27}$$

$$\mathcal{O}^{q,p}(\hat{\Psi}'_{\lambda'_N,\lambda_q},\hat{\Psi}_{\lambda_N,\lambda_q})$$

$$\equiv \int \mathcal{D} \sum_{\sigma\in S_3} \delta^{c,f,h}_{\sigma(c'),\sigma(f'),\sigma(h')} \sum_{l=1}^{3} \delta_{f_l,q} \Psi^*_{\lambda'_N,\lambda'_q}(\sigma(\kappa))|_l \Psi_{\lambda_N,\lambda_q} \mathrm{sign}^p(\lambda_{\text{active}})$$